Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.4-f-x-^m-d+e-x^2-^q-a+b-x^2+c-x^4-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [413]. This is test number [41].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (413)	% 0. (0)
Mathematica	% 96.85 (400)	% 3.15 (13)
Maple	% 91.04 (376)	% 8.96 (37)
Maxima	% 26.15 (108)	% 73.85 (305)
Fricas	% 60.53 (250)	% 39.47 (163)
Sympy	% 33.66 (139)	% 66.34 (274)
Giac	% 46. (190)	% 54. (223)

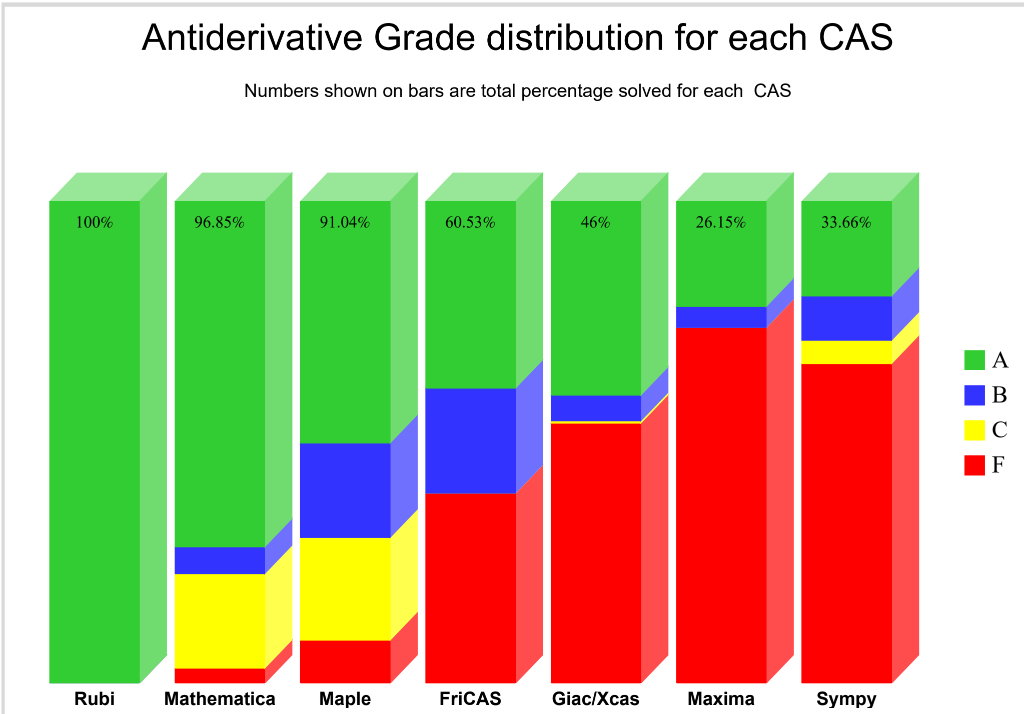
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

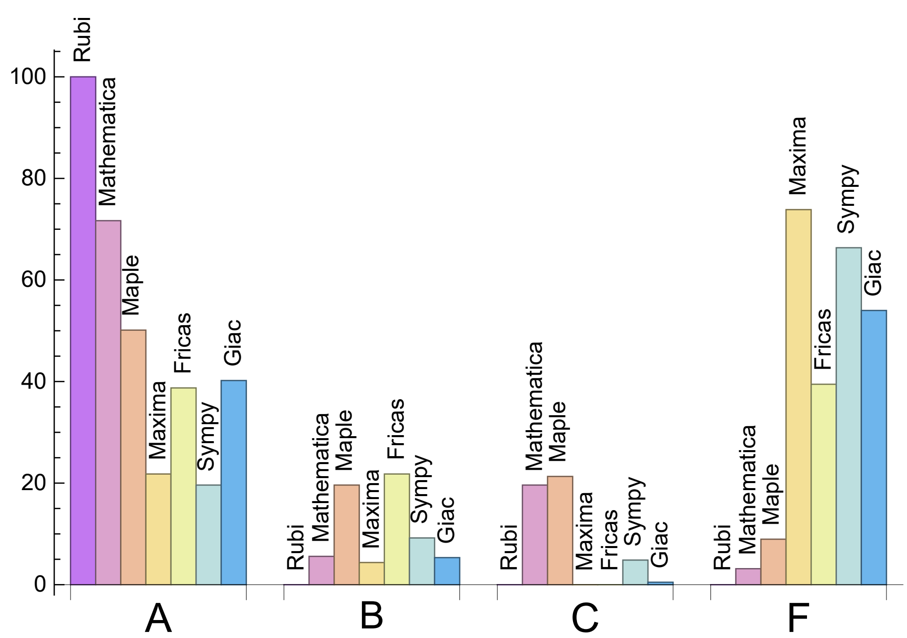
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	71.67	5.57	19.61	3.15
Maple	50.12	19.61	21.31	8.96
Maxima	21.79	4.36	0.	73.85
Fricas	38.74	21.79	0.	39.47
Sympy	19.61	9.2	4.84	66.34
Giac	40.19	5.33	0.48	54.

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.54	229.96	1.01	189.	1.
Mathematica	0.74	562.94	1.91	156.	0.95
Maple	0.02	386.39	1.61	203.	1.05
Maxima	1.17	132.73	1.66	120.	1.37
Fricas	12.96	2951.61	10.73	481.	4.63
Sympy	10.75	334.16	2.37	133.	1.23
Giac	3.02	357.99	2.13	168.	1.29

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {151, 152, 153, 154, 155, 163, 164, 166, 167, 168, 189, 190, 191, 192, 193, 199, 200, 202, 203, 204, 205, 206, 207, 209, 211, 212, 213, 214, 215, 217, 219, 224, 225, 226, 227, 228, 354, 361, 362, 364, 365, 366, 371, 372, 373, 374, 375, 381, 384, 385, 394, 395, 396, 397, 398, 399}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

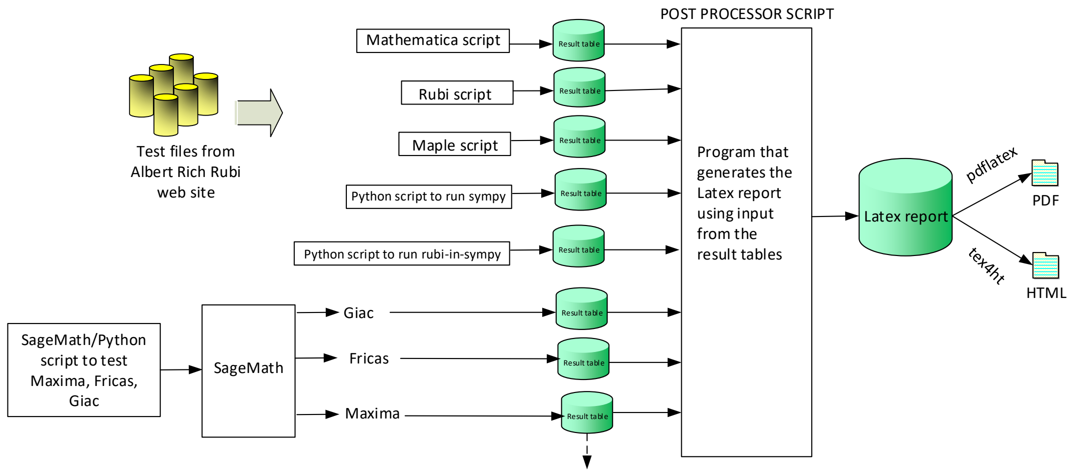
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 48, 55, 57, 59, 61, 62, 63, 64, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 175, 181, 182, 183, 184, 185, 186, 187, 188, 194, 195, 196, 197, 198, 204, 205, 206, 207, 209, 211, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 262, 263, 264, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 322, 323, 324, 325, 326, 332, 333, 334, 335, 336, 342, 343, 344, 345, 346, 347, 355, 356, 357, 358, 359, 360, 367, 368, 369, 370, 376, 377, 378, 379, 380, 386, 387, 388, 389, 390, 391, 392, 393, 401, 402, 403, 404, 405, 406, 413 }

B grade: { 56, 58, 60, 66, 68, 354, 361, 362, 363, 364, 365, 366, 371, 372, 373, 374, 375, 381, 382, 383, 384, 394, 395 }

C grade: { 15, 16, 17, 18, 19, 24, 25, 26, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 49, 50, 51, 52, 53, 54, 151, 152, 153, 154, 155, 163, 164, 166, 167, 168, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 202, 203, 224, 259, 260, 265, 266, 310, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 385, 396, 397, 398, 399 }

F grade: { 165, 201, 208, 210, 216, 218, 400, 407, 408, 409, 410, 411, 412}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 48, 49, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 114, 115, 129, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 263, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 297, 298, 299, 300, 332, 333, 335, 336 }

B grade: { 55, 56, 58, 60, 65, 66, 68, 70, 87, 88, 102, 107, 108, 109, 110, 111, 112, 113, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 169, 170, 176, 195, 220, 221, 222, 262, 264, 295, 296, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 322, 323, 324, 325, 326, 334, 342, 343, 344, 345, 346, 347, 376, 377, 378, 379, 380, 385 }

C grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 259, 260, 265, 266, 310, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 413 }

F grade: { 90, 91, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 12, 13, 14, 20, 25, 26, 32, 37, 38, 44, 45, 46, 47, 48, 49, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 76, 82, 87, 88, 89, 93, 94, 95, 96, 97, 98, 99, 100, 101, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 181, 182, 183, 184, 185, 186, 187, 188, 194, 195, 196, 197, 198, 268, 273, 274, 275, 276, 277, 278, 279

B grade: { 8, 9, 11, 21, 22, 23, 24, 33, 34, 35, 36, 56, 58, 60, 66, 68, 70, 92 }

C grade: { }

F grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 55, 65, 75, 77, 78, 79, 80, 81, 83, 84, 85, 86, 90, 91, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 269, 270, 271, 272, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 48, 49, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 175, 181, 182, 183, 184, 185, 186, 187, 188, 194, 195, 198, 229, 230, 231, 232, 233, 234, 235, 244, 245, 246, 247, 248, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 298, 299, 312, 313, 315, 336 }

B grade: { 55, 56, 58, 60, 65, 66, 68, 70, 87, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 196, 197, 220, 221, 222, 237, 238, 239, 240, 241, 242, 243, 252, 253, 254, 255, 256, 305, 306, 332, 333, 334, 335, 343, 344, 345, 346, 347, 357, 362, 363, 364, 365, 366, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 388, 389, 390, 391, 413 }

C grade: { }

F grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 90, 91, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 236, 249, 250, 251, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 295, 296, 297, 300, 301, 302, 303, 304, 307, 308, 309, 310, 311, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 342, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361, 367, 368, 369, 370, 371, 372, 373, 386, 387, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 92, 95, 96, 97, 98, 99, 100, 101, 109, 137, 138, 139, 140, 141, 220, 221, 222, 273, 274, 275, 276, 277, 278, 279, 281, 282, 284, 285, 288, 289, 290, 291, 292, 293 }

B grade: { 21, 22, 47, 48, 49, 56, 58, 60, 66, 68, 70, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 126, 127, 128, 129, 232, 280, 283, 286, 287, 294 }

C grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54 }

F grade: { 55, 65, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 116, 117, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 136, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 20, 21, 22, 32, 33, 34, 35, 37, 38, 44, 45, 46, 47, 48, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 112, 113, 114, 115, 116, 117, 124, 125, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 142, 143, 144, 156, 157, 158, 169, 170, 171, 181, 182, 183, 184, 194, 195, 196, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 334, 361, 362, 371, 372, 373, 386, 387, 388, 394, 413 }

B grade: { 55, 56, 58, 60, 65, 66, 68, 70, 87, 88, 89, 92, 93, 94, 126, 220, 221, 222, 343, 344, 345, 385 }

C grade: { 109, 110 }

F grade: { 11, 12, 13, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 39, 40, 41, 42, 43, 49, 50, 51, 52, 53, 54, 81, 82, 83, 84, 85, 86, 90, 91, 107, 108, 111, 118, 119, 120, 121, 122, 123, 132, 133, 134, 135, 136, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 172, 173, 174, 175, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 190, 191, 192, 193, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	149	126	169	306	151	177
normalized size	1	1.	1.	0.85	1.13	2.05	1.01	1.19
time (sec)	N/A	0.22	0.005	0.01	0.966	1.715	0.085	1.123

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	149	126	169	315	155	177
normalized size	1	1.	1.	0.85	1.13	2.11	1.04	1.19
time (sec)	N/A	0.098	0.004	0.001	0.953	1.277	0.086	1.163

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	146	125	167	300	150	176
normalized size	1	1.	1.64	1.4	1.88	3.37	1.69	1.98
time (sec)	N/A	0.077	0.004	0.001	0.971	1.216	0.083	1.16

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	141	122	163	298	148	171
normalized size	1	1.	1.	0.87	1.16	2.11	1.05	1.21
time (sec)	N/A	0.082	0.004	0.003	0.942	1.509	0.083	1.125

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	142	123	169	298	150	177
normalized size	1	1.	1.	0.87	1.19	2.1	1.06	1.25
time (sec)	N/A	0.111	0.008	0.015	0.955	1.722	0.416	1.129

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	139	122	163	373	143	171
normalized size	1	1.	1.	0.88	1.17	2.68	1.03	1.23
time (sec)	N/A	0.082	0.008	0.016	0.939	1.666	0.405	1.141

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	142	123	169	336	150	192
normalized size	1	1.	1.	0.87	1.19	2.37	1.06	1.35
time (sec)	N/A	0.121	0.008	0.006	0.952	1.68	0.436	1.145

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	50	53	138	128	97	70
normalized size	1	1.	0.75	0.79	2.06	1.91	1.45	1.04
time (sec)	N/A	0.05	0.034	0.04	1.434	1.607	5.557	1.159

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	44	46	126	116	70	62
normalized size	1	1.	0.86	0.9	2.47	2.27	1.37	1.22
time (sec)	N/A	0.031	0.029	0.004	1.447	1.558	4.081	1.149

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	36	34	90	90	53	50
normalized size	1	1.	0.82	0.77	2.05	2.05	1.2	1.14
time (sec)	N/A	0.021	0.032	0.01	1.435	1.512	2.775	1.126

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	49	134	149	83	0
normalized size	1	1.	0.98	0.84	2.31	2.57	1.43	0.
time (sec)	N/A	0.055	0.043	0.014	1.446	1.582	12.312	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	61	119	177	83	0
normalized size	1	1.	1.	1.03	2.02	3.	1.41	0.
time (sec)	N/A	0.056	0.048	0.012	1.454	1.573	6.003	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	75	123	181	76	0
normalized size	1	1.	0.94	1.19	1.95	2.87	1.21	0.
time (sec)	N/A	0.055	0.077	0.011	1.441	1.54	5.125	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	72	52	80	146	63	84
normalized size	1	1.	1.24	0.9	1.38	2.52	1.09	1.45
time (sec)	N/A	0.047	0.04	0.01	1.423	1.523	5.271	1.139

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	82	192	0	0	78	0
normalized size	1	1.	0.39	0.92	0.	0.	0.38	0.
time (sec)	N/A	0.124	0.039	0.083	0.	0.	2.025	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	68	180	0	0	78	0
normalized size	1	1.	0.35	0.94	0.	0.	0.41	0.
time (sec)	N/A	0.097	0.029	0.013	0.	0.	1.833	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	48	168	0	0	76	0
normalized size	1	1.	0.27	0.95	0.	0.	0.43	0.
time (sec)	N/A	0.064	0.011	0.011	0.	0.	1.733	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	53	167	0	0	78	0
normalized size	1	1.	0.31	0.98	0.	0.	0.46	0.
time (sec)	N/A	0.067	0.023	0.016	0.	0.	1.833	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	54	170	0	0	83	0
normalized size	1	1.	0.28	0.89	0.	0.	0.43	0.
time (sec)	N/A	0.089	0.024	0.016	0.	0.	1.994	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	72	73	171	166	131	88
normalized size	1	1.	0.87	0.88	2.06	2.	1.58	1.06
time (sec)	N/A	0.059	0.054	0.024	1.423	1.534	13.573	1.147

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	58	159	150	124	80
normalized size	1	1.	0.81	0.87	2.37	2.24	1.85	1.19
time (sec)	N/A	0.04	0.041	0.006	1.418	1.595	10.509	1.158

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	56	46	128	131	109	70
normalized size	1	1.	0.93	0.77	2.13	2.18	1.82	1.17
time (sec)	N/A	0.03	0.032	0.012	1.421	1.536	7.254	1.115

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	67	75	186	185	114	0
normalized size	1	1.	0.86	0.96	2.38	2.37	1.46	0.
time (sec)	N/A	0.075	0.045	0.016	1.437	1.547	25.638	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	71	75	165	200	114	0
normalized size	1	1.	0.88	0.93	2.04	2.47	1.41	0.
time (sec)	N/A	0.075	0.057	0.017	1.426	1.542	9.079	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	60	73	166	204	133	0
normalized size	1	1.	0.7	0.85	1.93	2.37	1.55	0.
time (sec)	N/A	0.077	0.032	0.017	1.428	1.559	10.516	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	60	73	151	209	148	0
normalized size	1	1.	0.73	0.89	1.84	2.55	1.8	0.
time (sec)	N/A	0.075	0.031	0.019	1.419	1.656	10.767	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	74	216	0	0	160	0
normalized size	1	1.	0.31	0.92	0.	0.	0.68	0.
time (sec)	N/A	0.134	0.046	0.023	0.	0.	4.019	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	68	204	0	0	160	0
normalized size	1	1.	0.31	0.93	0.	0.	0.73	0.
time (sec)	N/A	0.12	0.031	0.011	0.	0.	3.306	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	49	192	0	0	158	0
normalized size	1	1.	0.25	0.97	0.	0.	0.8	0.
time (sec)	N/A	0.082	0.013	0.013	0.	0.	2.929	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	53	192	0	0	160	0
normalized size	1	1.	0.27	0.96	0.	0.	0.8	0.
time (sec)	N/A	0.088	0.028	0.015	0.	0.	3.373	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	54	192	0	0	163	0
normalized size	1	1.	0.27	0.96	0.	0.	0.81	0.
time (sec)	N/A	0.086	0.027	0.016	0.	0.	3.777	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	44	51	140	120	85	62
normalized size	1	1.	0.66	0.76	2.09	1.79	1.27	0.93
time (sec)	N/A	0.058	0.035	0.017	1.457	1.543	6.513	1.136

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	35	39	103	92	66	50
normalized size	1	1.	0.69	0.76	2.02	1.8	1.29	0.98
time (sec)	N/A	0.042	0.023	0.011	1.464	1.513	4.787	1.12

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	32	88	86	53	45
normalized size	1	1.	0.97	0.91	2.51	2.46	1.51	1.29
time (sec)	N/A	0.026	0.02	0.006	1.422	1.525	3.522	1.136

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	20	57	63	22	35
normalized size	1	1.	1.	0.83	2.38	2.62	0.92	1.46
time (sec)	N/A	0.017	0.025	0.009	1.444	1.505	1.718	1.128

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	30	90	109	31	0
normalized size	1	1.	1.	0.79	2.37	2.87	0.82	0.
time (sec)	N/A	0.038	0.022	0.009	1.434	1.526	4.893	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	63	119	31	65
normalized size	1	1.	1.	0.74	1.5	2.83	0.74	1.55
time (sec)	N/A	0.037	0.025	0.01	1.429	1.55	3.04	1.184

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	49	43	80	132	88	72
normalized size	1	1.	0.84	0.74	1.38	2.28	1.52	1.24
time (sec)	N/A	0.051	0.03	0.013	1.429	1.543	7.571	1.172

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	74	168	0	0	75	0
normalized size	1	1.	0.4	0.91	0.	0.	0.41	0.
time (sec)	N/A	0.085	0.033	0.017	0.	0.	2.105	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	66	155	0	0	75	0
normalized size	1	1.	0.4	0.93	0.	0.	0.45	0.
time (sec)	N/A	0.066	0.025	0.013	0.	0.	1.91	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	48	146	0	0	73	0
normalized size	1	1.	0.31	0.94	0.	0.	0.47	0.
time (sec)	N/A	0.046	0.014	0.009	0.	0.	1.403	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	53	158	0	0	75	0
normalized size	1	1.	0.31	0.91	0.	0.	0.43	0.
time (sec)	N/A	0.062	0.027	0.015	0.	0.	1.547	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	54	170	0	0	80	0
normalized size	1	1.	0.29	0.9	0.	0.	0.42	0.
time (sec)	N/A	0.086	0.026	0.017	0.	0.	1.897	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	51	50	120	158	66	61
normalized size	1	1.	0.88	0.86	2.07	2.72	1.14	1.05
time (sec)	N/A	0.046	0.029	0.017	1.416	1.509	12.297	1.135

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	46	37	85	143	48	53
normalized size	1	1.	1.02	0.82	1.89	3.18	1.07	1.18
time (sec)	N/A	0.039	0.024	0.013	1.42	1.496	10.265	1.168

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	41	34	73	131	39	45
normalized size	1	1.	1.17	0.97	2.09	3.74	1.11	1.29
time (sec)	N/A	0.027	0.019	0.007	1.42	1.518	7.086	1.18

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	30	78	31	22
normalized size	1	1.	1.	0.85	1.5	3.9	1.55	1.1
time (sec)	N/A	0.016	0.01	0.005	1.411	1.509	4.292	1.13

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	40	76	159	212	72
normalized size	1	1.	1.	0.87	1.65	3.46	4.61	1.57
time (sec)	N/A	0.043	0.039	0.016	1.427	1.496	11.613	1.16

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	47	92	186	228	0
normalized size	1	1.	0.69	0.72	1.42	2.86	3.51	0.
time (sec)	N/A	0.055	0.028	0.013	1.426	1.533	10.205	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	70	168	0	0	75	0
normalized size	1	1.	0.36	0.86	0.	0.	0.38	0.
time (sec)	N/A	0.085	0.046	0.02	0.	0.	4.757	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	68	168	0	0	75	0
normalized size	1	1.	0.38	0.95	0.	0.	0.42	0.
time (sec)	N/A	0.069	0.032	0.013	0.	0.	4.286	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	66	168	0	0	73	0
normalized size	1	1.	0.37	0.93	0.	0.	0.41	0.
time (sec)	N/A	0.061	0.026	0.013	0.	0.	4.256	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	71	180	0	0	75	0
normalized size	1	1.	0.36	0.92	0.	0.	0.38	0.
time (sec)	N/A	0.082	0.065	0.018	0.	0.	6.071	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	54	192	0	0	80	0
normalized size	1	1.	0.25	0.9	0.	0.	0.37	0.
time (sec)	N/A	0.108	0.029	0.019	0.	0.	8.123	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	189	2295	0	5265	0	5065
normalized size	1	1.	0.7	8.53	0.	19.57	0.	18.83
time (sec)	N/A	0.161	0.171	0.019	0.	1.771	0.	1.308

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	153	130	174	390	134	193
normalized size	1	1.	2.43	2.06	2.76	6.19	2.13	3.06
time (sec)	N/A	0.197	0.024	0.002	0.941	1.28	0.098	1.116

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	153	130	174	420	141	194
normalized size	1	1.	1.	0.85	1.14	2.75	0.92	1.27
time (sec)	N/A	0.117	0.025	0.001	0.979	1.263	0.097	1.103

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	151	130	174	387	136	194
normalized size	1	1.	3.36	2.89	3.87	8.6	3.02	4.31
time (sec)	N/A	0.123	0.021	0.	0.984	1.244	0.096	1.111

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	153	130	174	414	139	194
normalized size	1	1.	1.	0.85	1.14	2.71	0.91	1.27
time (sec)	N/A	0.085	0.021	0.	0.966	1.272	0.098	1.123

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	149	130	174	381	133	194
normalized size	1	1.	5.14	4.48	6.	13.14	4.59	6.69
time (sec)	N/A	0.049	0.014	0.002	0.973	1.233	0.094	1.13

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	143	127	169	397	134	190
normalized size	1	1.	1.	0.89	1.18	2.78	0.94	1.33
time (sec)	N/A	0.074	0.018	0.001	0.965	1.271	0.097	1.105

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	149	132	176	344	131	196
normalized size	1	1.	1.6	1.42	1.89	3.7	1.41	2.11
time (sec)	N/A	0.055	0.028	0.003	0.956	1.427	0.321	1.114

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	141	129	169	401	124	188
normalized size	1	1.	1.	0.91	1.2	2.84	0.88	1.33
time (sec)	N/A	0.082	0.028	0.004	0.938	1.422	0.474	1.122

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	147	131	176	378	131	211
normalized size	1	1.	1.	0.89	1.2	2.57	0.89	1.44
time (sec)	N/A	0.136	0.04	0.006	0.946	1.434	0.534	1.11

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	122	1121	0	3202	0	2495
normalized size	1	1.	0.6	5.52	0.	15.77	0.	12.29
time (sec)	N/A	0.073	0.052	0.01	0.	1.631	0.	1.226

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	85	62	82	194	76	82
normalized size	1	1.	2.5	1.82	2.41	5.71	2.24	2.41
time (sec)	N/A	0.047	0.002	0.001	0.953	1.252	0.068	1.117

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	62	82	197	75	82
normalized size	1	1.	1.	0.75	0.99	2.37	0.9	0.99
time (sec)	N/A	0.031	0.002	0.001	0.964	1.255	0.07	1.109

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	83	62	82	186	75	82
normalized size	1	1.	3.61	2.7	3.57	8.09	3.26	3.57
time (sec)	N/A	0.022	0.002	0.002	0.941	1.255	0.072	1.112

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	62	82	196	75	82
normalized size	1	1.	1.	0.75	0.99	2.36	0.9	0.99
time (sec)	N/A	0.027	0.002	0.001	0.969	1.244	0.068	1.132

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	62	82	182	71	82
normalized size	1	1.	1.	5.64	7.45	16.55	6.45	7.45
time (sec)	N/A	0.002	0.002	0.001	0.982	1.465	0.068	1.111

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	77	180	68	77
normalized size	1	1.	1.	0.79	1.05	2.47	0.93	1.05
time (sec)	N/A	0.022	0.001	0.001	0.966	1.522	0.072	1.117

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	59	84	193	75	84
normalized size	1	1.	1.	0.74	1.05	2.41	0.94	1.05
time (sec)	N/A	0.033	0.003	0.001	0.974	1.667	0.104	1.108

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	60	80	232	66	80
normalized size	1	1.	1.	0.82	1.1	3.18	0.9	1.1
time (sec)	N/A	0.025	0.003	0.005	0.927	1.395	0.101	1.119

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	61	84	227	75	93
normalized size	1	1.	1.	0.76	1.05	2.84	0.94	1.16
time (sec)	N/A	0.04	0.003	0.007	0.964	1.462	0.117	1.119

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	80	90	0	277	90	136
normalized size	1	1.	0.55	0.62	0.	1.91	0.62	0.94
time (sec)	N/A	0.09	0.049	0.046	0.	1.557	0.511	1.105

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	51	55	92	65	27	57
normalized size	1	1.	0.61	0.66	1.11	0.78	0.33	0.69
time (sec)	N/A	0.072	0.021	0.007	0.993	1.522	0.432	1.136

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	69	62	0	223	82	80
normalized size	1	1.	0.71	0.64	0.	2.3	0.85	0.82
time (sec)	N/A	0.047	0.029	0.008	0.	1.512	0.466	1.131

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	54	57	0	74	26	82
normalized size	1	1.	0.59	0.62	0.	0.8	0.28	0.89
time (sec)	N/A	0.072	0.021	0.01	0.	1.498	0.732	1.116

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	72	67	0	230	82	84
normalized size	1	1.	0.71	0.66	0.	2.28	0.81	0.83
time (sec)	N/A	0.063	0.031	0.01	0.	1.574	0.515	1.119

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	70	79	0	109	41	177
normalized size	1	1.	0.51	0.58	0.	0.8	0.3	1.29
time (sec)	N/A	0.098	0.033	0.014	0.	1.536	0.879	1.139

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	108	188	0	621	0	0
normalized size	1	1.	0.71	1.23	0.	4.06	0.	0.
time (sec)	N/A	0.119	0.063	0.016	0.	1.566	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	45	38	96	86	0	0
normalized size	1	1.	0.58	0.49	1.25	1.12	0.	0.
time (sec)	N/A	0.066	0.02	0.009	0.985	1.492	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	108	186	0	621	0	0
normalized size	1	1.	0.69	1.19	0.	3.98	0.	0.
time (sec)	N/A	0.089	0.053	0.016	0.	1.521	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	92	133	0	250	0	0
normalized size	1	1.	0.57	0.83	0.	1.55	0.	0.
time (sec)	N/A	0.118	0.043	0.021	0.	1.563	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	124	206	0	686	0	0
normalized size	1	1.	0.65	1.08	0.	3.61	0.	0.
time (sec)	N/A	0.185	0.068	0.019	0.	1.596	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	130	249	0	413	0	0
normalized size	1	1.	0.58	1.12	0.	1.85	0.	0.
time (sec)	N/A	0.184	0.073	0.021	0.	1.566	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	160	1099	663	2005	0	2988
normalized size	1	1.	0.4	2.75	1.66	5.01	0.	7.47
time (sec)	N/A	0.243	0.233	0.009	1.04	1.636	0.	1.284

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	112	495	328	871	0	1368
normalized size	1	1.	0.41	1.79	1.19	3.16	0.	4.96
time (sec)	N/A	0.153	0.128	0.009	0.998	1.611	0.	1.179

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	86	131	101	216	0	363
normalized size	1	1.	0.56	0.86	0.66	1.41	0.	2.37
time (sec)	N/A	0.076	0.053	0.004	1.015	1.587	0.	1.165

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	78	0	0	0	0	0
normalized size	1	1.	0.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.073	0.059	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	101	0	0	0	0	0
normalized size	1	1.	0.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.084	0.028	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	25	40	116	99	165	119
normalized size	1	1.	0.74	1.18	3.41	2.91	4.85	3.5
time (sec)	N/A	0.029	0.009	0.003	0.994	1.608	11.997	1.123

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	45	62	182	185	0	265
normalized size	1	1.	0.52	0.72	2.12	2.15	0.	3.08
time (sec)	N/A	0.089	0.025	0.005	1.012	1.559	0.	1.142

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	68	99	265	284	0	447
normalized size	1	1.	0.53	0.77	2.07	2.22	0.	3.49
time (sec)	N/A	0.129	0.037	0.006	1.018	1.567	0.	1.131

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	166	226	224	501	202	261
normalized size	1	1.	1.	1.36	1.35	3.02	1.22	1.57
time (sec)	N/A	0.393	0.051	0.001	1.003	1.214	0.098	1.143

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	166	226	224	494	204	261
normalized size	1	1.	1.	1.36	1.35	2.98	1.23	1.57
time (sec)	N/A	0.154	0.057	0.	0.988	1.26	0.098	1.151

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	154	226	224	490	199	261
normalized size	1	1.	0.93	1.36	1.35	2.95	1.2	1.57
time (sec)	N/A	0.287	0.062	0.001	1.093	1.271	0.1	1.111

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	161	223	220	471	199	255
normalized size	1	1.	1.	1.39	1.37	2.93	1.24	1.58
time (sec)	N/A	0.118	0.05	0.001	1.124	1.234	0.102	1.108

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	162	191	225	381	199	261
normalized size	1	1.	1.	1.18	1.39	2.35	1.23	1.61
time (sec)	N/A	0.227	0.062	0.003	0.989	1.453	0.481	1.104

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	156	186	219	404	185	250
normalized size	1	1.	1.	1.19	1.4	2.59	1.19	1.6
time (sec)	N/A	0.108	0.094	0.005	0.987	1.481	0.479	1.136

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	162	190	225	390	197	286
normalized size	1	1.	1.	1.17	1.39	2.41	1.22	1.77
time (sec)	N/A	0.226	0.075	0.008	0.979	1.461	0.574	1.119

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	126	261	0	896	619	170
normalized size	1	1.	0.95	1.96	0.	6.74	4.65	1.28
time (sec)	N/A	0.207	0.061	0.005	0.	1.697	8.129	1.211

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	93	175	0	670	434	123
normalized size	1	1.	0.96	1.8	0.	6.91	4.47	1.27
time (sec)	N/A	0.116	0.068	0.001	0.	1.621	4.978	1.178

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	98	0	487	287	90
normalized size	1	1.	1.	1.38	0.	6.86	4.04	1.27
time (sec)	N/A	0.07	0.052	0.003	0.	1.539	2.512	1.185

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	128	105	0	567	330	105
normalized size	1	1.	1.64	1.35	0.	7.27	4.23	1.35
time (sec)	N/A	0.139	0.108	0.005	0.	1.74	53.625	1.195

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	186	191	0	834	495	167
normalized size	1	1.	1.66	1.71	0.	7.45	4.42	1.49
time (sec)	N/A	0.245	0.227	0.01	0.	2.702	170.625	1.167

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	327	825	0	10329	709	0
normalized size	1	1.	1.25	3.16	0.	39.57	2.72	0.
time (sec)	N/A	1.489	0.414	0.046	0.	9.021	32.406	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	251	560	0	5264	428	0
normalized size	1	1.	1.21	2.69	0.	25.31	2.06	0.
time (sec)	N/A	0.528	0.168	0.023	0.	2.964	11.205	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	173	328	0	3077	314	6435
normalized size	1	1.	1.01	1.91	0.	17.89	1.83	37.41
time (sec)	N/A	0.201	0.098	0.018	0.	2.24	4.975	2.432

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	206	353	0	5806	490	5257
normalized size	1	1.	1.09	1.87	0.	30.72	2.59	27.81
time (sec)	N/A	0.4	0.289	0.021	0.	3.562	12.052	2.548

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	267	611	0	10842	774	0
normalized size	1	1.	0.99	2.25	0.	40.01	2.86	0.
time (sec)	N/A	0.653	0.329	0.025	0.	10.184	34.436	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	208	689	0	2789	1266	323
normalized size	1	1.	0.98	3.25	0.	13.16	5.97	1.52
time (sec)	N/A	0.381	0.313	0.017	0.	2.241	37.16	19.541

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	160	286	0	1804	916	262
normalized size	1	1.	1.09	1.95	0.	12.27	6.23	1.78
time (sec)	N/A	0.175	0.206	0.013	0.	1.84	18.004	19.928

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	111	158	0	1131	394	162
normalized size	1	1.	1.04	1.48	0.	10.57	3.68	1.51
time (sec)	N/A	0.113	0.087	0.01	0.	1.529	6.086	19.652

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	101	127	0	1026	374	138
normalized size	1	1.	1.07	1.35	0.	10.91	3.98	1.47
time (sec)	N/A	0.088	0.084	0.006	0.	1.604	3.877	20.241

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	243	361	0	2160	0	271
normalized size	1	1.	1.62	2.41	0.	14.4	0.	1.81
time (sec)	N/A	0.331	0.351	0.017	0.	5.758	0.	19.867

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	379	622	0	3426	0	338
normalized size	1	1.	1.7	2.79	0.	15.36	0.	1.52
time (sec)	N/A	0.419	0.551	0.023	0.	11.263	0.	19.689

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	425	425	455	1507	0	15583	1448	0
normalized size	1	1.	1.07	3.55	0.	36.67	3.41	0.
time (sec)	N/A	3.675	1.311	0.04	0.	24.211	168.3	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	362	1030	0	9642	1129	0
normalized size	1	1.	1.08	3.07	0.	28.7	3.36	0.
time (sec)	N/A	1.717	0.939	0.033	0.	8.306	68.212	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	298	733	0	7073	923	0
normalized size	1	1.	1.08	2.66	0.	25.63	3.34	0.
time (sec)	N/A	0.553	0.707	0.03	0.	4.665	31.822	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	293	293	304	1761	0	10055	1180	0
normalized size	1	1.	1.04	6.01	0.	34.32	4.03	0.
time (sec)	N/A	0.846	0.844	0.084	0.	11.507	61.45	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	382	1252	0	16292	0	0
normalized size	1	1.	0.98	3.22	0.	41.88	0.	0.
time (sec)	N/A	1.22	1.177	0.039	0.	25.883	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	487	1653	0	23162	0	0
normalized size	1	1.	0.93	3.17	0.	44.37	0.	0.
time (sec)	N/A	1.365	1.32	0.046	0.	54.506	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	435	2054	0	6745	0	807
normalized size	1	1.	1.19	5.63	0.	18.48	0.	2.21
time (sec)	N/A	1.455	0.758	0.026	0.	3.347	0.	38.279

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	354	723	0	4617	0	629
normalized size	1	1.	1.39	2.85	0.	18.18	0.	2.48
time (sec)	N/A	0.404	0.526	0.02	0.	2.041	0.	39.023

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	261	398	0	2839	775	429
normalized size	1	1.	1.79	2.73	0.	19.45	5.31	2.94
time (sec)	N/A	0.139	0.294	0.017	0.	1.457	149.303	35.484

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	233	411	0	2824	833	362
normalized size	1	1.	1.26	2.22	0.	15.26	4.5	1.96
time (sec)	N/A	0.262	0.271	0.014	0.	1.46	63.236	31.829

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	172	379	0	2595	789	308
normalized size	1	1.	1.01	2.23	0.	15.26	4.64	1.81
time (sec)	N/A	0.163	0.237	0.013	0.	1.576	27.452	27.678

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	142	262	0	2367	661	281
normalized size	1	1.	1.02	1.88	0.	17.03	4.76	2.02
time (sec)	N/A	0.124	0.142	0.009	0.	1.41	15.486	26.209

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	396	1161	0	5311	0	568
normalized size	1	1.	1.57	4.61	0.	21.08	0.	2.25
time (sec)	N/A	0.543	0.677	0.025	0.	15.524	0.	26.115

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	363	363	642	1862	0	8294	0	875
normalized size	1	1.	1.77	5.13	0.	22.85	0.	2.41
time (sec)	N/A	0.771	1.5	0.033	0.	34.522	0.	27.415

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	554	554	644	2015	0	21535	0	0
normalized size	1	1.	1.16	3.64	0.	38.87	0.	0.
time (sec)	N/A	11.195	2.569	0.051	0.	49.246	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	461	461	543	1631	0	15486	0	0
normalized size	1	1.	1.18	3.54	0.	33.59	0.	0.
time (sec)	N/A	4.622	2.214	0.045	0.	19.85	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	447	1283	0	11958	0	0
normalized size	1	1.	1.18	3.38	0.	31.47	0.	0.
time (sec)	N/A	1.415	1.839	0.043	0.	14.693	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	438	438	436	1335	0	15784	0	0
normalized size	1	1.	1.	3.05	0.	36.04	0.	0.
time (sec)	N/A	1.09	1.666	0.043	0.	26.788	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	516	11936	0	22209	0	0
normalized size	1	1.	1.12	25.95	0.	48.28	0.	0.
time (sec)	N/A	1.353	2.402	0.194	0.	67.186	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	23	50	17	26
normalized size	1	1.	1.	0.72	0.92	2.	0.68	1.04
time (sec)	N/A	0.018	0.006	0.006	0.953	1.634	0.115	1.087

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	34	50	17	26
normalized size	1	1.	1.	0.72	1.36	2.	0.68	1.04
time (sec)	N/A	0.029	0.005	0.005	0.946	1.479	0.11	1.082

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	41	95	37	41
normalized size	1	1.	1.	0.84	1.11	2.57	1.	1.11
time (sec)	N/A	0.035	0.011	0.005	1.459	1.489	0.116	1.092

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	72	95	37	41
normalized size	1	1.	1.	0.84	1.95	2.57	1.	1.11
time (sec)	N/A	0.042	0.006	0.002	1.48	1.52	0.114	1.091

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	41	0	132	44	51
normalized size	1	1.	1.	0.91	0.	2.93	0.98	1.13
time (sec)	N/A	0.049	0.025	0.006	0.	1.48	0.153	1.13

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	71	91	140	185	0	90
normalized size	1	1.	0.7	0.89	1.37	1.81	0.	0.88
time (sec)	N/A	0.079	0.031	0.023	0.983	1.596	0.	1.115

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	66	74	117	161	0	81
normalized size	1	1.	0.81	0.91	1.44	1.99	0.	1.
time (sec)	N/A	0.056	0.021	0.014	0.978	1.639	0.	1.084

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	61	57	95	136	0	72
normalized size	1	1.	0.82	0.77	1.28	1.84	0.	0.97
time (sec)	N/A	0.044	0.017	0.012	1.227	1.7	0.	1.116

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	92	85	120	254	0	0
normalized size	1	1.	0.98	0.9	1.28	2.7	0.	0.
time (sec)	N/A	0.083	0.038	0.011	1.439	1.567	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	97	104	120	292	0	0
normalized size	1	1.	1.	1.07	1.24	3.01	0.	0.
time (sec)	N/A	0.083	0.04	0.014	1.467	1.53	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	97	121	143	293	0	0
normalized size	1	1.	0.98	1.22	1.44	2.96	0.	0.
time (sec)	N/A	0.083	0.04	0.014	1.474	1.64	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	74	118	134	234	0	0
normalized size	1	1.	0.82	1.31	1.49	2.6	0.	0.
time (sec)	N/A	0.068	0.026	0.013	1.431	1.391	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	82	135	157	261	0	0
normalized size	1	1.	0.74	1.22	1.41	2.35	0.	0.
time (sec)	N/A	0.086	0.028	0.016	1.464	1.412	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	84	152	180	292	0	0
normalized size	1	1.	0.64	1.15	1.36	2.21	0.	0.
time (sec)	N/A	0.109	0.037	0.018	1.504	1.331	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	322	322	237	260	0	0	0	0
normalized size	1	1.	0.74	0.81	0.	0.	0.	0.
time (sec)	N/A	0.274	0.325	0.098	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	305	305	234	243	0	0	0	0
normalized size	1	1.	0.77	0.8	0.	0.	0.	0.
time (sec)	N/A	0.203	0.287	0.013	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	229	226	0	0	0	0
normalized size	1	1.	0.82	0.81	0.	0.	0.	0.
time (sec)	N/A	0.122	0.291	0.012	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	284	284	231	225	0	0	0	0
normalized size	1	1.	0.81	0.79	0.	0.	0.	0.
time (sec)	N/A	0.128	0.304	0.018	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	305	305	237	228	0	0	0	0
normalized size	1	1.	0.78	0.75	0.	0.	0.	0.
time (sec)	N/A	0.158	0.313	0.017	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	81	138	182	240	0	109
normalized size	1	1.	0.64	1.09	1.43	1.89	0.	0.86
time (sec)	N/A	0.096	0.038	0.034	1.058	1.205	0.	1.125

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	76	121	159	204	0	100
normalized size	1	1.	0.72	1.14	1.5	1.92	0.	0.94
time (sec)	N/A	0.072	0.031	0.016	0.971	1.32	0.	1.094

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	71	104	136	180	0	90
normalized size	1	1.	0.72	1.05	1.37	1.82	0.	0.91
time (sec)	N/A	0.058	0.026	0.014	0.945	1.245	0.	1.104

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	104	117	162	300	0	0
normalized size	1	1.	0.87	0.98	1.36	2.52	0.	0.
time (sec)	N/A	0.106	0.064	0.016	1.483	1.298	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	107	117	162	325	0	0
normalized size	1	1.	0.88	0.96	1.33	2.66	0.	0.
time (sec)	N/A	0.107	0.054	0.015	1.468	1.348	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	107	117	185	320	0	0
normalized size	1	1.	0.84	0.92	1.46	2.52	0.	0.
time (sec)	N/A	0.109	0.065	0.017	1.483	1.38	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	107	117	208	321	0	0
normalized size	1	1.	0.84	0.92	1.64	2.53	0.	0.
time (sec)	N/A	0.107	0.052	0.017	1.49	1.314	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	356	356	249	294	0	0	0	0
normalized size	1	1.	0.7	0.83	0.	0.	0.	0.
time (sec)	N/A	0.256	0.317	0.02	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	331	331	244	277	0	0	0	0
normalized size	1	1.	0.74	0.84	0.	0.	0.	0.
time (sec)	N/A	0.214	0.308	0.015	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	308	308	0	260	0	0	0	0
normalized size	1	1.	0.	0.84	0.	0.	0.	0.
time (sec)	N/A	0.153	0.	0.012	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	312	312	235	260	0	0	0	0
normalized size	1	1.	0.75	0.83	0.	0.	0.	0.
time (sec)	N/A	0.151	0.308	0.016	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	314	314	247	260	0	0	0	0
normalized size	1	1.	0.79	0.83	0.	0.	0.	0.
time (sec)	N/A	0.164	0.335	0.016	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	331	331	244	259	0	0	0	0
normalized size	1	1.	0.74	0.78	0.	0.	0.	0.
time (sec)	N/A	0.203	0.317	0.02	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	139	286	0	725	0	203
normalized size	1	1.	0.91	1.87	0.	4.74	0.	1.33
time (sec)	N/A	0.203	0.107	0.027	0.	1.553	0.	1.2

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	101	176	0	545	0	132
normalized size	1	1.	1.01	1.76	0.	5.45	0.	1.32
time (sec)	N/A	0.093	0.048	0.015	0.	1.483	0.	1.153

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	78	93	0	421	0	93
normalized size	1	1.	1.03	1.22	0.	5.54	0.	1.22
time (sec)	N/A	0.064	0.025	0.01	0.	1.508	0.	1.178

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	89	76	0	1211	0	0
normalized size	1	1.	0.99	0.84	0.	13.46	0.	0.
time (sec)	N/A	0.094	0.029	0.013	0.	2.049	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	82	104	0	460	0	0
normalized size	1	1.	1.02	1.3	0.	5.75	0.	0.
time (sec)	N/A	0.083	0.036	0.016	0.	1.77	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	107	194	0	593	0	0
normalized size	1	1.	0.86	1.56	0.	4.78	0.	0.
time (sec)	N/A	0.145	0.076	0.018	0.	2.379	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	148	311	0	779	0	0
normalized size	1	1.	0.84	1.76	0.	4.4	0.	0.
time (sec)	N/A	0.239	0.116	0.019	0.	3.148	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	532	815	0	0	0	0
normalized size	1	1.	1.32	2.02	0.	0.	0.	0.
time (sec)	N/A	0.284	2.164	0.048	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	479	607	0	0	0	0
normalized size	1	1.	1.43	1.81	0.	0.	0.	0.
time (sec)	N/A	0.149	1.382	0.01	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	302	362	0	0	0	0
normalized size	1	1.	1.07	1.28	0.	0.	0.	0.
time (sec)	N/A	0.083	0.256	0.009	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	448	386	0	0	0	0
normalized size	1	1.	1.44	1.24	0.	0.	0.	0.
time (sec)	N/A	0.131	1.075	0.014	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	373	656	0	0	0	0
normalized size	1	1.	0.99	1.74	0.	0.	0.	0.
time (sec)	N/A	0.227	0.696	0.018	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	66	87	122	165	0	81
normalized size	1	1.	0.67	0.89	1.24	1.68	0.	0.83
time (sec)	N/A	0.087	0.031	0.016	0.95	1.374	0.	1.142

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	61	70	99	139	0	72
normalized size	1	1.	0.79	0.91	1.29	1.81	0.	0.94
time (sec)	N/A	0.066	0.022	0.012	0.96	1.315	0.	1.132

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	76	123	0	62
normalized size	1	1.	1.	0.95	1.36	2.2	0.	1.11
time (sec)	N/A	0.045	0.016	0.011	0.968	1.28	0.	1.114

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	36	53	103	0	53
normalized size	1	1.	1.	0.73	1.08	2.1	0.	1.08
time (sec)	N/A	0.032	0.01	0.01	0.953	1.399	0.	1.112

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	52	78	203	0	0
normalized size	1	1.	1.	0.75	1.13	2.94	0.	0.
time (sec)	N/A	0.062	0.016	0.01	1.43	1.467	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	62	49	69	200	0	0
normalized size	1	1.	1.	0.79	1.11	3.23	0.	0.
time (sec)	N/A	0.05	0.015	0.013	1.425	1.4	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	67	66	92	215	0	0
normalized size	1	1.	0.81	0.8	1.11	2.59	0.	0.
time (sec)	N/A	0.07	0.023	0.013	1.446	1.395	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	77	83	115	235	0	0
normalized size	1	1.	0.74	0.8	1.11	2.26	0.	0.
time (sec)	N/A	0.088	0.024	0.013	1.463	1.469	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	298	298	229	226	0	0	0	0
normalized size	1	1.	0.77	0.76	0.	0.	0.	0.
time (sec)	N/A	0.175	0.304	0.02	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	222	208	0	0	0	0
normalized size	1	1.	0.82	0.77	0.	0.	0.	0.
time (sec)	N/A	0.12	0.293	0.014	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	257	257	159	194	0	0	0	0
normalized size	1	1.	0.62	0.75	0.	0.	0.	0.
time (sec)	N/A	0.077	0.144	0.011	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	278	278	224	211	0	0	0	0
normalized size	1	1.	0.81	0.76	0.	0.	0.	0.
time (sec)	N/A	0.123	0.275	0.017	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	302	302	237	228	0	0	0	0
normalized size	1	1.	0.78	0.75	0.	0.	0.	0.
time (sec)	N/A	0.163	0.307	0.017	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	72	91	99	232	0	70
normalized size	1	1.	0.94	1.18	1.29	3.01	0.	0.91
time (sec)	N/A	0.058	0.024	0.019	0.966	1.452	0.	1.124

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	54	95	76	209	0	62
normalized size	1	1.	0.96	1.7	1.36	3.73	0.	1.11
time (sec)	N/A	0.043	0.125	0.011	0.96	1.234	0.	1.149

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	43	113	0	28
normalized size	1	1.	1.	0.88	1.72	4.52	0.	1.12
time (sec)	N/A	0.019	0.095	0.005	0.952	1.331	0.	1.16

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	66	67	88	281	0	0
normalized size	1	1.	1.	1.02	1.33	4.26	0.	0.
time (sec)	N/A	0.057	0.029	0.018	1.436	1.376	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	88	84	111	308	0	0
normalized size	1	1.	0.98	0.93	1.23	3.42	0.	0.
time (sec)	N/A	0.071	0.018	0.013	1.666	1.349	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	307	307	219	240	0	0	0	0
normalized size	1	1.	0.71	0.78	0.	0.	0.	0.
time (sec)	N/A	0.163	0.301	0.021	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	286	286	219	240	0	0	0	0
normalized size	1	1.	0.77	0.84	0.	0.	0.	0.
time (sec)	N/A	0.123	0.287	0.014	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	A	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	282	282	0	240	0	0	0	0
normalized size	1	1.	0.	0.85	0.	0.	0.	0.
time (sec)	N/A	0.109	0.	0.013	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	309	309	228	257	0	0	0	0
normalized size	1	1.	0.74	0.83	0.	0.	0.	0.
time (sec)	N/A	0.164	0.315	0.02	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	326	326	234	274	0	0	0	0
normalized size	1	1.	0.72	0.84	0.	0.	0.	0.
time (sec)	N/A	0.206	0.321	0.02	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	297	297	430	0	0	0	0	0
normalized size	1	1.	1.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.385	0.975	0.056	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	297	297	386	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.323	5.737	0.04	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	295	295	386	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.321	0.747	0.038	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	295	295	370	0	0	0	0	0
normalized size	1	1.	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.317	0.913	0.042	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	299	299	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.347	0.	0.036	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	299	299	490	0	0	0	0	0
normalized size	1	1.	1.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.354	6.194	0.036	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	297	297	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.351	0.	0.035	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	297	297	447	0	0	0	0	0
normalized size	1	1.	1.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.348	1.057	0.038	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	297	297	354	0	0	0	0	0
normalized size	1	1.	1.19	0.	0.	0.	0.	0.
time (sec)	N/A	0.328	0.625	0.031	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	297	297	242	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.322	5.153	0.02	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	295	295	241	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.325	0.222	0.022	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	295	295	356	0	0	0	0	0
normalized size	1	1.	1.21	0.	0.	0.	0.	0.
time (sec)	N/A	0.325	0.686	0.037	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	303	303	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.344	0.	0.026	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	303	397	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.349	5.751	0.019	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	301	301	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.35	0.	0.033	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	301	301	460	0	0	0	0	0
normalized size	1	1.	1.53	0.	0.	0.	0.	0.
time (sec)	N/A	0.351	1.053	0.054	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	191	1935	0	3271	11538	3802
normalized size	1	1.	0.79	7.96	0.	13.46	47.48	15.65
time (sec)	N/A	0.176	0.341	0.008	0.	1.478	14.784	1.19

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	117	783	0	1416	4190	1590
normalized size	1	1.	0.75	5.05	0.	9.14	27.03	10.26
time (sec)	N/A	0.099	0.148	0.008	0.	1.385	6.136	1.133

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	59	221	0	414	1056	473
normalized size	1	1.	0.71	2.66	0.	4.99	12.72	5.7
time (sec)	N/A	0.047	0.049	0.003	0.	1.665	2.087	1.107

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	156	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.299	0.269	0.024	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	392	358	160	0	0	0	0	0
normalized size	1	0.91	0.41	0.	0.	0.	0.	0.
time (sec)	N/A	2.646	0.217	0.025	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	319	319	466	0	0	0	0	0
normalized size	1	1.	1.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.399	0.541	0.01	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	317	317	267	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.361	0.256	0.008	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	317	317	267	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.352	0.276	0.012	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	323	323	307	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.388	0.428	0.01	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	134	122	0	563	0	163
normalized size	1	1.	1.	0.91	0.	4.2	0.	1.22
time (sec)	N/A	0.179	0.061	0.011	0.	24.976	0.	1.1

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	99	108	0	446	0	142
normalized size	1	1.	0.84	0.92	0.	3.78	0.	1.2
time (sec)	N/A	0.152	0.095	0.007	0.	9.134	0.	1.089

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	77	92	0	365	0	122
normalized size	1	1.	0.73	0.88	0.	3.48	0.	1.16
time (sec)	N/A	0.137	0.035	0.008	0.	5.375	0.	1.115

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	66	83	0	315	932	116
normalized size	1	1.	0.69	0.86	0.	3.28	9.71	1.21
time (sec)	N/A	0.094	0.039	0.007	0.	2.3	144.776	1.114

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	67	83	0	319	0	115
normalized size	1	1.	0.7	0.86	0.	3.32	0.	1.2
time (sec)	N/A	0.064	0.037	0.006	0.	2.819	0.	1.099

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	134	101	0	439	0	138
normalized size	1	1.	1.18	0.89	0.	3.85	0.	1.21
time (sec)	N/A	0.125	0.071	0.01	0.	36.287	0.	1.096

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	169	119	0	574	0	178
normalized size	1	1.	1.31	0.92	0.	4.45	0.	1.38
time (sec)	N/A	0.15	0.103	0.01	0.	178.174	0.	1.11

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	209	145	0	0	0	227
normalized size	1	1.	1.34	0.93	0.	0.	0.	1.46
time (sec)	N/A	0.183	0.095	0.013	0.	0.	0.	1.094

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	344	405	0	8402	0	490
normalized size	1	1.	0.96	1.13	0.	23.4	0.	1.36
time (sec)	N/A	0.346	0.379	0.013	0.	52.357	0.	1.109

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	373	387	0	8208	0	450
normalized size	1	1.	1.08	1.12	0.	23.79	0.	1.3
time (sec)	N/A	0.302	0.246	0.01	0.	8.307	0.	1.201

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	233	363	0	7710	0	441
normalized size	1	1.	0.69	1.08	0.	22.95	0.	1.31
time (sec)	N/A	0.274	0.156	0.007	0.	2.7	0.	1.156

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	232	351	0	7464	0	454
normalized size	1	1.	0.69	1.04	0.	22.15	0.	1.35
time (sec)	N/A	0.267	0.133	0.008	0.	1.914	0.	1.138

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	234	363	0	7792	0	458
normalized size	1	1.	0.7	1.08	0.	23.19	0.	1.36
time (sec)	N/A	0.273	0.144	0.007	0.	4.46	0.	1.153

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	389	390	0	8235	0	470
normalized size	1	1.	1.12	1.12	0.	23.66	0.	1.35
time (sec)	N/A	0.3	0.253	0.01	0.	13.505	0.	1.122

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	367	406	0	8462	0	491
normalized size	1	1.	1.02	1.13	0.	23.51	0.	1.36
time (sec)	N/A	0.301	0.428	0.012	0.	47.087	0.	1.142

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	135	305	0	1106	0	339
normalized size	1	1.	0.8	1.8	0.	6.54	0.	2.01
time (sec)	N/A	0.367	0.217	0.033	0.	81.27	0.	1.098

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	142	260	0	922	0	301
normalized size	1	1.	0.95	1.73	0.	6.15	0.	2.01
time (sec)	N/A	0.247	0.115	0.017	0.	39.678	0.	1.109

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	120	252	0	987	0	297
normalized size	1	1.	0.78	1.65	0.	6.45	0.	1.94
time (sec)	N/A	0.248	0.158	0.016	0.	16.317	0.	1.088

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	114	247	0	992	0	254
normalized size	1	1.	0.77	1.67	0.	6.7	0.	1.72
time (sec)	N/A	0.187	0.148	0.016	0.	16.473	0.	1.107

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	117	255	0	925	0	269
normalized size	1	1.	0.77	1.69	0.	6.13	0.	1.78
time (sec)	N/A	0.181	0.139	0.016	0.	39.405	0.	1.094

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	241	309	0	0	0	377
normalized size	1	1.	1.15	1.48	0.	0.	0.	1.8
time (sec)	N/A	0.239	0.195	0.022	0.	0.	0.	1.075

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	248	332	0	0	0	464
normalized size	1	1.	1.05	1.41	0.	0.	0.	1.97
time (sec)	N/A	0.261	0.451	0.025	0.	0.	0.	1.11

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	278	363	0	0	0	473
normalized size	1	1.	1.05	1.37	0.	0.	0.	1.78
time (sec)	N/A	0.327	0.427	0.025	0.	0.	0.	1.097

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	712	712	431	873	0	20569	0	784
normalized size	1	1.	0.61	1.23	0.	28.89	0.	1.1
time (sec)	N/A	0.671	0.319	0.014	0.	96.675	0.	1.133

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	687	687	428	852	0	20034	0	803
normalized size	1	1.	0.62	1.24	0.	29.16	0.	1.17
time (sec)	N/A	0.602	0.379	0.014	0.	59.362	0.	1.137

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	685	685	423	848	0	19637	0	791
normalized size	1	1.	0.62	1.24	0.	28.67	0.	1.15
time (sec)	N/A	0.609	0.285	0.016	0.	59.105	0.	1.151

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	685	685	428	852	0	19999	0	814
normalized size	1	1.	0.62	1.24	0.	29.2	0.	1.19
time (sec)	N/A	0.56	0.294	0.013	0.	69.43	0.	1.131

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	689	689	429	873	0	20650	0	814
normalized size	1	1.	0.62	1.27	0.	29.97	0.	1.18
time (sec)	N/A	0.601	0.308	0.016	0.	132.414	0.	1.128

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	745	745	499	911	0	0	0	863
normalized size	1	1.	0.67	1.22	0.	0.	0.	1.16
time (sec)	N/A	0.772	0.382	0.017	0.	0.	0.	1.122

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	751	751	513	932	0	0	0	848
normalized size	1	1.	0.68	1.24	0.	0.	0.	1.13
time (sec)	N/A	0.686	0.399	0.02	0.	0.	0.	1.118

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	40	110	0	0	0	0
normalized size	1	1.	0.57	1.57	0.	0.	0.	0.
time (sec)	N/A	0.059	0.09	0.058	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	36	112	0	0	0	0
normalized size	1	1.	0.51	1.6	0.	0.	0.	0.
time (sec)	N/A	0.061	0.089	0.017	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	46	96	0	0	0	0
normalized size	1	1.	0.46	0.97	0.	0.	0.	0.
time (sec)	N/A	0.055	0.107	0.017	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	37	143	0	0	0	0
normalized size	1	1.	0.61	2.34	0.	0.	0.	0.
time (sec)	N/A	0.041	0.081	0.024	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	54	99	0	0	0	0
normalized size	1	1.	0.48	0.88	0.	0.	0.	0.
time (sec)	N/A	0.065	0.079	0.015	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	35	134	0	0	0	0
normalized size	1	1.	0.61	2.35	0.	0.	0.	0.
time (sec)	N/A	0.053	0.076	0.02	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	60	168	0	0	0	0
normalized size	1	1.	0.81	2.27	0.	0.	0.	0.
time (sec)	N/A	0.064	0.093	0.033	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	56	115	0	0	0	0
normalized size	1	1.	0.76	1.55	0.	0.	0.	0.
time (sec)	N/A	0.065	0.086	0.014	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	142	159	0	475	0	211
normalized size	1	1.	0.58	0.65	0.	1.95	0.	0.87
time (sec)	N/A	0.13	0.177	0.012	0.	1.944	0.	1.108

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	56	51	68	113	0	85
normalized size	1	1.	0.52	0.47	0.63	1.05	0.	0.79
time (sec)	N/A	0.101	0.03	0.004	0.965	1.803	0.	1.1

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	121	119	0	370	0	147
normalized size	1	1.	0.68	0.67	0.	2.08	0.	0.83
time (sec)	N/A	0.076	0.112	0.007	0.	1.82	0.	1.118

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	83	80	0	300	0	113
normalized size	1	1.	0.55	0.53	0.	1.97	0.	0.74
time (sec)	N/A	0.095	0.046	0.01	0.	1.838	0.	1.11

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	122	128	0	320	0	157
normalized size	1	1.	0.69	0.72	0.	1.81	0.	0.89
time (sec)	N/A	0.091	0.119	0.01	0.	1.833	0.	1.106

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	90	133	0	332	0	135
normalized size	1	1.	0.51	0.75	0.	1.88	0.	0.76
time (sec)	N/A	0.119	0.045	0.01	0.	2.022	0.	1.105

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	72	73	97	200	76	107
normalized size	1	1.	0.92	0.94	1.24	2.56	0.97	1.37
time (sec)	N/A	0.135	0.026	0.001	0.955	1.502	0.074	1.095

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	73	97	196	82	107
normalized size	1	1.	1.	0.94	1.24	2.51	1.05	1.37
time (sec)	N/A	0.064	0.016	0.	0.95	1.489	0.078	1.098

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	72	73	97	196	76	107
normalized size	1	1.	0.96	0.97	1.29	2.61	1.01	1.43
time (sec)	N/A	0.131	0.022	0.	0.932	1.487	0.074	1.072

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	93	185	78	103
normalized size	1	1.	1.	0.96	1.27	2.53	1.07	1.41
time (sec)	N/A	0.044	0.016	0.001	0.947	1.528	0.074	1.117

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	77	99	165	73	107
normalized size	1	1.	1.	1.04	1.34	2.23	0.99	1.45
time (sec)	N/A	0.09	0.021	0.003	0.958	1.66	0.316	1.098

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	71	75	93	170	73	100
normalized size	1	1.	1.	1.06	1.31	2.39	1.03	1.41
time (sec)	N/A	0.048	0.033	0.004	0.947	1.663	0.315	1.08

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	71	76	99	173	71	131
normalized size	1	1.	0.96	1.03	1.34	2.34	0.96	1.77
time (sec)	N/A	0.096	0.042	0.007	0.935	1.705	0.41	1.084

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	165	214	0	922	313	216
normalized size	1	1.	0.98	1.27	0.	5.49	1.86	1.29
time (sec)	N/A	0.234	0.137	0.01	0.	1.918	1.501	1.098

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	133	176	0	756	184	169
normalized size	1	1.	0.99	1.3	0.	5.6	1.36	1.25
time (sec)	N/A	0.159	0.079	0.012	0.	1.826	1.405	1.112

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	102	141	0	648	160	123
normalized size	1	1.	0.96	1.33	0.	6.11	1.51	1.16
time (sec)	N/A	0.106	0.066	0.008	0.	1.851	1.212	1.11

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	88	118	0	541	153	101
normalized size	1	1.	1.06	1.42	0.	6.52	1.84	1.22
time (sec)	N/A	0.093	0.054	0.009	0.	1.844	1.007	1.125

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	86	89	121	0	555	155	112
normalized size	1	0.97	1.	1.36	0.	6.24	1.74	1.26
time (sec)	N/A	0.118	0.062	0.013	0.	1.893	1.337	1.095

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	105	146	0	672	167	127
normalized size	1	1.	0.99	1.38	0.	6.34	1.58	1.2
time (sec)	N/A	0.137	0.065	0.014	0.	1.87	1.733	1.09

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	135	183	0	775	284	177
normalized size	1	1.	0.99	1.35	0.	5.7	2.09	1.3
time (sec)	N/A	0.252	0.09	0.013	0.	1.904	2.496	1.102

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	166	221	0	938	328	221
normalized size	1	1.	0.99	1.32	0.	5.62	1.96	1.32
time (sec)	N/A	0.33	0.095	0.016	0.	1.811	3.366	1.091

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	170	239	0	1125	233	216
normalized size	1	1.	0.98	1.38	0.	6.5	1.35	1.25
time (sec)	N/A	0.321	0.111	0.011	0.	1.861	4.373	1.1

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	141	202	0	1010	211	169
normalized size	1	1.	0.99	1.41	0.	7.06	1.48	1.18
time (sec)	N/A	0.21	0.087	0.012	0.	1.856	3.94	1.073

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	122	179	0	873	201	144
normalized size	1	1.	0.98	1.44	0.	7.04	1.62	1.16
time (sec)	N/A	0.138	0.104	0.01	0.	1.799	3.097	1.089

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	110	131	0	813	196	136
normalized size	1	1.	0.96	1.14	0.	7.07	1.7	1.18
time (sec)	N/A	0.116	0.097	0.008	0.	1.839	1.772	1.105

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	124	124	182	0	887	202	149
normalized size	1	0.98	0.98	1.43	0.	6.98	1.59	1.17
time (sec)	N/A	0.204	0.14	0.012	0.	1.771	2.421	1.095

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	141	207	0	1031	214	173
normalized size	1	1.	0.99	1.46	0.	7.26	1.51	1.22
time (sec)	N/A	0.217	0.086	0.014	0.	1.913	3.407	1.115

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	173	245	0	1143	330	221
normalized size	1	1.	1.01	1.43	0.	6.68	1.93	1.29
time (sec)	N/A	0.373	0.114	0.016	0.	1.853	4.766	1.126

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	228	538	0	0	0	319
normalized size	1	1.	0.99	2.34	0.	0.	0.	1.39
time (sec)	N/A	0.487	0.251	0.014	0.	0.	0.	1.164

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	186	408	0	0	0	262
normalized size	1	1.	0.98	2.16	0.	0.	0.	1.39
time (sec)	N/A	0.329	0.179	0.01	0.	0.	0.	1.172

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	139	289	0	0	0	212
normalized size	1	1.	0.88	1.83	0.	0.	0.	1.34
time (sec)	N/A	0.261	0.108	0.007	0.	0.	0.	1.176

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	114	176	0	716	0	180
normalized size	1	1.	0.86	1.33	0.	5.42	0.	1.36
time (sec)	N/A	0.157	0.072	0.008	0.	112.082	0.	1.136

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	112	176	0	718	0	181
normalized size	1	1.	0.84	1.32	0.	5.4	0.	1.36
time (sec)	N/A	0.123	0.069	0.008	0.	78.36	0.	1.165

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	242	298	0	0	0	232
normalized size	1	1.	1.45	1.78	0.	0.	0.	1.39
time (sec)	N/A	0.306	0.324	0.012	0.	0.	0.	1.14

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	331	430	0	0	0	320
normalized size	1	1.	1.61	2.1	0.	0.	0.	1.56
time (sec)	N/A	0.47	0.339	0.016	0.	0.	0.	1.158

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	426	584	0	0	0	448
normalized size	1	1.	1.59	2.18	0.	0.	0.	1.67
time (sec)	N/A	0.597	0.426	0.019	0.	0.	0.	1.185

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	387	387	463	1449	0	0	0	0
normalized size	1	1.	1.2	3.74	0.	0.	0.	0.
time (sec)	N/A	4.032	0.635	0.039	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	385	1098	0	0	0	0
normalized size	1	1.	1.19	3.4	0.	0.	0.	0.
time (sec)	N/A	1.366	0.543	0.033	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	323	764	0	30765	0	0
normalized size	1	1.	1.15	2.73	0.	109.88	0.	0.
time (sec)	N/A	0.894	0.344	0.027	0.	27.464	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	277	478	0	24804	0	0
normalized size	1	1.	1.1	1.9	0.	98.82	0.	0.
time (sec)	N/A	0.451	0.507	0.023	0.	10.006	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	274	480	0	0	0	0
normalized size	1	1.	1.08	1.89	0.	0.	0.	0.
time (sec)	N/A	0.52	0.251	0.025	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	340	817	0	0	0	0
normalized size	1	1.	1.14	2.74	0.	0.	0.	0.
time (sec)	N/A	0.96	0.434	0.028	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	410	1160	0	0	0	0
normalized size	1	1.	1.18	3.33	0.	0.	0.	0.
time (sec)	N/A	1.549	0.544	0.035	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	866	866	267	336	0	0	0	0
normalized size	1	1.	0.31	0.39	0.	0.	0.	0.
time (sec)	N/A	2.509	0.374	0.092	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	267	1049	0	0	0	0
normalized size	1	1.	0.98	3.86	0.	0.	0.	0.
time (sec)	N/A	0.573	0.427	0.043	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	205	887	0	2718	0	0
normalized size	1	1.	0.99	4.26	0.	13.07	0.	0.
time (sec)	N/A	0.315	0.265	0.01	0.	119.521	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	167	757	0	2327	0	0
normalized size	1	1.	0.99	4.51	0.	13.85	0.	0.
time (sec)	N/A	0.217	0.119	0.004	0.	9.723	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	179	851	0	0	0	0
normalized size	1	1.	0.96	4.58	0.	0.	0.	0.
time (sec)	N/A	0.256	0.157	0.016	0.	0.	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	361	165	1009	0	2414	0	0
normalized size	1	1.	0.46	2.8	0.	6.69	0.	0.
time (sec)	N/A	0.509	0.311	0.016	0.	3.759	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	619	209	528	0	0	0	0
normalized size	1	1.46	0.49	1.25	0.	0.	0.	0.
time (sec)	N/A	0.561	0.26	0.069	0.	0.	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	591	204	509	0	0	0	0
normalized size	1	1.42	0.49	1.22	0.	0.	0.	0.
time (sec)	N/A	0.41	0.183	0.007	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	470	127	341	0	0	0	0
normalized size	1	1.23	0.33	0.9	0.	0.	0.	0.
time (sec)	N/A	0.202	0.112	0.003	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	208	511	0	0	0	0
normalized size	1	1.	0.52	1.28	0.	0.	0.	0.
time (sec)	N/A	0.242	0.2	0.013	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	154	448	0	0	0	0
normalized size	1	1.	0.43	1.24	0.	0.	0.	0.
time (sec)	N/A	0.194	0.181	0.015	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	546	546	224	549	0	0	0	0
normalized size	1	1.	0.41	1.01	0.	0.	0.	0.
time (sec)	N/A	0.544	0.247	0.018	0.	0.	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	482	482	545	2068	0	0	0	0
normalized size	1	1.	1.13	4.29	0.	0.	0.	0.
time (sec)	N/A	1.103	1.03	0.049	0.	0.	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	360	360	344	1696	0	0	0	0
normalized size	1	1.	0.96	4.71	0.	0.	0.	0.
time (sec)	N/A	0.697	0.604	0.01	0.	0.	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	269	269	255	1411	0	0	0	0
normalized size	1	1.	0.95	5.25	0.	0.	0.	0.
time (sec)	N/A	0.456	0.366	0.006	0.	0.	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	251	1270	0	0	0	0
normalized size	1	1.	0.72	3.63	0.	0.	0.	0.
time (sec)	N/A	0.573	0.521	0.021	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	562	562	240	1207	0	0	0	0
normalized size	1	1.	0.43	2.15	0.	0.	0.	0.
time (sec)	N/A	0.924	0.513	0.023	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	875	214	547	0	0	0	0
normalized size	1	1.89	0.46	1.18	0.	0.	0.	0.
time (sec)	N/A	0.672	0.276	0.021	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	428	602	209	377	0	0	0	0
normalized size	1	1.41	0.49	0.88	0.	0.	0.	0.
time (sec)	N/A	0.349	0.159	0.007	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	722	722	213	528	0	0	0	0
normalized size	1	1.	0.3	0.73	0.	0.	0.	0.
time (sec)	N/A	0.368	0.219	0.013	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	625	625	219	530	0	0	0	0
normalized size	1	1.	0.35	0.85	0.	0.	0.	0.
time (sec)	N/A	0.366	0.235	0.016	0.	0.	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	224	549	0	0	0	0
normalized size	1	1.	0.41	0.99	0.	0.	0.	0.
time (sec)	N/A	0.488	0.265	0.016	0.	0.	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	167	267	0	2921	0	0
normalized size	1	1.	0.97	1.54	0.	16.88	0.	0.
time (sec)	N/A	0.314	0.327	0.02	0.	130.366	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	133	204	0	2360	0	0
normalized size	1	1.	0.97	1.49	0.	17.23	0.	0.
time (sec)	N/A	0.157	0.116	0.01	0.	7.394	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	87	165	0	768	0	101
normalized size	1	1.	1.01	1.92	0.	8.93	0.	1.17
time (sec)	N/A	0.084	0.019	0.006	0.	1.797	0.	1.14

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	134	207	0	2381	0	0
normalized size	1	1.	0.97	1.5	0.	17.25	0.	0.
time (sec)	N/A	0.195	0.141	0.012	0.	2.694	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	176	276	0	3019	0	0
normalized size	1	1.	0.81	1.27	0.	13.85	0.	0.
time (sec)	N/A	0.266	0.27	0.012	0.	6.496	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	127	222	0	0	0	0
normalized size	1	1.	0.3	0.53	0.	0.	0.	0.
time (sec)	N/A	0.184	0.219	0.02	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	99	134	0	0	0	0
normalized size	1	1.	0.4	0.54	0.	0.	0.	0.
time (sec)	N/A	0.126	0.136	0.006	0.	0.	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	80	70	0	0	0	0
normalized size	1	1.	0.33	0.29	0.	0.	0.	0.
time (sec)	N/A	0.111	0.059	0.003	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	399	147	178	0	0	0	0
normalized size	1	1.	0.37	0.45	0.	0.	0.	0.
time (sec)	N/A	0.347	0.234	0.014	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	422	219	260	0	0	0	0
normalized size	1	1.	0.52	0.62	0.	0.	0.	0.
time (sec)	N/A	0.517	0.196	0.016	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	271	720	0	0	0	0
normalized size	1	1.	1.15	3.05	0.	0.	0.	0.
time (sec)	N/A	0.474	0.841	0.066	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	204	613	0	2815	0	536
normalized size	1	1.	1.22	3.67	0.	16.86	0.	3.21
time (sec)	N/A	0.294	0.653	0.011	0.	5.921	0.	1.305

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	162	506	0	2778	0	595
normalized size	1	1.	1.02	3.18	0.	17.47	0.	3.74
time (sec)	N/A	0.191	0.207	0.01	0.	5.578	0.	1.232

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	167	454	0	2815	0	613
normalized size	1	1.	1.01	2.73	0.	16.96	0.	3.69
time (sec)	N/A	0.171	0.165	0.006	0.	5.242	0.	1.217

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	236	612	0	9844	0	0
normalized size	1	1.	0.89	2.3	0.	37.01	0.	0.
time (sec)	N/A	0.388	0.87	0.02	0.	22.387	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	350	863	0	13005	0	0
normalized size	1	1.	0.84	2.06	0.	31.04	0.	0.
time (sec)	N/A	0.564	1.639	0.02	0.	55.353	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	566	199	603	0	0	0	0
normalized size	1	1.26	0.44	1.34	0.	0.	0.	0.
time (sec)	N/A	0.347	0.285	0.032	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	503	199	586	0	0	0	0
normalized size	1	1.19	0.47	1.39	0.	0.	0.	0.
time (sec)	N/A	0.256	0.218	0.009	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	501	199	561	0	0	0	0
normalized size	1	1.19	0.47	1.33	0.	0.	0.	0.
time (sec)	N/A	0.24	0.217	0.009	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	503	199	536	0	0	0	0
normalized size	1	1.19	0.47	1.27	0.	0.	0.	0.
time (sec)	N/A	0.242	0.198	0.01	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	501	199	366	0	0	0	0
normalized size	1	1.19	0.47	0.87	0.	0.	0.	0.
time (sec)	N/A	0.226	0.152	0.005	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	468	644	211	553	0	0	0	0
normalized size	1	1.38	0.45	1.18	0.	0.	0.	0.
time (sec)	N/A	0.445	0.237	0.016	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	406	406	943	496	0	0	0	0
normalized size	1	1.	2.32	1.22	0.	0.	0.	0.
time (sec)	N/A	8.593	10.828	0.049	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	591	332	0	0	0	0
normalized size	1	1.	1.82	1.02	0.	0.	0.	0.
time (sec)	N/A	3.529	7.818	0.026	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	308	275	0	0	0	0
normalized size	1	1.	1.05	0.94	0.	0.	0.	0.
time (sec)	N/A	3.6	0.571	0.023	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	179	177	0	2241	0	0
normalized size	1	1.	0.89	0.88	0.	11.09	0.	0.
time (sec)	N/A	0.363	0.33	0.013	0.	74.911	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	241	294	0	0	0	0
normalized size	1	1.	0.86	1.05	0.	0.	0.	0.
time (sec)	N/A	1.35	0.801	0.023	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	370	349	401	0	0	0	0
normalized size	1	0.97	0.91	1.05	0.	0.	0.	0.
time (sec)	N/A	4.134	1.407	0.028	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	552	552	466	655	0	0	0	0
normalized size	1	1.	0.84	1.19	0.	0.	0.	0.
time (sec)	N/A	4.244	2.045	0.033	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	390	390	10915	290	0	0	0	72
normalized size	1	1.	27.99	0.74	0.	0.	0.	0.18
time (sec)	N/A	2.919	6.418	0.033	0.	0.	0.	1.281

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	324	324	7768	224	0	6734	0	36
normalized size	1	1.	23.98	0.69	0.	20.78	0.	0.11
time (sec)	N/A	1.517	6.173	0.024	0.	29.939	0.	1.223

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	2585	161	0	2026	0	0
normalized size	1	1.	10.77	0.67	0.	8.44	0.	0.
time (sec)	N/A	0.318	5.597	0.015	0.	8.334	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	291	291	4644	272	0	4811	0	0
normalized size	1	1.	15.96	0.93	0.	16.53	0.	0.
time (sec)	N/A	0.671	6.336	0.026	0.	19.621	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	373	373	7777	322	0	8227	0	0
normalized size	1	1.	20.85	0.86	0.	22.06	0.	0.
time (sec)	N/A	2.527	6.409	0.03	0.	75.004	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	512	512	10933	503	0	11867	0	0
normalized size	1	1.	21.35	0.98	0.	23.18	0.	0.
time (sec)	N/A	4.943	6.597	0.034	0.	126.104	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	460	460	457	490	0	0	0	0
normalized size	1	1.	0.99	1.07	0.	0.	0.	0.
time (sec)	N/A	5.084	0.988	0.03	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	324	279	0	0	0	0
normalized size	1	1.	0.99	0.85	0.	0.	0.	0.
time (sec)	N/A	1.456	0.582	0.022	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	333	388	0	0	0	0
normalized size	1	1.	0.96	1.12	0.	0.	0.	0.
time (sec)	N/A	1.744	1.373	0.027	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	416	380	555	0	0	0	0
normalized size	1	1.	0.91	1.33	0.	0.	0.	0.
time (sec)	N/A	3.243	1.629	0.03	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	595	595	18689	516	0	0	0	140
normalized size	1	1.	31.41	0.87	0.	0.	0.	0.24
time (sec)	N/A	3.285	6.513	0.034	0.	0.	0.	1.434

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	491	491	14032	382	0	0	0	78
normalized size	1	1.	28.58	0.78	0.	0.	0.	0.16
time (sec)	N/A	1.802	6.275	0.029	0.	0.	0.	1.438

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	487	487	9290	217	0	0	0	36
normalized size	1	1.	19.08	0.45	0.	0.	0.	0.07
time (sec)	N/A	1.572	6.163	0.021	0.	0.	0.	1.352

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	260	432	7789	360	0	8124	0	0
normalized size	1	1.66	29.96	1.38	0.	31.25	0.	0.
time (sec)	N/A	0.855	6.322	0.03	0.	28.913	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	523	523	9321	511	0	15790	0	0
normalized size	1	1.	17.82	0.98	0.	30.19	0.	0.
time (sec)	N/A	2.617	6.43	0.031	0.	167.91	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	354	2134	0	7294	0	0
normalized size	1	1.	1.26	7.59	0.	25.96	0.	0.
time (sec)	N/A	7.336	0.535	0.081	0.	33.224	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	276	1223	0	4096	0	0
normalized size	1	1.	1.21	5.34	0.	17.89	0.	0.
time (sec)	N/A	1.751	0.395	0.043	0.	12.454	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	169	1167	0	1868	0	0
normalized size	1	1.	0.93	6.41	0.	10.26	0.	0.
time (sec)	N/A	0.268	0.245	0.036	0.	5.873	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	212	2099	0	2604	0	0
normalized size	1	1.	0.88	8.71	0.	10.8	0.	0.
time (sec)	N/A	1.645	0.442	0.049	0.	23.636	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	292	2770	0	5650	0	0
normalized size	1	1.	1.01	9.55	0.	19.48	0.	0.
time (sec)	N/A	2.362	0.77	0.059	0.	67.38	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	325	325	10606	222	0	5638	0	0
normalized size	1	1.	32.63	0.68	0.	17.35	0.	0.
time (sec)	N/A	5.391	6.304	0.033	0.	12.467	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	7543	175	0	2969	0	0
normalized size	1	1.	28.68	0.67	0.	11.29	0.	0.
time (sec)	N/A	2.135	6.148	0.02	0.	5.627	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	2959	130	0	1651	0	0
normalized size	1	1.	13.45	0.59	0.	7.5	0.	0.
time (sec)	N/A	0.294	6.066	0.011	0.	2.474	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	265	265	2661	217	0	4091	0	0
normalized size	1	1.	10.04	0.82	0.	15.44	0.	0.
time (sec)	N/A	0.78	5.479	0.023	0.	3.459	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	743	160	0	790	0	282
normalized size	1	1.	7.74	1.67	0.	8.23	0.	2.94
time (sec)	N/A	0.2	0.51	0.069	0.	1.707	0.	1.207

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	479	479	461	377	0	0	0	142
normalized size	1	1.	0.96	0.79	0.	0.	0.	0.3
time (sec)	N/A	1.858	1.939	0.029	0.	0.	0.	1.233

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	366	366	355	269	0	0	0	74
normalized size	1	1.	0.97	0.73	0.	0.	0.	0.2
time (sec)	N/A	1.173	1.149	0.023	0.	0.	0.	1.225

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	292	200	0	21835	0	36
normalized size	1	1.	0.98	0.67	0.	73.27	0.	0.12
time (sec)	N/A	0.724	0.633	0.02	0.	125.849	0.	1.201

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	227	161	0	6973	0	0
normalized size	1	1.	0.95	0.67	0.	29.05	0.	0.
time (sec)	N/A	0.304	0.474	0.017	0.	31.494	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	229	151	0	9038	0	0
normalized size	1	1.	0.94	0.62	0.	37.19	0.	0.
time (sec)	N/A	0.178	0.397	0.012	0.	75.449	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	271	197	0	12658	0	0
normalized size	1	1.	0.97	0.7	0.	45.21	0.	0.
time (sec)	N/A	0.602	1.044	0.023	0.	37.89	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	320	248	0	0	0	0
normalized size	1	1.	0.94	0.73	0.	0.	0.	0.
time (sec)	N/A	0.741	0.73	0.024	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	443	443	383	350	0	0	0	0
normalized size	1	1.	0.86	0.79	0.	0.	0.	0.
time (sec)	N/A	1.432	1.795	0.027	0.	0.	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	350	507	10968	480	0	0	0	101
normalized size	1	1.45	31.34	1.37	0.	0.	0.	0.29
time (sec)	N/A	4.328	11.276	0.034	0.	0.	0.	1.19

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	360	360	7792	338	0	0	0	0
normalized size	1	1.	21.64	0.94	0.	0.	0.	0.
time (sec)	N/A	1.267	11.195	0.03	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	333	333	2119	252	0	0	0	0
normalized size	1	1.	6.36	0.76	0.	0.	0.	0.
time (sec)	N/A	0.658	10.918	0.027	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	341	341	2112	246	0	0	0	0
normalized size	1	1.	6.19	0.72	0.	0.	0.	0.
time (sec)	N/A	0.774	10.001	0.02	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	339	462	2158	387	0	0	0	0
normalized size	1	1.36	6.37	1.14	0.	0.	0.	0.
time (sec)	N/A	2.837	8.775	0.032	0.	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	419	647	2218	541	0	0	0	0
normalized size	1	1.54	5.29	1.29	0.	0.	0.	0.
time (sec)	N/A	5.566	8.596	0.036	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	243	243	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.65	0.213	0.086	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	272	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.939	0.763	0.06	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	211	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.542	0.384	0.05	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	183	0	0	0	0	0
normalized size	1	1.	0.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.329	0.237	0.044	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	168	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.357	0.3	0.043	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	218	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.502	0.407	0.031	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	259	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.663	0.445	0.063	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	339	339	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.628	0.537	0.046	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	273	273	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.531	0.251	0.041	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	162	162	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.314	0.103	0.043	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	190	190	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.294	0.036	0.041	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	264	264	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.553	0.221	0.057	0.	0.	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	328	328	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.62	0.517	0.039	0.	0.	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	44	44	101	0	255	0	78
normalized size	1	1.1	1.1	2.52	0.	6.38	0.	1.95
time (sec)	N/A	0.072	0.056	0.061	0.	1.835	0.	1.11

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [252] had the largest ratio of [0.5]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	20	0.1
2	A	2	1	1.	20	0.05
3	A	4	3	1.	18	0.167
4	A	2	1	1.	17	0.059
5	A	3	2	1.	20	0.1
6	A	2	1	1.	20	0.05
7	A	3	2	1.	20	0.1
8	A	5	5	1.	20	0.25
9	A	4	4	1.	20	0.2
10	A	4	4	1.	18	0.222
11	A	7	7	1.	20	0.35
12	A	7	7	1.	20	0.35
13	A	7	7	1.	20	0.35
14	A	6	6	1.	20	0.3
15	A	6	5	1.	20	0.25
16	A	5	5	1.	20	0.25
17	A	4	4	1.	17	0.235
18	A	4	4	1.	20	0.2
19	A	5	5	1.	20	0.25
20	A	6	5	1.	20	0.25
21	A	5	4	1.	20	0.2
22	A	5	4	1.	18	0.222
23	A	8	7	1.	20	0.35
24	A	8	8	1.	20	0.4
25	A	8	7	1.	20	0.35
26	A	8	8	1.	20	0.4
27	A	7	5	1.	20	0.25
28	A	6	5	1.	20	0.25
29	A	5	4	1.	17	0.235
30	A	5	5	1.	20	0.25
31	A	5	4	1.	20	0.2
32	A	5	4	1.	20	0.2
33	A	4	4	1.	20	0.2
34	A	3	3	1.	20	0.15
35	A	3	3	1.	18	0.167
36	A	6	6	1.	20	0.3
37	A	5	5	1.	20	0.25
38	A	6	6	1.	20	0.3
39	A	5	4	1.	20	0.2
40	A	4	4	1.	20	0.2
41	A	3	3	1.	17	0.176
42	A	4	4	1.	20	0.2
43	A	5	4	1.	20	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	4	4	1.	20	0.2
45	A	4	4	1.	20	0.2
46	A	3	3	1.	20	0.15
47	A	2	2	1.	18	0.111
48	A	6	6	1.	20	0.3
49	A	6	6	1.	20	0.3
50	A	5	5	1.	20	0.25
51	A	4	4	1.	20	0.2
52	A	4	4	1.	17	0.235
53	A	5	5	1.	20	0.25
54	A	6	5	1.	20	0.25
55	A	3	2	1.	25	0.08
56	A	4	3	1.	23	0.13
57	A	3	2	1.	23	0.087
58	A	4	3	1.	23	0.13
59	A	3	2	1.	23	0.087
60	A	4	3	1.	21	0.143
61	A	3	2	1.	20	0.1
62	A	5	4	1.	23	0.174
63	A	3	2	1.	23	0.087
64	A	4	3	1.	23	0.13
65	A	3	2	1.	23	0.087
66	A	4	3	1.	21	0.143
67	A	3	2	1.	21	0.095
68	A	4	3	1.	21	0.143
69	A	3	2	1.	21	0.095
70	A	2	2	1.	19	0.105
71	A	3	2	1.	18	0.111
72	A	4	3	1.	21	0.143
73	A	3	2	1.	21	0.095
74	A	4	3	1.	21	0.143
75	A	4	4	1.	33	0.121
76	A	4	4	1.	31	0.129
77	A	3	3	1.	30	0.1
78	A	4	3	1.	33	0.091
79	A	3	3	1.	33	0.091
80	A	4	3	1.	33	0.091
81	A	4	4	1.	33	0.121
82	A	3	3	1.	31	0.097
83	A	4	4	1.	30	0.133
84	A	4	3	1.	33	0.091
85	A	5	4	1.	33	0.121
86	A	4	3	1.	33	0.091
87	A	3	2	1.	35	0.057

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
88	A	3	2	1.	35	0.057
89	A	3	2	1.	35	0.057
90	A	3	3	1.	35	0.086
91	A	3	3	1.	35	0.086
92	A	2	2	1.	29	0.069
93	A	5	4	1.	31	0.129
94	A	5	4	1.	31	0.129
95	A	3	2	1.	25	0.08
96	A	2	1	1.	25	0.04
97	A	3	2	1.	23	0.087
98	A	2	1	1.	22	0.045
99	A	3	2	1.	25	0.08
100	A	2	1	1.	25	0.04
101	A	3	2	1.	25	0.08
102	A	7	6	1.	25	0.24
103	A	6	6	1.	25	0.24
104	A	5	5	1.	23	0.217
105	A	7	6	1.	25	0.24
106	A	7	6	1.	25	0.24
107	A	5	3	1.	25	0.12
108	A	4	3	1.	25	0.12
109	A	3	2	1.	22	0.091
110	A	4	3	1.	25	0.12
111	A	5	3	1.	25	0.12
112	A	7	7	1.	25	0.28
113	A	6	6	1.	25	0.24
114	A	4	4	1.	25	0.16
115	A	4	4	1.	23	0.174
116	A	8	7	1.	25	0.28
117	A	8	7	1.	25	0.28
118	A	6	4	1.	25	0.16
119	A	5	4	1.	25	0.16
120	A	4	3	1.	25	0.12
121	A	4	3	1.	22	0.136
122	A	5	4	1.	25	0.16
123	A	6	4	1.	25	0.16
124	A	8	7	1.	25	0.28
125	A	7	6	1.	25	0.24
126	A	5	5	1.	25	0.2
127	A	5	5	1.	25	0.2
128	A	5	5	1.	25	0.2
129	A	5	5	1.	23	0.217
130	A	9	7	1.	25	0.28
131	A	9	7	1.	25	0.28

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
132	A	7	4	1.	25	0.16
133	A	6	4	1.	25	0.16
134	A	5	3	1.	25	0.12
135	A	5	4	1.	25	0.16
136	A	5	3	1.	22	0.136
137	A	4	3	1.	21	0.143
138	A	5	4	1.	22	0.182
139	A	5	5	1.	17	0.294
140	A	6	6	1.	18	0.333
141	A	5	5	1.	22	0.227
142	A	6	6	1.	25	0.24
143	A	5	5	1.	25	0.2
144	A	5	5	1.	23	0.217
145	A	7	6	1.	25	0.24
146	A	7	6	1.	25	0.24
147	A	7	6	1.	25	0.24
148	A	5	5	1.	25	0.2
149	A	6	6	1.	25	0.24
150	A	7	6	1.	25	0.24
151	A	6	5	1.	25	0.2
152	A	5	5	1.	25	0.2
153	A	4	4	1.	22	0.182
154	A	4	4	1.	25	0.16
155	A	5	5	1.	25	0.2
156	A	7	6	1.	25	0.24
157	A	6	5	1.	25	0.2
158	A	6	5	1.	23	0.217
159	A	8	6	1.	25	0.24
160	A	8	7	1.	25	0.28
161	A	8	6	1.	25	0.24
162	A	8	7	1.	25	0.28
163	A	7	5	1.	25	0.2
164	A	6	5	1.	25	0.2
165	A	5	4	1.	22	0.182
166	A	5	5	1.	25	0.2
167	A	5	4	1.	25	0.16
168	A	6	5	1.	25	0.2
169	A	5	5	1.	27	0.185
170	A	4	4	1.	27	0.148
171	A	4	4	1.	25	0.16
172	A	6	5	1.	27	0.185
173	A	4	4	1.	27	0.148
174	A	5	5	1.	27	0.185
175	A	6	5	1.	27	0.185

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	5	4	1.	27	0.148
177	A	4	4	1.	27	0.148
178	A	3	3	1.	24	0.125
179	A	4	4	1.	27	0.148
180	A	5	4	1.	27	0.148
181	A	6	5	1.	25	0.2
182	A	5	5	1.	25	0.2
183	A	4	4	1.	25	0.16
184	A	4	4	1.	23	0.174
185	A	6	5	1.	25	0.2
186	A	4	4	1.	25	0.16
187	A	5	5	1.	25	0.2
188	A	6	5	1.	25	0.2
189	A	5	4	1.	25	0.16
190	A	4	4	1.	25	0.16
191	A	3	3	1.	22	0.136
192	A	4	4	1.	25	0.16
193	A	5	4	1.	25	0.16
194	A	5	5	1.	25	0.2
195	A	4	4	1.	25	0.16
196	A	2	2	1.	23	0.087
197	A	5	5	1.	25	0.2
198	A	5	5	1.	25	0.2
199	A	5	5	1.	25	0.2
200	A	4	4	1.	25	0.16
201	A	4	4	1.	22	0.182
202	A	5	5	1.	25	0.2
203	A	6	5	1.	25	0.2
204	A	6	3	1.	31	0.097
205	A	6	3	1.	31	0.097
206	A	6	3	1.	31	0.097
207	A	6	3	1.	31	0.097
208	A	6	3	1.	31	0.097
209	A	6	3	1.	31	0.097
210	A	6	3	1.	31	0.097
211	A	6	3	1.	31	0.097
212	A	6	3	1.	31	0.097
213	A	6	3	1.	31	0.097
214	A	6	3	1.	31	0.097
215	A	6	3	1.	31	0.097
216	A	6	3	1.	31	0.097
217	A	6	3	1.	31	0.097
218	A	6	3	1.	31	0.097
219	A	6	3	1.	31	0.097

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
220	A	2	1	1.	27	0.037
221	A	2	1	1.	27	0.037
222	A	2	1	1.	25	0.04
223	A	3	2	1.	27	0.074
224	A	4	3	0.91	27	0.111
225	A	6	3	1.	29	0.103
226	A	6	3	1.	29	0.103
227	A	6	3	1.	29	0.103
228	A	6	3	1.	29	0.103
229	A	6	5	1.	22	0.227
230	A	6	5	1.	22	0.227
231	A	6	5	1.	22	0.227
232	A	6	5	1.	22	0.227
233	A	6	6	1.	20	0.3
234	A	6	5	1.	22	0.227
235	A	6	5	1.	22	0.227
236	A	6	5	1.	22	0.227
237	A	12	8	1.	22	0.364
238	A	12	8	1.	22	0.364
239	A	12	8	1.	22	0.364
240	A	12	8	1.	22	0.364
241	A	12	8	1.	19	0.421
242	A	12	8	1.	22	0.364
243	A	12	8	1.	22	0.364
244	A	7	6	1.	22	0.273
245	A	7	6	1.	22	0.273
246	A	7	6	1.	22	0.273
247	A	7	6	1.	22	0.273
248	A	7	6	1.	20	0.3
249	A	8	6	1.	22	0.273
250	A	8	6	1.	22	0.273
251	A	8	6	1.	22	0.273
252	A	24	11	1.	22	0.5
253	A	23	10	1.	22	0.454
254	A	23	10	1.	22	0.454
255	A	23	10	1.	22	0.454
256	A	22	9	1.	19	0.474
257	A	22	9	1.	22	0.409
258	A	22	9	1.	22	0.409
259	A	4	4	1.	20	0.2
260	A	4	4	1.	22	0.182
261	A	6	6	1.	22	0.273
262	A	3	3	1.	24	0.125
263	A	7	7	1.	20	0.35

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
264	A	4	4	1.	22	0.182
265	A	4	4	1.	22	0.182
266	A	4	4	1.	24	0.167
267	A	6	6	1.	37	0.162
268	A	4	3	1.	35	0.086
269	A	5	5	1.	34	0.147
270	A	6	6	1.	37	0.162
271	A	5	5	1.	37	0.135
272	A	6	6	1.	37	0.162
273	A	3	2	1.	25	0.08
274	A	2	1	1.	25	0.04
275	A	3	2	1.	23	0.087
276	A	2	1	1.	22	0.045
277	A	3	2	1.	25	0.08
278	A	2	1	1.	25	0.04
279	A	3	2	1.	25	0.08
280	A	4	3	1.	25	0.12
281	A	4	3	1.	25	0.12
282	A	4	3	1.	25	0.12
283	A	3	3	1.	22	0.136
284	A	3	3	0.97	25	0.12
285	A	4	3	1.	25	0.12
286	A	4	3	1.	25	0.12
287	A	4	3	1.	25	0.12
288	A	5	4	1.	25	0.16
289	A	5	4	1.	25	0.16
290	A	4	4	1.	25	0.16
291	A	3	3	1.	22	0.136
292	A	4	4	0.98	25	0.16
293	A	5	3	1.	25	0.12
294	A	5	4	1.	25	0.16
295	A	7	6	1.	27	0.222
296	A	7	6	1.	27	0.222
297	A	7	6	1.	27	0.222
298	A	7	6	1.	27	0.222
299	A	7	7	1.	25	0.28
300	A	7	6	1.	27	0.222
301	A	7	6	1.	27	0.222
302	A	7	6	1.	27	0.222
303	A	6	3	1.	27	0.111
304	A	6	3	1.	27	0.111
305	A	6	3	1.	27	0.111
306	A	6	3	1.	27	0.111
307	A	6	3	1.	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
308	A	6	3	1.	27	0.111
309	A	6	3	1.	27	0.111
310	A	19	12	1.	31	0.387
311	A	8	7	1.	29	0.241
312	A	7	6	1.	29	0.207
313	A	7	6	1.	27	0.222
314	A	9	6	1.	29	0.207
315	A	21	8	1.	29	0.276
316	A	17	9	1.46	29	0.31
317	A	13	8	1.42	29	0.276
318	A	7	6	1.23	26	0.231
319	A	8	7	1.	29	0.241
320	A	7	6	1.	29	0.207
321	A	13	9	1.	29	0.31
322	A	9	7	1.	29	0.241
323	A	8	6	1.	29	0.207
324	A	8	7	1.	27	0.259
325	A	14	8	1.	29	0.276
326	A	24	9	1.	29	0.31
327	A	19	9	1.89	29	0.31
328	A	12	7	1.41	26	0.269
329	A	13	10	1.	29	0.345
330	A	13	9	1.	29	0.31
331	A	15	9	1.	29	0.31
332	A	7	6	1.	29	0.207
333	A	6	5	1.	29	0.172
334	A	3	3	1.	27	0.111
335	A	7	4	1.	29	0.138
336	A	10	5	1.	29	0.172
337	A	4	4	1.	29	0.138
338	A	3	3	1.	29	0.103
339	A	3	3	1.	26	0.115
340	A	6	6	1.	29	0.207
341	A	7	7	1.	29	0.241
342	A	7	6	1.	29	0.207
343	A	5	5	1.	29	0.172
344	A	5	5	1.	29	0.172
345	A	5	5	1.	27	0.185
346	A	11	6	1.	29	0.207
347	A	15	7	1.	29	0.241
348	A	10	8	1.26	29	0.276
349	A	8	7	1.19	29	0.241
350	A	8	7	1.19	29	0.241
351	A	8	7	1.19	29	0.241

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	8	7	1.19	26	0.269
353	A	15	10	1.38	29	0.345
354	A	7	5	1.	29	0.172
355	A	7	5	1.	29	0.172
356	A	6	5	1.	29	0.172
357	A	5	4	1.	27	0.148
358	A	8	6	1.	29	0.207
359	A	10	7	0.97	29	0.241
360	A	13	7	1.	29	0.241
361	A	10	7	1.	29	0.241
362	A	9	6	1.	29	0.207
363	A	11	6	1.	26	0.231
364	A	8	5	1.	29	0.172
365	A	12	7	1.	29	0.241
366	A	15	7	1.	29	0.241
367	A	7	5	1.	29	0.172
368	A	6	5	1.	27	0.185
369	A	8	6	1.	29	0.207
370	A	10	7	1.	29	0.241
371	A	17	9	1.	29	0.31
372	A	16	8	1.	29	0.276
373	A	13	7	1.	26	0.269
374	A	16	8	1.66	29	0.276
375	A	19	10	1.	29	0.345
376	A	7	5	1.	29	0.172
377	A	6	5	1.	29	0.172
378	A	5	4	1.	27	0.148
379	A	8	6	1.	29	0.207
380	A	8	6	1.	29	0.207
381	A	9	6	1.	29	0.207
382	A	8	5	1.	29	0.172
383	A	9	5	1.	26	0.192
384	A	8	5	1.	29	0.172
385	A	8	6	1.	25	0.24
386	A	17	7	1.	29	0.241
387	A	13	7	1.	29	0.241
388	A	10	6	1.	29	0.207
389	A	6	3	1.	29	0.103
390	A	5	3	1.	26	0.115
391	A	9	5	1.	29	0.172
392	A	11	6	1.	29	0.207
393	A	14	6	1.	29	0.207
394	A	14	7	1.45	29	0.241
395	A	8	5	1.	29	0.172

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	8	5	1.	29	0.172
397	A	8	5	1.	26	0.192
398	A	12	8	1.36	29	0.276
399	A	15	8	1.54	29	0.276
400	A	6	3	1.	29	0.103
401	A	5	3	1.	27	0.111
402	A	5	3	1.	27	0.111
403	A	5	3	1.	27	0.111
404	A	5	3	1.	25	0.12
405	A	8	5	1.	27	0.185
406	A	9	5	1.	27	0.185
407	A	12	8	1.	27	0.296
408	A	10	6	1.	27	0.222
409	A	6	3	1.	27	0.111
410	A	5	3	1.	24	0.125
411	A	10	6	1.	27	0.222
412	A	12	6	1.	27	0.222
413	A	5	5	1.1	28	0.179

Chapter 3

Listing of integrals

3.1 $\int x^3 \left(d+ex^2\right) \left(a+cx^4\right)^5 \, dx$

Optimal. Leaf size=149

$$\frac{5}{8}a^2c^3dx^{16}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{9}a^2c^3ex^{18}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}$$

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10)/2 + (5*a
^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/8 + (5*a^2*c^3
*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^22)/22 + (c^5*d*x^24)/24 + (c^
5*e*x^26)/26
```

Rubi [A] time = 0.219689, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1252, 766}

$$\frac{5}{8}a^2c^3dx^{16}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{9}a^2c^3ex^{18}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10)/2 + (5*a
^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/8 + (5*a^2*c^3
*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^22)/22 + (c^5*d*x^24)/24 + (c^
5*e*x^26)/26
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3 (d+ex^2) (a+cx^4)^5 \, dx = \frac{1}{2}\text{Subst}\left(\int x(d+ex) (a+cx^2)^5 \, dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int (a^5dx + a^5ex^2 + 5a^4cdx^3 + 5a^4cex^4 + 10a^3c^2dx^5 + 10a^3c^2ex^6 + 10a^2c^3dx^7 +$$
$$= \frac{1}{4}a^5dx^4 + \frac{1}{6}a^5ex^6 + \frac{5}{8}a^4cdx^8 + \frac{1}{2}a^4cex^{10} + \frac{5}{6}a^3c^2dx^{12} + \frac{5}{7}a^3c^2ex^{14} + \frac{5}{8}a^2c^3dx^{16} + \frac{5}{9}a^2c^3$$

Mathematica [A] time = 0.0051757, size = 149, normalized size = 1.

$$\frac{5}{8}a^2c^3dx^{16} + \frac{5}{6}a^3c^2dx^{12} + \frac{5}{9}a^2c^3ex^{18} + \frac{5}{7}a^3c^2ex^{14} + \frac{5}{8}a^4cdx^8 + \frac{1}{2}a^4cex^{10} + \frac{1}{4}a^5dx^4 + \frac{1}{6}a^5ex^6 + \frac{1}{4}ac^4dx^{20} + \frac{5}{22}ac^4ex^{22} +$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10)/2 + (5*a
^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/8 + (5*a^2*c^3
*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^22)/22 + (c^5*d*x^24)/24 + (c^
5*e*x^26)/26
```

Maple [A] time = 0.01, size = 126, normalized size = 0.9

$$\frac{a^5dx^4}{4} + \frac{a^5ex^6}{6} + \frac{5\,a^4cdx^8}{8} + \frac{a^4cex^{10}}{2} + \frac{5\,a^3c^2dx^{12}}{6} + \frac{5\,a^3c^2ex^{14}}{7} + \frac{5\,a^2c^3dx^{16}}{8} + \frac{5\,a^2c^3ex^{18}}{9} + \frac{ac^4dx^{20}}{4} + \frac{5\,ac^4ex^{22}}{22} + \frac{c^5d}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] 1/4*a^5*d*x^4+1/6*a^5*e*x^6+5/8*a^4*c*d*x^8+1/2*a^4*c*e*x^10+5/6*a^3*c^2*d*
x^12+5/7*a^3*c^2*e*x^14+5/8*a^2*c^3*d*x^16+5/9*a^2*c^3*e*x^18+1/4*a*c^4*d*x
^20+5/22*a*c^4*e*x^22+1/24*c^5*d*x^24+1/26*c^5*e*x^26
```

Maxima [A] time = 0.965705, size = 169, normalized size = 1.13

$$\frac{1}{26}\,c^5ex^{26} + \frac{1}{24}\,c^5dx^{24} + \frac{5}{22}\,ac^4ex^{22} + \frac{1}{4}\,ac^4dx^{20} + \frac{5}{9}\,a^2c^3ex^{18} + \frac{5}{8}\,a^2c^3dx^{16} + \frac{5}{7}\,a^3c^2ex^{14} + \frac{5}{6}\,a^3c^2dx^{12} + \frac{1}{2}\,a^4cex^{10} + \frac{5}{8}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/26*c^5*e*x^26 + 1/24*c^5*d*x^24 + 5/22*a*c^4*e*x^22 + 1/4*a*c^4*d*x^20 +
5/9*a^2*c^3*e*x^18 + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*e*x^14 + 5/6*a^3*c^2*
d*x^12 + 1/2*a^4*c*e*x^10 + 5/8*a^4*c*d*x^8 + 1/6*a^5*e*x^6 + 1/4*a^5*d*x^4
```

Fricas [A] time = 1.7152, size = 306, normalized size = 2.05

$$\frac{1}{26}x^{26}ec^5 + \frac{1}{24}x^{24}dc^5 + \frac{5}{22}x^{22}ec^4a + \frac{1}{4}x^{20}dc^4a + \frac{5}{9}x^{18}ec^3a^2 + \frac{5}{8}x^{16}dc^3a^2 + \frac{5}{7}x^{14}ec^2a^3 + \frac{5}{6}x^{12}dc^2a^3 + \frac{1}{2}x^{10}eca^4 + \frac{5}{8}x^8dca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/26*x^26*e*c^5 + 1/24*x^24*d*c^5 + 5/22*x^22*e*c^4*a + 1/4*x^20*d*c^4*a +
5/9*x^18*e*c^3*a^2 + 5/8*x^16*d*c^3*a^2 + 5/7*x^14*e*c^2*a^3 + 5/6*x^12*d*c
^2*a^3 + 1/2*x^10*e*c*a^4 + 5/8*x^8*d*c*a^4 + 1/6*x^6*e*a^5 + 1/4*x^4*d*a^5
```

Sympy [A] time = 0.084633, size = 151, normalized size = 1.01

$$\frac{a^5dx^4}{4}+\frac{a^5ex^6}{6}+\frac{5a^4cdx^8}{8}+\frac{a^4cex^{10}}{2}+\frac{5a^3c^2dx^{12}}{6}+\frac{5a^3c^2ex^{14}}{7}+\frac{5a^2c^3dx^{16}}{8}+\frac{5a^2c^3ex^{18}}{9}+\frac{ac^4dx^{20}}{4}+\frac{5ac^4ex^{22}}{22}+\frac{c^5}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**4/4 + a**5*e*x**6/6 + 5*a**4*c*d*x**8/8 + a**4*c*e*x**10/2 + 5*a*
*3*c**2*d*x**12/6 + 5*a**3*c**2*e*x**14/7 + 5*a**2*c**3*d*x**16/8 + 5*a**2*
c**3*e*x**18/9 + a*c**4*d*x**20/4 + 5*a*c**4*e*x**22/22 + c**5*d*x**24/24 +
 c**5*e*x**26/26
```

Giac [A] time = 1.12323, size = 177, normalized size = 1.19

$$\frac{1}{26}c^5x^{26}e+\frac{1}{24}c^5dx^{24}+\frac{5}{22}ac^4x^{22}e+\frac{1}{4}ac^4dx^{20}+\frac{5}{9}a^2c^3x^{18}e+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{7}a^3c^2x^{14}e+\frac{5}{6}a^3c^2dx^{12}+\frac{1}{2}a^4cx^{10}e+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/26*c^5*x^26*e + 1/24*c^5*d*x^24 + 5/22*a*c^4*x^22*e + 1/4*a*c^4*d*x^20 +
5/9*a^2*c^3*x^18*e + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*x^14*e + 5/6*a^3*c^2*
d*x^12 + 1/2*a^4*c*x^10*e + 5/8*a^4*c*d*x^8 + 1/6*a^5*x^6*e + 1/4*a^5*d*x^4
```

3.2 $\int x^2 \left(d+ex^2\right) \left(a+cx^4\right)^5 \, dx$

Optimal. Leaf size=149

$$\frac{2}{3}a^2c^3dx^{15}+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{17}a^2c^3ex^{17}+\frac{10}{13}a^3c^2ex^{13}+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{2}$$

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^9)/9 + (10
*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c^3*d*x^15)/3 + (10*a
^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*a*c^4*e*x^21)/21 + (c^5*d*x^23
)/23 + (c^5*e*x^25)/25
```

Rubi [A] time = 0.0977514, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1262}

$$\frac{2}{3}a^2c^3dx^{15}+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{17}a^2c^3ex^{17}+\frac{10}{13}a^3c^2ex^{13}+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^9)/9 + (10
*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c^3*d*x^15)/3 + (10*a
^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*a*c^4*e*x^21)/21 + (c^5*d*x^23
)/23 + (c^5*e*x^25)/25
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(d+ex^2\right)\left(a+cx^4\right)^5 \, dx &= \int \left(a^5dx^2+a^5ex^4+5a^4cdx^6+5a^4cex^8+10a^3c^2dx^{10}+10a^3c^2ex^{12}+10a^2c^3dx^{14}+10a^2c^{3}\right.\\ &= \frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2\end{aligned}$$

Mathematica [A] time = 0.0041141, size = 149, normalized size = 1.

$$\frac{2}{3}a^2c^3dx^{15}+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{17}a^2c^3ex^{17}+\frac{10}{13}a^3c^2ex^{13}+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^9)/9 + (10
*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c^3*d*x^15)/3 + (10*a
^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*a*c^4*e*x^21)/21 + (c^5*d*x^23
)/23 + (c^5*e*x^25)/25
```

Maple [A] time = 0.001, size = 126, normalized size = 0.9

$$\frac{a^5dx^3}{3}+\frac{a^5ex^5}{5}+\frac{5\,a^4cdx^7}{7}+\frac{5\,a^4cex^9}{9}+\frac{10\,a^3c^2dx^{11}}{11}+\frac{10\,a^3c^2ex^{13}}{13}+\frac{2\,a^2c^3dx^{15}}{3}+\frac{10\,a^2c^3ex^{17}}{17}+\frac{5\,ac^4dx^{19}}{19}+\frac{5\,ac^4ex}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] 1/3*a^5*d*x^3+1/5*a^5*e*x^5+5/7*a^4*c*d*x^7+5/9*a^4*c*e*x^9+10/11*a^3*c^2*d
*x^11+10/13*a^3*c^2*e*x^13+2/3*a^2*c^3*d*x^15+10/17*a^2*c^3*e*x^17+5/19*a*c
^4*d*x^19+5/21*a*c^4*e*x^21+1/23*c^5*d*x^23+1/25*c^5*e*x^25
```

Maxima [A] time = 0.952864, size = 169, normalized size = 1.13

$$\frac{1}{25}\,c^5ex^{25}+\frac{1}{23}\,c^5dx^{23}+\frac{5}{21}\,ac^4ex^{21}+\frac{5}{19}\,ac^4dx^{19}+\frac{10}{17}\,a^2c^3ex^{17}+\frac{2}{3}\,a^2c^3dx^{15}+\frac{10}{13}\,a^3c^2ex^{13}+\frac{10}{11}\,a^3c^2dx^{11}+\frac{5}{9}\,a^4cex$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/25*c^5*e*x^25 + 1/23*c^5*d*x^23 + 5/21*a*c^4*e*x^21 + 5/19*a*c^4*d*x^19 +
 10/17*a^2*c^3*e*x^17 + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*c^2*e*x^13 + 10/11*a
^3*c^2*d*x^11 + 5/9*a^4*c*e*x^9 + 5/7*a^4*c*d*x^7 + 1/5*a^5*e*x^5 + 1/3*a^5
*d*x^3
```

Fricas [A] time = 1.27712, size = 315, normalized size = 2.11

$$\frac{1}{25}x^{25}ec^5+\frac{1}{23}x^{23}dc^5+\frac{5}{21}x^{21}ec^4a+\frac{5}{19}x^{19}dc^4a+\frac{10}{17}x^{17}ec^3a^2+\frac{2}{3}x^{15}dc^3a^2+\frac{10}{13}x^{13}ec^2a^3+\frac{10}{11}x^{11}dc^2a^3+\frac{5}{9}x^9eca^4+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*e*c^5 + 1/23*x^23*d*c^5 + 5/21*x^21*e*c^4*a + 5/19*x^19*d*c^4*a +
 10/17*x^17*e*c^3*a^2 + 2/3*x^15*d*c^3*a^2 + 10/13*x^13*e*c^2*a^3 + 10/11*x
^11*d*c^2*a^3 + 5/9*x^9*e*c*a^4 + 5/7*x^7*d*c*a^4 + 1/5*x^5*e*a^5 + 1/3*x^3
*d*a^5
```

Sympy [A] time = 0.086129, size = 155, normalized size = 1.04

$$\frac{a^5dx^3}{3}+\frac{a^5ex^5}{5}+\frac{5a^4cdx^7}{7}+\frac{5a^4cex^9}{9}+\frac{10a^3c^2dx^{11}}{11}+\frac{10a^3c^2ex^{13}}{13}+\frac{2a^2c^3dx^{15}}{3}+\frac{10a^2c^3ex^{17}}{17}+\frac{5ac^4dx^{19}}{19}+\frac{5ac^4ex^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**3/3 + a**5*e*x**5/5 + 5*a**4*c*d*x**7/7 + 5*a**4*c*e*x**9/9 + 10*
a**3*c**2*d*x**11/11 + 10*a**3*c**2*e*x**13/13 + 2*a**2*c**3*d*x**15/3 + 10
*a**2*c**3*e*x**17/17 + 5*a*c**4*d*x**19/19 + 5*a*c**4*e*x**21/21 + c**5*d*
x**23/23 + c**5*e*x**25/25
```

Giac [A] time = 1.16342, size = 177, normalized size = 1.19

$$\frac{1}{25}c^5x^{25}e + \frac{1}{23}c^5dx^{23} + \frac{5}{21}ac^4x^{21}e + \frac{5}{19}ac^4dx^{19} + \frac{10}{17}a^2c^3x^{17}e + \frac{2}{3}a^2c^3dx^{15} + \frac{10}{13}a^3c^2x^{13}e + \frac{10}{11}a^3c^2dx^{11} + \frac{5}{9}a^4cx^9e +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/25*c^5*x^25*e + 1/23*c^5*d*x^23 + 5/21*a*c^4*x^21*e + 5/19*a*c^4*d*x^19 +
 10/17*a^2*c^3*x^17*e + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*c^2*x^13*e + 10/11*a
^3*c^2*d*x^11 + 5/9*a^4*c*x^9*e + 5/7*a^4*c*d*x^7 + 1/5*a^5*x^5*e + 1/3*a^5
*d*x^3
```

3.3 $\int x\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx$

Optimal. Leaf size=89

$$\frac{5}{7}a^2c^3dx^{14}+a^3c^2dx^{10}+\frac{5}{6}a^4cdx^6+\frac{1}{2}a^5dx^2+\frac{5}{18}ac^4dx^{18}+\frac{e\left(a+cx^4\right)^6}{24c}+\frac{1}{22}c^5dx^{22}$$

```
[Out] (a^5*d*x^2)/2 + (5*a^4*c*d*x^6)/6 + a^3*c^2*d*x^10 + (5*a^2*c^3*d*x^14)/7 +
 (5*a*c^4*d*x^18)/18 + (c^5*d*x^22)/22 + (e*(a + c*x^4)^6)/(24*c)
```

Rubi [A] time = 0.0770721, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1248, 641, 194}

$$\frac{5}{7}a^2c^3dx^{14}+a^3c^2dx^{10}+\frac{5}{6}a^4cdx^6+\frac{1}{2}a^5dx^2+\frac{5}{18}ac^4dx^{18}+\frac{e\left(a+cx^4\right)^6}{24c}+\frac{1}{22}c^5dx^{22}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^2)/2 + (5*a^4*c*d*x^6)/6 + a^3*c^2*d*x^10 + (5*a^2*c^3*d*x^14)/7 +
 (5*a*c^4*d*x^18)/18 + (c^5*d*x^22)/22 + (e*(a + c*x^4)^6)/(24*c)
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx &= \frac{1}{2}\,\text{Subst}\left(\int (d+ex)\left(a+cx^2\right)^5\,dx,x,x^2\right)\\ &= \frac{e\left(a+cx^4\right)^6}{24c}+\frac{1}{2}d\,\text{Subst}\left(\int \left(a+cx^2\right)^5\,dx,x,x^2\right)\\ &= \frac{e\left(a+cx^4\right)^6}{24c}+\frac{1}{2}d\,\text{Subst}\left(\int \left(a^5+5a^4cx^2+10a^3c^2x^4+10a^2c^3x^6+5ac^4x^8+c^5x^{10}\right)\,dx\right.\\ &= \frac{1}{2}a^5dx^2+\frac{5}{6}a^4cdx^6+a^3c^2dx^{10}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{18}ac^4dx^{18}+\frac{1}{22}c^5dx^{22}+\frac{e\left(a+cx^4\right)^6}{24c}\end{aligned}$$

Mathematica [A] time = 0.0039613, size = 146, normalized size = 1.64

$$\frac{5}{7}a^2c^3dx^{14} + a^3c^2dx^{10} + \frac{5}{8}a^2c^3ex^{16} + \frac{5}{6}a^3c^2ex^{12} + \frac{5}{6}a^4cdx^6 + \frac{5}{8}a^4cex^8 + \frac{1}{2}a^5dx^2 + \frac{1}{4}a^5ex^4 + \frac{5}{18}ac^4dx^{18} + \frac{1}{4}ac^4ex^{20} + \frac{1}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^2)/2 + (a^5*e*x^4)/4 + (5*a^4*c*d*x^6)/6 + (5*a^4*c*e*x^8)/8 + a^3
*c^2*d*x^10 + (5*a^3*c^2*e*x^12)/6 + (5*a^2*c^3*d*x^14)/7 + (5*a^2*c^3*e*x^
16)/8 + (5*a*c^4*d*x^18)/18 + (a*c^4*e*x^20)/4 + (c^5*d*x^22)/22 + (c^5*e*x
^24)/24
```

Maple [A] time = 0.001, size = 125, normalized size = 1.4

$$\frac{c^5ex^{24}}{24} + \frac{c^5dx^{22}}{22} + \frac{ac^4ex^{20}}{4} + \frac{5\,ac^4dx^{18}}{18} + \frac{5\,a^2c^3ex^{16}}{8} + \frac{5\,a^2c^3dx^{14}}{7} + \frac{5\,a^3c^2ex^{12}}{6} + a^3c^2dx^{10} + \frac{5\,a^4cex^8}{8} + \frac{5\,a^4cdx^6}{6} + \frac{a^5e}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] 1/24*c^5*e*x^24+1/22*c^5*d*x^22+1/4*a*c^4*e*x^20+5/18*a*c^4*d*x^18+5/8*a^2*
c^3*e*x^16+5/7*a^2*c^3*d*x^14+5/6*a^3*c^2*e*x^12+a^3*c^2*d*x^10+5/8*a^4*c*e
*x^8+5/6*a^4*c*d*x^6+1/4*a^5*e*x^4+1/2*a^5*d*x^2
```

Maxima [A] time = 0.970514, size = 167, normalized size = 1.88

$$\frac{1}{24}\,c^5ex^{24} + \frac{1}{22}\,c^5dx^{22} + \frac{1}{4}\,ac^4ex^{20} + \frac{5}{18}\,ac^4dx^{18} + \frac{5}{8}\,a^2c^3ex^{16} + \frac{5}{7}\,a^2c^3dx^{14} + \frac{5}{6}\,a^3c^2ex^{12} + a^3c^2dx^{10} + \frac{5}{8}\,a^4cex^8 + \frac{5}{6}\,a^4cd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*c^5*e*x^24 + 1/22*c^5*d*x^22 + 1/4*a*c^4*e*x^20 + 5/18*a*c^4*d*x^18 +
5/8*a^2*c^3*e*x^16 + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*e*x^12 + a^3*c^2*d*x^
10 + 5/8*a^4*c*e*x^8 + 5/6*a^4*c*d*x^6 + 1/4*a^5*e*x^4 + 1/2*a^5*d*x^2
```

Fricas [A] time = 1.21557, size = 300, normalized size = 3.37

$$\frac{1}{24}x^{24}ec^5 + \frac{1}{22}x^{22}dc^5 + \frac{1}{4}x^{20}ec^4a + \frac{5}{18}x^{18}dc^4a + \frac{5}{8}x^{16}ec^3a^2 + \frac{5}{7}x^{14}dc^3a^2 + \frac{5}{6}x^{12}ec^2a^3 + x^{10}dc^2a^3 + \frac{5}{8}x^8eca^4 + \frac{5}{6}x^6dca^4 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*e*c^5 + 1/22*x^22*d*c^5 + 1/4*x^20*e*c^4*a + 5/18*x^18*d*c^4*a +
5/8*x^16*e*c^3*a^2 + 5/7*x^14*d*c^3*a^2 + 5/6*x^12*e*c^2*a^3 + x^10*d*c^2*a
^3 + 5/8*x^8*e*c*a^4 + 5/6*x^6*d*c*a^4 + 1/4*x^4*e*a^5 + 1/2*x^2*d*a^5
```

Sympy [A] time = 0.082939, size = 150, normalized size = 1.69

$$\frac{a^5dx^2}{2}+\frac{a^5ex^4}{4}+\frac{5a^4cdx^6}{6}+\frac{5a^4cex^8}{8}+a^3c^2dx^{10}+\frac{5a^3c^2ex^{12}}{6}+\frac{5a^2c^3dx^{14}}{7}+\frac{5a^2c^3ex^{16}}{8}+\frac{5ac^4dx^{18}}{18}+\frac{ac^4ex^{20}}{4}+\frac{c^5dx}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**2/2 + a**5*e*x**4/4 + 5*a**4*c*d*x**6/6 + 5*a**4*c*e*x**8/8 + a**
3*c**2*d*x**10 + 5*a**3*c**2*e*x**12/6 + 5*a**2*c**3*d*x**14/7 + 5*a**2*c**
3*e*x**16/8 + 5*a*c**4*d*x**18/18 + a*c**4*e*x**20/4 + c**5*d*x**22/22 + c*
*5*e*x**24/24
```

Giac [A] time = 1.15953, size = 176, normalized size = 1.98

$$\frac{1}{24}c^5x^{24}e+\frac{1}{22}c^5dx^{22}+\frac{1}{4}ac^4x^{20}e+\frac{5}{18}ac^4dx^{18}+\frac{5}{8}a^2c^3x^{16}e+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{6}a^3c^2x^{12}e+a^3c^2dx^{10}+\frac{5}{8}a^4cx^8e+\frac{5}{6}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/24*c^5*x^24*e + 1/22*c^5*d*x^22 + 1/4*a*c^4*x^20*e + 5/18*a*c^4*d*x^18 +
5/8*a^2*c^3*x^16*e + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*x^12*e + a^3*c^2*d*x^
10 + 5/8*a^4*c*x^8*e + 5/6*a^4*c*d*x^6 + 1/4*a^5*x^4*e + 1/2*a^5*d*x^2
```

3.4 $\int \left(d+ex^2\right)\left(a+cx^4\right)^5 \, dx$

Optimal. Leaf size=141

$$\frac{10}{13}a^2c^3dx^{13}+\frac{10}{9}a^3c^2dx^9+\frac{2}{3}a^2c^3ex^{15}+\frac{10}{11}a^3c^2ex^{11}+a^4cdx^5+\frac{5}{7}a^4cex^7+a^5dx+\frac{1}{3}a^5ex^3+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}$$

```
[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a^3*c^2*d*x
^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13 + (2*a^2*c^3*e*x^15
)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/19 + (c^5*d*x^21)/21 + (c^5*e*
x^23)/23
```

Rubi [A] time = 0.0819144, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1154}

$$\frac{10}{13}a^2c^3dx^{13}+\frac{10}{9}a^3c^2dx^9+\frac{2}{3}a^2c^3ex^{15}+\frac{10}{11}a^3c^2ex^{11}+a^4cdx^5+\frac{5}{7}a^4cex^7+a^5dx+\frac{1}{3}a^5ex^3+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a^3*c^2*d*x
^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13 + (2*a^2*c^3*e*x^15
)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/19 + (c^5*d*x^21)/21 + (c^5*e*
x^23)/23
```

Rule 1154

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)\left(a+cx^4\right)^5 \, dx &= \int \left(a^5d+a^5ex^2+5a^4cdx^4+5a^4cex^6+10a^3c^2dx^8+10a^3c^2ex^{10}+10a^2c^3dx^{12}+10a^2c^3ex^{14}\right.\\ &= a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2c^3ex^{15}+\end{aligned}$$

Mathematica [A] time = 0.0039031, size = 141, normalized size = 1.

$$\frac{10}{13}a^2c^3dx^{13}+\frac{10}{9}a^3c^2dx^9+\frac{2}{3}a^2c^3ex^{15}+\frac{10}{11}a^3c^2ex^{11}+a^4cdx^5+\frac{5}{7}a^4cex^7+a^5dx+\frac{1}{3}a^5ex^3+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a^3*c^2*d*x
^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13 + (2*a^2*c^3*e*x^15
)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/19 + (c^5*d*x^21)/21 + (c^5*e*
x^23)/23
```

Maple [A] time = 0.003, size = 122, normalized size = 0.9

$$a^5dx+\frac{a^5ex^3}{3}+a^4cdx^5+\frac{5\,a^4cex^7}{7}+\frac{10\,a^3c^2dx^9}{9}+\frac{10\,a^3c^2ex^{11}}{11}+\frac{10\,a^2c^3dx^{13}}{13}+\frac{2\,a^2c^3ex^{15}}{3}+\frac{5\,ac^4dx^{17}}{17}+\frac{5\,ac^4ex^{19}}{19}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5,x)
```

```
[Out] a^5*d*x+1/3*a^5*e*x^3+a^4*c*d*x^5+5/7*a^4*c*e*x^7+10/9*a^3*c^2*d*x^9+10/11*
a^3*c^2*e*x^11+10/13*a^2*c^3*d*x^13+2/3*a^2*c^3*e*x^15+5/17*a*c^4*d*x^17+5/
19*a*c^4*e*x^19+1/21*c^5*d*x^21+1/23*c^5*e*x^23
```

Maxima [A] time = 0.941618, size = 163, normalized size = 1.16

$$\frac{1}{23}\,c^5ex^{23}+\frac{1}{21}\,c^5dx^{21}+\frac{5}{19}\,ac^4ex^{19}+\frac{5}{17}\,ac^4dx^{17}+\frac{2}{3}\,a^2c^3ex^{15}+\frac{10}{13}\,a^2c^3dx^{13}+\frac{10}{11}\,a^3c^2ex^{11}+\frac{10}{9}\,a^3c^2dx^9+\frac{5}{7}\,a^4cex^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/23*c^5*e*x^23 + 1/21*c^5*d*x^21 + 5/19*a*c^4*e*x^19 + 5/17*a*c^4*d*x^17 +
 2/3*a^2*c^3*e*x^15 + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*c^2*e*x^11 + 10/9*a^
3*c^2*d*x^9 + 5/7*a^4*c*e*x^7 + a^4*c*d*x^5 + 1/3*a^5*e*x^3 + a^5*d*x
```

Fricas [A] time = 1.50945, size = 298, normalized size = 2.11

$$\frac{1}{23}x^{23}ec^5+\frac{1}{21}x^{21}dc^5+\frac{5}{19}x^{19}ec^4a+\frac{5}{17}x^{17}dc^4a+\frac{2}{3}x^{15}ec^3a^2+\frac{10}{13}x^{13}dc^3a^2+\frac{10}{11}x^{11}ec^2a^3+\frac{10}{9}x^9dc^2a^3+\frac{5}{7}x^7eca^4+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*e*c^5 + 1/21*x^21*d*c^5 + 5/19*x^19*e*c^4*a + 5/17*x^17*d*c^4*a +
 2/3*x^15*e*c^3*a^2 + 10/13*x^13*d*c^3*a^2 + 10/11*x^11*e*c^2*a^3 + 10/9*x^
9*d*c^2*a^3 + 5/7*x^7*e*c*a^4 + x^5*d*c*a^4 + 1/3*x^3*e*a^5 + x*d*a^5
```

Sympy [A] time = 0.082565, size = 148, normalized size = 1.05

$$a^5dx+\frac{a^5ex^3}{3}+a^4cdx^5+\frac{5a^4cex^7}{7}+\frac{10a^3c^2dx^9}{9}+\frac{10a^3c^2ex^{11}}{11}+\frac{10a^2c^3dx^{13}}{13}+\frac{2a^2c^3ex^{15}}{3}+\frac{5ac^4dx^{17}}{17}+\frac{5ac^4ex^{19}}{19}+\frac{c^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x + a**5*e*x**3/3 + a**4*c*d*x**5 + 5*a**4*c*e*x**7/7 + 10*a**3*c**2
*d*x**9/9 + 10*a**3*c**2*e*x**11/11 + 10*a**2*c**3*d*x**13/13 + 2*a**2*c**3
*e*x**15/3 + 5*a*c**4*d*x**17/17 + 5*a*c**4*e*x**19/19 + c**5*d*x**21/21 +
```

```
c**5*e*x**23/23
```

Giac [A] time = 1.12518, size = 171, normalized size = 1.21

$$\frac{1}{23}c^5x^{23}e + \frac{1}{21}c^5dx^{21} + \frac{5}{19}ac^4x^{19}e + \frac{5}{17}ac^4dx^{17} + \frac{2}{3}a^2c^3x^{15}e + \frac{10}{13}a^2c^3dx^{13} + \frac{10}{11}a^3c^2x^{11}e + \frac{10}{9}a^3c^2dx^9 + \frac{5}{7}a^4cx^7e + a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/23*c^5*x^23*e + 1/21*c^5*d*x^21 + 5/19*a*c^4*x^19*e + 5/17*a*c^4*d*x^17 +
 2/3*a^2*c^3*x^15*e + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*c^2*x^11*e + 10/9*a^
3*c^2*d*x^9 + 5/7*a^4*c*x^7*e + a^4*c*d*x^5 + 1/3*a^5*x^3*e + a^5*d*x
```

3.5 $\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx$

Optimal. Leaf size=142

$$\frac{5}{6}a^2c^3dx^{12}+\frac{5}{4}a^3c^2dx^8+\frac{5}{7}a^2c^3ex^{14}+a^3c^2ex^{10}+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+a^5d\log(x)+\frac{1}{2}a^5ex^2+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}$$

```
[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^2*d*x^8)/4
 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e*x^14)/7 + (5*a*c^4*
d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*
d*Log[x]
```

Rubi [A] time = 0.111289, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1252, 766}

$$\frac{5}{6}a^2c^3dx^{12}+\frac{5}{4}a^3c^2dx^8+\frac{5}{7}a^2c^3ex^{14}+a^3c^2ex^{10}+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+a^5d\log(x)+\frac{1}{2}a^5ex^2+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

```
[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^2*d*x^8)/4
 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e*x^14)/7 + (5*a*c^4*
d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*
d*Log[x]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)(a+cx^2)^5}{x}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(a^5e+\frac{a^5d}{x}+5a^4cdx+5a^4cex^2+10a^3c^2dx^3+10a^3c^2ex^4+10a^2c^3dx^5+10a\right.\right.\\ &= \frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4\end{aligned}$$

Mathematica [A] time = 0.0081567, size = 142, normalized size = 1.

$$\frac{5}{6}a^2c^3dx^{12}+\frac{5}{4}a^3c^2dx^8+\frac{5}{7}a^2c^3ex^{14}+a^3c^2ex^{10}+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+a^5d\log(x)+\frac{1}{2}a^5ex^2+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

```
[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^2*d*x^8)/4
 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e*x^14)/7 + (5*a*c^4*
d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*
d*Log[x]
```

Maple [A] time = 0.015, size = 123, normalized size = 0.9

$$\frac{a^5ex^2}{2}+\frac{5\,a^4cdx^4}{4}+\frac{5\,a^4cex^6}{6}+\frac{5\,a^3c^2dx^8}{4}+a^3c^2ex^{10}+\frac{5\,a^2c^3dx^{12}}{6}+\frac{5\,a^2c^3ex^{14}}{7}+\frac{5\,ac^4dx^{16}}{16}+\frac{5\,ac^4ex^{18}}{18}+\frac{c^5dx^{20}}{20}+\frac{c^5e}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x,x)
```

```
[Out] 1/2*a^5*e*x^2+5/4*a^4*c*d*x^4+5/6*a^4*c*e*x^6+5/4*a^3*c^2*d*x^8+a^3*c^2*e*x
^10+5/6*a^2*c^3*d*x^12+5/7*a^2*c^3*e*x^14+5/16*a*c^4*d*x^16+5/18*a*c^4*e*x^
18+1/20*c^5*d*x^20+1/22*c^5*e*x^22+a^5*d*ln(x)
```

Maxima [A] time = 0.954996, size = 169, normalized size = 1.19

$$\frac{1}{22}c^5ex^{22}+\frac{1}{20}c^5dx^{20}+\frac{5}{18}ac^4ex^{18}+\frac{5}{16}ac^4dx^{16}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{6}a^2c^3dx^{12}+a^3c^2ex^{10}+\frac{5}{4}a^3c^2dx^8+\frac{5}{6}a^4cex^6+\frac{5}{4}a^4cd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^4*d*x^16 +
 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^10 + 5/4*a^3*c^2*d*x
^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*a^5*e*x^2 + 1/2*a^5*d*log(x^2)
```

Fricas [A] time = 1.72184, size = 298, normalized size = 2.1

$$\frac{1}{22}c^5ex^{22}+\frac{1}{20}c^5dx^{20}+\frac{5}{18}ac^4ex^{18}+\frac{5}{16}ac^4dx^{16}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{6}a^2c^3dx^{12}+a^3c^2ex^{10}+\frac{5}{4}a^3c^2dx^8+\frac{5}{6}a^4cex^6+\frac{5}{4}a^4cd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/22*c^5*e*x^22 + 1/20*c^5*d*x^20 + 5/18*a*c^4*e*x^18 + 5/16*a*c^4*d*x^16 +
 5/7*a^2*c^3*e*x^14 + 5/6*a^2*c^3*d*x^12 + a^3*c^2*e*x^10 + 5/4*a^3*c^2*d*x
^8 + 5/6*a^4*c*e*x^6 + 5/4*a^4*c*d*x^4 + 1/2*a^5*e*x^2 + a^5*d*log(x)
```

Sympy [A] time = 0.41624, size = 150, normalized size = 1.06

$$a^5d\log{(x)}+\frac{a^5ex^2}{2}+\frac{5a^4cdx^4}{4}+\frac{5a^4cex^6}{6}+\frac{5a^3c^2dx^8}{4}+a^3c^2ex^{10}+\frac{5a^2c^3dx^{12}}{6}+\frac{5a^2c^3ex^{14}}{7}+\frac{5ac^4dx^{16}}{16}+\frac{5ac^4ex^{18}}{18}+\frac{c^5}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x,x)
```

```
[Out] a**5*d*log(x) + a**5*e*x**2/2 + 5*a**4*c*d*x**4/4 + 5*a**4*c*e*x**6/6 + 5*a
**3*c**2*d*x**8/4 + a**3*c**2*e*x**10 + 5*a**2*c**3*d*x**12/6 + 5*a**2*c**3
*e*x**14/7 + 5*a*c**4*d*x**16/16 + 5*a*c**4*e*x**18/18 + c**5*d*x**20/20 +
c**5*e*x**22/22
```

Giac [A] time = 1.1293, size = 177, normalized size = 1.25

$$\frac{1}{22}c^5x^{22}e + \frac{1}{20}c^5dx^{20} + \frac{5}{18}ac^4x^{18}e + \frac{5}{16}ac^4dx^{16} + \frac{5}{7}a^2c^3x^{14}e + \frac{5}{6}a^2c^3dx^{12} + a^3c^2x^{10}e + \frac{5}{4}a^3c^2dx^8 + \frac{5}{6}a^4cx^6e + \frac{5}{4}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*c^5*x^22*e + 1/20*c^5*d*x^20 + 5/18*a*c^4*x^18*e + 5/16*a*c^4*d*x^16 +
 5/7*a^2*c^3*x^14*e + 5/6*a^2*c^3*d*x^12 + a^3*c^2*x^10*e + 5/4*a^3*c^2*d*x
^8 + 5/6*a^4*c*x^6*e + 5/4*a^4*c*d*x^4 + 1/2*a^5*x^2*e + 1/2*a^5*d*log(x^2)
```

3.6 $\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx$

Optimal. Leaf size=139

$$\frac{10}{11}a^2c^3dx^{11}+\frac{10}{7}a^3c^2dx^7+\frac{10}{13}a^2c^3ex^{13}+\frac{10}{9}a^3c^2ex^9+\frac{5}{3}a^4cdx^3+a^4cex^5-\frac{a^5d}{x}+a^5ex+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+\frac{1}{19}c^5d$$

```
[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^3*c^2*d*x^
7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 + (10*a^2*c^3*e*x^13)/
13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 + (c^5*d*x^19)/19 + (c^5*e*x^21
)/21
```

Rubi [A] time = 0.081904, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1262}

$$\frac{10}{11}a^2c^3dx^{11}+\frac{10}{7}a^3c^2dx^7+\frac{10}{13}a^2c^3ex^{13}+\frac{10}{9}a^3c^2ex^9+\frac{5}{3}a^4cdx^3+a^4cex^5-\frac{a^5d}{x}+a^5ex+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+\frac{1}{19}c^5d$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

```
[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^3*c^2*d*x^
7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 + (10*a^2*c^3*e*x^13)/
13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 + (c^5*d*x^19)/19 + (c^5*e*x^21
)/21
```

Rule 1262

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x^2}\,dx=\int \left(a^5e+\frac{a^5d}{x^2}+5a^4cdx^2+5a^4cex^4+10a^3c^2dx^6+10a^3c^2ex^8+10a^2c^3dx^{10}+10a^2c^3ex^{12}+5\right.$$
$$=-\frac{a^5d}{x}+a^5ex+\frac{5}{3}a^4cdx^3+a^4cex^5+\frac{10}{7}a^3c^2dx^7+\frac{10}{9}a^3c^2ex^9+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{13}a^2c^3ex^{13}+$$

Mathematica [A] time = 0.0078724, size = 139, normalized size = 1.

$$\frac{10}{11}a^2c^3dx^{11}+\frac{10}{7}a^3c^2dx^7+\frac{10}{13}a^2c^3ex^{13}+\frac{10}{9}a^3c^2ex^9+\frac{5}{3}a^4cdx^3+a^4cex^5-\frac{a^5d}{x}+a^5ex+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+\frac{1}{19}c^5d$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

```
[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^3*c^2*d*x^
7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 + (10*a^2*c^3*e*x^13)/
```

```
13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 + (c^5*d*x^19)/19 + (c^5*e*x^21
)/21
```

Maple [A] time = 0.016, size = 122, normalized size = 0.9

$$-\frac{a^5 d}{x}+a^5 ex+\frac{5\,a^4 cdx^3}{3}+a^4 cex^5+\frac{10\,a^3c^2dx^7}{7}+\frac{10\,a^3c^2ex^9}{9}+\frac{10\,a^2c^3dx^{11}}{11}+\frac{10\,a^2c^3ex^{13}}{13}+\frac{ac^4dx^{15}}{3}+\frac{5\,ac^4ex^{17}}{17}+\frac{c^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x^2,x)
```

```
[Out] -a^5*d/x+a^5*e*x+5/3*a^4*c*d*x^3+a^4*c*e*x^5+10/7*a^3*c^2*d*x^7+10/9*a^3*c^
2*e*x^9+10/11*a^2*c^3*d*x^11+10/13*a^2*c^3*e*x^13+1/3*a*c^4*d*x^15+5/17*a*c
^4*e*x^17+1/19*c^5*d*x^19+1/21*c^5*e*x^21
```

Maxima [A] time = 0.939337, size = 163, normalized size = 1.17

$$\frac{1}{21}\,c^5ex^{21}+\frac{1}{19}\,c^5dx^{19}+\frac{5}{17}\,ac^4ex^{17}+\frac{1}{3}\,ac^4dx^{15}+\frac{10}{13}\,a^2c^3ex^{13}+\frac{10}{11}\,a^2c^3dx^{11}+\frac{10}{9}\,a^3c^2ex^9+\frac{10}{7}\,a^3c^2dx^7+a^4cex^5+\frac{5}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*c^5*e*x^21 + 1/19*c^5*d*x^19 + 5/17*a*c^4*e*x^17 + 1/3*a*c^4*d*x^15 +
10/13*a^2*c^3*e*x^13 + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*c^2*e*x^9 + 10/7*a^3
*c^2*d*x^7 + a^4*c*e*x^5 + 5/3*a^4*c*d*x^3 + a^5*e*x - a^5*d/x
```

Fricas [A] time = 1.66616, size = 373, normalized size = 2.68

$$\frac{138567\,c^5ex^{22}+153153\,c^5dx^{20}+855855\,ac^4ex^{18}+969969\,ac^4dx^{16}+2238390\,a^2c^3ex^{14}+2645370\,a^2c^3dx^{12}+323323}{2909907\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/2909907*(138567*c^5*e*x^22 + 153153*c^5*d*x^20 + 855855*a*c^4*e*x^18 + 96
9969*a*c^4*d*x^16 + 2238390*a^2*c^3*e*x^14 + 2645370*a^2*c^3*d*x^12 + 32332
30*a^3*c^2*e*x^10 + 4157010*a^3*c^2*d*x^8 + 2909907*a^4*c*e*x^6 + 4849845*a
^4*c*d*x^4 + 2909907*a^5*e*x^2 - 2909907*a^5*d)/x
```

Sympy [A] time = 0.405244, size = 143, normalized size = 1.03

$$-\frac{a^5d}{x}+a^5ex+\frac{5a^4cdx^3}{3}+a^4cex^5+\frac{10a^3c^2dx^7}{7}+\frac{10a^3c^2ex^9}{9}+\frac{10a^2c^3dx^{11}}{11}+\frac{10a^2c^3ex^{13}}{13}+\frac{ac^4dx^{15}}{3}+\frac{5ac^4ex^{17}}{17}+\frac{c^5d}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x**2,x)
```

```
[Out] -a**5*d/x + a**5*e*x + 5*a**4*c*d*x**3/3 + a**4*c*e*x**5 + 10*a**3*c**2*d*x
**7/7 + 10*a**3*c**2*e*x**9/9 + 10*a**2*c**3*d*x**11/11 + 10*a**2*c**3*e*x*
*13/13 + a*c**4*d*x**15/3 + 5*a*c**4*e*x**17/17 + c**5*d*x**19/19 + c**5*e*
x**21/21
```

Giac [A] time = 1.14078, size = 171, normalized size = 1.23

$$\frac{1}{21}c^5x^{21}e+\frac{1}{19}c^5dx^{19}+\frac{5}{17}ac^4x^{17}e+\frac{1}{3}ac^4dx^{15}+\frac{10}{13}a^2c^3x^{13}e+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{9}a^3c^2x^9e+\frac{10}{7}a^3c^2dx^7+a^4cx^5e+\frac{5}{3}a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/21*c^5*x^21*e + 1/19*c^5*d*x^19 + 5/17*a*c^4*x^17*e + 1/3*a*c^4*d*x^15 +
10/13*a^2*c^3*x^13*e + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*c^2*x^9*e + 10/7*a^3
*c^2*d*x^7 + a^4*c*x^5*e + 5/3*a^4*c*d*x^3 + a^5*x*e - a^5*d/x
```

3.7 $\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx$

Optimal. Leaf size=142

$$a^2c^3dx^{10}+\frac{5}{3}a^3c^2dx^6+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{4}a^3c^2ex^8+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4-\frac{a^5d}{2x^2}+a^5e\log(x)+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+$$

```
[Out] -(a^5*d)/(2*x^2) + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3*c^2*d*x^6
)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3*e*x^12)/6 + (5*a*c^
4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x^18)/18 + (c^5*e*x^20)/20 + a^
5*e*Log[x]
```

Rubi [A] time = 0.121243, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1252, 766}

$$a^2c^3dx^{10}+\frac{5}{3}a^3c^2dx^6+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{4}a^3c^2ex^8+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4-\frac{a^5d}{2x^2}+a^5e\log(x)+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

```
[Out] -(a^5*d)/(2*x^2) + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3*c^2*d*x^6
)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3*e*x^12)/6 + (5*a*c^
4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x^18)/18 + (c^5*e*x^20)/20 + a^
5*e*Log[x]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\left(a+cx^4\right)^5}{x^3}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)\left(a+cx^2\right)^5}{x^2}\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int \left(5a^4cd+\frac{a^5d}{x^2}+\frac{a^5e}{x}+5a^4cex+10a^3c^2dx^2+10a^3c^2ex^3+10a^2c^3dx^4+10a^2\right.\right.$$
$$=-\frac{a^5d}{2x^2}+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{14}ac^4dx^1$$

Mathematica [A] time = 0.0082041, size = 142, normalized size = 1.

$$a^2c^3dx^{10}+\frac{5}{3}a^3c^2dx^6+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{4}a^3c^2ex^8+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4-\frac{a^5d}{2x^2}+a^5e\log(x)+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

```
[Out] -(a^5*d)/(2*x^2) + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3*c^2*d*x^6
)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3*e*x^12)/6 + (5*a*c^
4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x^18)/18 + (c^5*e*x^20)/20 + a^
5*e*Log[x]
```

Maple [A] time = 0.006, size = 123, normalized size = 0.9

$$-\frac{a^5d}{2\,x^2}+\frac{5\,a^4cdx^2}{2}+\frac{5\,a^4cex^4}{4}+\frac{5\,a^3c^2dx^6}{3}+\frac{5\,a^3c^2ex^8}{4}+a^2c^3dx^{10}+\frac{5\,a^2c^3ex^{12}}{6}+\frac{5\,ac^4dx^{14}}{14}+\frac{5\,ac^4ex^{16}}{16}+\frac{c^5dx^{18}}{18}+\frac{c^5ex}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x^3,x)
```

```
[Out] -1/2*a^5*d/x^2+5/2*a^4*c*d*x^2+5/4*a^4*c*e*x^4+5/3*a^3*c^2*d*x^6+5/4*a^3*c^
2*e*x^8+a^2*c^3*d*x^10+5/6*a^2*c^3*e*x^12+5/14*a*c^4*d*x^14+5/16*a*c^4*e*x^
16+1/18*c^5*d*x^18+1/20*c^5*e*x^20+a^5*e*ln(x)
```

Maxima [A] time = 0.952209, size = 169, normalized size = 1.19

$$\frac{1}{20}\,c^5ex^{20}+\frac{1}{18}\,c^5dx^{18}+\frac{5}{16}\,ac^4ex^{16}+\frac{5}{14}\,ac^4dx^{14}+\frac{5}{6}\,a^2c^3ex^{12}+a^2c^3dx^{10}+\frac{5}{4}\,a^3c^2ex^8+\frac{5}{3}\,a^3c^2dx^6+\frac{5}{4}\,a^4cex^4+\frac{5}{2}\,a^4cdx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*c^5*e*x^20 + 1/18*c^5*d*x^18 + 5/16*a*c^4*e*x^16 + 5/14*a*c^4*d*x^14 +
 5/6*a^2*c^3*e*x^12 + a^2*c^3*d*x^10 + 5/4*a^3*c^2*e*x^8 + 5/3*a^3*c^2*d*x^
6 + 5/4*a^4*c*e*x^4 + 5/2*a^4*c*d*x^2 + 1/2*a^5*e*log(x^2) - 1/2*a^5*d/x^2
```

Fricas [A] time = 1.68025, size = 336, normalized size = 2.37

$$\frac{252\,c^5ex^{22}+280\,c^5dx^{20}+1575\,ac^4ex^{18}+1800\,ac^4dx^{16}+4200\,a^2c^3ex^{14}+5040\,a^2c^3dx^{12}+6300\,a^3c^2ex^{10}+8400\,a^3c^2dx^8}{5040\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*c^5*e*x^22 + 280*c^5*d*x^20 + 1575*a*c^4*e*x^18 + 1800*a*c^4*d*
x^16 + 4200*a^2*c^3*e*x^14 + 5040*a^2*c^3*d*x^12 + 6300*a^3*c^2*e*x^10 + 84
00*a^3*c^2*d*x^8 + 6300*a^4*c*e*x^6 + 12600*a^4*c*d*x^4 + 5040*a^5*e*x^2*lo
g(x) - 2520*a^5*d)/x^2
```

Sympy [A] time = 0.436451, size = 150, normalized size = 1.06

$$-\frac{a^5d}{2x^2} + a^5e\log(x) + \frac{5a^4cdx^2}{2} + \frac{5a^4cex^4}{4} + \frac{5a^3c^2dx^6}{3} + \frac{5a^3c^2ex^8}{4} + a^2c^3dx^{10} + \frac{5a^2c^3ex^{12}}{6} + \frac{5ac^4dx^{14}}{14} + \frac{5ac^4ex^{16}}{16} + \frac{c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x**3,x)
```

```
[Out] -a**5*d/(2*x**2) + a**5*e*log(x) + 5*a**4*c*d*x**2/2 + 5*a**4*c*e*x**4/4 +
5*a**3*c**2*d*x**6/3 + 5*a**3*c**2*e*x**8/4 + a**2*c**3*d*x**10 + 5*a**2*c*
*3*e*x**12/6 + 5*a*c**4*d*x**14/14 + 5*a*c**4*e*x**16/16 + c**5*d*x**18/18
+ c**5*e*x**20/20
```

Giac [A] time = 1.14471, size = 192, normalized size = 1.35

$$\frac{1}{20}c^5x^{20}e + \frac{1}{18}c^5dx^{18} + \frac{5}{16}ac^4x^{16}e + \frac{5}{14}ac^4dx^{14} + \frac{5}{6}a^2c^3x^{12}e + a^2c^3dx^{10} + \frac{5}{4}a^3c^2x^8e + \frac{5}{3}a^3c^2dx^6 + \frac{5}{4}a^4cx^4e + \frac{5}{2}a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*c^5*x^20*e + 1/18*c^5*d*x^18 + 5/16*a*c^4*x^16*e + 5/14*a*c^4*d*x^14 +
 5/6*a^2*c^3*x^12*e + a^2*c^3*d*x^10 + 5/4*a^3*c^2*x^8*e + 5/3*a^3*c^2*d*x^
6 + 5/4*a^4*c*x^4*e + 5/2*a^4*c*d*x^2 + 1/2*a^5*e*log(x^2) - 1/2*(a^5*x^2*e
 + a^5*d)/x^2
```

3.8 $\int x^5 \left(2+3x^2\right) \sqrt{5+x^4} \, dx$

Optimal. Leaf size=67

$$\frac{3}{10}\left(x^4+5\right)^{3/2}x^4-\frac{5}{8}\sqrt{x^4+5}x^2-\frac{1}{4}\left(4-x^2\right)\left(x^4+5\right)^{3/2}-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] (-5*x^2*Sqrt[5 + x^4])/8 + (3*x^4*(5 + x^4)^(3/2))/10 - ((4 - x^2)*(5 + x^4
)^(3/2))/4 - (25*ArcSinh[x^2/Sqrt[5]])/8
```

Rubi [A] time = 0.0501683, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1252, 833, 780, 195, 215}

$$\frac{3}{10}\left(x^4+5\right)^{3/2}x^4-\frac{5}{8}\sqrt{x^4+5}x^2-\frac{1}{4}\left(4-x^2\right)\left(x^4+5\right)^{3/2}-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (-5*x^2*Sqrt[5 + x^4])/8 + (3*x^4*(5 + x^4)^(3/2))/10 - ((4 - x^2)*(5 + x^4
)^(3/2))/4 - (25*ArcSinh[x^2/Sqrt[5]])/8
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int x^5\left(2+3x^2\right)\sqrt{5+x^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(2+3x)\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= \frac{3}{10}x^4\left(5+x^4\right)^{3/2}+\frac{1}{10}\operatorname{Subst}\left(\int x(-30+10x)\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= \frac{3}{10}x^4\left(5+x^4\right)^{3/2}-\frac{1}{4}\left(4-x^2\right)\left(5+x^4\right)^{3/2}-\frac{5}{4}\operatorname{Subst}\left(\int \sqrt{5+x^2}\,dx,x,x^2\right)\\ &= -\frac{5}{8}x^2\sqrt{5+x^4}+\frac{3}{10}x^4\left(5+x^4\right)^{3/2}-\frac{1}{4}\left(4-x^2\right)\left(5+x^4\right)^{3/2}-\frac{25}{8}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,\right.\\ &= -\frac{5}{8}x^2\sqrt{5+x^4}+\frac{3}{10}x^4\left(5+x^4\right)^{3/2}-\frac{1}{4}\left(4-x^2\right)\left(5+x^4\right)^{3/2}-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.0341664, size = 50, normalized size = 0.75

$$\frac{1}{40}\sqrt{x^4+5}\left(12x^8+10x^6+20x^4+25x^2-200\right)-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(-200 + 25*x^2 + 20*x^4 + 10*x^6 + 12*x^8))/40 - (25*ArcSinh
[x^2/Sqrt[5]])/8
```

Maple [A] time = 0.04, size = 53, normalized size = 0.8

$$\frac{3x^4-10}{10}\left(x^4+5\right)^{\frac{3}{2}}+\frac{x^2}{4}\left(x^4+5\right)^{\frac{3}{2}}-\frac{5x^2}{8}\sqrt{x^4+5}-\frac{25}{8}\operatorname{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 1/10*(x^4+5)^(3/2)*(3*x^4-10)+1/4*x^2*(x^4+5)^(3/2)-5/8*x^2*(x^4+5)^(1/2)-2
5/8*arcsinh(1/5*x^2*5^(1/2))
```

Maxima [B] time = 1.43369, size = 138, normalized size = 2.06

$$\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}-\frac{5}{2}\left(x^4+5\right)^{\frac{3}{2}}-\frac{25\left(\frac{\sqrt{x^4+5}}{x^2}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{8\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}-\frac{25}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{25}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5)^(5/2) - 5/2*(x^4 + 5)^(3/2) - 25/8*(sqrt(x^4 + 5)/x^2 + (x^4
 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 25/16*log(sqrt(x
^4 + 5)/x^2 + 1) + 25/16*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.60692, size = 128, normalized size = 1.91

$$\frac{1}{40}\left(12\,x^{8}+10\,x^{6}+20\,x^{4}+25\,x^{2}-200\right)\sqrt{x^{4}+5}+\frac{25}{8}\,\log\left(-x^{2}+\sqrt{x^{4}+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(12*x^8 + 10*x^6 + 20*x^4 + 25*x^2 - 200)*sqrt(x^4 + 5) + 25/8*log(-x^
2 + sqrt(x^4 + 5))
```

Sympy [A] time = 5.55715, size = 97, normalized size = 1.45

$$\frac{x^{10}}{4\sqrt{x^{4}+5}}+\frac{3x^{8}\sqrt{x^{4}+5}}{10}+\frac{15x^{6}}{8\sqrt{x^{4}+5}}+\frac{x^{4}\sqrt{x^{4}+5}}{2}+\frac{25x^{2}}{8\sqrt{x^{4}+5}}-5\sqrt{x^{4}+5}-\frac{25\operatorname{asinh}\left(\frac{\sqrt{5}x^{2}}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] x**10/(4*sqrt(x**4 + 5)) + 3*x**8*sqrt(x**4 + 5)/10 + 15*x**6/(8*sqrt(x**4
+ 5)) + x**4*sqrt(x**4 + 5)/2 + 25*x**2/(8*sqrt(x**4 + 5)) - 5*sqrt(x**4 +
5) - 25*asinh(sqrt(5)*x**2/5)/8
```

Giac [A] time = 1.15904, size = 70, normalized size = 1.04

$$\frac{1}{40}\,\sqrt{x^{4}+5}\left(\left(2\left(\left(6\,x^{2}+5\right)x^{2}+10\right)x^{2}+25\right)x^{2}-200\right)+\frac{25}{8}\,\log\left(-x^{2}+\sqrt{x^{4}+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/40*sqrt(x^4 + 5)*((2*((6*x^2 + 5)*x^2 + 10)*x^2 + 25)*x^2 - 200) + 25/8*l
og(-x^2 + sqrt(x^4 + 5))
```

3.9 $\int x^3 \left(2+3x^2\right) \sqrt{5+x^4} \, dx$

Optimal. Leaf size=51

$$-\frac{15}{16}\sqrt{x^4+5}x^2+\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}-\frac{75}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] (-15*x^2*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 - (75*ArcSinh
[x^2/Sqrt[5]])/16
```

Rubi [A] time = 0.0307754, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1252, 780, 195, 215}

$$-\frac{15}{16}\sqrt{x^4+5}x^2+\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}-\frac{75}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (-15*x^2*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 - (75*ArcSinh
[x^2/Sqrt[5]])/16
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int x^3\left(2+3x^2\right)\sqrt{5+x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int x(2+3x)\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= \frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}-\frac{15}{8}\,\text{Subst}\left(\int \sqrt{5+x^2}\,dx,x,x^2\right)\\ &= -\frac{15}{16}x^2\sqrt{5+x^4}+\frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}-\frac{75}{16}\,\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= -\frac{15}{16}x^2\sqrt{5+x^4}+\frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}-\frac{75}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.0288524, size = 44, normalized size = 0.86

$$\frac{1}{48}\left(\sqrt{x^4+5}\left(18x^6+16x^4+45x^2+80\right)-225\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(80 + 45*x^2 + 16*x^4 + 18*x^6) - 225*ArcSinh[x^2/Sqrt[5]])/
48
```

Maple [A] time = 0.004, size = 46, normalized size = 0.9

$$\frac{3x^2}{8}\left(x^4+5\right)^{\frac{3}{2}}-\frac{15x^2}{16}\sqrt{x^4+5}-\frac{75}{16}\text{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 3/8*x^2*(x^4+5)^(3/2)-15/16*x^2*(x^4+5)^(1/2)-75/16*arcsinh(1/5*x^2*5^(1/2)
)+1/3*(x^4+5)^(3/2)
```

Maxima [B] time = 1.44704, size = 126, normalized size = 2.47

$$\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}-\frac{75\left(\frac{\sqrt{x^4+5}}{x^2}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}-\frac{75}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{75}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) - 75/16*(sqrt(x^4 + 5)/x^2 + (x^4 + 5)^(3/2)/x^6)/(2*(x
^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 75/32*log(sqrt(x^4 + 5)/x^2 + 1) + 75/
32*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.55788, size = 116, normalized size = 2.27

$$\frac{1}{48}\left(18\,x^6+16\,x^4+45\,x^2+80\right)\sqrt{x^4+5}+\frac{75}{16}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(18*x^6 + 16*x^4 + 45*x^2 + 80)*sqrt(x^4 + 5) + 75/16*log(-x^2 + sqrt(
x^4 + 5))
```

Sympy [A] time = 4.08094, size = 70, normalized size = 1.37

$$\frac{3x^{10}}{8\sqrt{x^4+5}}+\frac{45x^6}{16\sqrt{x^4+5}}+\frac{75x^2}{16\sqrt{x^4+5}}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{3}-\frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) + 45*x**6/(16*sqrt(x**4 + 5)) + 75*x**2/(16*sqrt
(x**4 + 5)) + (x**4 + 5)**(3/2)/3 - 75*asinh(sqrt(5)*x**2/5)/16
```

Giac [A] time = 1.14915, size = 62, normalized size = 1.22

$$\frac{1}{48}\,\sqrt{x^4+5}\left(\left(2\left(9\,x^2+8\right)x^2+45\right)x^2+80\right)+\frac{75}{16}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 + 45)*x^2 + 80) + 75/16*log(-x^2 + s
qrt(x^4 + 5))
```

3.10 $\int x\left(2+3x^2\right)\sqrt{5+x^4}\,dx$

Optimal. Leaf size=44

$$\frac{1}{2}\sqrt{x^4+5}x^2+\frac{1}{2}\left(x^4+5\right)^{3/2}+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] (x^2*Sqrt[5 + x^4])/2 + (5 + x^4)^(3/2)/2 + (5*ArcSinh[x^2/Sqrt[5]])/2
```

Rubi [A] time = 0.0214319, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1248, 641, 195, 215}

$$\frac{1}{2}\sqrt{x^4+5}x^2+\frac{1}{2}\left(x^4+5\right)^{3/2}+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```



```
[Out] (x^2*Sqrt[5 + x^4])/2 + (5 + x^4)^(3/2)/2 + (5*ArcSinh[x^2/Sqrt[5]])/2
```


Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int x\left(2+3x^2\right)\sqrt{5+x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int(2+3x)\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\left(5+x^4\right)^{3/2}+\text{Subst}\left(\int\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}x^2\sqrt{5+x^4}+\frac{1}{2}\left(5+x^4\right)^{3/2}+\frac{5}{2}\,\text{Subst}\left(\int\frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}x^2\sqrt{5+x^4}+\frac{1}{2}\left(5+x^4\right)^{3/2}+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.032063, size = 36, normalized size = 0.82

$$\frac{1}{2}\sqrt{x^4+5}\left(x^4+x^2+5\right)+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(5 + x^2 + x^4))/2 + (5*ArcSinh[x^2/Sqrt[5]])/2
```

Maple [A] time = 0.01, size = 34, normalized size = 0.8

$$\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{5}{2}\text{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{x^2}{2}\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 1/2*(x^4+5)^(3/2)+5/2*arcsinh(1/5*x^2*5^(1/2))+1/2*x^2*(x^4+5)^(1/2)
```

Maxima [A] time = 1.43529, size = 90, normalized size = 2.05

$$\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{5\sqrt{x^4+5}}{2x^2\left(\frac{x^4+5}{x^4}-1\right)}+\frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 5/4*log
(sqrt(x^4 + 5)/x^2 + 1) - 5/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.51203, size = 90, normalized size = 2.05

$$\frac{1}{2}\left(x^4+x^2+5\right)\sqrt{x^4+5}-\frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x^4 + x^2 + 5)*sqrt(x^4 + 5) - 5/2*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A] time = 2.77543, size = 53, normalized size = 1.2

$$\frac{x^6}{2\sqrt{x^4+5}}+\frac{5x^2}{2\sqrt{x^4+5}}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{2}+\frac{5\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) + 5*x**2/(2*sqrt(x**4 + 5)) + (x**4 + 5)**(3/2)/2 +
 5*asinh(sqrt(5)*x**2/5)/2
```

Giac [A] time = 1.12636, size = 50, normalized size = 1.14

$$\frac{1}{2}\sqrt{x^4+5}\left(\left(x^2+1\right)x^2+5\right)-\frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 + 5) - 5/2*log(-x^2 + sqrt(x^4 + 5))
```

3.11 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx$

Optimal. Leaf size=58

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 + (15*ArcSinh[x^2/Sqrt[5]])/4 - Sqrt[5]*ArcTa
nh[Sqrt[5 + x^4]/Sqrt[5]]
```

Rubi [A] time = 0.05466, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1252, 815, 844, 215, 266, 63, 207}

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 + (15*ArcSinh[x^2/Sqrt[5]])/4 - Sqrt[5]*ArcTa
nh[Sqrt[5 + x^4]/Sqrt[5]]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x}\,dx, x, x^2\right)\\
&= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{1}{4}\operatorname{Subst}\left(\int \frac{20+15x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{15}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right) + 5\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{5}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + 5\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0428864, size = 57, normalized size = 0.98

$$\frac{1}{4}\left(\sqrt{x^4+5}\,(3x^2+4) + 15\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - 4\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4] + 15*ArcSinh[x^2/Sqrt[5]] - 4*Sqrt[5]*ArcTanh[Sq
rt[5 + x^4]/Sqrt[5]])/4
```

Maple [A] time = 0.014, size = 49, normalized size = 0.8

$$\frac{3x^2}{4}\sqrt{x^4+5} + \frac{15}{4}\operatorname{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right) + \sqrt{x^4+5} - \sqrt{5}\operatorname{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x,x)
```

```
[Out] 3/4*x^2*(x^4+5)^(1/2)+15/4*arcsinh(1/5*x^2*5^(1/2))+(x^4+5)^(1/2)-5^(1/2)*a
rctanh(5^(1/2)/(x^4+5)^(1/2))
```

Maxima [B] time = 1.44598, size = 134, normalized size = 2.31

$$\frac{1}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\sqrt{x^4+5}+\frac{15\sqrt{x^4+5}}{4x^2\left(\frac{x^4+5}{x^4}-1\right)}+\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + sqr
t(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 15/8*log(sqrt(x
^4 + 5)/x^2 + 1) - 15/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.58218, size = 149, normalized size = 2.57

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-\frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2)
 - 15/4*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A] time = 12.3123, size = 83, normalized size = 1.43

$$\frac{3x^6}{4\sqrt{x^4+5}}+\frac{15x^2}{4\sqrt{x^4+5}}+\sqrt{x^4+5}+\frac{\sqrt{5}\log\left(x^4\right)}{2}-\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)+\frac{15\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x,x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) + s
qrt(5)*log(x**4)/2 - sqrt(5)*log(sqrt(x**4/5 + 1) + 1) + 15*asinh(sqrt(5)*x
**2/5)/4
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+5}\left(3x^2+2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x, x)
```

3.12 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx$

Optimal. Leaf size=59

$$-\frac{\sqrt{x^4+5}\,(2-3x^2)}{2x^2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

```
[Out] -((2 - 3*x^2)*Sqrt[5 + x^4])/(2*x^2) + ArcSinh[x^2/Sqrt[5]] - (3*Sqrt[5]*Ar
cTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rubi [A] time = 0.0557464, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1252, 813, 844, 215, 266, 63, 207}

$$-\frac{\sqrt{x^4+5}\,(2-3x^2)}{2x^2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

```
[Out] -((2 - 3*x^2)*Sqrt[5 + x^4])/(2*x^2) + ArcSinh[x^2/Sqrt[5]] - (3*Sqrt[5]*Ar
cTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(2+3x^2\right)\sqrt{5+x^4}}{x^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{\left(2-3x^2\right)\sqrt{5+x^4}}{2x^2}-\frac{1}{4}\operatorname{Subst}\left(\int \frac{-30-4x}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\left(2-3x^2\right)\sqrt{5+x^4}}{2x^2}+\frac{15}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right)+\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\left(2-3x^2\right)\sqrt{5+x^4}}{2x^2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{15}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx,x,x^4\right)\\
&= -\frac{\left(2-3x^2\right)\sqrt{5+x^4}}{2x^2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{15}{2}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{5+x^4}\right)\\
&= -\frac{\left(2-3x^2\right)\sqrt{5+x^4}}{2x^2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0476781, size = 59, normalized size = 1.

$$\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\left(\frac{\left(3x^2-2\right)\sqrt{x^4+5}}{x^2}-3\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

```
[Out] ArcSinh[x^2/Sqrt[5]] + (((-2 + 3*x^2)*Sqrt[5 + x^4])/x^2 - 3*Sqrt[5]*ArcTan
h[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Maple [A] time = 0.012, size = 61, normalized size = 1.

$$\frac{3}{2}\sqrt{x^4+5}-\frac{3\sqrt{5}}{2}\operatorname{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)-\frac{1}{5x^2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{x^2}{5}\sqrt{x^4+5}+\operatorname{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^3,x)
```

```
[Out] 3/2*(x^4+5)^(1/2)-3/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-1/5/x^2*(x^4+5
)^(3/2)+1/5*x^2*(x^4+5)^(1/2)+arcsinh(1/5*x^2*5^(1/2))
```

Maxima [A] time = 1.45379, size = 119, normalized size = 2.02

$$\frac{3}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}-\frac{\sqrt{x^4+5}}{x^2}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/2
*sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*l
og(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.57317, size = 177, normalized size = 3.

$$\frac{3\sqrt{5}x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-2x^2\log\left(-x^2+\sqrt{x^4+5}\right)-2x^2+\sqrt{x^4+5}\left(3x^2-2\right)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 2*x^2*log(-x^2 + s
qrt(x^4 + 5)) - 2*x^2 + sqrt(x^4 + 5)*(3*x^2 - 2))/x^2
```

Sympy [A] time = 6.00339, size = 83, normalized size = 1.41

$$-\frac{x^2}{\sqrt{x^4+5}}+\frac{3\sqrt{x^4+5}}{2}+\frac{3\sqrt{5}\log\left(x^4\right)}{4}-\frac{3\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}+\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)-\frac{5}{x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**3,x)
```

```
[Out] -x**2/sqrt(x**4 + 5) + 3*sqrt(x**4 + 5)/2 + 3*sqrt(5)*log(x**4)/4 - 3*sqrt(
5)*log(sqrt(x**4/5 + 1) + 1)/2 + asinh(sqrt(5)*x**2/5) - 5/(x**2*sqrt(x**4
+ 5))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+5}\left(3x^2+2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^3, x)
```

3.13 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5} \, dx$

Optimal. Leaf size=63

$$-\frac{\sqrt{x^4+5}\,(3x^2+1)}{2x^4}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] -((1 + 3*x^2)*Sqrt[5 + x^4])/(2*x^4) + (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh
[Sqrt[5 + x^4]/Sqrt[5]]/(2*Sqrt[5])
```

Rubi [A] time = 0.0548039, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1252, 811, 844, 215, 266, 63, 207}

$$-\frac{\sqrt{x^4+5}\,(3x^2+1)}{2x^4}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
```

```
[Out] -((1 + 3*x^2)*Sqrt[5 + x^4])/(2*x^4) + (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh
[Sqrt[5 + x^4]/Sqrt[5]]/(2*Sqrt[5])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4}-\frac{1}{40}\operatorname{Subst}\left(\int \frac{-20-60x}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right)+\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx,x,x^4\right)\\
&= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{5+x^4}\right)\\
&= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.076863, size = 59, normalized size = 0.94

$$\frac{1}{10}\left(-\frac{5\sqrt{x^4+5}\,(3x^2+1)}{x^4}+15\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\sqrt{5}\tanh^{-1}\left(\sqrt{\frac{x^4}{5}+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
```

```
[Out] ((-5*(1 + 3*x^2)*Sqrt[5 + x^4])/x^4 + 15*ArcSinh[x^2/Sqrt[5]] - Sqrt[5]*Arc
Tanh[Sqrt[1 + x^4/5]])/10
```

Maple [A] time = 0.011, size = 75, normalized size = 1.2

$$-\frac{3}{10\,x^2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{3\,x^2}{10}\sqrt{x^4+5}+\frac{3}{2}\operatorname{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)-\frac{1}{10\,x^4}\left(x^4+5\right)^{\frac{3}{2}}+\frac{1}{10}\sqrt{x^4+5}-\frac{\sqrt{5}}{10}\operatorname{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^5,x)
```

```
[Out] -3/10/x^2*(x^4+5)^(3/2)+3/10*x^2*(x^4+5)^(1/2)+3/2*arcsinh(1/5*x^2*5^(1/2))
-1/10/x^4*(x^4+5)^(3/2)+1/10*(x^4+5)^(1/2)-1/10*5^(1/2)*arctanh(5^(1/2)/(x^
4+5)^(1/2))
```

Maxima [A] time = 1.44143, size = 123, normalized size = 1.95

$$\frac{1}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{2x^2}-\frac{\sqrt{x^4+5}}{2x^4}+\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 3/
2*sqrt(x^4 + 5)/x^2 - 1/2*sqrt(x^4 + 5)/x^4 + 3/4*log(sqrt(x^4 + 5)/x^2 + 1
) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.53979, size = 181, normalized size = 2.87

$$\frac{\sqrt{5}x^4\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-15x^4\log\left(-x^2+\sqrt{x^4+5}\right)-15x^4-5\sqrt{x^4+5}\left(3x^2+1\right)}{10x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/10*(sqrt(5)*x^4*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15*x^4*log(-x^2 + s
qrt(x^4 + 5)) - 15*x^4 - 5*sqrt(x^4 + 5)*(3*x^2 + 1))/x^4
```

Sympy [A] time = 5.12512, size = 76, normalized size = 1.21

$$-\frac{3x^2}{2\sqrt{x^4+5}}-\frac{\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}-\frac{\sqrt{1+\frac{5}{x^4}}}{2x^2}-\frac{15}{2x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**5,x)
```

```
[Out] -3*x**2/(2*sqrt(x**4 + 5)) - sqrt(5)*asinh(sqrt(5)/x**2)/10 + 3*asinh(sqrt(
5)*x**2/5)/2 - sqrt(1 + 5/x**4)/(2*x**2) - 15/(2*x**2*sqrt(x**4 + 5))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+5}\left(3x^2+2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^5, x)
```

3.14 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7} \, dx$

Optimal. Leaf size=58

$$-\frac{(x^4+5)^{3/2}}{15x^6} - \frac{3\sqrt{x^4+5}}{4x^4} - \frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{4\sqrt{5}}$$

```
[Out] (-3*Sqrt[5 + x^4])/(4*x^4) - (5 + x^4)^(3/2)/(15*x^6) - (3*ArcTanh[Sqrt[5 +
 x^4]/Sqrt[5]])/(4*Sqrt[5])
```

Rubi [A] time = 0.0466886, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1252, 807, 266, 47, 63, 207}

$$-\frac{(x^4+5)^{3/2}}{15x^6} - \frac{3\sqrt{x^4+5}}{4x^4} - \frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
```


```
[Out] (-3*Sqrt[5 + x^4])/(4*x^4) - (5 + x^4)^(3/2)/(15*x^6) - (3*ArcTanh[Sqrt[5 +
 x^4]/Sqrt[5]])/(4*Sqrt[5])
```


Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{2}\operatorname{Subst}\left(\int \frac{\sqrt{5+x^2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{4}\operatorname{Subst}\left(\int \frac{\sqrt{5+x}}{x^2}\,dx, x, x^4\right)\\
&= -\frac{3\sqrt{5+x^4}}{4x^4} - \frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= -\frac{3\sqrt{5+x^4}}{4x^4} - \frac{(5+x^4)^{3/2}}{15x^6} + \frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= -\frac{3\sqrt{5+x^4}}{4x^4} - \frac{(5+x^4)^{3/2}}{15x^6} - \frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{4\sqrt{5}}
\end{aligned}
$$

Mathematica [A] time = 0.0398948, size = 72, normalized size = 1.24

$$
-\frac{(x^4+5)^{3/2}}{15x^6} - \frac{3\left(5x^4+\sqrt{5}\sqrt{x^4+5}x^4\tanh^{-1}\left(\sqrt{\frac{x^4}{5}+1}\right)+25\right)}{20x^4\sqrt{x^4+5}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
```

```
[Out] -(5 + x^4)^(3/2)/(15*x^6) - (3*(25 + 5*x^4 + Sqrt[5]*x^4*Sqrt[5 + x^4]*ArcT
anh[Sqrt[1 + x^4/5]]))/(20*x^4*Sqrt[5 + x^4])
```

Maple [A] time = 0.01, size = 52, normalized size = 0.9

$$
-\frac{1}{15x^6}(x^4+5)^{\frac{3}{2}} - \frac{3}{20x^4}(x^4+5)^{\frac{3}{2}} + \frac{3}{20}\sqrt{x^4+5} - \frac{3\sqrt{5}}{20}\operatorname{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^7,x)
```

```
[Out] -1/15*(x^4+5)^(3/2)/x^6-3/20/x^4*(x^4+5)^(3/2)+3/20*(x^4+5)^(1/2)-3/20*5^(1
/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

Maxima [A] time = 1.42295, size = 80, normalized size = 1.38

$$\frac{3}{40}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{4x^4}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{15x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 3/
4*sqrt(x^4 + 5)/x^4 - 1/15*(x^4 + 5)^(3/2)/x^6
```

Fricas [A] time = 1.52349, size = 146, normalized size = 2.52

$$\frac{9\sqrt{5}x^6\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-4x^6-\left(4x^4+45x^2+20\right)\sqrt{x^4+5}}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(9*sqrt(5)*x^6*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 4*x^6 - (4*x^4 +
45*x^2 + 20)*sqrt(x^4 + 5))/x^6
```

Sympy [A] time = 5.27075, size = 63, normalized size = 1.09

$$-\frac{\sqrt{1+\frac{5}{x^4}}}{15}-\frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{20}-\frac{3\sqrt{1+\frac{5}{x^4}}}{4x^2}-\frac{\sqrt{1+\frac{5}{x^4}}}{3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**7,x)
```

```
[Out] -sqrt(1 + 5/x**4)/15 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/20 - 3*sqrt(1 + 5/x**4
)/(4*x**2) - sqrt(1 + 5/x**4)/(3*x**4)
```

Giac [A] time = 1.13902, size = 84, normalized size = 1.45

$$-\frac{1}{60}\left(\frac{5\left(\frac{4}{x^2}+9\right)}{x^2}+4\right)\sqrt{\frac{5}{x^4}+1}-\frac{3}{40}\sqrt{5}\log\left(\sqrt{5}+\sqrt{x^4+5}\right)+\frac{3}{40}\sqrt{5}\log\left(-\sqrt{5}+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/60*(5*(4/x^2 + 9)/x^2 + 4)*sqrt(5/x^4 + 1) - 3/40*sqrt(5)*log(sqrt(5) +
sqrt(x^4 + 5)) + 3/40*sqrt(5)*log(-sqrt(5) + sqrt(x^4 + 5))
```

3.15 $\int x^4 \left(2+3x^2\right) \sqrt{5+x^4}\,dx$

Optimal. Leaf size=208

$$-\frac{5\sqrt[4]{5}\left(21+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{21\sqrt{x^4+5}}+\frac{1}{21}\left(7x^2+6\right)\sqrt{x^4+5}x^5+\frac{2}{3}\sqrt{x^4+5}x^3-\frac{10\sqrt{x}}{x^2}$$

```
[Out] (20*x*Sqrt[5 + x^4])/21 + (2*x^3*Sqrt[5 + x^4])/3 - (10*x*Sqrt[5 + x^4])/(S
qrt[5] + x^2) + (x^5*(6 + 7*x^2)*Sqrt[5 + x^4])/21 + (10*5^(1/4)*(Sqrt[5] +
 x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2]
)/Sqrt[5 + x^4] - (5*5^(1/4)*(21 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4
)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(21*Sqrt[5 + x^4]
)
```

Rubi [A] time = 0.124185, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1274, 1280, 1198, 220, 1196}

$$\frac{1}{21}\left(7x^2+6\right)\sqrt{x^4+5}x^5+\frac{2}{3}\sqrt{x^4+5}x^3-\frac{10\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\frac{20}{21}\sqrt{x^4+5}x-\frac{5\sqrt[4]{5}\left(21+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan\right.}{21\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (20*x*Sqrt[5 + x^4])/21 + (2*x^3*Sqrt[5 + x^4])/3 - (10*x*Sqrt[5 + x^4])/(S
qrt[5] + x^2) + (x^5*(6 + 7*x^2)*Sqrt[5 + x^4])/21 + (10*5^(1/4)*(Sqrt[5] +
 x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2]
)/Sqrt[5 + x^4] - (5*5^(1/4)*(21 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4
)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(21*Sqrt[5 + x^4]
)
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int x^4\left(2+3x^2\right)\sqrt{5+x^4}\,dx &= \frac{1}{21}x^5\left(6+7x^2\right)\sqrt{5+x^4}+\frac{10}{63}\int\frac{x^4\left(18+21x^2\right)}{\sqrt{5+x^4}}\,dx\\ &= \frac{2}{3}x^3\sqrt{5+x^4}+\frac{1}{21}x^5\left(6+7x^2\right)\sqrt{5+x^4}-\frac{2}{63}\int\frac{x^2\left(315-90x^2\right)}{\sqrt{5+x^4}}\,dx\\ &= \frac{20}{21}x\sqrt{5+x^4}+\frac{2}{3}x^3\sqrt{5+x^4}+\frac{1}{21}x^5\left(6+7x^2\right)\sqrt{5+x^4}+\frac{2}{189}\int\frac{-450-945x^2}{\sqrt{5+x^4}}\,dx\\ &= \frac{20}{21}x\sqrt{5+x^4}+\frac{2}{3}x^3\sqrt{5+x^4}+\frac{1}{21}x^5\left(6+7x^2\right)\sqrt{5+x^4}+\left(10\sqrt{5}\right)\int\frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\frac{1}{21}\left(10\left(1\right.\right.\\ &= \frac{20}{21}x\sqrt{5+x^4}+\frac{2}{3}x^3\sqrt{5+x^4}-\frac{10x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{21}x^5\left(6+7x^2\right)\sqrt{5+x^4}+\frac{10\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\vphantom{x}}}{}\end{aligned}$$

Mathematica [C] time = 0.0387653, size = 82, normalized size = 0.39

$$\frac{1}{21}x\left(-35\sqrt{5}x^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)-30\sqrt{5}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)+7\left(x^4+5\right)^{3/2}x^2+6\left(x^4+5\right)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (x*(6*(5 + x^4)^(3/2) + 7*x^2*(5 + x^4)^(3/2) - 30*Sqrt[5]*Hypergeometric2F
1[-1/2, 1/4, 5/4, -x^4/5] - 35*Sqrt[5]*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4
, -x^4/5]))/21
```

Maple [C] time = 0.083, size = 192, normalized size = 0.9

$$\frac{x^7}{3}\sqrt{x^4+5}+\frac{2\,x^3}{3}\sqrt{x^4+5}-\frac{2\,i}{\sqrt{i\sqrt{5}}}\sqrt{25-5\,i\sqrt{5}x^2}\sqrt{25+5\,i\sqrt{5}x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\text{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 1/3*x^7*(x^4+5)^(1/2)+2/3*x^3*(x^4+5)^(1/2)-2*I/(I*5^(1/2))^(1/2)*(25-5*I*5
^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x
*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
+2/7*x^5*(x^4+5)^(1/2)+20/21*x*(x^4+5)^(1/2)-4/21*5^(1/2)/(I*5^(1/2))^(1/2)
*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*Ellipt
icF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5}\left(3\,x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(3\,x^6+2\,x^4\right)\sqrt{x^4+5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)*sqrt(x^4 + 5), x)
```

Sympy [C] time = 2.02494, size = 78, normalized size = 0.38

$$\frac{3\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**
4*exp_polar(I*pi)/5)/(2*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5}\left(3\,x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4, x)
```

3.16 $\int x^2 \left(2+3x^2\right) \sqrt{5+x^4} \, dx$

Optimal. Leaf size=192

$$\frac{\sqrt[4]{5}\left(14-5\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{7\sqrt{x^4+5}}+\frac{1}{35}\left(15x^2+14\right)\sqrt{x^4+5}x^3+\frac{4\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\frac{10}{7}\sqrt{x^4}$$

```
[Out] (10*x*Sqrt[5 + x^4])/7 + (4*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (x^3*(14 + 1
5*x^2)*Sqrt[5 + x^4])/35 - (4*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[
5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*
(14 - 5*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
F[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rubi [A] time = 0.096707, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1274, 1280, 1198, 220, 1196}

$$\frac{1}{35}\left(15x^2+14\right)\sqrt{x^4+5}x^3+\frac{4\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\frac{10}{7}\sqrt{x^4+5}x+\frac{\sqrt[4]{5}\left(14-5\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{7\sqrt{x^4+5}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (10*x*Sqrt[5 + x^4])/7 + (4*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (x^3*(14 + 1
5*x^2)*Sqrt[5 + x^4])/35 - (4*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[
5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*
(14 - 5*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
F[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
```

```
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int x^2\left(2+3x^2\right)\sqrt{5+x^4}\,dx &= \frac{1}{35}x^3\left(14+15x^2\right)\sqrt{5+x^4}+\frac{2}{7}\int\frac{x^2\left(14+15x^2\right)}{\sqrt{5+x^4}}\,dx\\ &= \frac{10}{7}x\sqrt{5+x^4}+\frac{1}{35}x^3\left(14+15x^2\right)\sqrt{5+x^4}-\frac{2}{21}\int\frac{75-42x^2}{\sqrt{5+x^4}}\,dx\\ &= \frac{10}{7}x\sqrt{5+x^4}+\frac{1}{35}x^3\left(14+15x^2\right)\sqrt{5+x^4}-\left(4\sqrt{5}\right)\int\frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\frac{1}{7}\left(2\left(25-14\sqrt{5}\right)\right)\int\frac{}{\sqrt{}}\\ &= \frac{10}{7}x\sqrt{5+x^4}+\frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{35}x^3\left(14+15x^2\right)\sqrt{5+x^4}-\frac{4\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan\right.}{\sqrt{5+x^4}}\end{aligned}$$

Mathematica [C] time = 0.0286933, size = 68, normalized size = 0.35

$$\frac{1}{21}x\left(14\sqrt{5}x^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)-45\sqrt{5}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)+9\left(x^4+5\right)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (x*(9*(5 + x^4)^(3/2) - 45*Sqrt[5]*Hypergeometric2F1[-1/2, 1/4, 5/4, -x^4/5
] + 14*Sqrt[5]*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -x^4/5]))/21
```

Maple [C] time = 0.013, size = 180, normalized size = 0.9

$$\frac{3x^5}{7}\sqrt{x^4+5}+\frac{10x}{7}\sqrt{x^4+5}-\frac{2\sqrt{5}}{7\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\mathrm{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}+\frac{2x^3}{5}\sqrt{x^4+5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 3/7*x^5*(x^4+5)^(1/2)+10/7*x*(x^4+5)^(1/2)-2/7*5^(1/2)/(I*5^(1/2))^(1/2)*(2
5-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF
(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)+2/5*x^3*(x^4+5)^(1/2)+4/5*I/(I*5^(1/2))
^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*
(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^
(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5}\left(3\,x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(3\,x^4+2\,x^2\right)\sqrt{x^4+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)*sqrt(x^4 + 5), x)
```

Sympy [C] time = 1.83294, size = 78, normalized size = 0.41

$$\frac{3\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*
exp_polar(I*pi)/5)/(2*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5}\left(3\,x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2, x)
```

3.17 $\int \left(2+3x^2\right)\sqrt{5+x^4}\,dx$

Optimal. Leaf size=176

$$\frac{\sqrt[4]{5}\left(9+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{3\sqrt{x^4+5}}+\frac{1}{15}\left(9x^2+10\right)\sqrt{x^4+5}x+\frac{6\sqrt{x^4+5}x}{x^2+\sqrt{5}}-\frac{6\sqrt[4]{5}\left(x^2+\right.}{}$$

```
[Out] (6*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (x*(10 + 9*x^2)*Sqrt[5 + x^4])/15 - (
6*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*Arc
Tan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*(9 + 2*Sqrt[5])*(Sqrt[5] + x
^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/
(3*Sqrt[5 + x^4])
```

Rubi [A] time = 0.0635536, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1177, 1198, 220, 1196}

$$\frac{1}{15}\left(9x^2+10\right)\sqrt{x^4+5}x+\frac{6\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\frac{\sqrt[4]{5}\left(9+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{x^4+5}}-\frac{6\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\cdots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (6*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (x*(10 + 9*x^2)*Sqrt[5 + x^4])/15 - (
6*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*Arc
Tan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*(9 + 2*Sqrt[5])*(Sqrt[5] + x
^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/
(3*Sqrt[5 + x^4])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \left(2+3x^2\right)\sqrt{5+x^4}\,dx = \frac{1}{15}x\left(10+9x^2\right)\sqrt{5+x^4}+\frac{1}{15}\int \frac{100+90x^2}{\sqrt{5+x^4}}\,dx$$
$$= \frac{1}{15}x\left(10+9x^2\right)\sqrt{5+x^4}-\left(6\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx+\frac{1}{3}\left(2\left(10+9\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$
$$= \frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{15}x\left(10+9x^2\right)\sqrt{5+x^4}-\frac{6\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}+\frac{\sqrt[4]{5}(}{}$$

Mathematica [C] time = 0.0114945, size = 48, normalized size = 0.27

$$\sqrt{5}x\left(x^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)+2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] Sqrt[5]*x*(2*Hypergeometric2F1[-1/2, 1/4, 5/4, -x^4/5] + x^2*Hypergeometric
2F1[-1/2, 3/4, 7/4, -x^4/5])
```

Maple [C] time = 0.011, size = 168, normalized size = 1.

$$\frac{3x^3}{5}\sqrt{x^4+5}+\frac{\frac{6i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\text{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}+\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2),x)
```

```
[Out] 3/5*x^3*(x^4+5)^(1/2)+6/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(2
5+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))
^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))+2/3*x*(x^4+5)^(1/2)
+4/15*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*
x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5}\left(3x^2+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^4+5}\left(3x^2+2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

Sympy [C] time = 1.73328, size = 76, normalized size = 0.43

$$\frac{3\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2),x)

[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp
_polar(I*pi)/5)/(2*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x^4+5}\left(3x^2+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

3.18 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx$

Optimal. Leaf size=171

$$\frac{\sqrt[4]{5}\left(2+\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{4\sqrt{x^4+5}x}{x^2+\sqrt{5}}-\frac{\left(2-x^2\right)\sqrt{x^4+5}}{x}-\frac{4\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4}{(x^2+}}}{\sqrt{x}}$$

```
[Out] -(((2 - x^2)*Sqrt[5 + x^4])/x) + (4*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (4*5
^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan
[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*(2 + Sqrt[5])*(Sqrt[5] + x^2)*S
qrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[
5 + x^4]
```

Rubi [A] time = 0.0670011, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1272, 1198, 220, 1196}

$$\frac{4\sqrt{x^4+5}x}{x^2+\sqrt{5}}-\frac{\left(2-x^2\right)\sqrt{x^4+5}}{x}+\frac{\sqrt[4]{5}\left(2+\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{4\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\right.}{\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^2,x]
```

```
[Out] -(((2 - x^2)*Sqrt[5 + x^4])/x) + (4*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (4*5
^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan
[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*(2 + Sqrt[5])*(Sqrt[5] + x^2)*S
qrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[
5 + x^4]
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx = -\frac{(2-x^2)\sqrt{5+x^4}}{x} - \frac{2}{3}\int \frac{-15-6x^2}{\sqrt{5+x^4}}\,dx$$
$$= -\frac{(2-x^2)\sqrt{5+x^4}}{x} - \left(4\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx + \left(2\left(5+2\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$
$$= -\frac{(2-x^2)\sqrt{5+x^4}}{x} + \frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{4\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{\sqrt[4]{5}(2\ldots}{\ldots}$$

Mathematica [C] time = 0.0226249, size = 53, normalized size = 0.31

$$3\sqrt{5}x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right) - \frac{2\sqrt{5}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^2,x]
```

```
[Out] (-2*Sqrt[5]*Hypergeometric2F1[-1/2, -1/4, 3/4, -x^4/5])/x + 3*Sqrt[5]*x*Hyp
ergeometric2F1[-1/2, 1/4, 5/4, -x^4/5]
```

Maple [C] time = 0.016, size = 167, normalized size = 1.

$$x\sqrt{x^4+5} + \frac{2\sqrt{5}}{5\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\text{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}} - 2\frac{\sqrt{x^4+5}}{x} + \frac{\frac{4i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^2,x)
```

```
[Out] x*(x^4+5)^(1/2)+2/5*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(2
5+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^
(1/2),I)-2*(x^4+5)^(1/2)/x+4/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/
2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(
1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5}\left(3x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

Sympy [C] time = 1.83293, size = 78, normalized size = 0.46

$$\frac{3\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**2,x)
```

```
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(I*pi)/5)/(
4*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), x**4*exp_po
lar(I*pi)/5)/(2*x*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

3.19 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx$

Optimal. Leaf size=192

$$\frac{\left(2+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{3\sqrt[4]{5}\sqrt{x^4+5}}+\frac{6\sqrt{x^4+5}x}{x^2+\sqrt{5}}-\frac{6\sqrt{x^4+5}}{x}-\frac{\left(2-9x^2\right)\sqrt{x^4+5}}{3x^3}-\frac{6\sqrt[4]{5}\left(x\right.}{}$$

```
[Out] (-6*Sqrt[5 + x^4])/x - ((2 - 9*x^2)*Sqrt[5 + x^4])/(3*x^3) + (6*x*Sqrt[5 +
x^4])/(Sqrt[5] + x^2) - (6*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5]
+ x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + ((2 + 9*Sqrt
[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x
/5^(1/4)], 1/2])/(3*5^(1/4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.0893237, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1272, 1282, 1198, 220, 1196}

$$\frac{6\sqrt{x^4+5}x}{x^2+\sqrt{5}}-\frac{6\sqrt{x^4+5}}{x}-\frac{\left(2-9x^2\right)\sqrt{x^4+5}}{3x^3}+\frac{\left(2+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{5}\sqrt{x^4+5}}-\frac{6\sqrt[4]{5}\left(x^2+\sqrt{5}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^4,x]
```


```
[Out] (-6*Sqrt[5 + x^4])/x - ((2 - 9*x^2)*Sqrt[5 + x^4])/(3*x^3) + (6*x*Sqrt[5 +
x^4])/(Sqrt[5] + x^2) - (6*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5]
+ x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + ((2 + 9*Sqrt
[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x
/5^(1/4)], 1/2])/(3*5^(1/4)*Sqrt[5 + x^4])
```


Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx &= -\frac{(2-9x^2)\sqrt{5+x^4}}{3x^3}-\frac{2}{3}\int \frac{-45-2x^2}{x^2\sqrt{5+x^4}}\,dx\\
&= -\frac{6\sqrt{5+x^4}}{x}-\frac{(2-9x^2)\sqrt{5+x^4}}{3x^3}+\frac{2}{15}\int \frac{10+45x^2}{\sqrt{5+x^4}}\,dx\\
&= -\frac{6\sqrt{5+x^4}}{x}-\frac{(2-9x^2)\sqrt{5+x^4}}{3x^3}-\left(6\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx+\frac{1}{3}\left(2\left(2+9\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx\\
&= -\frac{6\sqrt{5+x^4}}{x}-\frac{(2-9x^2)\sqrt{5+x^4}}{3x^3}+\frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{6\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right.}{\sqrt{5+x^4}}
\end{aligned}
$$

Mathematica [C] time = 0.0243619, size = 54, normalized size = 0.28

$$
-\frac{\sqrt{5}\left(9x^2\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)+2\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{x^4}{5}\right)\right)}{3x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^4,x]
```

```
[Out] -(Sqrt[5]*(2*Hypergeometric2F1[-3/4, -1/2, 1/4, -x^4/5] + 9*x^2*Hypergeomet
ric2F1[-1/2, -1/4, 3/4, -x^4/5]))/(3*x^3)
```

Maple [C] time = 0.016, size = 170, normalized size = 0.9

$$
-\frac{2}{3x^3}\sqrt{x^4+5}+\frac{4\sqrt{5}}{75\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\mathrm{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}-3\frac{\sqrt{x^4+5}}{x}+\frac{\frac{6i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^4,x)
```

```
[Out] -2/3*(x^4+5)^(1/2)/x^3+4/75*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^
(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5
^(1/2))^(1/2),I)-3*(x^4+5)^(1/2)/x+6/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*
x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2
)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5}\,(3\,x^2+2)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5}\,(3\,x^2+2)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

Sympy [C] time = 1.99396, size = 83, normalized size = 0.43

$$\frac{3\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4x\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt{5}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**4,x)
```

```
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), x**4*exp_polar(I*pi)/5)/(
4*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), x**4*exp_
polar(I*pi)/5)/(2*x**3*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5}\,(3\,x^2+2)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="giac")

[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

3.20 $\int x^5 \left(2+3x^2\right) \left(5+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=83

$$\frac{3}{14}\left(x^4+5\right)^{5/2}x^4-\frac{5}{24}\left(x^4+5\right)^{3/2}x^2-\frac{25}{16}\sqrt{x^4+5}x^2-\frac{1}{42}\left(18-7x^2\right)\left(x^4+5\right)^{5/2}-\frac{125}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] (-25*x^2*Sqrt[5 + x^4])/16 - (5*x^2*(5 + x^4)^(3/2))/24 + (3*x^4*(5 + x^4)^
(5/2))/14 - ((18 - 7*x^2)*(5 + x^4)^(5/2))/42 - (125*ArcSinh[x^2/Sqrt[5]])/
16
```

Rubi [A] time = 0.0588598, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1252, 833, 780, 195, 215}

$$\frac{3}{14}\left(x^4+5\right)^{5/2}x^4-\frac{5}{24}\left(x^4+5\right)^{3/2}x^2-\frac{25}{16}\sqrt{x^4+5}x^2-\frac{1}{42}\left(18-7x^2\right)\left(x^4+5\right)^{5/2}-\frac{125}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (-25*x^2*Sqrt[5 + x^4])/16 - (5*x^2*(5 + x^4)^(3/2))/24 + (3*x^4*(5 + x^4)^
(5/2))/14 - ((18 - 7*x^2)*(5 + x^4)^(5/2))/42 - (125*ArcSinh[x^2/Sqrt[5]])/
16
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(2+3x^2\right)\left(5+x^4\right)^{3/2} dx &= \frac{1}{2}\operatorname{Subst}\left(\int x^2(2+3x)\left(5+x^2\right)^{3/2} dx, x, x^2\right)\\
&= \frac{3}{14}x^4\left(5+x^4\right)^{5/2}+\frac{1}{14}\operatorname{Subst}\left(\int x(-30+14x)\left(5+x^2\right)^{3/2} dx, x, x^2\right)\\
&= \frac{3}{14}x^4\left(5+x^4\right)^{5/2}-\frac{1}{42}\left(18-7x^2\right)\left(5+x^4\right)^{5/2}-\frac{5}{6}\operatorname{Subst}\left(\int \left(5+x^2\right)^{3/2} dx, x, x^2\right)\\
&= -\frac{5}{24}x^2\left(5+x^4\right)^{3/2}+\frac{3}{14}x^4\left(5+x^4\right)^{5/2}-\frac{1}{42}\left(18-7x^2\right)\left(5+x^4\right)^{5/2}-\frac{25}{8}\operatorname{Subst}\left(\int \sqrt{5+x\right.\\
&= -\frac{25}{16}x^2\sqrt{5+x^4}-\frac{5}{24}x^2\left(5+x^4\right)^{3/2}+\frac{3}{14}x^4\left(5+x^4\right)^{5/2}-\frac{1}{42}\left(18-7x^2\right)\left(5+x^4\right)^{5/2}-\frac{125}{16}\\
&= -\frac{25}{16}x^2\sqrt{5+x^4}-\frac{5}{24}x^2\left(5+x^4\right)^{3/2}+\frac{3}{14}x^4\left(5+x^4\right)^{5/2}-\frac{1}{42}\left(18-7x^2\right)\left(5+x^4\right)^{5/2}-\frac{125}{16}
\end{aligned}$$

Mathematica [A] time = 0.0543012, size = 72, normalized size = 0.87

$$\frac{1}{6}x^2\left(x^4+5\right)^{5/2}+\frac{3}{14}\left(x^4-2\right)\left(x^4+5\right)^{5/2}-\frac{5}{48}\left(\sqrt{x^4+5}\left(2x^4+25\right)x^2+75\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (x^2*(5 + x^4)^(5/2))/6 + (3*(-2 + x^4)*(5 + x^4)^(5/2))/14 - (5*(x^2*Sqrt[
5 + x^4]*(25 + 2*x^4) + 75*ArcSinh[x^2/Sqrt[5]]))/48
```

Maple [A] time = 0.024, size = 73, normalized size = 0.9

$$\frac{\left(3x^4-6\right)\left(x^8+10x^4+25\right)}{14}\sqrt{x^4+5}+\frac{x^{10}}{6}\sqrt{x^4+5}+\frac{35x^6}{24}\sqrt{x^4+5}+\frac{25x^2}{16}\sqrt{x^4+5}-\frac{125}{16}\operatorname{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 3/14*(x^4+5)^(1/2)*(x^4-2)*(x^8+10*x^4+25)+1/6*x^10*(x^4+5)^(1/2)+35/24*x^6
*(x^4+5)^(1/2)+25/16*x^2*(x^4+5)^(1/2)-125/16*arcsinh(1/5*x^2*5^(1/2))
```

Maxima [A] time = 1.42308, size = 171, normalized size = 2.06

$$\frac{3}{14}\left(x^4+5\right)^{\frac{7}{2}}-\frac{3}{2}\left(x^4+5\right)^{\frac{5}{2}}-\frac{125\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{8\left(x^4+5\right)^{\frac{3}{2}}}{x^6}-\frac{3\left(x^4+5\right)^{\frac{5}{2}}}{x^{10}}\right)}{48\left(\frac{3\left(x^4+5\right)}{x^4}-\frac{3\left(x^4+5\right)^2}{x^8}+\frac{\left(x^4+5\right)^3}{x^{12}}-1\right)}-\frac{125}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{125}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/14*(x^4 + 5)^(7/2) - 3/2*(x^4 + 5)^(5/2) - 125/48*(3*sqrt(x^4 + 5)/x^2 -
8*(x^4 + 5)^(3/2)/x^6 - 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4 + 5)/x^4 - 3*(x^4 +
 5)^2/x^8 + (x^4 + 5)^3/x^12 - 1) - 125/32*log(sqrt(x^4 + 5)/x^2 + 1) + 125
/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.53446, size = 166, normalized size = 2.

$$\frac{1}{336}\left(72\,x^{12}+56\,x^{10}+576\,x^{8}+490\,x^{6}+360\,x^{4}+525\,x^{2}-3600\right)\sqrt{x^{4}+5}+\frac{125}{16}\,\log\left(-x^{2}+\sqrt{x^{4}+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/336*(72*x^12 + 56*x^10 + 576*x^8 + 490*x^6 + 360*x^4 + 525*x^2 - 3600)*sq
rt(x^4 + 5) + 125/16*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A] time = 13.5729, size = 131, normalized size = 1.58

$$\frac{x^{14}}{6\sqrt{x^{4}+5}}+\frac{3x^{12}\sqrt{x^{4}+5}}{14}+\frac{55x^{10}}{24\sqrt{x^{4}+5}}+\frac{12x^{8}\sqrt{x^{4}+5}}{7}+\frac{425x^{6}}{48\sqrt{x^{4}+5}}+\frac{15x^{4}\sqrt{x^{4}+5}}{14}+\frac{125x^{2}}{16\sqrt{x^{4}+5}}-\frac{75\sqrt{x^{4}+5}}{7}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**14/(6*sqrt(x**4 + 5)) + 3*x**12*sqrt(x**4 + 5)/14 + 55*x**10/(24*sqrt(x*
*4 + 5)) + 12*x**8*sqrt(x**4 + 5)/7 + 425*x**6/(48*sqrt(x**4 + 5)) + 15*x**
4*sqrt(x**4 + 5)/14 + 125*x**2/(16*sqrt(x**4 + 5)) - 75*sqrt(x**4 + 5)/7 -
125*asinh(sqrt(5)*x**2/5)/16
```

Giac [A] time = 1.14673, size = 88, normalized size = 1.06

$$\frac{1}{336}\,\sqrt{x^{4}+5}\left(\left(2\left(\left(4\left(\left(9\,x^{2}+7\right)x^{2}+72\right)x^{2}+245\right)x^{2}+180\right)x^{2}+525\right)x^{2}-3600\right)+\frac{125}{16}\,\log\left(-x^{2}+\sqrt{x^{4}+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/336*sqrt(x^4 + 5)*((2*((4*((9*x^2 + 7)*x^2 + 72)*x^2 + 245)*x^2 + 180)*x^
2 + 525)*x^2 - 3600) + 125/16*log(-x^2 + sqrt(x^4 + 5))
```

3.21 $\int x^3 \left(2+3x^2\right) \left(5+x^4\right)^{3/2} dx$

Optimal. Leaf size=67

$$\frac{1}{20}\left(5x^2+4\right)\left(x^4+5\right)^{5/2}-\frac{5}{16}x^2\left(x^4+5\right)^{3/2}-\frac{75}{32}x^2\sqrt{x^4+5}-\frac{375}{32}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] (-75*x^2*Sqrt[5 + x^4])/32 - (5*x^2*(5 + x^4)^(3/2))/16 + ((4 + 5*x^2)*(5 +
 x^4)^(5/2))/20 - (375*ArcSinh[x^2/Sqrt[5]])/32
```

Rubi [A] time = 0.039551, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1252, 780, 195, 215}

$$\frac{1}{20}\left(5x^2+4\right)\left(x^4+5\right)^{5/2}-\frac{5}{16}x^2\left(x^4+5\right)^{3/2}-\frac{75}{32}x^2\sqrt{x^4+5}-\frac{375}{32}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (-75*x^2*Sqrt[5 + x^4])/32 - (5*x^2*(5 + x^4)^(3/2))/16 + ((4 + 5*x^2)*(5 +
 x^4)^(5/2))/20 - (375*ArcSinh[x^2/Sqrt[5]])/32
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^3 \left(2+3x^2\right) \left(5+x^4\right)^{3/2} \, dx = \frac{1}{2} \text{Subst}\left(\int x(2+3x) \left(5+x^2\right)^{3/2} \, dx, x, x^2\right)$$
$$= \frac{1}{20} \left(4+5x^2\right) \left(5+x^4\right)^{5/2} - \frac{5}{4} \text{Subst}\left(\int \left(5+x^2\right)^{3/2} \, dx, x, x^2\right)$$
$$= -\frac{5}{16} x^2 \left(5+x^4\right)^{3/2} + \frac{1}{20} \left(4+5x^2\right) \left(5+x^4\right)^{5/2} - \frac{75}{16} \text{Subst}\left(\int \sqrt{5+x^2} \, dx, x, x^2\right)$$
$$= -\frac{75}{32} x^2 \sqrt{5+x^4} - \frac{5}{16} x^2 \left(5+x^4\right)^{3/2} + \frac{1}{20} \left(4+5x^2\right) \left(5+x^4\right)^{5/2} - \frac{375}{32} \text{Subst}\left(\int \frac{1}{\sqrt{5+ \cdots}}\right.$$
$$= -\frac{75}{32} x^2 \sqrt{5+x^4} - \frac{5}{16} x^2 \left(5+x^4\right)^{3/2} + \frac{1}{20} \left(4+5x^2\right) \left(5+x^4\right)^{5/2} - \frac{375}{32} \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Mathematica [A] time = 0.0414293, size = 54, normalized size = 0.81

$$\frac{1}{160} \left(\sqrt{x^4+5} \left(40x^{10}+32x^8+350x^6+320x^4+375x^2+800\right)-1875 \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(800 + 375*x^2 + 320*x^4 + 350*x^6 + 32*x^8 + 40*x^10) - 187
5*ArcSinh[x^2/Sqrt[5]])/160
```

Maple [A] time = 0.006, size = 58, normalized size = 0.9

$$\frac{x^{10}}{4} \sqrt{x^4+5} + \frac{35\, x^6}{16} \sqrt{x^4+5} + \frac{75\, x^2}{32} \sqrt{x^4+5} - \frac{375}{32} \text{Arcsinh}\left(\frac{x^2 \sqrt{5}}{5}\right) + \frac{1}{5} \left(x^4+5\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 1/4*x^10*(x^4+5)^(1/2)+35/16*x^6*(x^4+5)^(1/2)+75/32*x^2*(x^4+5)^(1/2)-375/
32*arcsinh(1/5*x^2*5^(1/2))+1/5*(x^4+5)^(5/2)
```

Maxima [B] time = 1.41751, size = 159, normalized size = 2.37

$$\frac{1}{5} \left(x^4+5\right)^{\frac{5}{2}} - \frac{125 \left(\frac{3\sqrt{x^4+5}}{x^2} - \frac{8\left(x^4+5\right)^{\frac{3}{2}}}{x^6} - \frac{3\left(x^4+5\right)^{\frac{5}{2}}}{x^{10}}\right)}{32 \left(\frac{3\left(x^4+5\right)}{x^4} - \frac{3\left(x^4+5\right)^2}{x^8} + \frac{\left(x^4+5\right)^3}{x^{12}} - 1\right)} - \frac{375}{64} \log\left(\frac{\sqrt{x^4+5}}{x^2} + 1\right) + \frac{375}{64} \log\left(\frac{\sqrt{x^4+5}}{x^2} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(x^4 + 5)^(5/2) - 125/32*(3*sqrt(x^4 + 5)/x^2 - 8*(x^4 + 5)^(3/2)/x^6 -
 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4 + 5)/x^4 - 3*(x^4 + 5)^2/x^8 + (x^4 + 5)^3
/x^12 - 1) - 375/64*log(sqrt(x^4 + 5)/x^2 + 1) + 375/64*log(sqrt(x^4 + 5)/x
```

```
^2 - 1)
```

Fricas [A] time = 1.5947, size = 150, normalized size = 2.24

$$\frac{1}{160}\left(40\,x^{10}+32\,x^{8}+350\,x^{6}+320\,x^{4}+375\,x^{2}+800\right)\sqrt{x^{4}+5}+\frac{375}{32}\,\log\left(-x^{2}+\sqrt{x^{4}+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/160*(40*x^10 + 32*x^8 + 350*x^6 + 320*x^4 + 375*x^2 + 800)*sqrt(x^4 + 5)
+ 375/32*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [B] time = 10.5095, size = 124, normalized size = 1.85

$$\frac{x^{14}}{4\sqrt{x^{4}+5}}+\frac{55x^{10}}{16\sqrt{x^{4}+5}}+\frac{x^{8}\sqrt{x^{4}+5}}{5}+\frac{425x^{6}}{32\sqrt{x^{4}+5}}+\frac{x^{4}\sqrt{x^{4}+5}}{3}+\frac{375x^{2}}{32\sqrt{x^{4}+5}}+\frac{5\left(x^{4}+5\right)^{\frac{3}{2}}}{3}-\frac{10\sqrt{x^{4}+5}}{3}-\frac{375\,\operatorname{asinh}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**14/(4*sqrt(x**4 + 5)) + 55*x**10/(16*sqrt(x**4 + 5)) + x**8*sqrt(x**4 +
5)/5 + 425*x**6/(32*sqrt(x**4 + 5)) + x**4*sqrt(x**4 + 5)/3 + 375*x**2/(32*
sqrt(x**4 + 5)) + 5*(x**4 + 5)**(3/2)/3 - 10*sqrt(x**4 + 5)/3 - 375*asinh(s
qrt(5)*x**2/5)/32
```

Giac [A] time = 1.15825, size = 80, normalized size = 1.19

$$\frac{1}{160}\,\sqrt{x^{4}+5}\left(\left(2\left(\left(4\left(5\,x^{2}+4\right)x^{2}+175\right)x^{2}+160\right)x^{2}+375\right)x^{2}+800\right)+\frac{375}{32}\,\log\left(-x^{2}+\sqrt{x^{4}+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/160*sqrt(x^4 + 5)*((2*((4*(5*x^2 + 4)*x^2 + 175)*x^2 + 160)*x^2 + 375)*x^
2 + 800) + 375/32*log(-x^2 + sqrt(x^4 + 5))
```

3.22 $\int x\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=60

$$\frac{3}{10}\left(x^4+5\right)^{5/2}+\frac{1}{4}x^2\left(x^4+5\right)^{3/2}+\frac{15}{8}x^2\sqrt{x^4+5}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] (15*x^2*Sqrt[5 + x^4])/8 + (x^2*(5 + x^4)^(3/2))/4 + (3*(5 + x^4)^(5/2))/10
 + (75*ArcSinh[x^2/Sqrt[5]])/8
```

Rubi [A] time = 0.030477, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1248, 641, 195, 215}

$$\frac{3}{10}\left(x^4+5\right)^{5/2}+\frac{1}{4}x^2\left(x^4+5\right)^{3/2}+\frac{15}{8}x^2\sqrt{x^4+5}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (15*x^2*Sqrt[5 + x^4])/8 + (x^2*(5 + x^4)^(3/2))/4 + (3*(5 + x^4)^(5/2))/10
 + (75*ArcSinh[x^2/Sqrt[5]])/8
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x\left(2+3x^2\right)\left(5+x^4\right)^{3/2}dx &= \frac{1}{2}\operatorname{Subst}\left(\int (2+3x)\left(5+x^2\right)^{3/2}dx,x,x^2\right)\\
&= \frac{3}{10}\left(5+x^4\right)^{5/2}+\operatorname{Subst}\left(\int \left(5+x^2\right)^{3/2}dx,x,x^2\right)\\
&= \frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{15}{4}\operatorname{Subst}\left(\int \sqrt{5+x^2}\,dx,x,x^2\right)\\
&= \frac{15}{8}x^2\sqrt{5+x^4}+\frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{75}{8}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}dx,x,x^2\right)\\
&= \frac{15}{8}x^2\sqrt{5+x^4}+\frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0324412, size = 56, normalized size = 0.93

$$
\frac{1}{2}\sqrt{x^4+5}\left(\frac{3x^8}{5}+\frac{x^6}{2}+6x^4+\frac{25x^2}{4}+15\right)+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(15 + (25*x^2)/4 + 6*x^4 + x^6/2 + (3*x^8)/5))/2 + (75*ArcSi
nh[x^2/Sqrt[5]])/8
```

Maple [A] time = 0.012, size = 46, normalized size = 0.8

$$
\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}+\frac{x^6}{4}\sqrt{x^4+5}+\frac{25\,x^2}{8}\sqrt{x^4+5}+\frac{75}{8}\operatorname{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 3/10*(x^4+5)^(5/2)+1/4*x^6*(x^4+5)^(1/2)+25/8*x^2*(x^4+5)^(1/2)+75/8*arcsin
h(1/5*x^2*5^(1/2))
```

Maxima [B] time = 1.4211, size = 128, normalized size = 2.13

$$
\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}+\frac{25\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{8\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}+\frac{75}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{75}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5)^(5/2) + 25/8*(3*sqrt(x^4 + 5)/x^2 - 5*(x^4 + 5)^(3/2)/x^6)/(
2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 75/16*log(sqrt(x^4 + 5)/x^2 + 1) -
 75/16*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.53592, size = 131, normalized size = 2.18

$$\frac{1}{40}\left(12\,x^{8}+10\,x^{6}+120\,x^{4}+125\,x^{2}+300\right)\sqrt{x^{4}+5}-\frac{75}{8}\,\log\left(-x^{2}+\sqrt{x^{4}+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(12*x^8 + 10*x^6 + 120*x^4 + 125*x^2 + 300)*sqrt(x^4 + 5) - 75/8*log(-
x^2 + sqrt(x^4 + 5))
```

Sympy [B] time = 7.25373, size = 109, normalized size = 1.82

$$\frac{x^{10}}{4\sqrt{x^{4}+5}}+\frac{3x^{8}\sqrt{x^{4}+5}}{10}+\frac{35x^{6}}{8\sqrt{x^{4}+5}}+\frac{x^{4}\sqrt{x^{4}+5}}{2}+\frac{125x^{2}}{8\sqrt{x^{4}+5}}+\frac{5\left(x^{4}+5\right)^{\frac{3}{2}}}{2}-5\sqrt{x^{4}+5}+\frac{75\,\operatorname{asinh}\left(\frac{\sqrt{5}x^{2}}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**10/(4*sqrt(x**4 + 5)) + 3*x**8*sqrt(x**4 + 5)/10 + 35*x**6/(8*sqrt(x**4
+ 5)) + x**4*sqrt(x**4 + 5)/2 + 125*x**2/(8*sqrt(x**4 + 5)) + 5*(x**4 + 5)*
*(3/2)/2 - 5*sqrt(x**4 + 5) + 75*asinh(sqrt(5)*x**2/5)/8
```

Giac [A] time = 1.11493, size = 70, normalized size = 1.17

$$\frac{1}{40}\,\sqrt{x^{4}+5}\left(\left(2\left(\left(6\,x^{2}+5\right)x^{2}+60\right)x^{2}+125\right)x^{2}+300\right)-\frac{75}{8}\,\log\left(-x^{2}+\sqrt{x^{4}+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/40*sqrt(x^4 + 5)*((2*((6*x^2 + 5)*x^2 + 60)*x^2 + 125)*x^2 + 300) - 75/8*
log(-x^2 + sqrt(x^4 + 5))
```

3.23 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x} dx$

Optimal. Leaf size=78

$$\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}+\frac{5}{16}\left(9x^2+16\right)\sqrt{x^4+5}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-5\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

```
[Out] (5*(16 + 9*x^2)*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 + (225
*ArcSinh[x^2/Sqrt[5]])/16 - 5*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]
```

Rubi [A] time = 0.0746282, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1252, 815, 844, 215, 266, 63, 207}

$$\frac{1}{24}\left(9x^2+8\right)\left(x^4+5\right)^{3/2}+\frac{5}{16}\left(9x^2+16\right)\sqrt{x^4+5}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-5\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
```

```
[Out] (5*(16 + 9*x^2)*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 + (225
*ArcSinh[x^2/Sqrt[5]])/16 - 5*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{1}{8}\text{Subst}\left(\int \frac{(40+45x)\sqrt{5+x^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{1}{16}\text{Subst}\left(\int \frac{400+225x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$

$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{225}{16}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right) +$$

$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{25}{2}\text{Subst}\left(\int\right.$$

$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + 25\,\text{Subst}\left(\int\right.$$

$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - 5\sqrt{5}\tanh^{-1}\left(\right.$$

Mathematica [A] time = 0.0452563, size = 67, normalized size = 0.86

$$\frac{1}{48}\left(\sqrt{x^4+5}\left(18x^6+16x^4+225x^2+320\right)+675\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-240\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
```

```
[Out] (Sqrt[5 + x^4]*(320 + 225*x^2 + 16*x^4 + 18*x^6) + 675*ArcSinh[x^2/Sqrt[5]]
 - 240*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/48
```

Maple [A] time = 0.016, size = 75, normalized size = 1.

$$\frac{3x^6}{8}\sqrt{x^4+5}+\frac{75x^2}{16}\sqrt{x^4+5}+\frac{225}{16}\text{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{x^4}{3}\sqrt{x^4+5}+\frac{20}{3}\sqrt{x^4+5}-5\sqrt{5}\text{Artanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x,x)
```

```
[Out] 3/8*x^6*(x^4+5)^(1/2)+75/16*x^2*(x^4+5)^(1/2)+225/16*arcsinh(1/5*x^2*5^(1/2
))+1/3*x^4*(x^4+5)^(1/2)+20/3*(x^4+5)^(1/2)-5*5^(1/2)*arctanh(5^(1/2)/(x^4+
5)^(1/2))
```

Maxima [B] time = 1.43695, size = 186, normalized size = 2.38

$$\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}} + \frac{5}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + 5\sqrt{x^4+5} + \frac{75\left(\frac{3\sqrt{x^4+5}}{x^2} - \frac{5\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4} - \frac{\left(x^4+5\right)^2}{x^8} - 1\right)} + \frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{225}{32}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) + 5/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) +
 sqrt(x^4 + 5))) + 5*sqrt(x^4 + 5) + 75/16*(3*sqrt(x^4 + 5)/x^2 - 5*(x^4 +
5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 225/32*log(sqrt(x^4
 + 5)/x^2 + 1) - 225/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.54698, size = 185, normalized size = 2.37

$$\frac{1}{48}\left(18\,x^6+16\,x^4+225\,x^2+320\right)\sqrt{x^4+5} + 5\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) - \frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/48*(18*x^6 + 16*x^4 + 225*x^2 + 320)*sqrt(x^4 + 5) + 5*sqrt(5)*log(-(sqrt
(5) - sqrt(x^4 + 5))/x^2) - 225/16*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A] time = 25.6382, size = 114, normalized size = 1.46

$$\frac{3x^{10}}{8\sqrt{x^4+5}} + \frac{105x^6}{16\sqrt{x^4+5}} + \frac{375x^2}{16\sqrt{x^4+5}} + \frac{\left(x^4+5\right)^{\frac{3}{2}}}{3} + 5\sqrt{x^4+5} + \frac{5\sqrt{5}\log\left(x^4\right)}{2} - 5\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right) + \frac{225\,\operatorname{asinh}\left(\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x,x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) + 105*x**6/(16*sqrt(x**4 + 5)) + 375*x**2/(16*sq
rt(x**4 + 5)) + (x**4 + 5)**(3/2)/3 + 5*sqrt(x**4 + 5) + 5*sqrt(5)*log(x**4
)/2 - 5*sqrt(5)*log(sqrt(x**4/5 + 1) + 1) + 225*asinh(sqrt(5)*x**2/5)/16
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x, x)
```

3.24 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=81

$$-\frac{(2-x^2)(x^4+5)^{3/2}}{2x^2}+\frac{3}{2}(x^2+5)\sqrt{x^4+5}+\frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{15}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

```
[Out] (3*(5 + x^2)*Sqrt[5 + x^4])/2 - ((2 - x^2)*(5 + x^4)^(3/2))/(2*x^2) + (15*A
rcSinh[x^2/Sqrt[5]])/2 - (15*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rubi [A] time = 0.0748548, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1252, 813, 815, 844, 215, 266, 63, 207}

$$-\frac{(2-x^2)(x^4+5)^{3/2}}{2x^2}+\frac{3}{2}(x^2+5)\sqrt{x^4+5}+\frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{15}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
```

```
[Out] (3*(5 + x^2)*Sqrt[5 + x^4])/2 - ((2 - x^2)*(5 + x^4)^(3/2))/(2*x^2) + (15*A
rcSinh[x^2/Sqrt[5]])/2 - (15*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 815

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
 + 1) + g*c*e*(m + 2*p + 1)*x)*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
 + 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{(-30-12x)\sqrt{5+x^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} - \frac{1}{8}\operatorname{Subst}\left(\int \frac{-300-60x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$

$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right) + \frac{75}{2}\operatorname{Subs}$$

$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{75}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\right.$$

$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{75}{2}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,d\right.$$

$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{15}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

Mathematica [C] time = 0.0568492, size = 71, normalized size = 0.88

$$\frac{1}{2}\left(\sqrt{x^4+5}\left(x^4+20\right)-15\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)\right)-\frac{5\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{x^4}{5}\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[5 + x^4]*(20 + x^4) - 15*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2 -
(5*Sqrt[5]*Hypergeometric2F1[-3/2, -1/2, 1/2, -x^4/5])/x^2
```

Maple [A] time = 0.017, size = 75, normalized size = 0.9

$$\frac{x^4}{2}\sqrt{x^4+5}+10\sqrt{x^4+5}-\frac{15\sqrt{5}}{2}\mathrm{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)+\frac{x^2}{2}\sqrt{x^4+5}+\frac{15}{2}\mathrm{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)-5\frac{\sqrt{x^4+5}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^3,x)
```

```
[Out] 1/2*x^4*(x^4+5)^(1/2)+10*(x^4+5)^(1/2)-15/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)
^(1/2))+1/2*x^2*(x^4+5)^(1/2)+15/2*arcsinh(1/5*x^2*5^(1/2))-5*(x^4+5)^(1/2)
/x^2
```

Maxima [B] time = 1.42607, size = 165, normalized size = 2.04

$$\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{15}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{15}{2}\sqrt{x^4+5}-\frac{5\sqrt{x^4+5}}{x^2}+\frac{5\sqrt{x^4+5}}{2x^2\left(\frac{x^4+5}{x^4}-1\right)}+\frac{15}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{15}{4}\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) + 15/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5)
+ sqrt(x^4 + 5))) + 15/2*sqrt(x^4 + 5) - 5*sqrt(x^4 + 5)/x^2 + 5/2*sqrt(x^4
 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 15/4*log(sqrt(x^4 + 5)/x^2 + 1) - 15/4*lo
g(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.54216, size = 200, normalized size = 2.47

$$\frac{15\sqrt{5}x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-15\,x^2\log\left(-x^2+\sqrt{x^4+5}\right)-10\,x^2+\left(x^6+x^4+20\,x^2-10\right)\sqrt{x^4+5}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(15*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15*x^2*log(-x^2 +
 sqrt(x^4 + 5)) - 10*x^2 + (x^6 + x^4 + 20*x^2 - 10)*sqrt(x^4 + 5))/x^2
```

Sympy [A] time = 9.07861, size = 114, normalized size = 1.41

$$\frac{x^{6}}{2\sqrt{x^{4}+5}}-\frac{5x^{2}}{2\sqrt{x^{4}+5}}+\frac{\left(x^{4}+5\right)^{\frac{3}{2}}}{2}+\frac{15\sqrt{x^{4}+5}}{2}+\frac{15\sqrt{5}\log\left(x^{4}\right)}{4}-\frac{15\sqrt{5}\log\left(\sqrt{\frac{x^{4}}{5}+1}+1\right)}{2}+\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}x^{2}}{5}\right)}{2}-\frac{}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**3,x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) - 5*x**2/(2*sqrt(x**4 + 5)) + (x**4 + 5)**(3/2)/2 +
 15*sqrt(x**4 + 5)/2 + 15*sqrt(5)*log(x**4)/4 - 15*sqrt(5)*log(sqrt(x**4/5
+ 1) + 1)/2 + 15*asinh(sqrt(5)*x**2/5)/2 - 25/(x**2*sqrt(x**4 + 5))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{4}+5\right)^{\frac{3}{2}}\left(3\,x^{2}+2\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^3, x)
```

3.25 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=86

$$-\frac{(2-3x^2)(x^4+5)^{3/2}}{4x^4}-\frac{3(15-2x^2)\sqrt{x^4+5}}{4x^2}+\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

```
[Out] (-3*(15 - 2*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((2 - 3*x^2)*(5 + x^4)^(3/2))/(4*
x^4) + (45*ArcSinh[x^2/Sqrt[5]])/4 - (3*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[
5]])/2
```

Rubi [A] time = 0.0773522, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1252, 813, 844, 215, 266, 63, 207}

$$-\frac{(2-3x^2)(x^4+5)^{3/2}}{4x^4}-\frac{3(15-2x^2)\sqrt{x^4+5}}{4x^2}+\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
```

```
[Out] (-3*(15 - 2*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((2 - 3*x^2)*(5 + x^4)^(3/2))/(4*
x^4) + (45*ArcSinh[x^2/Sqrt[5]])/4 - (3*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[
5]])/2
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x^3}\,dx,x,x^2\right)\\
&= -\frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} - \frac{3}{16}\,\text{Subst}\left(\int \frac{(-60-8x)\sqrt{5+x^2}}{x^2}\,dx,x,x^2\right)\\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{3}{32}\,\text{Subst}\left(\int \frac{80+120x}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{15}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right) + \frac{45}{4}\ldots\\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{4}\,\text{Subst}\left(\int \frac{1}{x\sqrt{5}\ldots}\right.\\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{2}\,\text{Subst}\left(\int \frac{1}{-5+\ldots}\right.\\
&= -\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4} + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+\ldots}}{\sqrt{5}\ldots}\right.
\end{aligned}$$

Mathematica [C] time = 0.0317762, size = 60, normalized size = 0.7

$$\frac{1}{125}\left(x^4+5\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{x^4}{5}+1\right)-\frac{15\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{x^4}{5}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
```

```
[Out] (-15*Sqrt[5]*Hypergeometric2F1[-3/2, -1/2, 1/2, -x^4/5])/(2*x^2) + ((5 + x^
4)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + x^4/5])/125
```

Maple [A] time = 0.017, size = 73, normalized size = 0.9

$$\frac{3x^2}{4}\sqrt{x^4+5}+\frac{45}{4}\mathrm{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)-\frac{15}{2x^2}\sqrt{x^4+5}+\sqrt{x^4+5}-\frac{5}{2x^4}\sqrt{x^4+5}-\frac{3\sqrt{5}}{2}\mathrm{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^5,x)
```

```
[Out] 3/4*x^2*(x^4+5)^(1/2)+45/4*arcsinh(1/5*x^2*5^(1/2))-15/2*(x^4+5)^(1/2)/x^2+
(x^4+5)^(1/2)-5/2*(x^4+5)^(1/2)/x^4-3/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/
2))
```

Maxima [A] time = 1.42842, size = 166, normalized size = 1.93

$$\frac{3}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\sqrt{x^4+5}-\frac{15\sqrt{x^4+5}}{2x^2}+\frac{15\sqrt{x^4+5}}{4x^2\left(\frac{x^4+5}{x^4}-1\right)}-\frac{5\sqrt{x^4+5}}{2x^4}+\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{45}{8}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + sqr
t(x^4 + 5) - 15/2*sqrt(x^4 + 5)/x^2 + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^
4 - 1)) - 5/2*sqrt(x^4 + 5)/x^4 + 45/8*log(sqrt(x^4 + 5)/x^2 + 1) - 45/8*lo
g(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.55938, size = 204, normalized size = 2.37

$$\frac{6\sqrt{5}x^4\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-45x^4\log\left(-x^2+\sqrt{x^4+5}\right)-30x^4+\left(3x^6+4x^4-30x^2-10\right)\sqrt{x^4+5}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(6*sqrt(5)*x^4*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 45*x^4*log(-x^2 +
sqrt(x^4 + 5)) - 30*x^4 + (3*x^6 + 4*x^4 - 30*x^2 - 10)*sqrt(x^4 + 5))/x^4
```

Sympy [A] time = 10.5165, size = 133, normalized size = 1.55

$$\frac{3x^6}{4\sqrt{x^4+5}}-\frac{15x^2}{4\sqrt{x^4+5}}+\sqrt{x^4+5}+\frac{\sqrt{5}\log\left(x^4\right)}{2}-\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)-\frac{\sqrt{5}\,\mathrm{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{2}+\frac{45\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}-\frac{5\sqrt{1+}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**5,x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) - 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) + s
qrt(5)*log(x**4)/2 - sqrt(5)*log(sqrt(x**4/5 + 1) + 1) - sqrt(5)*asinh(sqrt
(5)/x**2)/2 + 45*asinh(sqrt(5)*x**2/5)/4 - 5*sqrt(1 + 5/x**4)/(2*x**2) - 75
/(2*x**2*sqrt(x**4 + 5))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^5, x)
```

3.26 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7} dx$

Optimal. Leaf size=82

$$-\frac{(9x^2+4)(x^4+5)^{3/2}}{12x^6}-\frac{(4-9x^2)\sqrt{x^4+5}}{4x^2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{9}{4}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

```
[Out] -((4 - 9*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((4 + 9*x^2)*(5 + x^4)^(3/2))/(12*x^
6) + ArcSinh[x^2/Sqrt[5]] - (9*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/4
```

Rubi [A] time = 0.0749648, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {1252, 811, 813, 844, 215, 266, 63, 207}

$$-\frac{(9x^2+4)(x^4+5)^{3/2}}{12x^6}-\frac{(4-9x^2)\sqrt{x^4+5}}{4x^2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{9}{4}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

```
[Out] -((4 - 9*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((4 + 9*x^2)*(5 + x^4)^(3/2))/(12*x^
6) + ArcSinh[x^2/Sqrt[5]] - (9*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/4
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 811

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p*((d*g - e*f*(m + 2
))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) +
 2*c*d*p*(e*f - d*g))*x))/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), x] - Dist[
p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2)^
(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*(m
 + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0]
 &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 813

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1
) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} - \frac{1}{40}\operatorname{Subst}\left(\int \frac{(-40-90x)\sqrt{5+x^2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \frac{1}{80}\operatorname{Subst}\left(\int \frac{900+80x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \frac{45}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right) + \text{Sub}$$

$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{45}{8}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,d\right.$$

$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{45}{4}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx\right.$$

$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{9}{4}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

Mathematica [C] time = 0.0310294, size = 60, normalized size = 0.73

$$\frac{3}{250}\left(x^4+5\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{x^4}{5}+1\right)-\frac{5\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{x^4}{5}\right)}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

```
[Out] (-5*Sqrt[5]*Hypergeometric2F1[-3/2, -3/2, -1/2, -x^4/5])/(3*x^6) + (3*(5 +
x^4)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + x^4/5])/250
```

Maple [A] time = 0.019, size = 73, normalized size = 0.9

$$\frac{3}{2}\sqrt{x^4+5}-\frac{15}{4x^4}\sqrt{x^4+5}-\frac{9\sqrt{5}}{4}\mathrm{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)+\mathrm{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)-\frac{4}{3x^2}\sqrt{x^4+5}-\frac{5}{3x^6}\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^7,x)
```

```
[Out] 3/2*(x^4+5)^(1/2)-15/4*(x^4+5)^(1/2)/x^4-9/4*5^(1/2)*arctanh(5^(1/2)/(x^4+5
)^(1/2))+arcsinh(1/5*x^2*5^(1/2))-4/3*(x^4+5)^(1/2)/x^2-5/3*(x^4+5)^(1/2)/x
^6
```

Maxima [A] time = 1.41917, size = 151, normalized size = 1.84

$$\frac{9}{8}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}-\frac{\sqrt{x^4+5}}{x^2}-\frac{15\sqrt{x^4+5}}{4x^4}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{3x^6}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/2
*sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 - 15/4*sqrt(x^4 + 5)/x^4 - 1/3*(x^4 + 5)
^(3/2)/x^6 + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1
)
```

Fricas [A] time = 1.65575, size = 209, normalized size = 2.55

$$\frac{27\sqrt{5}x^6\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-12x^6\log\left(-x^2+\sqrt{x^4+5}\right)-16x^6+\left(18x^6-16x^4-45x^2-20\right)\sqrt{x^4+5}}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(27*sqrt(5)*x^6*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 12*x^6*log(-x^2
+ sqrt(x^4 + 5)) - 16*x^6 + (18*x^6 - 16*x^4 - 45*x^2 - 20)*sqrt(x^4 + 5))/
x^6
```

Sympy [A] time = 10.7666, size = 148, normalized size = 1.8

$$-\frac{x^2}{\sqrt{x^4+5}} - \frac{\sqrt{1+\frac{5}{x^4}}}{3} + \frac{3\sqrt{x^4+5}}{2} + \frac{3\sqrt{5}\log\left(x^4\right)}{4} - \frac{3\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2} - \frac{3\sqrt{5}\,\mathrm{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{4} + \mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right) - \frac{15}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**7,x)
```

```
[Out] -x**2/sqrt(x**4 + 5) - sqrt(1 + 5/x**4)/3 + 3*sqrt(x**4 + 5)/2 + 3*sqrt(5)*
log(x**4)/4 - 3*sqrt(5)*log(sqrt(x**4/5 + 1) + 1)/2 - 3*sqrt(5)*asinh(sqrt(
5)/x**2)/4 + asinh(sqrt(5)*x**2/5) - 15*sqrt(1 + 5/x**4)/(4*x**2) - 5/(x**2
*sqrt(x**4 + 5)) - 5*sqrt(1 + 5/x**4)/(3*x**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^7, x)
```

3.27 $\int x^4 \left(2+3x^2\right) \left(5+x^4\right)^{3/2} dx$

Optimal. Leaf size=235

$$-\frac{50\sqrt[4]{5}\left(231+26\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{1001\sqrt{x^4+5}}+\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5+\frac{10\left(77x^2+7\right.}{10}$$

```
[Out] (200*x*Sqrt[5 + x^4])/77 + (20*x^3*Sqrt[5 + x^4])/13 - (300*x*Sqrt[5 + x^4]
)/(13*(Sqrt[5] + x^2)) + (10*x^5*(78 + 77*x^2)*Sqrt[5 + x^4])/1001 + (x^5*(
26 + 33*x^2)*(5 + x^4)^(3/2))/143 + (300*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(13*Sqrt[5 + x
^4]) - (50*5^(1/4)*(231 + 26*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[
5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(1001*Sqrt[5 + x^4])
```

Rubi [A] time = 0.134444, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1274, 1280, 1198, 220, 1196}

$$\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5+\frac{10\left(77x^2+78\right)\sqrt{x^4+5}x^5}{1001}+\frac{20}{13}\sqrt{x^4+5}x^3-\frac{300\sqrt{x^4+5}x}{13\left(x^2+\sqrt{5}\right)}+\frac{200}{77}\sqrt{x^4+5}x-\frac{50\sqrt[4]{5}\left(23\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (200*x*Sqrt[5 + x^4])/77 + (20*x^3*Sqrt[5 + x^4])/13 - (300*x*Sqrt[5 + x^4]
)/(13*(Sqrt[5] + x^2)) + (10*x^5*(78 + 77*x^2)*Sqrt[5 + x^4])/1001 + (x^5*(
26 + 33*x^2)*(5 + x^4)^(3/2))/143 + (300*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(13*Sqrt[5 + x
^4]) - (50*5^(1/4)*(231 + 26*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[
5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(1001*Sqrt[5 + x^4])
```

Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x^4\left(2+3x^2\right)\left(5+x^4\right)^{3/2}dx=\frac{1}{143}x^5\left(26+33x^2\right)\left(5+x^4\right)^{3/2}+\frac{30}{143}\int x^4\left(26+33x^2\right)\sqrt{5+x^4}\,dx$$

$$=\frac{10x^5\left(78+77x^2\right)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5\left(26+33x^2\right)\left(5+x^4\right)^{3/2}+\frac{100\int\frac{x^4\left(234+231x^2\right)}{\sqrt{5+x^4}}dx}{3003}$$

$$=\frac{20}{13}x^3\sqrt{5+x^4}+\frac{10x^5\left(78+77x^2\right)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5\left(26+33x^2\right)\left(5+x^4\right)^{3/2}-\frac{20\int\frac{x^2}{\cdots}}{\cdots}$$

$$=\frac{200}{77}x\sqrt{5+x^4}+\frac{20}{13}x^3\sqrt{5+x^4}+\frac{10x^5\left(78+77x^2\right)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5\left(26+33x^2\right)\left(5+\cdots\right.$$

$$=\frac{200}{77}x\sqrt{5+x^4}+\frac{20}{13}x^3\sqrt{5+x^4}+\frac{10x^5\left(78+77x^2\right)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5\left(26+33x^2\right)\left(5+\cdots\right.$$

$$=\frac{200}{77}x\sqrt{5+x^4}+\frac{20}{13}x^3\sqrt{5+x^4}-\frac{300x\sqrt{5+x^4}}{13\left(\sqrt{5}+x^2\right)}+\frac{10x^5\left(78+77x^2\right)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5\cdots$$

Mathematica [C] time = 0.0458187, size = 74, normalized size = 0.31

$$\frac{1}{143}x\left(-650\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)-825\sqrt{5}x^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)+\left(33x^2+26\right)\left(x^4+5\right)^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (x*((26 + 33*x^2)*(5 + x^4)^(5/2) - 650*Sqrt[5]*Hypergeometric2F1[-3/2, 1/4
, 5/4, -x^4/5] - 825*Sqrt[5]*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, -x^4/5])
)/143
```

Maple [C] time = 0.023, size = 216, normalized size = 0.9

$$\frac{3x^{11}\sqrt{x^4+5}}{13}+\frac{25x^7\sqrt{x^4+5}}{13}+\frac{20x^3\sqrt{x^4+5}}{13}-\frac{\frac{60i}{13}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\text{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 3/13*x^11*(x^4+5)^(1/2)+25/13*x^7*(x^4+5)^(1/2)+20/13*x^3*(x^4+5)^(1/2)-60/
13*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2
)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5
*x*5^(1/2)*(I*5^(1/2))^(1/2),I))+2/11*x^9*(x^4+5)^(1/2)+130/77*x^5*(x^4+5)^
(1/2)+200/77*x*(x^4+5)^(1/2)-40/77*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2
)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/
2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^4+5\right)^{\frac{3}{2}}\left(3x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(3x^{10}+2x^8+15x^6+10x^4\right)\sqrt{x^4+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^10 + 2*x^8 + 15*x^6 + 10*x^4)*sqrt(x^4 + 5), x)
```

Sympy [C] time = 4.01917, size = 160, normalized size = 0.68

$$\frac{3\sqrt{5}x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{15}{4}\right)}+\frac{\sqrt{5}x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{13}{4}\right)}+\frac{15\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{5\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\\\frac{9}{4}\end{matrix}\right.}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), x**4*exp_polar(I*p
i)/5)/(4*gamma(15/4)) + sqrt(5)*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,),
 x**4*exp_polar(I*pi)/5)/(2*gamma(13/4)) + 15*sqrt(5)*x**7*gamma(7/4)*hyper
((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(11/4)) + 5*sqrt(5)*
x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)/(2*gamma
(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+5\right)^{\frac{3}{2}} \left(3x^2+2\right) x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4, x)
```

3.28 $\int x^2 \left(2+3x^2\right) \left(5+x^4\right)^{3/2} \, dx$

Optimal. Leaf size=219

$$\frac{10\sqrt[4]{5}\left(154-45\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{231\sqrt{x^4+5}}+\frac{1}{99}\left(27x^2+22\right)\left(x^4+5\right)^{3/2}x^3+\frac{2}{231}\left(135x^2+15\right.$$

```
[Out] (300*x*Sqrt[5 + x^4])/77 + (40*x*Sqrt[5 + x^4])/(3*(Sqrt[5] + x^2)) + (2*x^
3*(154 + 135*x^2)*Sqrt[5 + x^4])/231 + (x^3*(22 + 27*x^2)*(5 + x^4)^(3/2))/
99 - (40*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
E[2*ArcTan[x/5^(1/4)], 1/2])/(3*Sqrt[5 + x^4]) + (10*5^(1/4)*(154 - 45*Sqrt
[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x
/5^(1/4)], 1/2])/(231*Sqrt[5 + x^4])
```

Rubi [A] time = 0.119878, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1274, 1280, 1198, 220, 1196}

$$\frac{1}{99}\left(27x^2+22\right)\left(x^4+5\right)^{3/2}x^3+\frac{2}{231}\left(135x^2+154\right)\sqrt{x^4+5}x^3+\frac{40\sqrt{x^4+5}x}{3\left(x^2+\sqrt{5}\right)}+\frac{300}{77}\sqrt{x^4+5}x+\frac{10\sqrt[4]{5}\left(154-45\sqrt{5}\right)\left(x^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```


```
[Out] (300*x*Sqrt[5 + x^4])/77 + (40*x*Sqrt[5 + x^4])/(3*(Sqrt[5] + x^2)) + (2*x^
3*(154 + 135*x^2)*Sqrt[5 + x^4])/231 + (x^3*(22 + 27*x^2)*(5 + x^4)^(3/2))/
99 - (40*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
E[2*ArcTan[x/5^(1/4)], 1/2])/(3*Sqrt[5 + x^4]) + (10*5^(1/4)*(154 - 45*Sqrt
[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x
/5^(1/4)], 1/2])/(231*Sqrt[5 + x^4])
```


Rule 1274

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*p + 3)), x] + Dist[(4*a*p)/((4*p
+ m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int x^2\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx=\frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}+\frac{10}{33}\int x^2\left(22+27x^2\right)\sqrt{5+x^4}\,dx$$

$$=\frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}+\frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}+\frac{20}{231}\int\frac{x^2\left(154+135\right.}{\sqrt{5+x^4}}$$

$$=\frac{300}{77}x\sqrt{5+x^4}+\frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}+\frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}-\frac{20}{693}$$

$$=\frac{300}{77}x\sqrt{5+x^4}+\frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}+\frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}-\frac{1}{3}\left(4\right.$$

$$=\frac{300}{77}x\sqrt{5+x^4}+\frac{40x\sqrt{5+x^4}}{3\left(\sqrt{5}+x^2\right)}+\frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}+\frac{1}{99}x^3\left(22+27x^2\right)\left(5\right.$$

Mathematica [C] time = 0.0314883, size = 68, normalized size = 0.31

$$\frac{1}{33}x\left(-225\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)+110\sqrt{5}x^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)+9\left(x^4+5\right)^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (x*(9*(5 + x^4)^(5/2) - 225*Sqrt[5]*Hypergeometric2F1[-3/2, 1/4, 5/4, -x^4/
5] + 110*Sqrt[5]*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, -x^4/5]))/33
```

Maple [C] time = 0.011, size = 204, normalized size = 0.9

$$\frac{3x^9}{11}\sqrt{x^4+5}+\frac{195x^5}{77}\sqrt{x^4+5}+\frac{300x}{77}\sqrt{x^4+5}-\frac{60\sqrt{5}}{77\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\text{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 3/11*x^9*(x^4+5)^(1/2)+195/77*x^5*(x^4+5)^(1/2)+300/77*x*(x^4+5)^(1/2)-60/7
7*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)
^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)+2/9*x^7*(
x^4+5)^(1/2)+22/9*x^3*(x^4+5)^(1/2)+8/3*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)
*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/
2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(3\,x^8+2\,x^6+15\,x^4+10\,x^2\right)\sqrt{x^4+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^8 + 2*x^6 + 15*x^4 + 10*x^2)*sqrt(x^4 + 5), x)
```

Sympy [C] time = 3.30631, size = 160, normalized size = 0.73

$$\frac{3\sqrt{5}x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{11}{4}\right)}+\frac{15\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{5\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(13/4)) + sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x*
*4*exp_polar(I*pi)/5)/(2*gamma(11/4)) + 15*sqrt(5)*x**5*gamma(5/4)*hyper((-
1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(9/4)) + 5*sqrt(5)*x**3*
gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(7/4)
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2, x)
```

3.29 $\int \left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=197

$$\frac{10\sqrt[4]{5}\left(7+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{7\sqrt{x^4+5}}+\frac{1}{21}x\left(7x^2+6\right)\left(x^4+5\right)^{3/2}+\frac{2}{7}x\left(7x^2+10\right)\sqrt{x^4+5}$$

```
[Out] (20*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (2*x*(10 + 7*x^2)*Sqrt[5 + x^4])/7 +
 (x*(6 + 7*x^2)*(5 + x^4)^(3/2))/21 - (20*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
 x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]
 + (10*5^(1/4)*(7 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^
2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rubi [A] time = 0.0817294, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1177, 1198, 220, 1196}

$$\frac{1}{21}x\left(7x^2+6\right)\left(x^4+5\right)^{3/2}+\frac{2}{7}x\left(7x^2+10\right)\sqrt{x^4+5}+\frac{20x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{10\sqrt[4]{5}\left(7+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right.\right.}{7\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (20*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (2*x*(10 + 7*x^2)*Sqrt[5 + x^4])/7 +
 (x*(6 + 7*x^2)*(5 + x^4)^(3/2))/21 - (20*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
 x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]
 + (10*5^(1/4)*(7 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^
2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx = \frac{1}{21}x\left(6+7x^2\right)\left(5+x^4\right)^{3/2}+\frac{1}{21}\int \left(180+210x^2\right)\sqrt{5+x^4}\,dx$$
$$= \frac{2}{7}x\left(10+7x^2\right)\sqrt{5+x^4}+\frac{1}{21}x\left(6+7x^2\right)\left(5+x^4\right)^{3/2}+\frac{1}{315}\int \frac{9000+6300x^2}{\sqrt{5+x^4}}\,dx$$
$$= \frac{2}{7}x\left(10+7x^2\right)\sqrt{5+x^4}+\frac{1}{21}x\left(6+7x^2\right)\left(5+x^4\right)^{3/2}-\left(20\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx+\frac{1}{7}\left(20\left(1\right.\right.$$
$$= \frac{20x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{2}{7}x\left(10+7x^2\right)\sqrt{5+x^4}+\frac{1}{21}x\left(6+7x^2\right)\left(5+x^4\right)^{3/2}-\frac{20\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{}}{}$$

Mathematica [C] time = 0.0129397, size = 49, normalized size = 0.25

$$5\sqrt{5}x\left(x^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)+2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] 5*Sqrt[5]*x*(2*Hypergeometric2F1[-3/2, 1/4, 5/4, -x^4/5] + x^2*Hypergeometr
ic2F1[-3/2, 3/4, 7/4, -x^4/5])
```

Maple [C] time = 0.013, size = 192, normalized size = 1.

$$\frac{x^7}{3}\sqrt{x^4+5}+\frac{11\,x^3}{3}\sqrt{x^4+5}+\frac{4\,i}{\sqrt{i\sqrt{5}}}\sqrt{25-5\,i\sqrt{5}x^2}\sqrt{25+5\,i\sqrt{5}x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\text{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{}}}{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2),x)
```

```
[Out] 1/3*x^7*(x^4+5)^(1/2)+11/3*x^3*(x^4+5)^(1/2)+4*I/(I*5^(1/2))^(1/2)*(25-5*I*
5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*
x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
)+2/7*x^5*(x^4+5)^(1/2)+30/7*x*(x^4+5)^(1/2)+8/7*5^(1/2)/(I*5^(1/2))^(1/2)*
(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*Ellipti
cF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(3\,x^6+2\,x^4+15\,x^2+10\right)\sqrt{x^4+5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5), x)
```

Sympy [C] time = 2.92852, size = 158, normalized size = 0.8

$$\frac{3\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{9}{4}\right)}+\frac{15\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{5\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**
4*exp_polar(I*pi)/5)/(2*gamma(9/4)) + 15*sqrt(5)*x**3*gamma(3/4)*hyper((-1/
2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(7/4)) + 5*sqrt(5)*x*gamma
(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

3.30 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=199

$$\frac{6\sqrt[4]{5}\left(14+5\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{7\sqrt{x^4+5}}-\frac{\left(14-3x^2\right)\left(x^4+5\right)^{3/2}}{7x}+\frac{6}{35}x\left(14x^2+25\right)\sqrt{x^4+}$$

```
[Out] (24*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (6*x*(25 + 14*x^2)*Sqrt[5 + x^4])/35
 - ((14 - 3*x^2)*(5 + x^4)^(3/2))/(7*x) - (24*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[
(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 +
x^4] + (6*5^(1/4)*(14 + 5*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5]
+ x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rubi [A] time = 0.0883871, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1272, 1177, 1198, 220, 1196}

$$-\frac{\left(14-3x^2\right)\left(x^4+5\right)^{3/2}}{7x}+\frac{6}{35}x\left(14x^2+25\right)\sqrt{x^4+5}+\frac{24x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{6\sqrt[4]{5}\left(14+5\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\right.}{7\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^2,x]
```

```
[Out] (24*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (6*x*(25 + 14*x^2)*Sqrt[5 + x^4])/35
 - ((14 - 3*x^2)*(5 + x^4)^(3/2))/(7*x) - (24*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[
(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 +
x^4] + (6*5^(1/4)*(14 + 5*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5]
+ x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1177

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*
(d*(4*p + 3) + e*(4*p + 1)*x^2)*(a + c*x^4)^p)/((4*p + 1)*(4*p + 3)), x] +
Dist[(2*p)/((4*p + 1)*(4*p + 3)), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(5+x^4\right)^{3/2}}{x^2}\,dx = -\frac{\left(14-3x^2\right)\left(5+x^4\right)^{3/2}}{7x} - \frac{6}{7}\int \left(-15-14x^2\right)\sqrt{5+x^4}\,dx$$

$$= \frac{6}{35}x\left(25+14x^2\right)\sqrt{5+x^4} - \frac{\left(14-3x^2\right)\left(5+x^4\right)^{3/2}}{7x} - \frac{2}{35}\int \frac{-750-420x^2}{\sqrt{5+x^4}}\,dx$$

$$= \frac{6}{35}x\left(25+14x^2\right)\sqrt{5+x^4} - \frac{\left(14-3x^2\right)\left(5+x^4\right)^{3/2}}{7x} - \left(24\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx + \frac{1}{7}\left(12\left(25+1\right.\right.$$

$$= \frac{24x\sqrt{5+x^4}}{\sqrt{5}+x^2} + \frac{6}{35}x\left(25+14x^2\right)\sqrt{5+x^4} - \frac{\left(14-3x^2\right)\left(5+x^4\right)^{3/2}}{7x} - \frac{24\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+}{(\sqrt{5}}}}{\sqrt{5}}$$

Mathematica [C] time = 0.0279748, size = 53, normalized size = 0.27

$$15\sqrt{5}x\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right) - \frac{10\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^2,x]
```

```
[Out] (-10*Sqrt[5]*Hypergeometric2F1[-3/2, -1/4, 3/4, -x^4/5])/x + 15*Sqrt[5]*x*H
ypergeometric2F1[-3/2, 1/4, 5/4, -x^4/5]
```

Maple [C] time = 0.015, size = 192, normalized size = 1.

$$\frac{3x^5}{7}\sqrt{x^4+5} + \frac{45x}{7}\sqrt{x^4+5} + \frac{12\sqrt{5}}{7\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}} - 10\frac{\sqrt{x^4+5}}{x} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^2,x)
```

```
[Out] 3/7*x^5*(x^4+5)^(1/2)+45/7*x*(x^4+5)^(1/2)+12/7*5^(1/2)/(I*5^(1/2))^(1/2)*(
25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*Elliptic
F(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-10*(x^4+5)^(1/2)/x+2/5*x^3*(x^4+5)^(1/
2)+24/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)
^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-Elliptic
E(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3\,x^6+2\,x^4+15\,x^2+10\right)\sqrt{x^4+5}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5)/x^2, x)
```

Sympy [C] time = 3.37316, size = 160, normalized size = 0.8

$$\frac{3\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{7}{4}\right)}+\frac{15\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{5\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\\\frac{3}{4}\end{matrix}\right.}{2x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**2,x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*
exp_polar(I*pi)/5)/(2*gamma(7/4)) + 15*sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/
4), (5/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(5/4)) + 5*sqrt(5)*gamma(-1/4)*
hyper((-1/2, -1/4), (3/4,), x**4*exp_polar(I*pi)/5)/(2*x*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2, x)
```

3.31 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=201

$$\frac{2\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{3\sqrt{x^4+5}}-\frac{\left(10-9x^2\right)\left(x^4+5\right)^{3/2}}{15x^3}-\frac{2\left(27-2x^2\right)\sqrt{x^4+5}}{3x}+$$

```
[Out] (-2*(27 - 2*x^2)*Sqrt[5 + x^4])/(3*x) + (36*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2
) - ((10 - 9*x^2)*(5 + x^4)^(3/2))/(15*x^3) - (36*5^(1/4)*(Sqrt[5] + x^2)*S
qrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[
5 + x^4] + (2*5^(1/4)*(27 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt
[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(3*Sqrt[5 + x^4])
```

Rubi [A] time = 0.0859568, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1272, 1198, 220, 1196}

$$-\frac{\left(10-9x^2\right)\left(x^4+5\right)^{3/2}}{15x^3}-\frac{2\left(27-2x^2\right)\sqrt{x^4+5}}{3x}+\frac{36x\sqrt{x^4+5}}{x^2+\sqrt{5}}+\frac{2\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{}}\right.\right.}{3\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^4,x]
```

```
[Out] (-2*(27 - 2*x^2)*Sqrt[5 + x^4])/(3*x) + (36*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2
) - ((10 - 9*x^2)*(5 + x^4)^(3/2))/(15*x^3) - (36*5^(1/4)*(Sqrt[5] + x^2)*S
qrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[
5 + x^4] + (2*5^(1/4)*(27 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt
[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(3*Sqrt[5 + x^4])
```

Rule 1272

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[((f*x)^(m + 1)*(a + c*x^4)^p*(d*(m + 4*p + 3) + e*(m + 1)*
x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(4*p)/(f^2*(m + 1)*(m + 4*p + 3)
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx = -\frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} - \frac{2}{5}\int \frac{(-45-10x^2)\sqrt{5+x^4}}{x^2}\,dx$$
$$= -\frac{2(27-2x^2)\sqrt{5+x^4}}{3x} - \frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} + \frac{4}{15}\int \frac{50+135x^2}{\sqrt{5+x^4}}\,dx$$
$$= -\frac{2(27-2x^2)\sqrt{5+x^4}}{3x} - \frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} - (36\sqrt{5})\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx + \frac{1}{3}\left(4\left(10+27\sqrt{\right.\right.$$
$$= -\frac{2(27-2x^2)\sqrt{5+x^4}}{3x} + \frac{36x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} - \frac{36\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2}}}{\sqrt{5}+}$$

Mathematica [C] time = 0.0268571, size = 54, normalized size = 0.27

$$-\frac{5\sqrt{5}\left(9x^2\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{x^4}{5}\right)+2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{x^4}{5}\right)\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^4,x]
```

```
[Out] (-5*Sqrt[5]*(2*Hypergeometric2F1[-3/2, -3/4, 1/4, -x^4/5] + 9*x^2*Hypergeom
etric2F1[-3/2, -1/4, 3/4, -x^4/5]))/(3*x^3)
```

Maple [C] time = 0.016, size = 192, normalized size = 1.

$$-\frac{10}{3x^3}\sqrt{x^4+5}+\frac{2x}{3}\sqrt{x^4+5}+\frac{8\sqrt{5}}{15\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\mathrm{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}-15\frac{\sqrt{x^4+5}}{x}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^4,x)
```

```
[Out] -10/3*(x^4+5)^(1/2)/x^3+2/3*x*(x^4+5)^(1/2)+8/15*5^(1/2)/(I*5^(1/2))^(1/2)*
(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*Ellipti
cF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-15*(x^4+5)^(1/2)/x+3/5*x^3*(x^4+5)^(1
/2)+36/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2
)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-Ellipti
cE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+5)^{\frac{3}{2}}(3x^2+2)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(3x^6+2x^4+15x^2+10)\sqrt{x^4+5}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5)/x^4, x)
```

Sympy [C] time = 3.77657, size = 163, normalized size = 0.81

$$\frac{3\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)}+\frac{15\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4x\Gamma\left(\frac{3}{4}\right)}+\frac{5\sqrt{5}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**4,x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp
_polar(I*pi)/5)/(2*gamma(5/4)) + 15*sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4),
 (3/4,), x**4*exp_polar(I*pi)/5)/(4*x*gamma(3/4)) + 5*sqrt(5)*gamma(-3/4)*h
yper((-3/4, -1/2), (1/4,), x**4*exp_polar(I*pi)/5)/(2*x**3*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+5)^{\frac{3}{2}}(3x^2+2)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4, x)
```

3.32 $\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{3}{8}\sqrt{x^4+5}x^6+\frac{1}{3}\sqrt{x^4+5}x^4-\frac{5}{48}\left(27x^2+32\right)\sqrt{x^4+5}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] (x^4*Sqrt[5 + x^4])/3 + (3*x^6*Sqrt[5 + x^4])/8 - (5*(32 + 27*x^2)*Sqrt[5 +
 x^4])/48 + (225*ArcSinh[x^2/Sqrt[5]])/16
```

Rubi [A] time = 0.0575545, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1252, 833, 780, 215}

$$\frac{3}{8}\sqrt{x^4+5}x^6+\frac{1}{3}\sqrt{x^4+5}x^4-\frac{5}{48}\left(27x^2+32\right)\sqrt{x^4+5}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x^4*Sqrt[5 + x^4])/3 + (3*x^6*Sqrt[5 + x^4])/8 - (5*(32 + 27*x^2)*Sqrt[5 +
 x^4])/48 + (225*ArcSinh[x^2/Sqrt[5]])/16
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3(2+3x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= \frac{3}{8}x^6\sqrt{5+x^4} + \frac{1}{8}\,\text{Subst}\left(\int \frac{x^2(-45+8x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{3}x^4\sqrt{5+x^4} + \frac{3}{8}x^6\sqrt{5+x^4} + \frac{1}{24}\,\text{Subst}\left(\int \frac{(-80-135x)x}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{3}x^4\sqrt{5+x^4} + \frac{3}{8}x^6\sqrt{5+x^4} - \frac{5}{48}\left(32+27x^2\right)\sqrt{5+x^4} + \frac{225}{16}\,\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{3}x^4\sqrt{5+x^4} + \frac{3}{8}x^6\sqrt{5+x^4} - \frac{5}{48}\left(32+27x^2\right)\sqrt{5+x^4} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.0348459, size = 44, normalized size = 0.66

$$\frac{1}{48}\left(\sqrt{x^4+5}\left(18x^6+16x^4-135x^2-160\right)+675\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(-160 - 135*x^2 + 16*x^4 + 18*x^6) + 675*ArcSinh[x^2/Sqrt[5]
])/48
```

Maple [A] time = 0.017, size = 51, normalized size = 0.8

$$\frac{3\,x^6}{8}\sqrt{x^4+5}-\frac{45\,x^2}{16}\sqrt{x^4+5}+\frac{225}{16}\text{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{x^4-10}{3}\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 3/8*x^6*(x^4+5)^(1/2)-45/16*x^2*(x^4+5)^(1/2)+225/16*arcsinh(1/5*x^2*5^(1/2
))+1/3*(x^4+5)^(1/2)*(x^4-10)
```

Maxima [A] time = 1.45685, size = 140, normalized size = 2.09

$$\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}-5\sqrt{x^4+5}-\frac{75\left(\frac{5\sqrt{x^4+5}}{x^2}-\frac{3\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}+\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) - 5*sqrt(x^4 + 5) - 75/16*(5*sqrt(x^4 + 5)/x^2 - 3*(x^4
 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 225/32*log(sqrt(
```

```
x^4 + 5)/x^2 + 1) - 225/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.54282, size = 120, normalized size = 1.79

$$\frac{1}{48}\left(18\,x^{6}+16\,x^{4}-135\,x^{2}-160\right)\sqrt{x^{4}+5}-\frac{225}{16}\,\log\left(-x^{2}+\sqrt{x^{4}+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(18*x^6 + 16*x^4 - 135*x^2 - 160)*sqrt(x^4 + 5) - 225/16*log(-x^2 + sq
rt(x^4 + 5))
```

Sympy [A] time = 6.5133, size = 85, normalized size = 1.27

$$\frac{3x^{10}}{8\sqrt{x^{4}+5}}-\frac{15x^{6}}{16\sqrt{x^{4}+5}}+\frac{x^{4}\sqrt{x^{4}+5}}{3}-\frac{225x^{2}}{16\sqrt{x^{4}+5}}-\frac{10\sqrt{x^{4}+5}}{3}+\frac{225\operatorname{asinh}\left(\frac{\sqrt{5}x^{2}}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) - 15*x**6/(16*sqrt(x**4 + 5)) + x**4*sqrt(x**4 +
 5)/3 - 225*x**2/(16*sqrt(x**4 + 5)) - 10*sqrt(x**4 + 5)/3 + 225*asinh(sqrt
(5)*x**2/5)/16
```

Giac [A] time = 1.13552, size = 62, normalized size = 0.93

$$\frac{1}{48}\,\sqrt{x^{4}+5}\left(\left(2\left(9\,x^{2}+8\right)x^{2}-135\right)x^{2}-160\right)-\frac{225}{16}\,\log\left(-x^{2}+\sqrt{x^{4}+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 - 135)*x^2 - 160) - 225/16*log(-x^2
+ sqrt(x^4 + 5))
```

3.33 $\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=51

$$\frac{1}{2}\sqrt{x^4+5}x^4-\frac{1}{2}\left(10-x^2\right)\sqrt{x^4+5}-\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] (x^4*Sqrt[5 + x^4])/2 - ((10 - x^2)*Sqrt[5 + x^4])/2 - (5*ArcSinh[x^2/Sqrt[
5]])/2
```

Rubi [A] time = 0.0417361, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1252, 833, 780, 215}

$$\frac{1}{2}\sqrt{x^4+5}x^4-\frac{1}{2}\left(10-x^2\right)\sqrt{x^4+5}-\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x^4*Sqrt[5 + x^4])/2 - ((10 - x^2)*Sqrt[5 + x^4])/2 - (5*ArcSinh[x^2/Sqrt[
5]])/2
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^5\left(2+3x^2\right)}{\sqrt{5+x^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(2+3x)}{\sqrt{5+x^2}}\,dx,x,x^2\right)$$
$$= \frac{1}{2}x^4\sqrt{5+x^4}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{x(-30+6x)}{\sqrt{5+x^2}}\,dx,x,x^2\right)$$
$$= \frac{1}{2}x^4\sqrt{5+x^4}-\frac{1}{2}\left(10-x^2\right)\sqrt{5+x^4}-\frac{5}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)$$
$$= \frac{1}{2}x^4\sqrt{5+x^4}-\frac{1}{2}\left(10-x^2\right)\sqrt{5+x^4}-\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Mathematica [A] time = 0.0231208, size = 35, normalized size = 0.69

$$\frac{1}{2}\left(\sqrt{x^4+5}\left(x^4+x^2-10\right)-5\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(-10 + x^2 + x^4) - 5*ArcSinh[x^2/Sqrt[5]])/2
```

Maple [A] time = 0.011, size = 39, normalized size = 0.8

$$\frac{x^4-10}{2}\sqrt{x^4+5}+\frac{x^2}{2}\sqrt{x^4+5}-\frac{5}{2}\operatorname{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 1/2*(x^4+5)^(1/2)*(x^4-10)+1/2*x^2*(x^4+5)^(1/2)-5/2*arcsinh(1/5*x^2*5^(1/2
))
```

Maxima [B] time = 1.46399, size = 103, normalized size = 2.02

$$\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}}-\frac{15}{2}\sqrt{x^4+5}+\frac{5\sqrt{x^4+5}}{2x^2\left(\frac{x^4+5}{x^4}-1\right)}-\frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) - 15/2*sqrt(x^4 + 5) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5
)/x^4 - 1)) - 5/4*log(sqrt(x^4 + 5)/x^2 + 1) + 5/4*log(sqrt(x^4 + 5)/x^2 -
1)
```

Fricas [A] time = 1.51328, size = 92, normalized size = 1.8

$$\frac{1}{2}\left(x^4+x^2-10\right)\sqrt{x^4+5}+\frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x^4 + x^2 - 10)*sqrt(x^4 + 5) + 5/2*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A] time = 4.78655, size = 66, normalized size = 1.29

$$\frac{x^6}{2\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{2}+\frac{5x^2}{2\sqrt{x^4+5}}-5\sqrt{x^4+5}-\frac{5\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) + x**4*sqrt(x**4 + 5)/2 + 5*x**2/(2*sqrt(x**4 + 5))
 - 5*sqrt(x**4 + 5) - 5*asinh(sqrt(5)*x**2/5)/2
```

Giac [A] time = 1.11976, size = 50, normalized size = 0.98

$$\frac{1}{2}\sqrt{x^4+5}\left(\left(x^2+1\right)x^2-10\right)+\frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 - 10) + 5/2*log(-x^2 + sqrt(x^4 + 5))
```

3.34 $\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}} \, dx$

Optimal. Leaf size=35

$$\frac{1}{4}\left(3x^2+4\right)\sqrt{x^4+5}-\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 - (15*ArcSinh[x^2/Sqrt[5]])/4
```

Rubi [A] time = 0.0258202, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1252, 780, 215}

$$\frac{1}{4}\left(3x^2+4\right)\sqrt{x^4+5}-\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 - (15*ArcSinh[x^2/Sqrt[5]])/4
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(2+3x^2\right)}{\sqrt{5+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(2+3x)}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}-\frac{15}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{1}{4}\left(4+3x^2\right)\sqrt{5+x^4}-\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0203834, size = 34, normalized size = 0.97

$$\frac{1}{4}\left(\left(3x^2+4\right)\sqrt{x^4+5}-15\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4] - 15*ArcSinh[x^2/Sqrt[5]])/4
```

Maple [A] time = 0.006, size = 32, normalized size = 0.9

$$\frac{3x^2}{4}\sqrt{x^4+5}-\frac{15}{4}\mathrm{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 3/4*x^2*(x^4+5)^(1/2)-15/4*arcsinh(1/5*x^2*5^(1/2))+(x^4+5)^(1/2)
```

Maxima [B] time = 1.42222, size = 88, normalized size = 2.51

$$\sqrt{x^4+5}+\frac{15\sqrt{x^4+5}}{4x^2\left(\frac{x^4+5}{x^4}-1\right)}-\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) - 15/8*log(sqr
t(x^4 + 5)/x^2 + 1) + 15/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.52503, size = 86, normalized size = 2.46

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A] time = 3.52178, size = 53, normalized size = 1.51

$$\frac{3x^6}{4\sqrt{x^4+5}}+\frac{15x^2}{4\sqrt{x^4+5}}+\sqrt{x^4+5}-\frac{15\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) - 1
5*asinh(sqrt(5)*x**2/5)/4
```

Giac [A] time = 1.13631, size = 45, normalized size = 1.29

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*log(-x^2 + sqrt(x^4 + 5))
```

3.35 $\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=24

$$\frac{3\sqrt{x^4+5}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] (3*Sqrt[5 + x^4])/2 + ArcSinh[x^2/Sqrt[5]]
```

Rubi [A] time = 0.0168086, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1248, 641, 215}

$$\frac{3\sqrt{x^4+5}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]

[Out] (3*Sqrt[5 + x^4])/2 + ArcSinh[x^2/Sqrt[5]]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(2+3x^2\right)}{\sqrt{5+x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{2+3x}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= \frac{3\sqrt{5+x^4}}{2}+\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= \frac{3\sqrt{5+x^4}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.0252598, size = 24, normalized size = 1.

$$\frac{3\sqrt{x^4+5}}{2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (3*Sqrt[5 + x^4])/2 + ArcSinh[x^2/Sqrt[5]]
```

Maple [A] time = 0.009, size = 20, normalized size = 0.8

$$\mathrm{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3}{2}\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] arcsinh(1/5*x^2*5^(1/2))+3/2*(x^4+5)^(1/2)
```

Maxima [B] time = 1.44408, size = 57, normalized size = 2.38

$$\frac{3}{2}\sqrt{x^4+5}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(x^4 + 5) + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/
x^2 - 1)
```

Fricas [A] time = 1.5046, size = 63, normalized size = 2.62

$$\frac{3}{2}\sqrt{x^4+5}-\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A] time = 1.71826, size = 22, normalized size = 0.92

$$\frac{3\sqrt{x^4+5}}{2}+\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(x**4 + 5)/2 + asinh(sqrt(5)*x**2/5)
```

Giac [A] time = 1.12754, size = 35, normalized size = 1.46

$$\frac{3}{2}\sqrt{x^4+5} - \log\left(-x^2 + \sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

3.36 $\int \frac{2+3x^2}{x\sqrt{5+x^4}} \, dx$

Optimal. Leaf size=38

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[5]
```

Rubi [A] time = 0.0382031, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1252, 844, 215, 266, 63, 207}

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]
```

```
[Out] (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[5]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)+\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.0219835, size = 38, normalized size = 1.

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]

[Out] (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[5]
```

Maple [A] time = 0.009, size = 30, normalized size = 0.8

$$\frac{3}{2}\operatorname{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)-\frac{\sqrt{5}}{5}\operatorname{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5)^(1/2),x)

[Out] 3/2*arcsinh(1/5*x^2*5^(1/2))-1/5*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

Maxima [B] time = 1.43429, size = 90, normalized size = 2.37

$$\frac{1}{10}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/
4*log(sqrt(x^4 + 5)/x^2 + 1) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.52611, size = 109, normalized size = 2.87

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-\frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 3/2*log(-x^2 + sqrt(x^4 +
 5))
```

Sympy [A] time = 4.8927, size = 31, normalized size = 0.82

$$-\frac{\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{5}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5)**(1/2),x)
```

```
[Out] -sqrt(5)*asinh(sqrt(5)/x**2)/5 + 3*asinh(sqrt(5)*x**2/5)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x), x)
```

3.37 $\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{x^4+5}}{5x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] -Sqrt[5 + x^4]/(5*x^2) - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(2*Sqrt[5])
```

Rubi [A] time = 0.0372282, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1252, 807, 266, 63, 207}

$$-\frac{\sqrt{x^4+5}}{5x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
```

```
[Out] -Sqrt[5 + x^4]/(5*x^2) - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(2*Sqrt[5])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x^2\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{5x^2}+\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{5x^2}+\frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx,x,x^4\right)\\
&= -\frac{\sqrt{5+x^4}}{5x^2}+\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{5+x^4}\right)\\
&= -\frac{\sqrt{5+x^4}}{5x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}
\end{aligned}
$$

Mathematica [A] time = 0.0248819, size = 42, normalized size = 1.

$$-\frac{\sqrt{x^4+5}}{5x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
```

```
[Out] -Sqrt[5 + x^4]/(5*x^2) - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(2*Sqrt[5])
```

Maple [A] time = 0.01, size = 31, normalized size = 0.7

$$-\frac{3\sqrt{5}}{10}\operatorname{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)-\frac{1}{5x^2}\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5)^(1/2),x)
```

```
[Out] -3/10*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-1/5*(x^4+5)^(1/2)/x^2
```

Maxima [A] time = 1.42862, size = 63, normalized size = 1.5

$$\frac{3}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{\sqrt{x^4+5}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 1/
5*sqrt(x^4 + 5)/x^2
```

Fricas [A] time = 1.55037, size = 119, normalized size = 2.83

$$\frac{3\sqrt{5}x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-2x^2-2\sqrt{x^4+5}}{10x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(3*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 2*x^2 - 2*sqrt(x^
4 + 5))/x^2
```

Sympy [A] time = 3.03978, size = 31, normalized size = 0.74

$$-\frac{\sqrt{1+\frac{5}{x^4}}}{5}-\frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5)**(1/2),x)
```

```
[Out] -sqrt(1 + 5/x**4)/5 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/10
```

Giac [A] time = 1.18384, size = 65, normalized size = 1.55

$$-\frac{3}{20}\sqrt{5}\log\left(\sqrt{5}+\sqrt{x^4+5}\right)+\frac{3}{20}\sqrt{5}\log\left(-\sqrt{5}+\sqrt{x^4+5}\right)-\frac{1}{5}\sqrt{\frac{5}{x^4}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] -3/20*sqrt(5)*log(sqrt(5) + sqrt(x^4 + 5)) + 3/20*sqrt(5)*log(-sqrt(5) + sq
rt(x^4 + 5)) - 1/5*sqrt(5/x^4 + 1)
```

3.38 $\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sqrt{x^4+5}}{10x^2}-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

```
[Out] -Sqrt[5 + x^4]/(10*x^4) - (3*Sqrt[5 + x^4])/(10*x^2) + ArcTanh[Sqrt[5 + x^4
]/Sqrt[5]]/(10*Sqrt[5])
```

Rubi [A] time = 0.0512227, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1252, 835, 807, 266, 63, 207}

$$-\frac{3\sqrt{x^4+5}}{10x^2}-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
```

```
[Out] -Sqrt[5 + x^4]/(10*x^4) - (3*Sqrt[5 + x^4])/(10*x^2) + ArcTanh[Sqrt[5 + x^4
]/Sqrt[5]]/(10*Sqrt[5])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x^3\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{1}{20}\operatorname{Subst}\left(\int \frac{-30+2x}{x^2\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{20}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}
\end{aligned}
$$

Mathematica [A] time = 0.0303463, size = 49, normalized size = 0.84

$$\frac{\sqrt{5}x^4\tanh^{-1}\left(\sqrt{\frac{x^4}{5}+1}\right)-5\left(3x^2+1\right)\sqrt{x^4+5}}{50x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
```

```
[Out] (-5*(1 + 3*x^2)*Sqrt[5 + x^4] + Sqrt[5]*x^4*ArcTanh[Sqrt[1 + x^4/5]])/(50*x
^4)
```

Maple [A] time = 0.013, size = 43, normalized size = 0.7

$$-\frac{1}{10\,x^4}\sqrt{x^4+5}+\frac{\sqrt{5}}{50}\operatorname{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)-\frac{3}{10\,x^2}\sqrt{x^4+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^5/(x^4+5)^(1/2),x)
```

```
[Out] -1/10*(x^4+5)^(1/2)/x^4+1/50*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-3/10*(x
^4+5)^(1/2)/x^2
```

Maxima [A] time = 1.4286, size = 80, normalized size = 1.38

$$-\frac{1}{100}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{10\,x^2}-\frac{\sqrt{x^4+5}}{10\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/100*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) -
3/10*sqrt(x^4 + 5)/x^2 - 1/10*sqrt(x^4 + 5)/x^4
```

Fricas [A] time = 1.5432, size = 132, normalized size = 2.28

$$\frac{\sqrt{5}x^4\log\left(\frac{\sqrt{5}+\sqrt{x^4+5}}{x^2}\right)-15\,x^4-5\sqrt{x^4+5}\left(3\,x^2+1\right)}{50\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/50*(sqrt(5)*x^4*log((sqrt(5) + sqrt(x^4 + 5))/x^2) - 15*x^4 - 5*sqrt(x^4
+ 5)*(3*x^2 + 1))/x^4
```

Sympy [A] time = 7.57052, size = 88, normalized size = 1.52

$$\frac{\sqrt{5}\left(-\frac{\log\left(\sqrt{\frac{x^4}{5}+1}-1\right)}{4}+\frac{\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{4}-\frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}+1\right)}-\frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}-1\right)}\right)}{25}-\frac{3\sqrt{5}\sqrt{5x^4+25}}{50x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**5/(x**4+5)**(1/2),x)
```

```
[Out] sqrt(5)*(-log(sqrt(x**4/5 + 1) - 1)/4 + log(sqrt(x**4/5 + 1) + 1)/4 - 1/(4*
(sqrt(x**4/5 + 1) + 1)) - 1/(4*(sqrt(x**4/5 + 1) - 1)))/25 - 3*sqrt(5)*sqrt
(5*x**4 + 25)/(50*x**2)
```

Giac [A] time = 1.17223, size = 72, normalized size = 1.24

$$-\frac{1}{10}\left(\frac{1}{x^2}+3\right)\sqrt{\frac{5}{x^4}+1}+\frac{1}{100}\sqrt{5}\log\left(\sqrt{5}+\sqrt{x^4+5}\right)-\frac{1}{100}\sqrt{5}\log\left(-\sqrt{5}+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/10*(1/x^2 + 3)*sqrt(5/x^4 + 1) + 1/100*sqrt(5)*log(sqrt(5) + sqrt(x^4 +
5)) - 1/100*sqrt(5)*log(-sqrt(5) + sqrt(x^4 + 5))
```

3.39 $\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=185

$$-\frac{\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{6\sqrt{x^4+5}}+\frac{3}{5}\sqrt{x^4+5}x^3-\frac{9\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\frac{2}{3}\sqrt{x^4+5}x+\frac{9\sqrt[4]{5}\left(x^2\right.}{}$$

```
[Out] (2*x*Sqrt[5 + x^4])/3 + (3*x^3*Sqrt[5 + x^4])/5 - (9*x*Sqrt[5 + x^4])/(Sqrt
[5] + x^2) + (9*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*E
llipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] - (5^(1/4)*(27 + 2*Sqrt[5
])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5
^(1/4)], 1/2])/(6*Sqrt[5 + x^4])
```

Rubi [A] time = 0.0850326, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1280, 1198, 220, 1196}

$$\frac{3}{5}\sqrt{x^4+5}x^3-\frac{9\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\frac{2}{3}\sqrt{x^4+5}x-\frac{\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{6\sqrt{x^4+5}}+\frac{9\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (2*x*Sqrt[5 + x^4])/3 + (3*x^3*Sqrt[5 + x^4])/5 - (9*x*Sqrt[5 + x^4])/(Sqrt
[5] + x^2) + (9*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*E
llipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] - (5^(1/4)*(27 + 2*Sqrt[5
])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5
^(1/4)], 1/2])/(6*Sqrt[5 + x^4])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(2+3x^2\right)}{\sqrt{5+x^4}}\,dx &= \frac{3}{5}x^3\sqrt{5+x^4}-\frac{1}{5}\int \frac{x^2\left(45-10x^2\right)}{\sqrt{5+x^4}}\,dx\\ &= \frac{2}{3}x\sqrt{5+x^4}+\frac{3}{5}x^3\sqrt{5+x^4}+\frac{1}{15}\int \frac{-50-135x^2}{\sqrt{5+x^4}}\,dx\\ &= \frac{2}{3}x\sqrt{5+x^4}+\frac{3}{5}x^3\sqrt{5+x^4}+\left(9\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\frac{1}{3}\left(10+27\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx\\ &= \frac{2}{3}x\sqrt{5+x^4}+\frac{3}{5}x^3\sqrt{5+x^4}-\frac{9x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{9\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}-\frac{\sqrt[4]{5}\left(\cdots\right.}{\cdots}\end{aligned}$$

Mathematica [C] time = 0.0325322, size = 74, normalized size = 0.4

$$\frac{1}{15}x\left(-9\sqrt{5}x^2\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)-10\sqrt{5}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)+\left(9x^2+10\right)\sqrt{x^4+5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x*((10 + 9*x^2)*Sqrt[5 + x^4] - 10*Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/4
, -x^4/5] - 9*Sqrt[5]*x^2*Hypergeometric2F1[1/2, 3/4, 7/4, -x^4/5]))/15
```

Maple [C] time = 0.017, size = 168, normalized size = 0.9

$$\frac{3x^3}{5}\sqrt{x^4+5}-\frac{\frac{9i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 3/5*x^3*(x^4+5)^(1/2)-9/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(2
5+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))
^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))+2/3*x*(x^4+5)^(1/2)
-2/15*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*
x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)x^4}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3x^6+2x^4}{\sqrt{x^4+5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)/sqrt(x^4 + 5), x)
```

Sympy [C] time = 2.1053, size = 75, normalized size = 0.41

$$\frac{3\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/5
)/(20*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4
*exp_polar(I*pi)/5)/(10*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)x^4}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5), x)
```

3.40 $\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=166

$$\frac{\sqrt[4]{5}\left(2-\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt{x^4+5}}+\frac{2\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\sqrt{x^4+5}x-\frac{2\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\cdots}{\sqrt{x^4+\cdots}}$$

```
[Out] x*Sqrt[5 + x^4] + (2*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (2*5^(1/4)*(Sqrt[5]
 + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/
2])/Sqrt[5 + x^4] + (5^(1/4)*(2 - Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(
Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*Sqrt[5 + x^4])
```

Rubi [A] time = 0.0663092, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1280, 1198, 220, 1196}

$$\frac{2\sqrt{x^4+5}x}{x^2+\sqrt{5}}+\sqrt{x^4+5}x+\frac{\sqrt[4]{5}\left(2-\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{x^4+5}}-\frac{2\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan\cdots\right)}{\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] x*Sqrt[5 + x^4] + (2*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (2*5^(1/4)*(Sqrt[5]
 + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/
2])/Sqrt[5 + x^4] + (5^(1/4)*(2 - Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(
Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*Sqrt[5 + x^4])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2\left(2+3x^2\right)}{\sqrt{5+x^4}}\,dx = x\sqrt{5+x^4}-\frac{1}{3}\int \frac{15-6x^2}{\sqrt{5+x^4}}\,dx$$
$$= x\sqrt{5+x^4}-\left(2\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\left(5-2\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$
$$= x\sqrt{5+x^4}+\frac{2x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{2\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}+\frac{\sqrt[4]{5}\left(2-\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\ldots}{2\ldots}$$

Mathematica [C] time = 0.0245802, size = 66, normalized size = 0.4

$$\frac{2x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)}{3\sqrt{5}}-\sqrt{5}x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)+\sqrt{x^4+5}x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] x*Sqrt[5 + x^4] - Sqrt[5]*x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4/5] + (2*x
^3*Hypergeometric2F1[1/2, 3/4, 7/4, -x^4/5])/(3*Sqrt[5])
```

Maple [C] time = 0.013, size = 155, normalized size = 0.9

$$x\sqrt{x^4+5}-\frac{\sqrt{5}}{5\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\mathrm{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}+\frac{\frac{2i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] x*(x^4+5)^(1/2)-1/5*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(2
5+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^
(1/2),I)+2/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)
*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-Ell
ipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3\,x^2+2\right)x^2}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^4+2\,x^2}{\sqrt{x^4+5}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)/sqrt(x^4 + 5), x)
```

Sympy [C] time = 1.91011, size = 75, normalized size = 0.45

$$\frac{3\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)
/(20*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*e
xp_polar(I*pi)/5)/(10*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(3\,x^2+2\right)x^2}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5), x)
```

3.41 $\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=155

$$\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}+\frac{3\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}-\frac{3\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)\right|}{\sqrt{x^4+5}}$$

```
[Out] (3*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (3*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]
+ ((2 + 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Ellipt
icF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.0455755, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1198, 220, 1196}

$$\frac{3\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{5}\sqrt{x^4+5}}-\frac{3\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/Sqrt[5 + x^4],x]
```

```
[Out] (3*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (3*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]
+ ((2 + 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Ellipt
icF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = -\left(\left(3\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx\right) + \left(2+3\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= \frac{3x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{3\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{\left(2+3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}F\left(2\mathrm{t}\right.}{2\sqrt[4]{5}\sqrt{5+x^4}}$$

Mathematica [C] time = 0.0138126, size = 48, normalized size = 0.31

$$\frac{x\left(x^2\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)+2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/Sqrt[5 + x^4],x]
```

```
[Out] (x*(2*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4/5] + x^2*Hypergeometric2F1[1/2,
 3/4, 7/4, -x^4/5]))/Sqrt[5]
```

Maple [C] time = 0.009, size = 146, normalized size = 0.9

$$\frac{\frac{3i}{5}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}+\frac{2\sqrt{5}}{25\sqrt{i\sqrt{5}}}\sqrt{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(x^4+5)^(1/2),x)
```

```
[Out] 3/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/
2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/
5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))+2/25*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^
(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5
^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3x^2+2}{\sqrt{x^4+5}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/sqrt(x^4 + 5), x)
```

Sympy [C] time = 1.40257, size = 73, normalized size = 0.47

$$\frac{3\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5)
/(20*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_
polar(I*pi)/5)/(10*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5), x)
```

3.42 $\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=173

$$\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{2\,5^{3/4}\sqrt{x^4+5}}+\frac{2\sqrt{x^4+5}x}{5\left(x^2+\sqrt{5}\right)}-\frac{2\sqrt{x^4+5}}{5x}-\frac{2\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\right.}{5^{3/4}\sqrt{x^4+5}}$$

```
[Out] (-2*Sqrt[5 + x^4])/(5*x) + (2*x*Sqrt[5 + x^4])/(5*(Sqrt[5] + x^2)) - (2*(Sq
rt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)
], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) + ((2 + 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5
 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(3/4)*
Sqrt[5 + x^4])
```

Rubi [A] time = 0.062236, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1282, 1198, 220, 1196}

$$\frac{2\sqrt{x^4+5}x}{5\left(x^2+\sqrt{5}\right)}-\frac{2\sqrt{x^4+5}}{5x}+\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\,5^{3/4}\sqrt{x^4+5}}-\frac{2\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{5^{3/4}\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^2*Sqrt[5 + x^4]),x]
```

```
[Out] (-2*Sqrt[5 + x^4])/(5*x) + (2*x*Sqrt[5 + x^4])/(5*(Sqrt[5] + x^2)) - (2*(Sq
rt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)
], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) + ((2 + 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5
 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(3/4)*
Sqrt[5 + x^4])
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = -\frac{2\sqrt{5+x^4}}{5x} - \frac{1}{5}\int \frac{-15-2x^2}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{2\sqrt{5+x^4}}{5x} - \frac{2\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{\sqrt{5}} + \frac{1}{5}\left(15+2\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{2\sqrt{5+x^4}}{5x} + \frac{2x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)} - \frac{2\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}} + \frac{\left(2+3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{\cdots}{(\sqrt{\cdots}}}}{2\ 5^{3/4}\sqrt{\cdots}}$$

Mathematica [C] time = 0.0267974, size = 53, normalized size = 0.31

$$\frac{3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{\sqrt{5}} - \frac{2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{x^4}{5}\right)}{\sqrt{5}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^2*Sqrt[5 + x^4]),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/4, 1/2, 3/4, -x^4/5])/(Sqrt[5]*x) + (3*x*Hypergeom
etric2F1[1/4, 1/2, 5/4, -x^4/5])/Sqrt[5]
```

Maple [C] time = 0.015, size = 158, normalized size = 0.9

$$\frac{3\sqrt{5}}{25\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\text{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}} - \frac{2}{5x}\sqrt{x^4+5} + \frac{\frac{2i}{25}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^2/(x^4+5)^(1/2),x)
```

```
[Out] 3/25*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x
^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-2/5*(x
^4+5)^(1/2)/x+2/25*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5
^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),
I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\sqrt{x^4+5x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^6+5\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/(x^6 + 5*x^2), x)
```

Sympy [C] time = 1.54663, size = 75, normalized size = 0.43

$$\frac{3\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\\frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**2/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi)/5)/(2
0*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_pol
ar(I*pi)/5)/(10*x*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2), x)
```

3.43 $\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=189

$$-\frac{\left(2-9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{30\sqrt[4]{5}\sqrt{x^4+5}}+\frac{3\sqrt{x^4+5}\,x}{5\left(x^2+\sqrt{5}\right)}-\frac{3\sqrt{x^4+5}}{5x}-\frac{2\sqrt{x^4+5}}{15x^3}-\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\cdots}}{\cdots}$$

```
[Out] (-2*Sqrt[5 + x^4])/(15*x^3) - (3*Sqrt[5 + x^4])/(5*x) + (3*x*Sqrt[5 + x^4])
/(5*(Sqrt[5] + x^2)) - (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]
*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) - ((2 - 9*Sqr
t[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[
x/5^(1/4)], 1/2])/(30*5^(1/4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.0860867, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1282, 1198, 220, 1196}

$$\frac{3\sqrt{x^4+5}\,x}{5\left(x^2+\sqrt{5}\right)}-\frac{3\sqrt{x^4+5}}{5x}-\frac{2\sqrt{x^4+5}}{15x^3}-\frac{\left(2-9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{30\sqrt[4]{5}\sqrt{x^4+5}}-\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(\cdots\right)}{5^{3/4}\sqrt{x^4+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^4*Sqrt[5 + x^4]),x]
```

```
[Out] (-2*Sqrt[5 + x^4])/(15*x^3) - (3*Sqrt[5 + x^4])/(5*x) + (3*x*Sqrt[5 + x^4])
/(5*(Sqrt[5] + x^2)) - (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]
*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) - ((2 - 9*Sqr
t[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[
x/5^(1/4)], 1/2])/(30*5^(1/4)*Sqrt[5 + x^4])
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx &= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{1}{15}\int \frac{-45+2x^2}{x^2\sqrt{5+x^4}}\,dx\\
&= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}+\frac{1}{75}\int \frac{-10+45x^2}{\sqrt{5+x^4}}\,dx\\
&= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}-\frac{3\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{\sqrt{5}}+\frac{1}{15}\left(-2+9\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx\\
&= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}+\frac{3x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)}-\frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}}-\frac{\left(2-9\sqrt{5}\right)}{}
\end{aligned}$$

Mathematica [C] time = 0.0255959, size = 54, normalized size = 0.29

$$-\frac{9x^2\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{x^4}{5}\right)+2\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{x^4}{5}\right)}{3\sqrt{5}x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^4*Sqrt[5 + x^4]),x]
```

```
[Out] -(2*Hypergeometric2F1[-3/4, 1/2, 1/4, -x^4/5] + 9*x^2*Hypergeometric2F1[-1/
4, 1/2, 3/4, -x^4/5])/(3*Sqrt[5]*x^3)
```

Maple [C] time = 0.017, size = 170, normalized size = 0.9

$$-\frac{3}{5x}\sqrt{x^4+5}+\frac{\frac{3i}{25}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^4/(x^4+5)^(1/2),x)
```

```
[Out] -3/5*(x^4+5)^(1/2)/x+3/25*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(2
5+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))
^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))-2/15*(x^4+5)^(1/2)/
x^3-2/375*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1
/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\sqrt{x^4+5}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+5}(3x^2+2)}{x^8+5x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/(x^8 + 5*x^4), x)
```

Sympy [C] time = 1.89739, size = 80, normalized size = 0.42

$$\frac{3\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20x\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt{5}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**4/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_polar(I*pi)/5)/(2
0*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4*exp_p
olar(I*pi)/5)/(10*x**3*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\sqrt{x^4+5}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4), x)
```

3.44 $\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{(3x^2+2)\,x^4}{2\sqrt{x^4+5}}+\frac{1}{4}\left(9x^2+8\right)\sqrt{x^4+5}-\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] -(x^4*(2 + 3*x^2))/(2*Sqrt[5 + x^4]) + ((8 + 9*x^2)*Sqrt[5 + x^4])/4 - (45*
ArcSinh[x^2/Sqrt[5]])/4
```

Rubi [A] time = 0.0457434, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1252, 819, 780, 215}

$$-\frac{(3x^2+2)\,x^4}{2\sqrt{x^4+5}}+\frac{1}{4}\left(9x^2+8\right)\sqrt{x^4+5}-\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -(x^4*(2 + 3*x^2))/(2*Sqrt[5 + x^4]) + ((8 + 9*x^2)*Sqrt[5 + x^4])/4 - (45*
ArcSinh[x^2/Sqrt[5]])/4
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3(2+3x)}{\left(5+x^2\right)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{x^4\left(2+3x^2\right)}{2\sqrt{5+x^4}}+\frac{1}{10}\operatorname{Subst}\left(\int \frac{x(20+45x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= -\frac{x^4\left(2+3x^2\right)}{2\sqrt{5+x^4}}+\frac{1}{4}\left(8+9x^2\right)\sqrt{5+x^4}-\frac{45}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= -\frac{x^4\left(2+3x^2\right)}{2\sqrt{5+x^4}}+\frac{1}{4}\left(8+9x^2\right)\sqrt{5+x^4}-\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.029022, size = 51, normalized size = 0.88

$$\frac{3x^6+4x^4+45x^2-45\sqrt{x^4+5}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+40}{4\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (40 + 45*x^2 + 4*x^4 + 3*x^6 - 45*Sqrt[5 + x^4]*ArcSinh[x^2/Sqrt[5]])/(4*Sq
rt[5 + x^4])
```

Maple [A] time = 0.017, size = 50, normalized size = 0.9

$$\frac{3\,x^6}{4}\frac{1}{\sqrt{x^4+5}}+\frac{45\,x^2}{4}\frac{1}{\sqrt{x^4+5}}-\frac{45}{4}\operatorname{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\left(x^4+10\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] 3/4*x^6/(x^4+5)^(1/2)+45/4*x^2/(x^4+5)^(1/2)-45/4*arcsinh(1/5*x^2*5^(1/2))+
1/(x^4+5)^(1/2)*(x^4+10)
```

Maxima [A] time = 1.41641, size = 120, normalized size = 2.07

$$\sqrt{x^4+5}-\frac{15\left(\frac{3\left(x^4+5\right)}{x^4}-2\right)}{4\left(\frac{\sqrt{x^4+5}}{x^2}-\frac{\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}+\frac{5}{\sqrt{x^4+5}}-\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^4 + 5) - 15/4*(3*(x^4 + 5)/x^4 - 2)/(sqrt(x^4 + 5)/x^2 - (x^4 + 5)^(
3/2)/x^6) + 5/sqrt(x^4 + 5) - 45/8*log(sqrt(x^4 + 5)/x^2 + 1) + 45/8*log(sq
```

```
rt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.50927, size = 158, normalized size = 2.72

$$\frac{30\,x^4 + 45\left(x^4+5\right)\log\left(-x^2+\sqrt{x^4+5}\right) + \left(3\,x^6+4\,x^4+45\,x^2+40\right)\sqrt{x^4+5} + 150}{4\left(x^4+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(30*x^4 + 45*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) + (3*x^6 + 4*x^4 + 45*
x^2 + 40)*sqrt(x^4 + 5) + 150)/(x^4 + 5)
```

Sympy [A] time = 12.297, size = 66, normalized size = 1.14

$$\frac{3x^6}{4\sqrt{x^4+5}} + \frac{x^4}{\sqrt{x^4+5}} + \frac{45x^2}{4\sqrt{x^4+5}} - \frac{45\,\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4} + \frac{10}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + x**4/sqrt(x**4 + 5) + 45*x**2/(4*sqrt(x**4 + 5)
) - 45*asinh(sqrt(5)*x**2/5)/4 + 10/sqrt(x**4 + 5)
```

Giac [A] time = 1.13531, size = 61, normalized size = 1.05

$$\frac{\left(\left(3\,x^2+4\right)x^2+45\right)x^2+40}{4\,\sqrt{x^4+5}} + \frac{45}{4}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((3*x^2 + 4)*x^2 + 45)*x^2 + 40)/sqrt(x^4 + 5) + 45/4*log(-x^2 + sqrt(
x^4 + 5))
```

3.45 $\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{(3x^2+2)x^2}{2\sqrt{x^4+5}}+3\sqrt{x^4+5}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] -(x^2*(2 + 3*x^2))/(2*Sqrt[5 + x^4]) + 3*Sqrt[5 + x^4] + ArcSinh[x^2/Sqrt[5
]]
```

Rubi [A] time = 0.0386738, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1252, 819, 641, 215}

$$-\frac{(3x^2+2)x^2}{2\sqrt{x^4+5}}+3\sqrt{x^4+5}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -(x^2*(2 + 3*x^2))/(2*Sqrt[5 + x^4]) + 3*Sqrt[5 + x^4] + ArcSinh[x^2/Sqrt[5
]]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g
) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^5\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(2+3x)}{\left(5+x^2\right)^{3/2}}\,dx,x,x^2\right)$$
$$= -\frac{x^2\left(2+3x^2\right)}{2\sqrt{5+x^4}} + \frac{1}{10}\,\text{Subst}\left(\int \frac{10+30x}{\sqrt{5+x^2}}\,dx,x,x^2\right)$$
$$= -\frac{x^2\left(2+3x^2\right)}{2\sqrt{5+x^4}} + 3\sqrt{5+x^4} + \text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)$$
$$= -\frac{x^2\left(2+3x^2\right)}{2\sqrt{5+x^4}} + 3\sqrt{5+x^4} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Mathematica [A] time = 0.0237748, size = 46, normalized size = 1.02

$$\frac{3x^4 - 2x^2 + 2\sqrt{x^4+5}\,\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + 30}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (30 - 2*x^2 + 3*x^4 + 2*Sqrt[5 + x^4]*ArcSinh[x^2/Sqrt[5]])/(2*Sqrt[5 + x^4
])
```

Maple [A] time = 0.013, size = 37, normalized size = 0.8

$$\frac{3\,x^4+30}{2}\frac{1}{\sqrt{x^4+5}} - x^2\frac{1}{\sqrt{x^4+5}} + \text{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] 3/2/(x^4+5)^(1/2)*(x^4+10)-x^2/(x^4+5)^(1/2)+arcsinh(1/5*x^2*5^(1/2))
```

Maxima [A] time = 1.41993, size = 85, normalized size = 1.89

$$-\frac{x^2}{\sqrt{x^4+5}} + \frac{3}{2}\sqrt{x^4+5} + \frac{15}{2\sqrt{x^4+5}} + \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) - \frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -x^2/sqrt(x^4 + 5) + 3/2*sqrt(x^4 + 5) + 15/2/sqrt(x^4 + 5) + 1/2*log(sqrt(
x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.49555, size = 143, normalized size = 3.18

$$-\frac{2\,x^4+2\,(x^4+5)\log\left(-x^2+\sqrt{x^4+5}\right)-(3\,x^4-2\,x^2+30)\sqrt{x^4+5}+10}{2\,(x^4+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^4 + 2*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) - (3*x^4 - 2*x^2 + 30)*
sqrt(x^4 + 5) + 10)/(x^4 + 5)
```

Sympy [A] time = 10.2649, size = 48, normalized size = 1.07

$$\frac{3x^4}{2\sqrt{x^4+5}}-\frac{x^2}{\sqrt{x^4+5}}+\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)+\frac{15}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**4/(2*sqrt(x**4 + 5)) - x**2/sqrt(x**4 + 5) + asinh(sqrt(5)*x**2/5) + 1
5/sqrt(x**4 + 5)
```

Giac [A] time = 1.16797, size = 53, normalized size = 1.18

$$\frac{(3\,x^2-2)x^2+30}{2\,\sqrt{x^4+5}}-\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((3*x^2 - 2)*x^2 + 30)/sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

3.46 $\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3x^2+2}{2\sqrt{x^4+5}}$$

```
[Out] -(2 + 3*x^2)/(2*Sqrt[5 + x^4]) + (3*ArcSinh[x^2/Sqrt[5]])/2
```

Rubi [A] time = 0.0265763, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1252, 778, 215}

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3x^2+2}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -(2 + 3*x^2)/(2*Sqrt[5 + x^4]) + (3*ArcSinh[x^2/Sqrt[5]])/2
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 778

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((a*(e*f + d*g) - (c*d*f - a*e*g)*x)*(a + c*x^2)^(p + 1))/
(2*a*c*(p + 1)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(2+3x)}{\left(5+x^2\right)^{3/2}}\,dx,x,x^2\right)\\ &= -\frac{2+3x^2}{2\sqrt{5+x^4}}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= -\frac{2+3x^2}{2\sqrt{5+x^4}}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.0187678, size = 41, normalized size = 1.17

$$\frac{-3x^2+3\sqrt{x^4+5}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-2}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (-2 - 3*x^2 + 3*Sqrt[5 + x^4]*ArcSinh[x^2/Sqrt[5]])/(2*Sqrt[5 + x^4])
```

Maple [A] time = 0.007, size = 34, normalized size = 1.

$$-\frac{3x^2}{2}\frac{1}{\sqrt{x^4+5}}+\frac{3}{2}\text{Arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)-\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] -3/2*x^2/(x^4+5)^(1/2)+3/2*arcsinh(1/5*x^2*5^(1/2))-1/(x^4+5)^(1/2)
```

Maxima [A] time = 1.41979, size = 73, normalized size = 2.09

$$-\frac{3x^2}{2\sqrt{x^4+5}}-\frac{1}{\sqrt{x^4+5}}+\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*x^2/sqrt(x^4 + 5) - 1/sqrt(x^4 + 5) + 3/4*log(sqrt(x^4 + 5)/x^2 + 1) -
 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A] time = 1.51848, size = 131, normalized size = 3.74

$$-\frac{3x^4+3\left(x^4+5\right)\log\left(-x^2+\sqrt{x^4+5}\right)+\sqrt{x^4+5}\left(3x^2+2\right)+15}{2\left(x^4+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(3*x^4 + 3*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) + sqrt(x^4 + 5)*(3*x^2
+ 2) + 15)/(x^4 + 5)
```

Sympy [A] time = 7.08634, size = 39, normalized size = 1.11

$$-\frac{3x^2}{2\sqrt{x^4+5}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}-\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] -3*x**2/(2*sqrt(x**4 + 5)) + 3*asinh(sqrt(5)*x**2/5)/2 - 1/sqrt(x**4 + 5)
```

Giac [A] time = 1.18008, size = 45, normalized size = 1.29

$$-\frac{3\,x^2+2}{2\,\sqrt{x^4+5}}-\frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*x^2 + 2)/sqrt(x^4 + 5) - 3/2*log(-x^2 + sqrt(x^4 + 5))
```

3.47 $\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}} \, dx$

Optimal. Leaf size=20

$$-\frac{15-2x^2}{10\sqrt{x^4+5}}$$

[Out] -(15 - 2*x^2)/(10*Sqrt[5 + x^4])

Rubi [A] time = 0.0156476, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1248, 637}

$$-\frac{15-2x^2}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

[In] Int[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]

[Out] -(15 - 2*x^2)/(10*Sqrt[5 + x^4])

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 637

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a
*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{\left(5+x^2\right)^{3/2}} \, dx, x, x^2\right) \\ &= -\frac{15-2x^2}{10\sqrt{5+x^4}} \end{aligned}$$

Mathematica [A] time = 0.0099904, size = 20, normalized size = 1.

$$\frac{2x^2-15}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]

[Out] (-15 + 2*x^2)/(10*Sqrt[5 + x^4])

Maple [A] time = 0.005, size = 17, normalized size = 0.9

$$\frac{2x^2-15}{10}\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] 1/10*(2*x^2-15)/(x^4+5)^(1/2)
```

Maxima [A] time = 1.41095, size = 30, normalized size = 1.5

$$\frac{x^2}{5\sqrt{x^4+5}}-\frac{3}{2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*x^2/sqrt(x^4 + 5) - 3/2/sqrt(x^4 + 5)
```

Fricas [A] time = 1.50904, size = 78, normalized size = 3.9

$$\frac{2x^4+\sqrt{x^4+5}\left(2x^2-15\right)+10}{10\left(x^4+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*(2*x^4 + sqrt(x^4 + 5)*(2*x^2 - 15) + 10)/(x^4 + 5)
```

Sympy [B] time = 4.29222, size = 31, normalized size = 1.55

$$\frac{\sqrt{5}x^2}{5\sqrt{5x^4+25}}-\frac{3}{2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] sqrt(5)*x**2/(5*sqrt(5*x**4 + 25)) - 3/(2*sqrt(x**4 + 5))
```

Giac [A] time = 1.13046, size = 22, normalized size = 1.1

$$\frac{2x^2-15}{10\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/10*(2*x^2 - 15)/sqrt(x^4 + 5)
```

3.48 $\int \frac{2+3x^2}{x(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{3x^2+2}{10\sqrt{x^4+5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

```
[Out] (2 + 3*x^2)/(10*Sqrt[5 + x^4]) - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/(5*Sqrt[5])
```

Rubi [A] time = 0.0432416, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1252, 823, 12, 266, 63, 207}

$$\frac{3x^2+2}{10\sqrt{x^4+5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*Sqrt[5 + x^4]) - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/(5*Sqrt[5])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x\left(5+x^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} - \frac{1}{50}\operatorname{Subst}\left(\int -\frac{10}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} + \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} + \frac{1}{10}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} + \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{5\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.0388873, size = 46, normalized size = 1.

$$\frac{1}{50}\left(\frac{5\left(3x^2+2\right)}{\sqrt{x^4+5}} - 2\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
```

```
[Out] ((5*(2 + 3*x^2))/Sqrt[5 + x^4] - 2*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/
50
```

Maple [A] time = 0.016, size = 40, normalized size = 0.9

$$\frac{3x^2}{10}\frac{1}{\sqrt{x^4+5}} + \frac{1}{5}\frac{1}{\sqrt{x^4+5}} - \frac{\sqrt{5}}{25}\operatorname{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5)^(3/2),x)
```

```
[Out] 3/10*x^2/(x^4+5)^(1/2)+1/5/(x^4+5)^(1/2)-1/25*5^(1/2)*arctanh(5^(1/2)/(x^4+
5)^(1/2))
```

Maxima [A] time = 1.42682, size = 76, normalized size = 1.65

$$\frac{3\,x^2}{10\,\sqrt{x^4+5}}+\frac{1}{50}\,\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{1}{5\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/10*x^2/sqrt(x^4 + 5) + 1/50*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(
5) + sqrt(x^4 + 5))) + 1/5/sqrt(x^4 + 5)
```

Fricas [A] time = 1.49631, size = 159, normalized size = 3.46

$$\frac{15\,x^4+2\,\sqrt{5}\left(x^4+5\right)\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)+5\,\sqrt{x^4+5}\left(3\,x^2+2\right)+75}{50\,\left(x^4+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50*(15*x^4 + 2*sqrt(5)*(x^4 + 5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) + 5*
sqrt(x^4 + 5)*(3*x^2 + 2) + 75)/(x^4 + 5)
```

Sympy [B] time = 11.6134, size = 212, normalized size = 4.61

$$\frac{2x^4\log\left(x^4\right)}{20\sqrt{5}x^4+100\sqrt{5}}-\frac{4x^4\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}x^4+100\sqrt{5}}-\frac{2x^4\log\left(5\right)}{20\sqrt{5}x^4+100\sqrt{5}}+\frac{3x^2}{10\sqrt{x^4+5}}+\frac{4\sqrt{5}\sqrt{x^4+5}}{20\sqrt{5}x^4+100\sqrt{5}}+\frac{10\log\left(x^4\right)}{20\sqrt{5}x^4+100\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5)**(3/2),x)
```

```
[Out] 2*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 4*x**4*log(sqrt(x**4/5 +
 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 2*x**4*log(5)/(20*sqrt(5)*x**4 +
 100*sqrt(5)) + 3*x**2/(10*sqrt(x**4 + 5)) + 4*sqrt(5)*sqrt(x**4 + 5)/(20*s
qrt(5)*x**4 + 100*sqrt(5)) + 10*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) -
 20*log(sqrt(x**4/5 + 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 10*log(5)/(
20*sqrt(5)*x**4 + 100*sqrt(5))
```

Giac [A] time = 1.16022, size = 72, normalized size = 1.57

$$-\frac{1}{50}\,\sqrt{5}\log\left(\sqrt{5}+\sqrt{x^4+5}\right)+\frac{1}{50}\,\sqrt{5}\log\left(-\sqrt{5}+\sqrt{x^4+5}\right)+\frac{3\,x^2+2}{10\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] -1/50*sqrt(5)*log(sqrt(5) + sqrt(x^4 + 5)) + 1/50*sqrt(5)*log(-sqrt(5) + sq
rt(x^4 + 5)) + 1/10*(3*x^2 + 2)/sqrt(x^4 + 5)
```

3.49 $\int \frac{2+3x^2}{x^3(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{3x^2+2}{10x^2\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

```
[Out] (2 + 3*x^2)/(10*x^2*Sqrt[5 + x^4]) - (2*Sqrt[5 + x^4])/(25*x^2) - (3*ArcTan
h[Sqrt[5 + x^4]/Sqrt[5]])/(10*Sqrt[5])
```

Rubi [A] time = 0.055294, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1252, 823, 807, 266, 63, 207}

$$\frac{3x^2+2}{10x^2\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x^2*Sqrt[5 + x^4]) - (2*Sqrt[5 + x^4])/(25*x^2) - (3*ArcTan
h[Sqrt[5 + x^4]/Sqrt[5]])/(10*Sqrt[5])
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x^2\left(5+x^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{1}{50}\operatorname{Subst}\left(\int \frac{-20-15x}{x^2\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}+\frac{3}{10}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}+\frac{3}{20}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx,x,x^4\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}+\frac{3}{10}\operatorname{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{5+x^4}\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}
\end{aligned}$$

Mathematica [C] time = 0.0284293, size = 45, normalized size = 0.69

$$\frac{15x^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{x^4}{5}+1\right)-4x^4-10}{50x^2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
```

```
[Out] (-10 - 4*x^4 + 15*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + x^4/5])/(50*x^2*S
qrt[5 + x^4])
```

Maple [A] time = 0.013, size = 47, normalized size = 0.7

$$-\frac{2x^4+5}{25x^2}\frac{1}{\sqrt{x^4+5}}+\frac{3}{10}\frac{1}{\sqrt{x^4+5}}-\frac{3\sqrt{5}}{50}\operatorname{Artanh}\left(\sqrt{5}\frac{1}{\sqrt{x^4+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5)^(3/2),x)
```

```
[Out] -1/25/x^2*(2*x^4+5)/(x^4+5)^(1/2)+3/10/(x^4+5)^(1/2)-3/50*5^(1/2)*arctanh(5
^(1/2)/(x^4+5)^(1/2))
```

Maxima [A] time = 1.42585, size = 92, normalized size = 1.42

$$-\frac{x^2}{25\sqrt{x^4+5}} + \frac{3}{100}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right) + \frac{3}{10\sqrt{x^4+5}} - \frac{\sqrt{x^4+5}}{25x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/25*x^2/sqrt(x^4 + 5) + 3/100*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqr
t(5) + sqrt(x^4 + 5))) + 3/10/sqrt(x^4 + 5) - 1/25*sqrt(x^4 + 5)/x^2
```

Fricas [A] time = 1.53314, size = 186, normalized size = 2.86

$$-\frac{4x^6 - 3\sqrt{5}\left(x^6+5x^2\right)\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right) + 20x^2 + \left(4x^4-15x^2+10\right)\sqrt{x^4+5}}{50\left(x^6+5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/50*(4*x^6 - 3*sqrt(5)*(x^6 + 5*x^2)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2)
+ 20*x^2 + (4*x^4 - 15*x^2 + 10)*sqrt(x^4 + 5))/(x^6 + 5*x^2)
```

Sympy [B] time = 10.2049, size = 228, normalized size = 3.51

$$\frac{3x^4\log\left(x^4\right)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{6x^4\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{3x^4\log\left(5\right)}{20\sqrt{5}x^4+100\sqrt{5}} + \frac{6\sqrt{5}\sqrt{x^4+5}}{20\sqrt{5}x^4+100\sqrt{5}} + \frac{15\log\left(x^4\right)}{20\sqrt{5}x^4+100\sqrt{5}} - \frac{30\log\left(\sqrt{\right.}}{20\sqrt{5}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 6*x**4*log(sqrt(x**4/5 +
 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 3*x**4*log(5)/(20*sqrt(5)*x**4 +
 100*sqrt(5)) + 6*sqrt(5)*sqrt(x**4 + 5)/(20*sqrt(5)*x**4 + 100*sqrt(5)) +
15*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 30*log(sqrt(x**4/5 + 1) + 1)
/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 15*log(5)/(20*sqrt(5)*x**4 + 100*sqrt(5)
) - 2/(25*sqrt(1 + 5/x**4)) - 1/(5*x**4*sqrt(1 + 5/x**4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^3), x)
```

3.50 $\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=196

$$\frac{(2+9\sqrt{5})(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{4\sqrt[4]{5}\sqrt{x^4+5}}-\frac{(15-2x^2)x^3}{10\sqrt{x^4+5}}+\frac{9\sqrt{x^4+5}x}{2(x^2+\sqrt{5})}-\frac{1}{5}\sqrt{x^4+5}x-\frac{9\sqrt[4]{5}(x^2$$

```
[Out] -(x^3*(15 - 2*x^2))/(10*Sqrt[5 + x^4]) - (x*Sqrt[5 + x^4])/5 + (9*x*Sqrt[5
+ x^4])/(2*(Sqrt[5] + x^2)) - (9*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sq
rt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(2*Sqrt[5 + x^4]) + ((
2 + 9*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[
2*ArcTan[x/5^(1/4)], 1/2])/(4*5^(1/4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.0852553, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1276, 1280, 1198, 220, 1196}

$$-\frac{(15-2x^2)x^3}{10\sqrt{x^4+5}}+\frac{9\sqrt{x^4+5}x}{2(x^2+\sqrt{5})}-\frac{1}{5}\sqrt{x^4+5}x+\frac{(2+9\sqrt{5})(x^2+\sqrt{5})\sqrt{\frac{x^4+5}{(x^2+\sqrt{5})^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{5}\sqrt{x^4+5}}-\frac{9\sqrt[4]{5}(x^2+\sqrt{5})}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```


```
[Out] -(x^3*(15 - 2*x^2))/(10*Sqrt[5 + x^4]) - (x*Sqrt[5 + x^4])/5 + (9*x*Sqrt[5
+ x^4])/(2*(Sqrt[5] + x^2)) - (9*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sq
rt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(2*Sqrt[5 + x^4]) + ((
2 + 9*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[
2*ArcTan[x/5^(1/4)], 1/2])/(4*5^(1/4)*Sqrt[5 + x^4])
```


Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*(a*e - c*d*x^2))/(4*a
*c*(p + 1)), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
```

```
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx &= -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}}+\frac{1}{10}\int \frac{x^2(45-6x^2)}{\sqrt{5+x^4}}\,dx\\ &= -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}-\frac{1}{30}\int \frac{-30-135x^2}{\sqrt{5+x^4}}\,dx\\ &= -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}-\frac{1}{2}(9\sqrt{5})\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\frac{1}{2}(-2-9\sqrt{5})\int \frac{1}{\sqrt{5+x^4}}\,dx\\ &= -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}+\frac{9x\sqrt{5+x^4}}{2(\sqrt{5}+x^2)}-\frac{9\sqrt[4]{5}(\sqrt{5}+x^2)\sqrt{\frac{5+x^4}{(\sqrt{5}+x^2)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{5+x^4}}+\frac{(2\end{aligned}$$

Mathematica [C] time = 0.0459446, size = 70, normalized size = 0.36

$$-\frac{3x^3\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{x^4}{5}\right)}{\sqrt{5}}+\frac{x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{\sqrt{5}}+\frac{(3x^2-1)x}{\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(-1 + 3*x^2))/Sqrt[5 + x^4] + (x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4/5
])/Sqrt[5] - (3*x^3*Hypergeometric2F1[3/4, 3/2, 7/4, -x^4/5])/Sqrt[5]
```

Maple [C] time = 0.02, size = 168, normalized size = 0.9

$$-\frac{3x^3}{2}\frac{1}{\sqrt{x^4+5}}+\frac{\frac{9i}{10}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] -3/2*x^3/(x^4+5)^(1/2)+9/10*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*
(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2
))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))-x/(x^4+5)^(1/2)+1
/25*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^
2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)x^4}{(x^4+5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(3x^6+2x^4)\sqrt{x^4+5}}{x^8+10x^4+25}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)*sqrt(x^4 + 5)/(x^8 + 10*x^4 + 25), x)
```

Sympy [C] time = 4.75707, size = 75, normalized size = 0.38

$$\frac{3\sqrt{5}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/5
)/(100*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**
4*exp_polar(I*pi)/5)/(50*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)x^4}{(x^4+5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2), x)
```

3.51 $\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{4\ 5^{3/4}\sqrt{x^4+5}}-\frac{\sqrt{x^4+5}x}{5\left(x^2+\sqrt{5}\right)}-\frac{\left(15-2x^2\right)x}{10\sqrt{x^4+5}}+\frac{\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E}{5^{3/4}\sqrt{x^4}}$$

```
[Out] -(x*(15 - 2*x^2))/(10*Sqrt[5 + x^4]) - (x*Sqrt[5 + x^4])/(5*(Sqrt[5] + x^2)
) + ((Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x
/5^(1/4)], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) - ((2 - 3*Sqrt[5])*(Sqrt[5] + x^2)
*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(4*
5^(3/4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.0691724, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1276, 1198, 220, 1196}

$$-\frac{\sqrt{x^4+5}x}{5\left(x^2+\sqrt{5}\right)}-\frac{\left(15-2x^2\right)x}{10\sqrt{x^4+5}}-\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{4\ 5^{3/4}\sqrt{x^4+5}}+\frac{\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\right.}{5^{3/4}\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -(x*(15 - 2*x^2))/(10*Sqrt[5 + x^4]) - (x*Sqrt[5 + x^4])/(5*(Sqrt[5] + x^2)
) + ((Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x
/5^(1/4)], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) - ((2 - 3*Sqrt[5])*(Sqrt[5] + x^2)
*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(4*
5^(3/4)*Sqrt[5 + x^4])
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*(a*e - c*d*x^2))/(4*a
*c*(p + 1)), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2\left(2+3x^2\right)}{\left(5+x^4\right)^{3/2}}\,dx = -\frac{x\left(15-2x^2\right)}{10\sqrt{5+x^4}}+\frac{1}{10}\int \frac{15-2x^2}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{x\left(15-2x^2\right)}{10\sqrt{5+x^4}}+\frac{\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{\sqrt{5}}+\frac{1}{10}\left(15-2\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{x\left(15-2x^2\right)}{10\sqrt{5+x^4}}-\frac{x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)}+\frac{\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}}-\frac{\left(2-3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{}}{4\ 5^{3/}}$$

Mathematica [C] time = 0.0321569, size = 68, normalized size = 0.38

$$\frac{1}{150}x\left(4\sqrt{5}x^2\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{x^4}{5}\right)+45\sqrt{5}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)-\frac{225}{\sqrt{x^4+5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(-225/Sqrt[5 + x^4] + 45*Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4/5
] + 4*Sqrt[5]*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, -x^4/5]))/150
```

Maple [C] time = 0.013, size = 168, normalized size = 1.

$$-\frac{3x}{2}\frac{1}{\sqrt{x^4+5}}+\frac{3\sqrt{5}}{50\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}+\frac{x^3}{5}\frac{1}{\sqrt{x^4+5}}-\frac{\frac{i}{25}}{\sqrt{i\sqrt{5}}}\sqrt{25-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] -3/2*x/(x^4+5)^(1/2)+3/50*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1
/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(
1/2))^(1/2),I)+1/5*x^3/(x^4+5)^(1/2)-1/25*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/
2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(
1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)x^2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(3x^4+2x^2)\sqrt{x^4+5}}{x^8+10x^4+25}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)*sqrt(x^4 + 5)/(x^8 + 10*x^4 + 25), x)
```

Sympy [C] time = 4.28565, size = 75, normalized size = 0.42

$$\frac{3\sqrt{5}x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi)/5)
/(100*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*
exp_polar(I*pi)/5)/(50*gamma(7/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)x^2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2), x)
```

3.52 $\int \frac{2+3x^2}{\left(5+x^4\right)^{3/2}} \, dx$

Optimal. Leaf size=180

$$\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{20\sqrt[4]{5}\sqrt{x^4+5}}-\frac{3\sqrt{x^4+5}\,x}{10\left(x^2+\sqrt{5}\right)}+\frac{\left(3x^2+2\right)x}{10\sqrt{x^4+5}}+\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\right.}{2\ 5^{3/4}\sqrt{x^4+}}$$

```
[Out] (x*(2 + 3*x^2))/(10*Sqrt[5 + x^4]) - (3*x*Sqrt[5 + x^4])/(10*(Sqrt[5] + x^2
)) + (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTa
n[x/5^(1/4)], 1/2])/(2*5^(3/4)*Sqrt[5 + x^4]) + ((2 - 3*Sqrt[5])*(Sqrt[5] +
 x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2]
)/(20*5^(1/4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.0614915, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1179, 1198, 220, 1196}

$$-\frac{3\sqrt{x^4+5}\,x}{10\left(x^2+\sqrt{5}\right)}+\frac{\left(3x^2+2\right)x}{10\sqrt{x^4+5}}+\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{20\sqrt[4]{5}\sqrt{x^4+5}}+\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\tan^{-1}\right.}{2\ 5^{3/4}\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(2 + 3*x^2))/(10*Sqrt[5 + x^4]) - (3*x*Sqrt[5 + x^4])/(10*(Sqrt[5] + x^2
)) + (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTa
n[x/5^(1/4)], 1/2])/(2*5^(3/4)*Sqrt[5 + x^4]) + ((2 - 3*Sqrt[5])*(Sqrt[5] +
 x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2]
)/(20*5^(1/4)*Sqrt[5 + x^4])
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{2+3x^2}{\left(5+x^4\right)^{3/2}}\,dx = \frac{x\left(2+3x^2\right)}{10\sqrt{5+x^4}} - \frac{1}{10}\int \frac{-2+3x^2}{\sqrt{5+x^4}}\,dx$$

$$= \frac{x\left(2+3x^2\right)}{10\sqrt{5+x^4}} + \frac{3\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{2\sqrt{5}} - \frac{1}{10}\left(-2+3\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= \frac{x\left(2+3x^2\right)}{10\sqrt{5+x^4}} - \frac{3x\sqrt{5+x^4}}{10\left(\sqrt{5}+x^2\right)} + \frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\,5^{3/4}\sqrt{5+x^4}} + \frac{\left(2-3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)}{20}$$

Mathematica [C] time = 0.0256387, size = 66, normalized size = 0.37

$$\frac{1}{25}x\left(\sqrt{5}x^2\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{x^4}{5}\right)+\sqrt{5}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)+\frac{5}{\sqrt{x^4+5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(5/Sqrt[5 + x^4] + Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4/5] + Sq
rt[5]*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, -x^4/5]))/25
```

Maple [C] time = 0.013, size = 168, normalized size = 0.9

$$\frac{3x^3}{10}\frac{1}{\sqrt{x^4+5}} - \frac{\frac{3i}{50}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\right)\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(x^4+5)^(3/2),x)
```

```
[Out] 3/10*x^3/(x^4+5)^(1/2)-3/50*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*
(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2
))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))+1/5*x/(x^4+5)^(1/
2)+1/125*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/
2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+5}\,(3x^2+2)}{x^8+10x^4+25}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/(x^8 + 10*x^4 + 25), x)
```

Sympy [C] time = 4.25619, size = 73, normalized size = 0.41

$$\frac{3\sqrt{5}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*exp_polar(I*pi)/5)
/(100*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp
_polar(I*pi)/5)/(50*gamma(5/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{(x^4+5)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

3.53 $\int \frac{2+3x^2}{x^2\left(5+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=196

$$\frac{3\left(2+\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{20\ 5^{3/4}\sqrt{x^4+5}}+\frac{3\sqrt{x^4+5}x}{25\left(x^2+\sqrt{5}\right)}-\frac{3\sqrt{x^4+5}}{25x}+\frac{3x^2+2}{10\sqrt{x^4+5}x}-\frac{3\left(x^2+\sqrt{\cdots}\right.}{\cdots}$$

```
[Out] (2 + 3*x^2)/(10*x*Sqrt[5 + x^4]) - (3*Sqrt[5 + x^4])/(25*x) + (3*x*Sqrt[5 +
 x^4])/(25*(Sqrt[5] + x^2)) - (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] +
x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(5*5^(3/4)*Sqrt[5 + x^4]) + (3
*(2 + Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[
2*ArcTan[x/5^(1/4)], 1/2])/(20*5^(3/4)*Sqrt[5 + x^4])
```

Rubi [A] time = 0.081502, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1278, 1282, 1198, 220, 1196}

$$\frac{3\sqrt{x^4+5}x}{25\left(x^2+\sqrt{5}\right)}-\frac{3\sqrt{x^4+5}}{25x}+\frac{3x^2+2}{10\sqrt{x^4+5}x}+\frac{3\left(2+\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{20\ 5^{3/4}\sqrt{x^4+5}}-\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4}{\left(x^2+\cdots\right.}}}{5\ 5^{3\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^2*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x*Sqrt[5 + x^4]) - (3*Sqrt[5 + x^4])/(25*x) + (3*x*Sqrt[5 +
 x^4])/(25*(Sqrt[5] + x^2)) - (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] +
x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(5*5^(3/4)*Sqrt[5 + x^4]) + (3
*(2 + Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[
2*ArcTan[x/5^(1/4)], 1/2])/(20*5^(3/4)*Sqrt[5 + x^4])
```

Rule 1278

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> -Simp[((f*x)^(m + 1)*(a + c*x^4)^(p + 1)*(d + e*x^2))/(4*a*f*(p
+ 1)), x] + Dist[1/(4*a*(p + 1)), Int[(f*x)^m*(a + c*x^4)^(p + 1)*Simp[d*(m
 + 4*(p + 1) + 1) + e*(m + 2*(2*p + 3) + 1)*x^2, x], x], x] /; FreeQ[{a, c,
 d, e, f, m}, x] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ
[m])
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
```

```
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^2\left(5+x^4\right)^{3/2}}\,dx &= \frac{2+3x^2}{10x\sqrt{5+x^4}}-\frac{1}{10}\int \frac{-6-3x^2}{x^2\sqrt{5+x^4}}\,dx\\
&= \frac{2+3x^2}{10x\sqrt{5+x^4}}-\frac{3\sqrt{5+x^4}}{25x}+\frac{1}{50}\int \frac{15+6x^2}{\sqrt{5+x^4}}\,dx\\
&= \frac{2+3x^2}{10x\sqrt{5+x^4}}-\frac{3\sqrt{5+x^4}}{25x}-\frac{3\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{5\sqrt{5}}+\frac{1}{50}\left(3\left(5+2\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx\\
&= \frac{2+3x^2}{10x\sqrt{5+x^4}}-\frac{3\sqrt{5+x^4}}{25x}+\frac{3x\sqrt{5+x^4}}{25\left(\sqrt{5}+x^2\right)}-\frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5\,5^{3/4}\sqrt{5+x^4}}+\frac{3\left(2+\sqrt{}\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.0652715, size = 71, normalized size = 0.36

$$\frac{3x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{10\sqrt{5}}-\frac{2\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{x^4}{5}\right)}{5\sqrt{5}x}+\frac{3x}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^2*(5 + x^4)^(3/2)),x]
```

```
[Out] (3*x)/(10*Sqrt[5 + x^4]) - (2*Hypergeometric2F1[-1/4, 3/2, 3/4, -x^4/5])/(5
*Sqrt[5]*x) + (3*x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^4/5])/(10*Sqrt[5])
```

Maple [C] time = 0.018, size = 180, normalized size = 0.9

$$\frac{3x}{10}\frac{1}{\sqrt{x^4+5}}+\frac{3\sqrt{5}}{250\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\mathrm{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)\frac{1}{\sqrt{x^4+5}}-\frac{2}{25x}\sqrt{x^4+5}-\frac{x^3}{25}\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^2/(x^4+5)^(3/2),x)
```

```
[Out] 3/10*x/(x^4+5)^(1/2)+3/250*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(
1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^
(1/2))^(1/2),I)-2/25*(x^4+5)^(1/2)/x-1/25*x^3/(x^4+5)^(1/2)+3/125*I/(I*5^(1
/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1
/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(
I*5^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5}\left(3\,x^2+2\right)}{x^{10}+10\,x^6+25\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/(x^10 + 10*x^6 + 25*x^2), x)
```

Sympy [C] time = 6.07129, size = 75, normalized size = 0.38

$$\frac{3\sqrt{5}x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**2/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi)/5)/(1
00*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_po
lar(I*pi)/5)/(50*x*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2), x)
```

3.54 $\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=214

$$\frac{\left(27-2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right),\frac{1}{2}\right)}{60\ 5^{3/4}\sqrt{x^4+5}}+\frac{9\sqrt{x^4+5}\,x}{50\left(x^2+\sqrt{5}\right)}-\frac{9\sqrt{x^4+5}}{50x}-\frac{\sqrt{x^4+5}}{15x^3}+\frac{3x^2+2}{10\sqrt{x^4+5}\,x^3}$$

```
[Out] (2 + 3*x^2)/(10*x^3*Sqrt[5 + x^4]) - Sqrt[5 + x^4]/(15*x^3) - (9*Sqrt[5 + x
^4])/(50*x) + (9*x*Sqrt[5 + x^4])/(50*(Sqrt[5] + x^2)) - (9*(Sqrt[5] + x^2)
*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(10
*5^(3/4)*Sqrt[5 + x^4]) + ((27 - 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/
(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(60*5^(3/4)*Sqrt[5
+ x^4])
```

Rubi [A] time = 0.107847, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1278, 1282, 1198, 220, 1196}

$$\frac{9\sqrt{x^4+5}\,x}{50\left(x^2+\sqrt{5}\right)}-\frac{9\sqrt{x^4+5}}{50x}-\frac{\sqrt{x^4+5}}{15x^3}+\frac{3x^2+2}{10\sqrt{x^4+5}\,x^3}+\frac{\left(27-2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{60\ 5^{3/4}\sqrt{x^4+5}}-\frac{9\left(x^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^4*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x^3*Sqrt[5 + x^4]) - Sqrt[5 + x^4]/(15*x^3) - (9*Sqrt[5 + x
^4])/(50*x) + (9*x*Sqrt[5 + x^4])/(50*(Sqrt[5] + x^2)) - (9*(Sqrt[5] + x^2)
*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(10
*5^(3/4)*Sqrt[5 + x^4]) + ((27 - 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/
(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(60*5^(3/4)*Sqrt[5
+ x^4])
```

Rule 1278

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> -Simp[((f*x)^(m + 1)*(a + c*x^4)^(p + 1)*(d + e*x^2))/(4*a*f*(p
+ 1)), x] + Dist[1/(4*a*(p + 1)), Int[(f*x)^m*(a + c*x^4)^(p + 1)*Simp[d*(m
 + 4*(p + 1) + 1) + e*(m + 2*(2*p + 3) + 1)*x^2, x], x], x] /; FreeQ[{a, c,
 d, e, f, m}, x] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ
[m])
```

Rule 1282

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + c*x^4)^(p + 1))/(a*f*(m + 1)), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1198

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx &= \frac{2+3x^2}{10x^3\sqrt{5+x^4}}-\frac{1}{10}\int \frac{-10-9x^2}{x^4\sqrt{5+x^4}}\,dx\\
&= \frac{2+3x^2}{10x^3\sqrt{5+x^4}}-\frac{\sqrt{5+x^4}}{15x^3}+\frac{1}{150}\int \frac{135-10x^2}{x^2\sqrt{5+x^4}}\,dx\\
&= \frac{2+3x^2}{10x^3\sqrt{5+x^4}}-\frac{\sqrt{5+x^4}}{15x^3}-\frac{9\sqrt{5+x^4}}{50x}-\frac{1}{750}\int \frac{50-135x^2}{\sqrt{5+x^4}}\,dx\\
&= \frac{2+3x^2}{10x^3\sqrt{5+x^4}}-\frac{\sqrt{5+x^4}}{15x^3}-\frac{9\sqrt{5+x^4}}{50x}-\frac{9\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{10\sqrt{5}}-\frac{1}{150}\left(10-27\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx\\
&= \frac{2+3x^2}{10x^3\sqrt{5+x^4}}-\frac{\sqrt{5+x^4}}{15x^3}-\frac{9\sqrt{5+x^4}}{50x}+\frac{9x\sqrt{5+x^4}}{50\left(\sqrt{5}+x^2\right)}-\frac{9\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{10\ 5^{3/4}\sqrt{5+x^4}}
\end{aligned}
$$

Mathematica [C] time = 0.0287845, size = 54, normalized size = 0.25

$$
-\frac{9x^2\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{x^4}{5}\right)+2\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{x^4}{5}\right)}{15\sqrt{5}x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^4*(5 + x^4)^(3/2)),x]
```

```
[Out] -(2*Hypergeometric2F1[-3/4, 3/2, 1/4, -x^4/5] + 9*x^2*Hypergeometric2F1[-1/
4, 3/2, 3/4, -x^4/5])/(15*Sqrt[5]*x^3)
```

Maple [C] time = 0.019, size = 192, normalized size = 0.9

$$
-\frac{3}{25x}\sqrt{x^4+5}-\frac{3x^3}{50}\frac{1}{\sqrt{x^4+5}}+\frac{\frac{9i}{250}}{\sqrt{i\sqrt{5}}}\sqrt{25-5i\sqrt{5}x^2}\sqrt{25+5i\sqrt{5}x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\sqrt{i\sqrt{5}}}{5}\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^4/(x^4+5)^(3/2),x)
```

```
[Out] -3/25*(x^4+5)^(1/2)/x-3/50*x^3/(x^4+5)^(1/2)+9/250*I/(I*5^(1/2))^(1/2)*(25-
5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(
1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2
),I))-1/25*x/(x^4+5)^(1/2)-2/75*(x^4+5)^(1/2)/x^3-1/375*5^(1/2)/(I*5^(1/2))
^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*
EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+5}\left(3x^2+2\right)}{x^{12}+10x^8+25x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/(x^12 + 10*x^8 + 25*x^4), x)
```

Sympy [C] time = 8.12264, size = 80, normalized size = 0.37

$$\frac{3\sqrt{5}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100x\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt{5}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**4/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_polar(I*pi)/5)/(1
00*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), x**4*exp_
polar(I*pi)/5)/(50*x**3*gamma(1/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\left(x^4+5\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4), x)
```

3.55 $\int (fx)^m \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=269

$$\frac{(10d+e)(fx)^{m+3}}{f^3(m+3)}+\frac{5(9d+2e)(fx)^{m+5}}{f^5(m+5)}+\frac{15(8d+3e)(fx)^{m+7}}{f^7(m+7)}+\frac{30(7d+4e)(fx)^{m+9}}{f^9(m+9)}+\frac{42(6d+5e)(fx)^{m+11}}{f^{11}(m+11)}+\frac{42(5d}{f}$$

```
[Out] (d*(f*x)^(1 + m))/(f*(1 + m)) + ((10*d + e)*(f*x)^(3 + m))/(f^3*(3 + m)) +
(5*(9*d + 2*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (15*(8*d + 3*e)*(f*x)^(7 + m)
)/(f^7*(7 + m)) + (30*(7*d + 4*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (42*(6*d +
 5*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (42*(5*d + 6*e)*(f*x)^(13 + m))/(f^
13*(13 + m)) + (30*(4*d + 7*e)*(f*x)^(15 + m))/(f^15*(15 + m)) + (15*(3*d +
 8*e)*(f*x)^(17 + m))/(f^17*(17 + m)) + (5*(2*d + 9*e)*(f*x)^(19 + m))/(f^1
9*(19 + m)) + ((d + 10*e)*(f*x)^(21 + m))/(f^21*(21 + m)) + (e*(f*x)^(23 +
m))/(f^23*(23 + m))
```

Rubi [A] time = 0.160783, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {28, 448}

$$\frac{(10d+e)(fx)^{m+3}}{f^3(m+3)}+\frac{5(9d+2e)(fx)^{m+5}}{f^5(m+5)}+\frac{15(8d+3e)(fx)^{m+7}}{f^7(m+7)}+\frac{30(7d+4e)(fx)^{m+9}}{f^9(m+9)}+\frac{42(6d+5e)(fx)^{m+11}}{f^{11}(m+11)}+\frac{42(5d}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```


```
[Out] (d*(f*x)^(1 + m))/(f*(1 + m)) + ((10*d + e)*(f*x)^(3 + m))/(f^3*(3 + m)) +
(5*(9*d + 2*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (15*(8*d + 3*e)*(f*x)^(7 + m)
)/(f^7*(7 + m)) + (30*(7*d + 4*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (42*(6*d +
 5*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (42*(5*d + 6*e)*(f*x)^(13 + m))/(f^
13*(13 + m)) + (30*(4*d + 7*e)*(f*x)^(15 + m))/(f^15*(15 + m)) + (15*(3*d +
 8*e)*(f*x)^(17 + m))/(f^17*(17 + m)) + (5*(2*d + 9*e)*(f*x)^(19 + m))/(f^1
9*(19 + m)) + ((d + 10*e)*(f*x)^(21 + m))/(f^21*(21 + m)) + (e*(f*x)^(23 +
m))/(f^23*(23 + m))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5 dx = \int (fx)^m \left(1+x^2\right)^{10}\left(d+ex^2\right) dx$$
$$= \int \left(d(fx)^m + \frac{(10d+e)(fx)^{2+m}}{f^2} + \frac{5(9d+2e)(fx)^{4+m}}{f^4} + \frac{15(8d+3e)(fx)^{6+m}}{f^6} + 30\right.$$
$$= \frac{d(fx)^{1+m}}{f(1+m)} + \frac{(10d+e)(fx)^{3+m}}{f^3(3+m)} + \frac{5(9d+2e)(fx)^{5+m}}{f^5(5+m)} + \frac{15(8d+3e)(fx)^{7+m}}{f^7(7+m)} + 30($$

Mathematica [A] time = 0.170736, size = 189, normalized size = 0.7

$$x(fx)^m \left(\frac{x^{20}(d+10e)}{m+21} + \frac{5x^{18}(2d+9e)}{m+19} + \frac{15x^{16}(3d+8e)}{m+17} + \frac{30x^{14}(4d+7e)}{m+15} + \frac{42x^{12}(5d+6e)}{m+13} + \frac{42x^{10}(6d+5e)}{m+11} + \frac{30x^8(7d}{m+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x*(f*x)^m*(d/(1 + m) + ((10*d + e)*x^2)/(3 + m) + (5*(9*d + 2*e)*x^4)/(5 +
m) + (15*(8*d + 3*e)*x^6)/(7 + m) + (30*(7*d + 4*e)*x^8)/(9 + m) + (42*(6*d
 + 5*e)*x^10)/(11 + m) + (42*(5*d + 6*e)*x^12)/(13 + m) + (30*(4*d + 7*e)*x
^14)/(15 + m) + (15*(3*d + 8*e)*x^16)/(17 + m) + (5*(2*d + 9*e)*x^18)/(19 +
 m) + ((d + 10*e)*x^20)/(21 + m) + (e*x^22)/(23 + m))
```

Maple [B] time = 0.019, size = 2295, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] (f*x)^m*(e*m^11*x^22+121*e*m^10*x^22+d*m^11*x^20+10*e*m^11*x^20+6435*e*m^9*
x^22+123*d*m^10*x^20+1230*e*m^10*x^20+197835*e*m^8*x^22+10*d*m^11*x^18+6635
*d*m^9*x^20+45*e*m^11*x^18+66350*e*m^9*x^20+3889578*e*m^7*x^22+1250*d*m^10*
x^18+206505*d*m^8*x^20+5625*e*m^10*x^18+2065050*e*m^8*x^20+51069018*e*m^6*x
^22+45*d*m^11*x^16+68430*d*m^9*x^18+4103178*d*m^7*x^20+120*e*m^11*x^16+3079
35*e*m^9*x^18+41031780*e*m^7*x^20+453714470*e*m^5*x^22+5715*d*m^10*x^16+215
8230*d*m^8*x^18+54362574*d*m^6*x^20+15240*e*m^10*x^16+9712035*e*m^8*x^18+54
3625740*e*m^6*x^20+2702025590*e*m^4*x^22+120*d*m^11*x^14+317655*d*m^9*x^16+
43391460*d*m^7*x^18+486687830*d*m^5*x^20+210*e*m^11*x^14+847080*e*m^9*x^16+
195261570*e*m^7*x^18+4866878300*e*m^5*x^20+10431670821*e*m^3*x^22+15480*d*m
^10*x^14+10162665*d*m^8*x^16+580855380*d*m^6*x^18+2917013970*d*m^4*x^20+270
90*e*m^10*x^14+27100440*e*m^8*x^16+2613849210*e*m^6*x^18+29170139700*e*m^4*
x^20+24372200061*e*m^2*x^22+210*d*m^11*x^12+873960*d*m^9*x^14+207024930*d*m
^7*x^16+5246766620*d*m^5*x^18+11320966021*d*m^3*x^20+252*e*m^11*x^12+152943
0*e*m^9*x^14+552066480*e*m^7*x^16+23610449790*e*m^5*x^18+113209660210*e*m^3
*x^20+29985521895*e*m*x^22+27510*d*m^10*x^12+28391400*d*m^8*x^14+2804395230
*d*m^6*x^16+31686018220*d*m^4*x^18+26560342503*d*m^2*x^20+33012*e*m^10*x^12
+49684950*e*m^8*x^14+7478387280*e*m^6*x^16+142587081990*e*m^4*x^18+26560342
5030*e*m^2*x^20+13749310575*e*x^22+252*d*m^11*x^10+1578150*d*m^9*x^12+58690
2960*d*m^7*x^14+25598865870*d*m^5*x^16+123748247730*d*m^3*x^18+32778930735*
d*m*x^20+210*e*m^11*x^10+1893780*e*m^9*x^12+1027080180*e*m^7*x^14+682636423
20*e*m^5*x^16+556867114785*e*m^3*x^18+327789307350*e*m*x^20+33516*d*m^10*x^
10+52110450*d*m^8*x^12+8059973040*d*m^6*x^14+156004908210*d*m^4*x^16+291789
```

```
582570*d*m^2*x^18+15058768725*d*x^20+27930*e*m^10*x^10+62532540*e*m^8*x^12+
14104952820*e*m^6*x^14+416013088560*e*m^4*x^16+1313053121565*e*m^2*x^18+150
587687250*e*x^20+210*d*m^11*x^8+1954260*d*m^9*x^10+1094918580*d*m^7*x^12+74
496630480*d*m^5*x^14+613938233025*d*m^3*x^16+361459164150*d*m*x^18+120*e*m^
11*x^8+1628550*e*m^9*x^10+1313902296*e*m^7*x^12+130369103340*e*m^5*x^14+163
7168621400*e*m^3*x^16+1626566238675*e*m*x^18+28350*d*m^10*x^8+65654820*d*m^
8*x^10+15277213980*d*m^6*x^12+459045550800*d*m^4*x^14+1456578341055*d*m^2*x
^16+166439022750*d*x^18+16200*e*m^10*x^8+54712350*e*m^8*x^10+18332656776*e*
m^6*x^12+803329713900*e*m^4*x^14+3884208909480*e*m^2*x^16+748975602375*e*x^
18+120*d*m^11*x^6+1680630*d*m^9*x^8+1404622296*d*m^7*x^10+143339613900*d*m^
5*x^12+1823707864920*d*m^3*x^14+1812743750475*d*m*x^16+45*e*m^11*x^6+960360
*e*m^9*x^8+1170518580*e*m^7*x^10+172007536680*e*m^5*x^12+3191488763610*e*m^
3*x^14+4833983334600*e*m*x^16+16440*d*m^10*x^6+57500730*d*m^8*x^8+199625413
68*d*m^6*x^10+895451283300*d*m^4*x^12+4360457499480*d*m^2*x^14+837090379125
*d*x^16+6165*e*m^10*x^6+32857560*e*m^8*x^8+16635451140*e*m^6*x^10+107454153
9960*e*m^4*x^12+7630800624090*e*m^2*x^14+2232241011000*e*x^16+45*d*m^11*x^4
+991080*d*m^9*x^6+1254847860*d*m^7*x^8+190744119720*d*m^5*x^10+360056778921
0*d*m^3*x^12+5458672303560*d*m*x^14+10*e*m^11*x^4+371655*e*m^9*x^6+71705592
0*e*m^7*x^8+158953433100*e*m^5*x^10+4320681347052*e*m^3*x^12+9552676531230*
e*m*x^14+6255*d*m^10*x^4+34563240*d*m^8*x^6+18217524780*d*m^6*x^8+121245419
9880*d*m^4*x^10+8695750818510*d*m^2*x^12+2529873145800*d*x^14+1390*e*m^10*x
^4+12961215*e*m^8*x^6+10410014160*e*m^6*x^8+1010378499900*e*m^4*x^10+104349
00982212*e*m^2*x^12+4427278005150*e*x^14+10*d*m^11*x^2+383535*d*m^9*x^4+770
831280*d*m^7*x^6+177985672620*d*m^5*x^8+4952725167852*d*m^3*x^10+1096992525
1950*d*m*x^12+e*m^11*x^2+85230*e*m^9*x^4+289061730*e*m^7*x^6+101706098640*e
*m^5*x^8+4127270973210*e*m^3*x^10+13163910302340*e*m*x^12+1410*d*m^10*x^2+1
3645125*d*m^8*x^4+11467698480*d*m^6*x^6+1156995210420*d*m^4*x^8+12123781647
516*d*m^2*x^10+5108397698250*d*x^12+141*e*m^10*x^2+3032250*e*m^8*x^4+430038
6930*e*m^6*x^6+661140120240*e*m^4*x^8+10103151372930*e*m^2*x^10+61300772379
00*e*x^12+d*m^11+87950*d*m^9*x^2+311564610*d*m^7*x^4+115122336720*d*m^5*x^6
+4828477578330*d*m^3*x^8+15456024948420*d*m*x^10+8795*e*m^9*x^2+69236580*e*
m^7*x^4+43170876270*e*m^5*x^6+2759130044760*e*m^3*x^8+12880020790350*e*m*x^
10+143*d*m^10+3194550*d*m^8*x^2+4765995990*d*m^6*x^4+770638650960*d*m^4*x^6
+12046833873270*d*m^2*x^8+7244636735700*d*x^10+319455*e*m^8*x^2+1059110220*
e*m^6*x^4+288989494110*e*m^4*x^6+6883905070440*e*m^2*x^8+6037197279750*e*x^
10+9075*d*m^9+74814180*d*m^7*x^2+49443604830*d*m^5*x^4+3314920570200*d*m^3*
x^6+15593181033150*d*m*x^8+7481418*e*m^7*x^2+10987467740*e*m^5*x^4+12430952
13825*e*m^3*x^6+8910389161800*e*m*x^8+336765*d*m^8+1180850580*d*m^6*x^2+343
967603850*d*m^4*x^4+8511631481880*d*m^2*x^6+7378796675250*d*x^8+118085058*e
*m^6*x^2+76437245300*e*m^4*x^4+3191861805705*e*m^2*x^6+4216455243000*e*x^8+
8103018*d*m^7+12740467100*d*m^5*x^2+1546183653345*d*m^3*x^4+11284114422600*
d*m*x^6+1274046710*e*m^5*x^2+343596367410*e*m^3*x^4+4231542908475*e*m*x^6+1
32426294*d*m^6+93153182700*d*m^4*x^2+4162610035755*d*m^2*x^4+5421156741000*
d*x^6+9315318270*e*m^4*x^2+925024452390*e*m^2*x^4+2032933777875*e*x^6+14958
75590*d*m^5+446323045810*d*m^3*x^2+5761525369635*d*m*x^4+44632304581*e*m^3*
x^2+1280338971030*e*m*x^4+11641582810*d*m^4+1304037152010*d*m^2*x^2+2846107
289025*d*x^4+130403715201*e*m^2*x^2+632468286450*e*x^4+60936676581*d*m^3+19
93349776950*d*m*x^2+199334977695*e*m*x^2+203363952363*d*m^2+1054113810750*d
*x^2+105411381075*e*x^2+387182170935*d*m+316234143225*d)*x/(1+m)/(3+m)/(5+m
)/(7+m)/(9+m)/(11+m)/(13+m)/(15+m)/(17+m)/(19+m)/(21+m)/(23+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.77115, size = 5265, normalized size = 19.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] ((e*m^11 + 121*e*m^10 + 6435*e*m^9 + 197835*e*m^8 + 3889578*e*m^7 + 5106901
8*e*m^6 + 453714470*e*m^5 + 2702025590*e*m^4 + 10431670821*e*m^3 + 24372200
061*e*m^2 + 29985521895*e*m + 13749310575*e)*x^23 + ((d + 10*e)*m^11 + 123*
(d + 10*e)*m^10 + 6635*(d + 10*e)*m^9 + 206505*(d + 10*e)*m^8 + 4103178*(d
+ 10*e)*m^7 + 54362574*(d + 10*e)*m^6 + 486687830*(d + 10*e)*m^5 + 29170139
70*(d + 10*e)*m^4 + 11320966021*(d + 10*e)*m^3 + 26560342503*(d + 10*e)*m^2
 + 32778930735*(d + 10*e)*m + 15058768725*d + 150587687250*e)*x^21 + 5*((2*
d + 9*e)*m^11 + 125*(2*d + 9*e)*m^10 + 6843*(2*d + 9*e)*m^9 + 215823*(2*d +
 9*e)*m^8 + 4339146*(2*d + 9*e)*m^7 + 58085538*(2*d + 9*e)*m^6 + 524676662*
(2*d + 9*e)*m^5 + 3168601822*(2*d + 9*e)*m^4 + 12374824773*(2*d + 9*e)*m^3
+ 29178958257*(2*d + 9*e)*m^2 + 36145916415*(2*d + 9*e)*m + 33287804550*d +
 149795120475*e)*x^19 + 15*((3*d + 8*e)*m^11 + 127*(3*d + 8*e)*m^10 + 7059*
(3*d + 8*e)*m^9 + 225837*(3*d + 8*e)*m^8 + 4600554*(3*d + 8*e)*m^7 + 623198
94*(3*d + 8*e)*m^6 + 568863686*(3*d + 8*e)*m^5 + 3466775738*(3*d + 8*e)*m^4
 + 13643071845*(3*d + 8*e)*m^3 + 32368407579*(3*d + 8*e)*m^2 + 40283194455*
(3*d + 8*e)*m + 55806025275*d + 148816067400*e)*x^17 + 30*((4*d + 7*e)*m^11
 + 129*(4*d + 7*e)*m^10 + 7283*(4*d + 7*e)*m^9 + 236595*(4*d + 7*e)*m^8 + 4
890858*(4*d + 7*e)*m^7 + 67166442*(4*d + 7*e)*m^6 + 620805254*(4*d + 7*e)*m
^5 + 3825379590*(4*d + 7*e)*m^4 + 15197565541*(4*d + 7*e)*m^3 + 36337145829
*(4*d + 7*e)*m^2 + 45488935863*(4*d + 7*e)*m + 84329104860*d + 147575933505
*e)*x^15 + 42*((5*d + 6*e)*m^11 + 131*(5*d + 6*e)*m^10 + 7515*(5*d + 6*e)*m
^9 + 248145*(5*d + 6*e)*m^8 + 5213898*(5*d + 6*e)*m^7 + 72748638*(5*d + 6*e
)*m^6 + 682569590*(5*d + 6*e)*m^5 + 4264053730*(5*d + 6*e)*m^4 + 1714556090
1*(5*d + 6*e)*m^3 + 41408337231*(5*d + 6*e)*m^2 + 52237739295*(5*d + 6*e)*m
 + 121628516625*d + 145954219950*e)*x^13 + 42*((6*d + 5*e)*m^11 + 133*(6*d
+ 5*e)*m^10 + 7755*(6*d + 5*e)*m^9 + 260535*(6*d + 5*e)*m^8 + 5573898*(6*d
+ 5*e)*m^7 + 79216434*(6*d + 5*e)*m^6 + 756921110*(6*d + 5*e)*m^5 + 4811326
190*(6*d + 5*e)*m^4 + 19653671301*(6*d + 5*e)*m^3 + 48110244633*(6*d + 5*e)
*m^2 + 61333432335*(6*d + 5*e)*m + 172491350850*d + 143742792375*e)*x^11 +
30*((7*d + 4*e)*m^11 + 135*(7*d + 4*e)*m^10 + 8003*(7*d + 4*e)*m^9 + 273813
*(7*d + 4*e)*m^8 + 5975466*(7*d + 4*e)*m^7 + 86750118*(7*d + 4*e)*m^6 + 847
550822*(7*d + 4*e)*m^5 + 5509501002*(7*d + 4*e)*m^4 + 22992750373*(7*d + 4*
e)*m^3 + 57365875587*(7*d + 4*e)*m^2 + 74253243015*(7*d + 4*e)*m + 24595988
9175*d + 140548508100*e)*x^9 + 15*((8*d + 3*e)*m^11 + 137*(8*d + 3*e)*m^10
+ 8259*(8*d + 3*e)*m^9 + 288027*(8*d + 3*e)*m^8 + 6423594*(8*d + 3*e)*m^7 +
 95564154*(8*d + 3*e)*m^6 + 959352806*(8*d + 3*e)*m^5 + 6421988758*(8*d + 3
*e)*m^4 + 27624338085*(8*d + 3*e)*m^3 + 70930262349*(8*d + 3*e)*m^2 + 94034
286855*(8*d + 3*e)*m + 361410449400*d + 135528918525*e)*x^7 + 5*((9*d + 2*e
)*m^11 + 139*(9*d + 2*e)*m^10 + 8523*(9*d + 2*e)*m^9 + 303225*(9*d + 2*e)*m
^8 + 6923658*(9*d + 2*e)*m^7 + 105911022*(9*d + 2*e)*m^6 + 1098746774*(9*d
+ 2*e)*m^5 + 7643724530*(9*d + 2*e)*m^4 + 34359636741*(9*d + 2*e)*m^3 + 925
02445239*(9*d + 2*e)*m^2 + 128033897103*(9*d + 2*e)*m + 569221457805*d + 12
6493657290*e)*x^5 + ((10*d + e)*m^11 + 141*(10*d + e)*m^10 + 8795*(10*d + e
)*m^9 + 319455*(10*d + e)*m^8 + 7481418*(10*d + e)*m^7 + 118085058*(10*d +
e)*m^6 + 1274046710*(10*d + e)*m^5 + 9315318270*(10*d + e)*m^4 + 4463230458
1*(10*d + e)*m^3 + 130403715201*(10*d + e)*m^2 + 199334977695*(10*d + e)*m
```

```
+ 1054113810750*d + 105411381075*e)*x^3 + (d*m^11 + 143*d*m^10 + 9075*d*m^9
 + 336765*d*m^8 + 8103018*d*m^7 + 132426294*d*m^6 + 1495875590*d*m^5 + 1164
1582810*d*m^4 + 60936676581*d*m^3 + 203363952363*d*m^2 + 387182170935*d*m +
 316234143225*d)*x)*(f*x)^m/(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 843
9783*m^8 + 140529312*m^7 + 1628301884*m^6 + 13137458400*m^5 + 72578259391*m
^4 + 264300628944*m^3 + 590546123298*m^2 + 703416314160*m + 316234143225)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30771, size = 5065, normalized size = 18.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] ((f*x)^m*m^11*x^23*e + 121*(f*x)^m*m^10*x^23*e + (f*x)^m*d*m^11*x^21 + 10*(
f*x)^m*m^11*x^21*e + 6435*(f*x)^m*m^9*x^23*e + 123*(f*x)^m*d*m^10*x^21 + 12
30*(f*x)^m*m^10*x^21*e + 197835*(f*x)^m*m^8*x^23*e + 10*(f*x)^m*d*m^11*x^19
 + 6635*(f*x)^m*d*m^9*x^21 + 45*(f*x)^m*m^11*x^19*e + 66350*(f*x)^m*m^9*x^2
1*e + 3889578*(f*x)^m*m^7*x^23*e + 1250*(f*x)^m*d*m^10*x^19 + 206505*(f*x)^
m*d*m^8*x^21 + 5625*(f*x)^m*m^10*x^19*e + 2065050*(f*x)^m*m^8*x^21*e + 5106
9018*(f*x)^m*m^6*x^23*e + 45*(f*x)^m*d*m^11*x^17 + 68430*(f*x)^m*d*m^9*x^19
 + 4103178*(f*x)^m*d*m^7*x^21 + 120*(f*x)^m*m^11*x^17*e + 307935*(f*x)^m*m^
9*x^19*e + 41031780*(f*x)^m*m^7*x^21*e + 453714470*(f*x)^m*m^5*x^23*e + 571
5*(f*x)^m*d*m^10*x^17 + 2158230*(f*x)^m*d*m^8*x^19 + 54362574*(f*x)^m*d*m^6
*x^21 + 15240*(f*x)^m*m^10*x^17*e + 9712035*(f*x)^m*m^8*x^19*e + 543625740*
(f*x)^m*m^6*x^21*e + 2702025590*(f*x)^m*m^4*x^23*e + 120*(f*x)^m*d*m^11*x^1
5 + 317655*(f*x)^m*d*m^9*x^17 + 43391460*(f*x)^m*d*m^7*x^19 + 486687830*(f*
x)^m*d*m^5*x^21 + 210*(f*x)^m*m^11*x^15*e + 847080*(f*x)^m*m^9*x^17*e + 195
261570*(f*x)^m*m^7*x^19*e + 4866878300*(f*x)^m*m^5*x^21*e + 10431670821*(f*
x)^m*m^3*x^23*e + 15480*(f*x)^m*d*m^10*x^15 + 10162665*(f*x)^m*d*m^8*x^17 +
 580855380*(f*x)^m*d*m^6*x^19 + 2917013970*(f*x)^m*d*m^4*x^21 + 27090*(f*x)
^m*m^10*x^15*e + 27100440*(f*x)^m*m^8*x^17*e + 2613849210*(f*x)^m*m^6*x^19*
e + 29170139700*(f*x)^m*m^4*x^21*e + 24372200061*(f*x)^m*m^2*x^23*e + 210*(
f*x)^m*d*m^11*x^13 + 873960*(f*x)^m*d*m^9*x^15 + 207024930*(f*x)^m*d*m^7*x^
17 + 5246766620*(f*x)^m*d*m^5*x^19 + 11320966021*(f*x)^m*d*m^3*x^21 + 252*(
f*x)^m*m^11*x^13*e + 1529430*(f*x)^m*m^9*x^15*e + 552066480*(f*x)^m*m^7*x^1
7*e + 23610449790*(f*x)^m*m^5*x^19*e + 113209660210*(f*x)^m*m^3*x^21*e + 29
985521895*(f*x)^m*m*x^23*e + 27510*(f*x)^m*d*m^10*x^13 + 28391400*(f*x)^m*d
*m^8*x^15 + 2804395230*(f*x)^m*d*m^6*x^17 + 31686018220*(f*x)^m*d*m^4*x^19
+ 26560342503*(f*x)^m*d*m^2*x^21 + 33012*(f*x)^m*m^10*x^13*e + 49684950*(f*
x)^m*m^8*x^15*e + 7478387280*(f*x)^m*m^6*x^17*e + 142587081990*(f*x)^m*m^4*
x^19*e + 265603425030*(f*x)^m*m^2*x^21*e + 13749310575*(f*x)^m*x^23*e + 252
*(f*x)^m*d*m^11*x^11 + 1578150*(f*x)^m*d*m^9*x^13 + 586902960*(f*x)^m*d*m^7
```

x^15 + 25598865870(f*x)^m*d*m^5*x^17 + 123748247730*(f*x)^m*d*m^3*x^19 + 32778930735*(f*x)^m*d*m*x^21 + 210*(f*x)^m*m^11*x^11*e + 1893780*(f*x)^m*m^9*x^13*e + 1027080180*(f*x)^m*m^7*x^15*e + 68263642320*(f*x)^m*m^5*x^17*e + 556867114785*(f*x)^m*m^3*x^19*e + 327789307350*(f*x)^m*m*x^21*e + 33516*(f*x)^m*d*m^10*x^11 + 52110450*(f*x)^m*d*m^8*x^13 + 8059973040*(f*x)^m*d*m^6*x^15 + 156004908210*(f*x)^m*d*m^4*x^17 + 291789582570*(f*x)^m*d*m^2*x^19 + 15058768725*(f*x)^m*d*x^21 + 27930*(f*x)^m*m^10*x^11*e + 62532540*(f*x)^m*m^8*x^13*e + 14104952820*(f*x)^m*m^6*x^15*e + 416013088560*(f*x)^m*m^4*x^17*e + 1313053121565*(f*x)^m*m^2*x^19*e + 150587687250*(f*x)^m*x^21*e + 210*(f*x)^m*d*m^11*x^9 + 1954260*(f*x)^m*d*m^9*x^11 + 1094918580*(f*x)^m*d*m^7*x^13 + 74496630480*(f*x)^m*d*m^5*x^15 + 613938233025*(f*x)^m*d*m^3*x^17 + 361459164150*(f*x)^m*d*m*x^19 + 120*(f*x)^m*m^11*x^9*e + 1628550*(f*x)^m*m^9*x^11*e + 1313902296*(f*x)^m*m^7*x^13*e + 130369103340*(f*x)^m*m^5*x^15*e + 1637168621400*(f*x)^m*m^3*x^17*e + 1626566238675*(f*x)^m*m*x^19*e + 28350*(f*x)^m*d*m^10*x^9 + 65654820*(f*x)^m*d*m^8*x^11 + 15277213980*(f*x)^m*d*m^6*x^13 + 459045550800*(f*x)^m*d*m^4*x^15 + 1456578341055*(f*x)^m*d*m^2*x^17 + 166439022750*(f*x)^m*d*x^19 + 16200*(f*x)^m*m^10*x^9*e + 54712350*(f*x)^m*m^8*x^11*e + 18332656776*(f*x)^m*m^6*x^13*e + 803329713900*(f*x)^m*m^4*x^15*e + 3884208909480*(f*x)^m*m^2*x^17*e + 748975602375*(f*x)^m*x^19*e + 120*(f*x)^m*d*m^11*x^7 + 1680630*(f*x)^m*d*m^9*x^9 + 1404622296*(f*x)^m*d*m^7*x^11 + 143339613900*(f*x)^m*d*m^5*x^13 + 1823707864920*(f*x)^m*d*m^3*x^15 + 1812743750475*(f*x)^m*d*m*x^17 + 45*(f*x)^m*m^11*x^7*e + 960360*(f*x)^m*m^9*x^9*e + 1170518580*(f*x)^m*m^7*x^11*e + 172007536680*(f*x)^m*m^5*x^13*e + 3191488763610*(f*x)^m*m^3*x^15*e + 4833983334600*(f*x)^m*m*x^17*e + 16440*(f*x)^m*d*m^10*x^7 + 57500730*(f*x)^m*d*m^8*x^9 + 19962541368*(f*x)^m*d*m^6*x^11 + 895451283300*(f*x)^m*d*m^4*x^13 + 4360457499480*(f*x)^m*d*m^2*x^15 + 837090379125*(f*x)^m*d*x^17 + 6165*(f*x)^m*m^10*x^7*e + 32857560*(f*x)^m*m^8*x^9*e + 16635451140*(f*x)^m*m^6*x^11*e + 1074541539960*(f*x)^m*m^4*x^13*e + 7630800624090*(f*x)^m*m^2*x^15*e + 2232241011000*(f*x)^m*x^17*e + 45*(f*x)^m*d*m^11*x^5 + 991080*(f*x)^m*d*m^9*x^7 + 1254847860*(f*x)^m*d*m^7*x^9 + 190744119720*(f*x)^m*d*m^5*x^11 + 3600567789210*(f*x)^m*d*m^3*x^13 + 5458672303560*(f*x)^m*d*m*x^15 + 10*(f*x)^m*m^11*x^5*e + 371655*(f*x)^m*m^9*x^7*e + 717055920*(f*x)^m*m^7*x^9*e + 158953433100*(f*x)^m*m^5*x^11*e + 4320681347052*(f*x)^m*m^3*x^13*e + 9552676531230*(f*x)^m*m*x^15*e + 6255*(f*x)^m*d*m^10*x^5 + 34563240*(f*x)^m*d*m^8*x^7 + 18217524780*(f*x)^m*d*m^6*x^9 + 1212454199880*(f*x)^m*d*m^4*x^11 + 8695750818510*(f*x)^m*d*m^2*x^13 + 2529873145800*(f*x)^m*d*x^15 + 1390*(f*x)^m*m^10*x^5*e + 12961215*(f*x)^m*m^8*x^7*e + 10410014160*(f*x)^m*m^6*x^9*e + 1010378499900*(f*x)^m*m^4*x^11*e + 10434900982212*(f*x)^m*m^2*x^13*e + 4427278005150*(f*x)^m*x^15*e + 10*(f*x)^m*d*m^11*x^3 + 383535*(f*x)^m*d*m^9*x^5 + 770831280*(f*x)^m*d*m^7*x^7 + 177985672620*(f*x)^m*d*m^5*x^9 + 4952725167852*(f*x)^m*d*m^3*x^11 + 10969925251950*(f*x)^m*d*m*x^13 + (f*x)^m*m^11*x^3*e + 85230*(f*x)^m*m^9*x^5*e + 289061730*(f*x)^m*m^7*x^7*e + 101706098640*(f*x)^m*m^5*x^9*e + 4127270973210*(f*x)^m*m^3*x^11*e + 13163910302340*(f*x)^m*m*x^13*e + 1410*(f*x)^m*d*m^10*x^3 + 13645125*(f*x)^m*d*m^8*x^5 + 11467698480*(f*x)^m*d*m^6*x^7 + 1156995210420*(f*x)^m*d*m^4*x^9 + 12123781647516*(f*x)^m*d*m^2*x^11 + 5108397698250*(f*x)^m*d*x^13 + 141*(f*x)^m*m^10*x^3*e + 3032250*(f*x)^m*m^8*x^5*e + 4300386930*(f*x)^m*m^6*x^7*e + 661140120240*(f*x)^m*m^4*x^9*e + 10103151372930*(f*x)^m*m^2*x^11*e + 6130077237900*(f*x)^m*x^13*e + (f*x)^m*d*m^11*x + 87950*(f*x)^m*d*m^9*x^3 + 311564610*(f*x)^m*d*m^7*x^5 + 115122336720*(f*x)^m*d*m^5*x^7 + 4828477578330*(f*x)^m*d*m^3*x^9 + 15456024948420*(f*x)^m*d*m*x^11 + 8795*(f*x)^m*m^9*x^3*e + 69236580*(f*x)^m*m^7*x^5*e + 43170876270*(f*x)^m*m^5*x^7*e + 2759130044760*(f*x)^m*m^3*x^9*e + 12880020790350*(f*x)^m*m*x^11*e + 143*(f*x)^m*d*m^10*x + 3194550*(f*x)^m*d*m^8*x^3 + 4765995990*(f*x)^m*d*m^6*x^5 + 770638650960*(f*x)^m*d*m^4*x^7 + 12046833873270*(f*x)^m*d*m^2*x^9 + 7244636735700*(f*x)^m*d*x^11 + 319455*(f*x)^m*m^8*x^3*e + 1059110220*(f*x)^m*m^6*x^5*e + 288989494110*(f*x)^m*m^4*x^7*e + 6883905070440*(f*x)^m*m^2*x^9*e + 6037197279750*(f*x)^m*x^11*e + 9075*(f*x)^m*d*m^9*x + 74814180*(f*x)^m*d*m^7*x^3 + 49443604830*(f*x)^m*d*m^5*x^5 + 3314920570200*(f*x)^m*d*m^3

```
*x^7 + 15593181033150*(f*x)^m*d*m*x^9 + 7481418*(f*x)^m*m^7*x^3*e + 1098746
7740*(f*x)^m*m^5*x^5*e + 1243095213825*(f*x)^m*m^3*x^7*e + 8910389161800*(f
*x)^m*m*x^9*e + 336765*(f*x)^m*d*m^8*x + 1180850580*(f*x)^m*d*m^6*x^3 + 343
967603850*(f*x)^m*d*m^4*x^5 + 8511631481880*(f*x)^m*d*m^2*x^7 + 73787966752
50*(f*x)^m*d*x^9 + 118085058*(f*x)^m*m^6*x^3*e + 76437245300*(f*x)^m*m^4*x^
5*e + 3191861805705*(f*x)^m*m^2*x^7*e + 4216455243000*(f*x)^m*x^9*e + 81030
18*(f*x)^m*d*m^7*x + 12740467100*(f*x)^m*d*m^5*x^3 + 1546183653345*(f*x)^m*
d*m^3*x^5 + 11284114422600*(f*x)^m*d*m*x^7 + 1274046710*(f*x)^m*m^5*x^3*e +
 343596367410*(f*x)^m*m^3*x^5*e + 4231542908475*(f*x)^m*m*x^7*e + 132426294
*(f*x)^m*d*m^6*x + 93153182700*(f*x)^m*d*m^4*x^3 + 4162610035755*(f*x)^m*d*
m^2*x^5 + 5421156741000*(f*x)^m*d*x^7 + 9315318270*(f*x)^m*m^4*x^3*e + 9250
24452390*(f*x)^m*m^2*x^5*e + 2032933777875*(f*x)^m*x^7*e + 1495875590*(f*x)
^m*d*m^5*x + 446323045810*(f*x)^m*d*m^3*x^3 + 5761525369635*(f*x)^m*d*m*x^5
 + 44632304581*(f*x)^m*m^3*x^3*e + 1280338971030*(f*x)^m*m*x^5*e + 11641582
810*(f*x)^m*d*m^4*x + 1304037152010*(f*x)^m*d*m^2*x^3 + 2846107289025*(f*x)
^m*d*x^5 + 130403715201*(f*x)^m*m^2*x^3*e + 632468286450*(f*x)^m*x^5*e + 60
936676581*(f*x)^m*d*m^3*x + 1993349776950*(f*x)^m*d*m*x^3 + 199334977695*(f
*x)^m*m*x^3*e + 203363952363*(f*x)^m*d*m^2*x + 1054113810750*(f*x)^m*d*x^3
+ 105411381075*(f*x)^m*x^3*e + 387182170935*(f*x)^m*d*m*x + 316234143225*(f
*x)^m*d*x)/(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529
312*m^7 + 1628301884*m^6 + 13137458400*m^5 + 72578259391*m^4 + 264300628944
*m^3 + 590546123298*m^2 + 703416314160*m + 316234143225)
```

3.56 $\int x^5 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=63

$$\frac{1}{26}\left(x^2+1\right)^{13}(d-3e)-\frac{1}{24}\left(x^2+1\right)^{12}(2d-3e)+\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{28}e\left(x^2+1\right)^{14}$$

```
[Out] ((d - e)*(1 + x^2)^11)/22 - ((2*d - 3*e)*(1 + x^2)^12)/24 + ((d - 3*e)*(1 +
 x^2)^13)/26 + (e*(1 + x^2)^14)/28
```

Rubi [A] time = 0.196701, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {28, 446, 76}

$$\frac{1}{26}\left(x^2+1\right)^{13}(d-3e)-\frac{1}{24}\left(x^2+1\right)^{12}(2d-3e)+\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{28}e\left(x^2+1\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] ((d - e)*(1 + x^2)^11)/22 - ((2*d - 3*e)*(1 + x^2)^12)/24 + ((d - 3*e)*(1 +
 x^2)^13)/26 + (e*(1 + x^2)^14)/28
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\int x^5\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x^5\left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int x^2(1+x)^{10}(d+ex)\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left((d-e)(1+x)^{10}+(-2d+3e)(1+x)^{11}+(d-3e)(1+x)^{12}+e(1+x)^{13}\right)\right.\\
&= \frac{1}{22}(d-e)\left(1+x^2\right)^{11}-\frac{1}{24}(2d-3e)\left(1+x^2\right)^{12}+\frac{1}{26}(d-3e)\left(1+x^2\right)^{13}+\frac{1}{28}e\left(1+x^2\right.
\end{aligned}$$

Mathematica [B] time = 0.0243831, size = 153, normalized size = 2.43

$$\frac{1}{26}x^{26}(d+10e)+\frac{5}{24}x^{24}(2d+9e)+\frac{15}{22}x^{22}(3d+8e)+\frac{3}{2}x^{20}(4d+7e)+\frac{7}{3}x^{18}(5d+6e)+\frac{21}{8}x^{16}(6d+5e)+\frac{15}{7}x^{14}(7d+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^6)/6 + ((10*d + e)*x^8)/8 + ((9*d + 2*e)*x^10)/2 + (5*(8*d + 3*e)*x^12
)/4 + (15*(7*d + 4*e)*x^14)/7 + (21*(6*d + 5*e)*x^16)/8 + (7*(5*d + 6*e)*x^
18)/3 + (3*(4*d + 7*e)*x^20)/2 + (15*(3*d + 8*e)*x^22)/22 + (5*(2*d + 9*e)*
x^24)/24 + ((d + 10*e)*x^26)/26 + (e*x^28)/28
```

Maple [B] time = 0.002, size = 130, normalized size = 2.1

$$\frac{ex^{28}}{28}+\frac{(d+10\,e)\,x^{26}}{26}+\frac{(10\,d+45\,e)\,x^{24}}{24}+\frac{(45\,d+120\,e)\,x^{22}}{22}+\frac{(120\,d+210\,e)\,x^{20}}{20}+\frac{(210\,d+252\,e)\,x^{18}}{18}+\frac{(252\,d+2}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/28*e*x^28+1/26*(d+10*e)*x^26+1/24*(10*d+45*e)*x^24+1/22*(45*d+120*e)*x^22
+1/20*(120*d+210*e)*x^20+1/18*(210*d+252*e)*x^18+1/16*(252*d+210*e)*x^16+1/
14*(210*d+120*e)*x^14+1/12*(120*d+45*e)*x^12+1/10*(45*d+10*e)*x^10+1/8*(10*
d+e)*x^8+1/6*d*x^6
```

Maxima [B] time = 0.941249, size = 174, normalized size = 2.76

$$\frac{1}{28}\,ex^{28}+\frac{1}{26}\,(d+10\,e)x^{26}+\frac{5}{24}\,(2\,d+9\,e)x^{24}+\frac{15}{22}\,(3\,d+8\,e)x^{22}+\frac{3}{2}\,(4\,d+7\,e)x^{20}+\frac{7}{3}\,(5\,d+6\,e)x^{18}+\frac{21}{8}\,(6\,d+5\,e)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/28*e*x^28 + 1/26*(d + 10*e)*x^26 + 5/24*(2*d + 9*e)*x^24 + 15/22*(3*d + 8
*e)*x^22 + 3/2*(4*d + 7*e)*x^20 + 7/3*(5*d + 6*e)*x^18 + 21/8*(6*d + 5*e)*x
^16 + 15/7*(7*d + 4*e)*x^14 + 5/4*(8*d + 3*e)*x^12 + 1/2*(9*d + 2*e)*x^10 +
 1/8*(10*d + e)*x^8 + 1/6*d*x^6
```

Fricas [B] time = 1.27951, size = 390, normalized size = 6.19

$$\frac{1}{28}x^{28}e+\frac{5}{13}x^{26}e+\frac{1}{26}x^{26}d+\frac{15}{8}x^{24}e+\frac{5}{12}x^{24}d+\frac{60}{11}x^{22}e+\frac{45}{22}x^{22}d+\frac{21}{2}x^{20}e+6x^{20}d+14x^{18}e+\frac{35}{3}x^{18}d+\frac{105}{8}x^{16}e+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/28*x^28*e + 5/13*x^26*e + 1/26*x^26*d + 15/8*x^24*e + 5/12*x^24*d + 60/11
*x^22*e + 45/22*x^22*d + 21/2*x^20*e + 6*x^20*d + 14*x^18*e + 35/3*x^18*d +
```

```
 105/8*x^16*e + 63/4*x^16*d + 60/7*x^14*e + 15*x^14*d + 15/4*x^12*e + 10*x^
12*d + x^10*e + 9/2*x^10*d + 1/8*x^8*e + 5/4*x^8*d + 1/6*x^6*d
```

Sympy [B] time = 0.09754, size = 134, normalized size = 2.13

$$\frac{dx^6}{6} + \frac{ex^{28}}{28} + x^{26}\left(\frac{d}{26} + \frac{5e}{13}\right) + x^{24}\left(\frac{5d}{12} + \frac{15e}{8}\right) + x^{22}\left(\frac{45d}{22} + \frac{60e}{11}\right) + x^{20}\left(6d + \frac{21e}{2}\right) + x^{18}\left(\frac{35d}{3} + 14e\right) + x^{16}\left(\frac{63d}{4} + \frac{105}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**6/6 + e*x**28/28 + x**26*(d/26 + 5*e/13) + x**24*(5*d/12 + 15*e/8) + x
**22*(45*d/22 + 60*e/11) + x**20*(6*d + 21*e/2) + x**18*(35*d/3 + 14*e) + x
**16*(63*d/4 + 105*e/8) + x**14*(15*d + 60*e/7) + x**12*(10*d + 15*e/4) + x
**10*(9*d/2 + e) + x**8*(5*d/4 + e/8)
```

Giac [B] time = 1.11646, size = 193, normalized size = 3.06

$$\frac{1}{28}x^{28}e + \frac{1}{26}dx^{26} + \frac{5}{13}x^{26}e + \frac{5}{12}dx^{24} + \frac{15}{8}x^{24}e + \frac{45}{22}dx^{22} + \frac{60}{11}x^{22}e + 6\,dx^{20} + \frac{21}{2}x^{20}e + \frac{35}{3}dx^{18} + 14\,x^{18}e + \frac{63}{4}dx^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/28*x^28*e + 1/26*d*x^26 + 5/13*x^26*e + 5/12*d*x^24 + 15/8*x^24*e + 45/22
*d*x^22 + 60/11*x^22*e + 6*d*x^20 + 21/2*x^20*e + 35/3*d*x^18 + 14*x^18*e +
 63/4*d*x^16 + 105/8*x^16*e + 15*d*x^14 + 60/7*x^14*e + 10*d*x^12 + 15/4*x^
12*e + 9/2*d*x^10 + x^10*e + 5/4*d*x^8 + 1/8*x^8*e + 1/6*d*x^6
```

3.57 $\int x^4 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=153

$$\frac{1}{25}x^{25}(d+10e)+\frac{5}{23}x^{23}(2d+9e)+\frac{5}{7}x^{21}(3d+8e)+\frac{30}{19}x^{19}(4d+7e)+\frac{42}{17}x^{17}(5d+6e)+\frac{14}{5}x^{15}(6d+5e)+\frac{30}{13}x^{13}(7d$$

```
[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x^
11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d + 6*
e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*d +
 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27
```

Rubi [A] time = 0.11655, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {28, 448}

$$\frac{1}{25}x^{25}(d+10e)+\frac{5}{23}x^{23}(2d+9e)+\frac{5}{7}x^{21}(3d+8e)+\frac{30}{19}x^{19}(4d+7e)+\frac{42}{17}x^{17}(5d+6e)+\frac{14}{5}x^{15}(6d+5e)+\frac{30}{13}x^{13}(7d$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x^
11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d + 6*
e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*d +
 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5 \, dx &= \int x^4 \left(1+x^2\right)^{10} \left(d+ex^2\right) dx\\ &= \int \left(dx^4+(10d+e)x^6+5(9d+2e)x^8+15(8d+3e)x^{10}+30(7d+4e)x^{12}+42(6d\right.\\ &= \frac{dx^5}{5}+\frac{1}{7}(10d+e)x^7+\frac{5}{9}(9d+2e)x^9+\frac{15}{11}(8d+3e)x^{11}+\frac{30}{13}(7d+4e)x^{13}+\frac{14}{5}(6d\end{aligned}$$

Mathematica [A] time = 0.0245074, size = 153, normalized size = 1.

$$\frac{1}{25}x^{25}(d+10e)+\frac{5}{23}x^{23}(2d+9e)+\frac{5}{7}x^{21}(3d+8e)+\frac{30}{19}x^{19}(4d+7e)+\frac{42}{17}x^{17}(5d+6e)+\frac{14}{5}x^{15}(6d+5e)+\frac{30}{13}x^{13}(7d$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x^
11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d + 6*
e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*d +
 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27
```

Maple [A] time = 0.001, size = 130, normalized size = 0.9

$$\frac{ex^{27}}{27}+\frac{(d+10\,e)\,x^{25}}{25}+\frac{(10\,d+45\,e)\,x^{23}}{23}+\frac{(45\,d+120\,e)\,x^{21}}{21}+\frac{(120\,d+210\,e)\,x^{19}}{19}+\frac{(210\,d+252\,e)\,x^{17}}{17}+\frac{(252\,d+210\,e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/27*e*x^27+1/25*(d+10*e)*x^25+1/23*(10*d+45*e)*x^23+1/21*(45*d+120*e)*x^21
+1/19*(120*d+210*e)*x^19+1/17*(210*d+252*e)*x^17+1/15*(252*d+210*e)*x^15+1/
13*(210*d+120*e)*x^13+1/11*(120*d+45*e)*x^11+1/9*(45*d+10*e)*x^9+1/7*(10*d+
e)*x^7+1/5*d*x^5
```

Maxima [A] time = 0.979051, size = 174, normalized size = 1.14

$$\frac{1}{27}\,ex^{27}+\frac{1}{25}\,(d+10\,e)x^{25}+\frac{5}{23}\,(2\,d+9\,e)x^{23}+\frac{5}{7}\,(3\,d+8\,e)x^{21}+\frac{30}{19}\,(4\,d+7\,e)x^{19}+\frac{42}{17}\,(5\,d+6\,e)x^{17}+\frac{14}{5}\,(6\,d+5\,e)x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/27*e*x^27 + 1/25*(d + 10*e)*x^25 + 5/23*(2*d + 9*e)*x^23 + 5/7*(3*d + 8*e
)*x^21 + 30/19*(4*d + 7*e)*x^19 + 42/17*(5*d + 6*e)*x^17 + 14/5*(6*d + 5*e)
*x^15 + 30/13*(7*d + 4*e)*x^13 + 15/11*(8*d + 3*e)*x^11 + 5/9*(9*d + 2*e)*x
^9 + 1/7*(10*d + e)*x^7 + 1/5*d*x^5
```

Fricas [A] time = 1.26298, size = 420, normalized size = 2.75

$$\frac{1}{27}x^{27}e+\frac{2}{5}x^{25}e+\frac{1}{25}x^{25}d+\frac{45}{23}x^{23}e+\frac{10}{23}x^{23}d+\frac{40}{7}x^{21}e+\frac{15}{7}x^{21}d+\frac{210}{19}x^{19}e+\frac{120}{19}x^{19}d+\frac{252}{17}x^{17}e+\frac{210}{17}x^{17}d+14x^{15}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/27*x^27*e + 2/5*x^25*e + 1/25*x^25*d + 45/23*x^23*e + 10/23*x^23*d + 40/7
*x^21*e + 15/7*x^21*d + 210/19*x^19*e + 120/19*x^19*d + 252/17*x^17*e + 210
/17*x^17*d + 14*x^15*e + 84/5*x^15*d + 120/13*x^13*e + 210/13*x^13*d + 45/1
1*x^11*e + 120/11*x^11*d + 10/9*x^9*e + 5*x^9*d + 1/7*x^7*e + 10/7*x^7*d +
1/5*x^5*d
```

Sympy [A] time = 0.09657, size = 141, normalized size = 0.92

$$\frac{dx^5}{5} + \frac{ex^{27}}{27} + x^{25}\left(\frac{d}{25} + \frac{2e}{5}\right) + x^{23}\left(\frac{10d}{23} + \frac{45e}{23}\right) + x^{21}\left(\frac{15d}{7} + \frac{40e}{7}\right) + x^{19}\left(\frac{120d}{19} + \frac{210e}{19}\right) + x^{17}\left(\frac{210d}{17} + \frac{252e}{17}\right) + x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**5/5 + e*x**27/27 + x**25*(d/25 + 2*e/5) + x**23*(10*d/23 + 45*e/23) +
x**21*(15*d/7 + 40*e/7) + x**19*(120*d/19 + 210*e/19) + x**17*(210*d/17 + 2
52*e/17) + x**15*(84*d/5 + 14*e) + x**13*(210*d/13 + 120*e/13) + x**11*(120
*d/11 + 45*e/11) + x**9*(5*d + 10*e/9) + x**7*(10*d/7 + e/7)
```

Giac [A] time = 1.10341, size = 194, normalized size = 1.27

$$\frac{1}{27}x^{27}e + \frac{1}{25}dx^{25} + \frac{2}{5}x^{25}e + \frac{10}{23}dx^{23} + \frac{45}{23}x^{23}e + \frac{15}{7}dx^{21} + \frac{40}{7}x^{21}e + \frac{120}{19}dx^{19} + \frac{210}{19}x^{19}e + \frac{210}{17}dx^{17} + \frac{252}{17}x^{17}e +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/27*x^27*e + 1/25*d*x^25 + 2/5*x^25*e + 10/23*d*x^23 + 45/23*x^23*e + 15/7
*d*x^21 + 40/7*x^21*e + 120/19*d*x^19 + 210/19*x^19*e + 210/17*d*x^17 + 252
/17*x^17*e + 84/5*d*x^15 + 14*x^15*e + 210/13*d*x^13 + 120/13*x^13*e + 120/
11*d*x^11 + 45/11*x^11*e + 5*d*x^9 + 10/9*x^9*e + 10/7*d*x^7 + 1/7*x^7*e +
1/5*d*x^5
```

3.58 $\int x^3 \left(d+e x^2\right) \left(1+2 x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=45

$$\frac{1}{24} \left(x^2+1\right)^{12} (d-2 e)-\frac{1}{22} \left(x^2+1\right)^{11} (d-e)+\frac{1}{26} e \left(x^2+1\right)^{13}$$

```
[Out] -((d - e)*(1 + x^2)^11)/22 + ((d - 2*e)*(1 + x^2)^12)/24 + (e*(1 + x^2)^13)
/26
```

Rubi [A] time = 0.123396, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {28, 446, 76}

$$\frac{1}{24} \left(x^2+1\right)^{12} (d-2 e)-\frac{1}{22} \left(x^2+1\right)^{11} (d-e)+\frac{1}{26} e \left(x^2+1\right)^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] -((d - e)*(1 + x^2)^11)/22 + ((d - 2*e)*(1 + x^2)^12)/24 + (e*(1 + x^2)^13)
/26
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(d+e x^2\right) \left(1+2 x^2+x^4\right)^5 \, dx &= \int x^3 \left(1+x^2\right)^{10} \left(d+e x^2\right) \, dx\\
&= \frac{1}{2} \text{Subst}\left(\int x (1+x)^{10} (d+e x) \, dx,x,x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left((-d+e) (1+x)^{10}+(d-2 e) (1+x)^{11}+e (1+x)^{12}\right) \, dx,x,x^2\right)\\
&= -\frac{1}{22} (d-e) \left(1+x^2\right)^{11}+\frac{1}{24} (d-2 e) \left(1+x^2\right)^{12}+\frac{1}{26} e \left(1+x^2\right)^{13}
\end{aligned}$$

Mathematica [B] time = 0.0211536, size = 151, normalized size = 3.36

$$\frac{1}{24}x^{24}(d+10e)+\frac{5}{22}x^{22}(2d+9e)+\frac{3}{4}x^{20}(3d+8e)+\frac{5}{3}x^{18}(4d+7e)+\frac{21}{8}x^{16}(5d+6e)+3x^{14}(6d+5e)+\frac{5}{2}x^{12}(7d+4e)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^4)/4 + ((10*d + e)*x^6)/6 + (5*(9*d + 2*e)*x^8)/8 + (3*(8*d + 3*e)*x^1
0)/2 + (5*(7*d + 4*e)*x^12)/2 + 3*(6*d + 5*e)*x^14 + (21*(5*d + 6*e)*x^16)/
8 + (5*(4*d + 7*e)*x^18)/3 + (3*(3*d + 8*e)*x^20)/4 + (5*(2*d + 9*e)*x^22)/
22 + ((d + 10*e)*x^24)/24 + (e*x^26)/26
```

Maple [B] time = 0., size = 130, normalized size = 2.9

$$\frac{ex^{26}}{26}+\frac{(d+10\,e)\,x^{24}}{24}+\frac{(10\,d+45\,e)\,x^{22}}{22}+\frac{(45\,d+120\,e)\,x^{20}}{20}+\frac{(120\,d+210\,e)\,x^{18}}{18}+\frac{(210\,d+252\,e)\,x^{16}}{16}+\frac{(252\,d+2}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/26*e*x^26+1/24*(d+10*e)*x^24+1/22*(10*d+45*e)*x^22+1/20*(45*d+120*e)*x^20
+1/18*(120*d+210*e)*x^18+1/16*(210*d+252*e)*x^16+1/14*(252*d+210*e)*x^14+1/
12*(210*d+120*e)*x^12+1/10*(120*d+45*e)*x^10+1/8*(45*d+10*e)*x^8+1/6*(10*d+
e)*x^6+1/4*d*x^4
```

Maxima [B] time = 0.984403, size = 174, normalized size = 3.87

$$\frac{1}{26}\,ex^{26}+\frac{1}{24}\,(d+10\,e)x^{24}+\frac{5}{22}\,(2\,d+9\,e)x^{22}+\frac{3}{4}\,(3\,d+8\,e)x^{20}+\frac{5}{3}\,(4\,d+7\,e)x^{18}+\frac{21}{8}\,(5\,d+6\,e)x^{16}+3\,(6\,d+5\,e)x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/26*e*x^26 + 1/24*(d + 10*e)*x^24 + 5/22*(2*d + 9*e)*x^22 + 3/4*(3*d + 8*e
)*x^20 + 5/3*(4*d + 7*e)*x^18 + 21/8*(5*d + 6*e)*x^16 + 3*(6*d + 5*e)*x^14
+ 5/2*(7*d + 4*e)*x^12 + 3/2*(8*d + 3*e)*x^10 + 5/8*(9*d + 2*e)*x^8 + 1/6*(
10*d + e)*x^6 + 1/4*d*x^4
```

Fricas [B] time = 1.24393, size = 387, normalized size = 8.6

$$\frac{1}{26}x^{26}e+\frac{5}{12}x^{24}e+\frac{1}{24}x^{24}d+\frac{45}{22}x^{22}e+\frac{5}{11}x^{22}d+6x^{20}e+\frac{9}{4}x^{20}d+\frac{35}{3}x^{18}e+\frac{20}{3}x^{18}d+\frac{63}{4}x^{16}e+\frac{105}{8}x^{16}d+15x^{14}e+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/26*x^26*e + 5/12*x^24*e + 1/24*x^24*d + 45/22*x^22*e + 5/11*x^22*d + 6*x^
20*e + 9/4*x^20*d + 35/3*x^18*e + 20/3*x^18*d + 63/4*x^16*e + 105/8*x^16*d
```

```
+ 15*x^14*e + 18*x^14*d + 10*x^12*e + 35/2*x^12*d + 9/2*x^10*e + 12*x^10*d
+ 5/4*x^8*e + 45/8*x^8*d + 1/6*x^6*e + 5/3*x^6*d + 1/4*x^4*d
```

Sympy [B] time = 0.095724, size = 136, normalized size = 3.02

$$\frac{dx^4}{4} + \frac{ex^{26}}{26} + x^{24}\left(\frac{d}{24} + \frac{5e}{12}\right) + x^{22}\left(\frac{5d}{11} + \frac{45e}{22}\right) + x^{20}\left(\frac{9d}{4} + 6e\right) + x^{18}\left(\frac{20d}{3} + \frac{35e}{3}\right) + x^{16}\left(\frac{105d}{8} + \frac{63e}{4}\right) + x^{14}(18d + 15e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**4/4 + e*x**26/26 + x**24*(d/24 + 5*e/12) + x**22*(5*d/11 + 45*e/22) +
x**20*(9*d/4 + 6*e) + x**18*(20*d/3 + 35*e/3) + x**16*(105*d/8 + 63*e/4) +
x**14*(18*d + 15*e) + x**12*(35*d/2 + 10*e) + x**10*(12*d + 9*e/2) + x**8*(
45*d/8 + 5*e/4) + x**6*(5*d/3 + e/6)
```

Giac [B] time = 1.1107, size = 194, normalized size = 4.31

$$\frac{1}{26}x^{26}e + \frac{1}{24}dx^{24} + \frac{5}{12}x^{24}e + \frac{5}{11}dx^{22} + \frac{45}{22}x^{22}e + \frac{9}{4}dx^{20} + 6x^{20}e + \frac{20}{3}dx^{18} + \frac{35}{3}x^{18}e + \frac{105}{8}dx^{16} + \frac{63}{4}x^{16}e + 18dx^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/26*x^26*e + 1/24*d*x^24 + 5/12*x^24*e + 5/11*d*x^22 + 45/22*x^22*e + 9/4*
d*x^20 + 6*x^20*e + 20/3*d*x^18 + 35/3*x^18*e + 105/8*d*x^16 + 63/4*x^16*e
+ 18*d*x^14 + 15*x^14*e + 35/2*d*x^12 + 10*x^12*e + 12*d*x^10 + 9/2*x^10*e
+ 45/8*d*x^8 + 5/4*x^8*e + 5/3*d*x^6 + 1/6*x^6*e + 1/4*d*x^4
```

3.59 $\int x^2 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 dx$

Optimal. Leaf size=153

$$\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d$$

```
[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^9
)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*e)
*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d +
9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25
```

Rubi [A] time = 0.0845725, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {28, 448}

$$\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^9
)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*e)
*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d +
9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^2 \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 dx = \int x^2 \left(1+x^2\right)^{10} \left(d+ex^2\right) dx$$
$$= \int \left(dx^2+(10d+e)x^4+5(9d+2e)x^6+15(8d+3e)x^8+30(7d+4e)x^{10}+42(6d\right.$$
$$= \frac{dx^3}{3}+\frac{1}{5}(10d+e)x^5+\frac{5}{7}(9d+2e)x^7+\frac{5}{3}(8d+3e)x^9+\frac{30}{11}(7d+4e)x^{11}+\frac{42}{13}(6d+$$

Mathematica [A] time = 0.0205113, size = 153, normalized size = 1.

$$\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^9
)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*e)
*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d +
9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25
```

Maple [A] time = 0., size = 130, normalized size = 0.9

$$\frac{ex^{25}}{25}+\frac{(d+10\,e)\,x^{23}}{23}+\frac{(10\,d+45\,e)\,x^{21}}{21}+\frac{(45\,d+120\,e)\,x^{19}}{19}+\frac{(120\,d+210\,e)\,x^{17}}{17}+\frac{(210\,d+252\,e)\,x^{15}}{15}+\frac{(252\,d+210\,e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/25*e*x^25+1/23*(d+10*e)*x^23+1/21*(10*d+45*e)*x^21+1/19*(45*d+120*e)*x^19
+1/17*(120*d+210*e)*x^17+1/15*(210*d+252*e)*x^15+1/13*(252*d+210*e)*x^13+1/
11*(210*d+120*e)*x^11+1/9*(120*d+45*e)*x^9+1/7*(45*d+10*e)*x^7+1/5*(10*d+e)
*x^5+1/3*d*x^3
```

Maxima [A] time = 0.965705, size = 174, normalized size = 1.14

$$\frac{1}{25}\,ex^{25}+\frac{1}{23}\,(d+10\,e)x^{23}+\frac{5}{21}\,(2\,d+9\,e)x^{21}+\frac{15}{19}\,(3\,d+8\,e)x^{19}+\frac{30}{17}\,(4\,d+7\,e)x^{17}+\frac{14}{5}\,(5\,d+6\,e)x^{15}+\frac{42}{13}\,(6\,d+5\,e)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/25*e*x^25 + 1/23*(d + 10*e)*x^23 + 5/21*(2*d + 9*e)*x^21 + 15/19*(3*d + 8
*e)*x^19 + 30/17*(4*d + 7*e)*x^17 + 14/5*(5*d + 6*e)*x^15 + 42/13*(6*d + 5*
e)*x^13 + 30/11*(7*d + 4*e)*x^11 + 5/3*(8*d + 3*e)*x^9 + 5/7*(9*d + 2*e)*x^
7 + 1/5*(10*d + e)*x^5 + 1/3*d*x^3
```

Fricas [A] time = 1.27225, size = 414, normalized size = 2.71

$$\frac{1}{25}x^{25}e+\frac{10}{23}x^{23}e+\frac{1}{23}x^{23}d+\frac{15}{7}x^{21}e+\frac{10}{21}x^{21}d+\frac{120}{19}x^{19}e+\frac{45}{19}x^{19}d+\frac{210}{17}x^{17}e+\frac{120}{17}x^{17}d+\frac{84}{5}x^{15}e+14x^{15}d+\frac{210}{13}x^{13}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*e + 10/23*x^23*e + 1/23*x^23*d + 15/7*x^21*e + 10/21*x^21*d + 120
/19*x^19*e + 45/19*x^19*d + 210/17*x^17*e + 120/17*x^17*d + 84/5*x^15*e + 1
4*x^15*d + 210/13*x^13*e + 252/13*x^13*d + 120/11*x^11*e + 210/11*x^11*d +
5*x^9*e + 40/3*x^9*d + 10/7*x^7*e + 45/7*x^7*d + 1/5*x^5*e + 2*x^5*d + 1/3*
x^3*d
```

Sympy [A] time = 0.097982, size = 139, normalized size = 0.91

$$\frac{dx^3}{3} + \frac{ex^{25}}{25} + x^{23}\left(\frac{d}{23} + \frac{10e}{23}\right) + x^{21}\left(\frac{10d}{21} + \frac{15e}{7}\right) + x^{19}\left(\frac{45d}{19} + \frac{120e}{19}\right) + x^{17}\left(\frac{120d}{17} + \frac{210e}{17}\right) + x^{15}\left(14d + \frac{84e}{5}\right) + x^{13}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**3/3 + e*x**25/25 + x**23*(d/23 + 10*e/23) + x**21*(10*d/21 + 15*e/7) +
 x**19*(45*d/19 + 120*e/19) + x**17*(120*d/17 + 210*e/17) + x**15*(14*d + 8
4*e/5) + x**13*(252*d/13 + 210*e/13) + x**11*(210*d/11 + 120*e/11) + x**9*(
40*d/3 + 5*e) + x**7*(45*d/7 + 10*e/7) + x**5*(2*d + e/5)
```

Giac [A] time = 1.12289, size = 194, normalized size = 1.27

$$\frac{1}{25}x^{25}e + \frac{1}{23}dx^{23} + \frac{10}{23}x^{23}e + \frac{10}{21}dx^{21} + \frac{15}{7}x^{21}e + \frac{45}{19}dx^{19} + \frac{120}{19}x^{19}e + \frac{120}{17}dx^{17} + \frac{210}{17}x^{17}e + 14\,dx^{15} + \frac{84}{5}x^{15}e +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/25*x^25*e + 1/23*d*x^23 + 10/23*x^23*e + 10/21*d*x^21 + 15/7*x^21*e + 45/
19*d*x^19 + 120/19*x^19*e + 120/17*d*x^17 + 210/17*x^17*e + 14*d*x^15 + 84/
5*x^15*e + 252/13*d*x^13 + 210/13*x^13*e + 210/11*d*x^11 + 120/11*x^11*e +
40/3*d*x^9 + 5*x^9*e + 45/7*d*x^7 + 10/7*x^7*e + 2*d*x^5 + 1/5*x^5*e + 1/3*
d*x^3
```

3.60 $\int x\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=29

$$\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{24}e\left(x^2+1\right)^{12}$$

```
[Out] ((d - e)*(1 + x^2)^11)/22 + (e*(1 + x^2)^12)/24
```

Rubi [A] time = 0.0493032, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 444, 43}

$$\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{24}e\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] ((d - e)*(1 + x^2)^11)/22 + (e*(1 + x^2)^12)/24
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x\left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int (1+x)^{10}(d+ex)\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left((d-e)(1+x)^{10}+e(1+x)^{11}\right)\,dx,x,x^2\right)\\ &= \frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24}e\left(1+x^2\right)^{12}\end{aligned}$$

Mathematica [B] time = 0.0135391, size = 149, normalized size = 5.14

$$\frac{1}{22}x^{22}(d+10e)+\frac{1}{4}x^{20}(2d+9e)+\frac{5}{6}x^{18}(3d+8e)+\frac{15}{8}x^{16}(4d+7e)+3x^{14}(5d+6e)+\frac{7}{2}x^{12}(6d+5e)+3x^{10}(7d+4e)+\frac{15}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^2)/2 + ((10*d + e)*x^4)/4 + (5*(9*d + 2*e)*x^6)/6 + (15*(8*d + 3*e)*x^
8)/8 + 3*(7*d + 4*e)*x^10 + (7*(6*d + 5*e)*x^12)/2 + 3*(5*d + 6*e)*x^14 + (
15*(4*d + 7*e)*x^16)/8 + (5*(3*d + 8*e)*x^18)/6 + ((2*d + 9*e)*x^20)/4 + ((
d + 10*e)*x^22)/22 + (e*x^24)/24
```

Maple [B] time = 0.002, size = 130, normalized size = 4.5

$$\frac{ex^{24}}{24}+\frac{(d+10\,e)\,x^{22}}{22}+\frac{(10\,d+45\,e)\,x^{20}}{20}+\frac{(45\,d+120\,e)\,x^{18}}{18}+\frac{(120\,d+210\,e)\,x^{16}}{16}+\frac{(210\,d+252\,e)\,x^{14}}{14}+\frac{(252\,d+2}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/24*e*x^24+1/22*(d+10*e)*x^22+1/20*(10*d+45*e)*x^20+1/18*(45*d+120*e)*x^18
+1/16*(120*d+210*e)*x^16+1/14*(210*d+252*e)*x^14+1/12*(252*d+210*e)*x^12+1/
10*(210*d+120*e)*x^10+1/8*(120*d+45*e)*x^8+1/6*(45*d+10*e)*x^6+1/4*(10*d+e)
*x^4+1/2*d*x^2
```

Maxima [B] time = 0.973006, size = 174, normalized size = 6.

$$\frac{1}{24}\,ex^{24}+\frac{1}{22}\,(d+10\,e)x^{22}+\frac{1}{4}\,(2\,d+9\,e)x^{20}+\frac{5}{6}\,(3\,d+8\,e)x^{18}+\frac{15}{8}\,(4\,d+7\,e)x^{16}+3\,(5\,d+6\,e)x^{14}+\frac{7}{2}\,(6\,d+5\,e)x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/24*e*x^24 + 1/22*(d + 10*e)*x^22 + 1/4*(2*d + 9*e)*x^20 + 5/6*(3*d + 8*e)
*x^18 + 15/8*(4*d + 7*e)*x^16 + 3*(5*d + 6*e)*x^14 + 7/2*(6*d + 5*e)*x^12 +
 3*(7*d + 4*e)*x^10 + 15/8*(8*d + 3*e)*x^8 + 5/6*(9*d + 2*e)*x^6 + 1/4*(10*
d + e)*x^4 + 1/2*d*x^2
```

Fricas [B] time = 1.23295, size = 381, normalized size = 13.14

$$\frac{1}{24}x^{24}e+\frac{5}{11}x^{22}e+\frac{1}{22}x^{22}d+\frac{9}{4}x^{20}e+\frac{1}{2}x^{20}d+\frac{20}{3}x^{18}e+\frac{5}{2}x^{18}d+\frac{105}{8}x^{16}e+\frac{15}{2}x^{16}d+18x^{14}e+15x^{14}d+\frac{35}{2}x^{12}e+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24*e + 5/11*x^22*e + 1/22*x^22*d + 9/4*x^20*e + 1/2*x^20*d + 20/3*x^
18*e + 5/2*x^18*d + 105/8*x^16*e + 15/2*x^16*d + 18*x^14*e + 15*x^14*d + 35
/2*x^12*e + 21*x^12*d + 12*x^10*e + 21*x^10*d + 45/8*x^8*e + 15*x^8*d + 5/3
*x^6*e + 15/2*x^6*d + 1/4*x^4*e + 5/2*x^4*d + 1/2*x^2*d
```

Sympy [B] time = 0.094275, size = 133, normalized size = 4.59

$$\frac{dx^2}{2} + \frac{ex^{24}}{24} + x^{22}\left(\frac{d}{22} + \frac{5e}{11}\right) + x^{20}\left(\frac{d}{2} + \frac{9e}{4}\right) + x^{18}\left(\frac{5d}{2} + \frac{20e}{3}\right) + x^{16}\left(\frac{15d}{2} + \frac{105e}{8}\right) + x^{14}(15d + 18e) + x^{12}\left(21d + \frac{35e}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**2/2 + e*x**24/24 + x**22*(d/22 + 5*e/11) + x**20*(d/2 + 9*e/4) + x**18
*(5*d/2 + 20*e/3) + x**16*(15*d/2 + 105*e/8) + x**14*(15*d + 18*e) + x**12*
(21*d + 35*e/2) + x**10*(21*d + 12*e) + x**8*(15*d + 45*e/8) + x**6*(15*d/2
 + 5*e/3) + x**4*(5*d/2 + e/4)
```

Giac [B] time = 1.13029, size = 194, normalized size = 6.69

$$\frac{1}{24}x^{24}e + \frac{1}{22}dx^{22} + \frac{5}{11}x^{22}e + \frac{1}{2}dx^{20} + \frac{9}{4}x^{20}e + \frac{5}{2}dx^{18} + \frac{20}{3}x^{18}e + \frac{15}{2}dx^{16} + \frac{105}{8}x^{16}e + 15\,dx^{14} + 18\,x^{14}e + 21\,dx^{12} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/24*x^24*e + 1/22*d*x^22 + 5/11*x^22*e + 1/2*d*x^20 + 9/4*x^20*e + 5/2*d*x
^18 + 20/3*x^18*e + 15/2*d*x^16 + 105/8*x^16*e + 15*d*x^14 + 18*x^14*e + 21
*d*x^12 + 35/2*x^12*e + 21*d*x^10 + 12*x^10*e + 15*d*x^8 + 45/8*x^8*e + 15/
2*d*x^6 + 5/3*x^6*e + 5/2*d*x^4 + 1/4*x^4*e + 1/2*d*x^2
```

3.61 $\int \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=143

$$\frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e)+2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e)+\frac{10}{3}x^9(7d+$$

```
[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*(
7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 + 2
*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 + ((
d + 10*e)*x^21)/21 + (e*x^23)/23
```

Rubi [A] time = 0.0742943, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {28, 373}

$$\frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e)+2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e)+\frac{10}{3}x^9(7d+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*(
7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 + 2
*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 + ((
d + 10*e)*x^21)/21 + (e*x^23)/23
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int \left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx\\
&= \int \left(d+(10d+e)x^2+5(9d+2e)x^4+15(8d+3e)x^6+30(7d+4e)x^8+42(6d+5e)x\right.\\
&= dx+\frac{1}{3}(10d+e)x^3+(9d+2e)x^5+\frac{15}{7}(8d+3e)x^7+\frac{10}{3}(7d+4e)x^9+\frac{42}{11}(6d+5e)x^1
\end{aligned}$$

Mathematica [A] time = 0.0181159, size = 143, normalized size = 1.

$$\frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e)+2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e)+\frac{10}{3}x^9(7d+$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*(
7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 + 2
*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 + ((
d + 10*e)*x^21)/21 + (e*x^23)/23
```

Maple [A] time = 0.001, size = 127, normalized size = 0.9

$$\frac{ex^{23}}{23} + \frac{(d+10\,e)\,x^{21}}{21} + \frac{(10\,d+45\,e)\,x^{19}}{19} + \frac{(45\,d+120\,e)\,x^{17}}{17} + \frac{(120\,d+210\,e)\,x^{15}}{15} + \frac{(210\,d+252\,e)\,x^{13}}{13} + \frac{(252\,d+210\,e}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/23*e*x^23+1/21*(d+10*e)*x^21+1/19*(10*d+45*e)*x^19+1/17*(45*d+120*e)*x^17
+1/15*(120*d+210*e)*x^15+1/13*(210*d+252*e)*x^13+1/11*(252*d+210*e)*x^11+1/
9*(210*d+120*e)*x^9+1/7*(120*d+45*e)*x^7+1/5*(45*d+10*e)*x^5+1/3*(10*d+e)*x
^3+d*x
```

Maxima [A] time = 0.96456, size = 169, normalized size = 1.18

$$\frac{1}{23}\,ex^{23} + \frac{1}{21}\,(d+10\,e)x^{21} + \frac{5}{19}\,(2\,d+9\,e)x^{19} + \frac{15}{17}\,(3\,d+8\,e)x^{17} + 2\,(4\,d+7\,e)x^{15} + \frac{42}{13}\,(5\,d+6\,e)x^{13} + \frac{42}{11}\,(6\,d+5\,e)x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/23*e*x^23 + 1/21*(d + 10*e)*x^21 + 5/19*(2*d + 9*e)*x^19 + 15/17*(3*d + 8
*e)*x^17 + 2*(4*d + 7*e)*x^15 + 42/13*(5*d + 6*e)*x^13 + 42/11*(6*d + 5*e)*
x^11 + 10/3*(7*d + 4*e)*x^9 + 15/7*(8*d + 3*e)*x^7 + (9*d + 2*e)*x^5 + 1/3*
(10*d + e)*x^3 + d*x
```

Fricas [A] time = 1.27074, size = 397, normalized size = 2.78

$$\frac{1}{23}x^{23}e + \frac{10}{21}x^{21}e + \frac{1}{21}x^{21}d + \frac{45}{19}x^{19}e + \frac{10}{19}x^{19}d + \frac{120}{17}x^{17}e + \frac{45}{17}x^{17}d + 14x^{15}e + 8x^{15}d + \frac{252}{13}x^{13}e + \frac{210}{13}x^{13}d + \frac{210}{11}x^{11}e -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*e + 10/21*x^21*e + 1/21*x^21*d + 45/19*x^19*e + 10/19*x^19*d + 12
0/17*x^17*e + 45/17*x^17*d + 14*x^15*e + 8*x^15*d + 252/13*x^13*e + 210/13*
x^13*d + 210/11*x^11*e + 252/11*x^11*d + 40/3*x^9*e + 70/3*x^9*d + 45/7*x^7
*e + 120/7*x^7*d + 2*x^5*e + 9*x^5*d + 1/3*x^3*e + 10/3*x^3*d + x*d
```

Sympy [A] time = 0.096666, size = 134, normalized size = 0.94

$$dx + \frac{ex^{23}}{23} + x^{21}\left(\frac{d}{21} + \frac{10e}{21}\right) + x^{19}\left(\frac{10d}{19} + \frac{45e}{19}\right) + x^{17}\left(\frac{45d}{17} + \frac{120e}{17}\right) + x^{15}(8d + 14e) + x^{13}\left(\frac{210d}{13} + \frac{252e}{13}\right) + x^{11}\left(\frac{252}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x + e*x**23/23 + x**21*(d/21 + 10*e/21) + x**19*(10*d/19 + 45*e/19) + x**
17*(45*d/17 + 120*e/17) + x**15*(8*d + 14*e) + x**13*(210*d/13 + 252*e/13)
+ x**11*(252*d/11 + 210*e/11) + x**9*(70*d/3 + 40*e/3) + x**7*(120*d/7 + 45
*e/7) + x**5*(9*d + 2*e) + x**3*(10*d/3 + e/3)
```

Giac [A] time = 1.10528, size = 190, normalized size = 1.33

$$\frac{1}{23}x^{23}e + \frac{1}{21}dx^{21} + \frac{10}{21}x^{21}e + \frac{10}{19}dx^{19} + \frac{45}{19}x^{19}e + \frac{45}{17}dx^{17} + \frac{120}{17}x^{17}e + 8\,dx^{15} + 14\,x^{15}e + \frac{210}{13}dx^{13} + \frac{252}{13}x^{13}e + \frac{25}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/23*x^23*e + 1/21*d*x^21 + 10/21*x^21*e + 10/19*d*x^19 + 45/19*x^19*e + 45
/17*d*x^17 + 120/17*x^17*e + 8*d*x^15 + 14*x^15*e + 210/13*d*x^13 + 252/13*
x^13*e + 252/11*d*x^11 + 210/11*x^11*e + 70/3*d*x^9 + 40/3*x^9*e + 120/7*d*
x^7 + 45/7*x^7*e + 9*d*x^5 + 2*x^5*e + 10/3*d*x^3 + 1/3*x^3*e + d*x
```

3.62 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x} \, dx$

Optimal. Leaf size=93

$$\frac{dx^{20}}{20}+\frac{5dx^{18}}{9}+\frac{45dx^{16}}{16}+\frac{60dx^{14}}{7}+\frac{35dx^{12}}{2}+\frac{126dx^{10}}{5}+\frac{105dx^8}{4}+20dx^6+\frac{45dx^4}{4}+5dx^2+d\log(x)+\frac{1}{22}e\left(x^2+1\right)^{11}$$

```
[Out] 5*d*x^2 + (45*d*x^4)/4 + 20*d*x^6 + (105*d*x^8)/4 + (126*d*x^10)/5 + (35*d*
x^12)/2 + (60*d*x^14)/7 + (45*d*x^16)/16 + (5*d*x^18)/9 + (d*x^20)/20 + (e*
(1 + x^2)^11)/22 + d*Log[x]
```

Rubi [A] time = 0.0547076, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {28, 446, 80, 43}

$$\frac{dx^{20}}{20}+\frac{5dx^{18}}{9}+\frac{45dx^{16}}{16}+\frac{60dx^{14}}{7}+\frac{35dx^{12}}{2}+\frac{126dx^{10}}{5}+\frac{105dx^8}{4}+20dx^6+\frac{45dx^4}{4}+5dx^2+d\log(x)+\frac{1}{22}e\left(x^2+1\right)^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] 5*d*x^2 + (45*d*x^4)/4 + 20*d*x^6 + (105*d*x^8)/4 + (126*d*x^10)/5 + (35*d*
x^12)/2 + (60*d*x^14)/7 + (45*d*x^16)/16 + (5*d*x^18)/9 + (d*x^20)/20 + (e*
(1 + x^2)^11)/22 + d*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x}\,dx = \int \frac{(1+x^2)^{10}(d+ex^2)}{x}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(1+x)^{10}(d+ex)}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{22}e(1+x^2)^{11} + \frac{1}{2}d\operatorname{Subst}\left(\int \frac{(1+x)^{10}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{22}e(1+x^2)^{11} + \frac{1}{2}d\operatorname{Subst}\left(\int \left(10+\frac{1}{x}+45x+120x^2+210x^3+252x^4+210x^5+\right.\right.$$
$$= 5dx^2 + \frac{45dx^4}{4} + 20dx^6 + \frac{105dx^8}{4} + \frac{126dx^{10}}{5} + \frac{35dx^{12}}{2} + \frac{60dx^{14}}{7} + \frac{45dx^{16}}{16} + \frac{5dx^{18}}{9}$$

Mathematica [A] time = 0.0276129, size = 149, normalized size = 1.6

$$\frac{1}{20}x^{20}(d+10e) + \frac{5}{18}x^{18}(2d+9e) + \frac{15}{16}x^{16}(3d+8e) + \frac{15}{7}x^{14}(4d+7e) + \frac{7}{2}x^{12}(5d+6e) + \frac{21}{5}x^{10}(6d+5e) + \frac{15}{4}x^8(7d+$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] ((10*d + e)*x^2)/2 + (5*(9*d + 2*e)*x^4)/4 + (5*(8*d + 3*e)*x^6)/2 + (15*(7
*d + 4*e)*x^8)/4 + (21*(6*d + 5*e)*x^10)/5 + (7*(5*d + 6*e)*x^12)/2 + (15*(
4*d + 7*e)*x^14)/7 + (15*(3*d + 8*e)*x^16)/16 + (5*(2*d + 9*e)*x^18)/18 + (
(d + 10*e)*x^20)/20 + (e*x^22)/22 + d*Log[x]
```

Maple [A] time = 0.003, size = 132, normalized size = 1.4

$$\frac{ex^{22}}{22} + \frac{dx^{20}}{20} + \frac{ex^{20}}{2} + \frac{5\,dx^{18}}{9} + \frac{5\,x^{18}e}{2} + \frac{45\,dx^{16}}{16} + \frac{15\,x^{16}e}{2} + \frac{60\,dx^{14}}{7} + 15\,x^{14}e + \frac{35\,dx^{12}}{2} + 21\,x^{12}e + \frac{126\,dx^{10}}{5} + 21\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x,x)
```

```
[Out] 1/22*e*x^22+1/20*d*x^20+1/2*e*x^20+5/9*d*x^18+5/2*x^18*e+45/16*d*x^16+15/2*
x^16*e+60/7*d*x^14+15*x^14*e+35/2*d*x^12+21*x^12*e+126/5*d*x^10+21*x^10*e+1
05/4*d*x^8+15*x^8*e+20*d*x^6+15/2*x^6*e+45/4*d*x^4+5/2*x^4*e+5*d*x^2+1/2*e*
x^2+d*ln(x)
```

Maxima [A] time = 0.955849, size = 176, normalized size = 1.89

$$\frac{1}{22}ex^{22} + \frac{1}{20}(d+10\,e)x^{20} + \frac{5}{18}(2\,d+9\,e)x^{18} + \frac{15}{16}(3\,d+8\,e)x^{16} + \frac{15}{7}(4\,d+7\,e)x^{14} + \frac{7}{2}(5\,d+6\,e)x^{12} + \frac{21}{5}(6\,d+5\,e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*e*x^22 + 1/20*(d + 10*e)*x^20 + 5/18*(2*d + 9*e)*x^18 + 15/16*(3*d + 8
*e)*x^16 + 15/7*(4*d + 7*e)*x^14 + 7/2*(5*d + 6*e)*x^12 + 21/5*(6*d + 5*e)*
x^10 + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)*x^6 + 5/4*(9*d + 2*e)*x^4 + 1
```

```
/2*(10*d + e)*x^2 + 1/2*d*log(x^2)
```

Fricas [A] time = 1.42698, size = 344, normalized size = 3.7

$$\frac{1}{22}ex^{22} + \frac{1}{20}(d + 10e)x^{20} + \frac{5}{18}(2d + 9e)x^{18} + \frac{15}{16}(3d + 8e)x^{16} + \frac{15}{7}(4d + 7e)x^{14} + \frac{7}{2}(5d + 6e)x^{12} + \frac{21}{5}(6d + 5e)x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="fricas")
```

```
[Out] 1/22*e*x^22 + 1/20*(d + 10*e)*x^20 + 5/18*(2*d + 9*e)*x^18 + 15/16*(3*d + 8
*e)*x^16 + 15/7*(4*d + 7*e)*x^14 + 7/2*(5*d + 6*e)*x^12 + 21/5*(6*d + 5*e)*
x^10 + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)*x^6 + 5/4*(9*d + 2*e)*x^4 + 1
/2*(10*d + e)*x^2 + d*log(x)
```

Sympy [A] time = 0.321105, size = 131, normalized size = 1.41

$$d\log(x) + \frac{ex^{22}}{22} + x^{20}\left(\frac{d}{20} + \frac{e}{2}\right) + x^{18}\left(\frac{5d}{9} + \frac{5e}{2}\right) + x^{16}\left(\frac{45d}{16} + \frac{15e}{2}\right) + x^{14}\left(\frac{60d}{7} + 15e\right) + x^{12}\left(\frac{35d}{2} + 21e\right) + x^{10}\left(\frac{126d}{5} +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x,x)
```

```
[Out] d*log(x) + e*x**22/22 + x**20*(d/20 + e/2) + x**18*(5*d/9 + 5*e/2) + x**16*
(45*d/16 + 15*e/2) + x**14*(60*d/7 + 15*e) + x**12*(35*d/2 + 21*e) + x**10*
(126*d/5 + 21*e) + x**8*(105*d/4 + 15*e) + x**6*(20*d + 15*e/2) + x**4*(45*
d/4 + 5*e/2) + x**2*(5*d + e/2)
```

Giac [A] time = 1.11416, size = 196, normalized size = 2.11

$$\frac{1}{22}x^{22}e + \frac{1}{20}dx^{20} + \frac{1}{2}x^{20}e + \frac{5}{9}dx^{18} + \frac{5}{2}x^{18}e + \frac{45}{16}dx^{16} + \frac{15}{2}x^{16}e + \frac{60}{7}dx^{14} + 15x^{14}e + \frac{35}{2}dx^{12} + 21x^{12}e + \frac{126}{5}dx^{10} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*x^22*e + 1/20*d*x^20 + 1/2*x^20*e + 5/9*d*x^18 + 5/2*x^18*e + 45/16*d*
x^16 + 15/2*x^16*e + 60/7*d*x^14 + 15*x^14*e + 35/2*d*x^12 + 21*x^12*e + 12
6/5*d*x^10 + 21*x^10*e + 105/4*d*x^8 + 15*x^8*e + 20*d*x^6 + 15/2*x^6*e + 4
5/4*d*x^4 + 5/2*x^4*e + 5*d*x^2 + 1/2*x^2*e + 1/2*d*log(x^2)
```

3.63 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2} dx$

Optimal. Leaf size=141

$$\frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e)+\frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e)+\frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e$$

```
[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7*
d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30*(
4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d + 10
*e)*x^19)/19 + (e*x^21)/21
```

Rubi [A] time = 0.0821492, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {28, 448}

$$\frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e)+\frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e)+\frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7*
d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30*(
4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d + 10
*e)*x^19)/19 + (e*x^21)/21
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx &= \int \frac{(1+x^2)^{10}(d+ex^2)}{x^2}\,dx\\ &= \int \left(10d\left(1+\frac{e}{10d}\right)+\frac{d}{x^2}+5(9d+2e)x^2+15(8d+3e)x^4+30(7d+4e)x^6+42(6d+\right.\\ &= -\frac{d}{x}+(10d+e)x+\frac{5}{3}(9d+2e)x^3+3(8d+3e)x^5+\frac{30}{7}(7d+4e)x^7+\frac{14}{3}(6d+5e)x^9\end{aligned}$$

Mathematica [A] time = 0.0281304, size = 141, normalized size = 1.

$$\frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e)+\frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e)+\frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7*
d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30*(
4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d + 10
*e)*x^19)/19 + (e*x^21)/21
```

Maple [A] time = 0.004, size = 129, normalized size = 0.9

$$\frac{ex^{21}}{21}+\frac{x^{19}d}{19}+\frac{10\,x^{19}e}{19}+\frac{10\,x^{17}d}{17}+\frac{45\,x^{17}e}{17}+3\,x^{15}d+8\,x^{15}e+\frac{120\,x^{13}d}{13}+\frac{210\,x^{13}e}{13}+\frac{210\,x^{11}d}{11}+\frac{252\,x^{11}e}{11}+28\,x^{9}d+\frac{7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x)
```

```
[Out] 1/21*e*x^21+1/19*x^19*d+10/19*x^19*e+10/17*x^17*d+45/17*x^17*e+3*x^15*d+8*x
^15*e+120/13*x^13*d+210/13*x^13*e+210/11*x^11*d+252/11*x^11*e+28*x^9*d+70/3
*x^9*e+30*x^7*d+120/7*x^7*e+24*d*x^5+9*x^5*e+15*d*x^3+10/3*x^3*e+10*d*x+e*x
-d/x
```

Maxima [A] time = 0.938122, size = 169, normalized size = 1.2

$$\frac{1}{21}\,ex^{21}+\frac{1}{19}\,(d+10\,e)x^{19}+\frac{5}{17}\,(2\,d+9\,e)x^{17}+(3\,d+8\,e)x^{15}+\frac{30}{13}\,(4\,d+7\,e)x^{13}+\frac{42}{11}\,(5\,d+6\,e)x^{11}+\frac{14}{3}\,(6\,d+5\,e)x^{9}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*e*x^21 + 1/19*(d + 10*e)*x^19 + 5/17*(2*d + 9*e)*x^17 + (3*d + 8*e)*x^
15 + 30/13*(4*d + 7*e)*x^13 + 42/11*(5*d + 6*e)*x^11 + 14/3*(6*d + 5*e)*x^9
 + 30/7*(7*d + 4*e)*x^7 + 3*(8*d + 3*e)*x^5 + 5/3*(9*d + 2*e)*x^3 + (10*d +
 e)*x - d/x
```

Fricas [A] time = 1.4223, size = 401, normalized size = 2.84

$$\frac{46189\,ex^{22}+51051\,(d+10\,e)x^{20}+285285\,(2\,d+9\,e)x^{18}+969969\,(3\,d+8\,e)x^{16}+2238390\,(4\,d+7\,e)x^{14}+3703518\,(5\,d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/969969*(46189*e*x^22 + 51051*(d + 10*e)*x^20 + 285285*(2*d + 9*e)*x^18 +
969969*(3*d + 8*e)*x^16 + 2238390*(4*d + 7*e)*x^14 + 3703518*(5*d + 6*e)*x^
12 + 4526522*(6*d + 5*e)*x^10 + 4157010*(7*d + 4*e)*x^8 + 2909907*(8*d + 3*
e)*x^6 + 1616615*(9*d + 2*e)*x^4 + 969969*(10*d + e)*x^2 - 969969*d)/x
```

Sympy [A] time = 0.473909, size = 124, normalized size = 0.88

$$-\frac{d}{x} + \frac{ex^{21}}{21} + x^{19}\left(\frac{d}{19} + \frac{10e}{19}\right) + x^{17}\left(\frac{10d}{17} + \frac{45e}{17}\right) + x^{15}(3d + 8e) + x^{13}\left(\frac{120d}{13} + \frac{210e}{13}\right) + x^{11}\left(\frac{210d}{11} + \frac{252e}{11}\right) + x^{9}\left(28d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**2,x)
```

```
[Out] -d/x + e*x**21/21 + x**19*(d/19 + 10*e/19) + x**17*(10*d/17 + 45*e/17) + x*
*15*(3*d + 8*e) + x**13*(120*d/13 + 210*e/13) + x**11*(210*d/11 + 252*e/11)
 + x**9*(28*d + 70*e/3) + x**7*(30*d + 120*e/7) + x**5*(24*d + 9*e) + x**3*
(15*d + 10*e/3) + x*(10*d + e)
```

Giac [A] time = 1.12163, size = 188, normalized size = 1.33

$$\frac{1}{21}x^{21}e + \frac{1}{19}dx^{19} + \frac{10}{19}x^{19}e + \frac{10}{17}dx^{17} + \frac{45}{17}x^{17}e + 3\,dx^{15} + 8\,x^{15}e + \frac{120}{13}dx^{13} + \frac{210}{13}x^{13}e + \frac{210}{11}dx^{11} + \frac{252}{11}x^{11}e + 28$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/21*x^21*e + 1/19*d*x^19 + 10/19*x^19*e + 10/17*d*x^17 + 45/17*x^17*e + 3*
d*x^15 + 8*x^15*e + 120/13*d*x^13 + 210/13*x^13*e + 210/11*d*x^11 + 252/11*
x^11*e + 28*d*x^9 + 70/3*x^9*e + 30*d*x^7 + 120/7*x^7*e + 24*d*x^5 + 9*x^5*
e + 15*d*x^3 + 10/3*x^3*e + 10*d*x + x*e - d/x
```

3.64 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3} dx$

Optimal. Leaf size=147

$$\frac{1}{18}x^{18}(d+10e)+\frac{5}{16}x^{16}(2d+9e)+\frac{15}{14}x^{14}(3d+8e)+\frac{5}{2}x^{12}(4d+7e)+\frac{21}{5}x^{10}(5d+6e)+\frac{21}{4}x^{8}(6d+5e)+5x^{6}(7d+4e)+$$

```
[Out] -d/(2*x^2) + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(7*d + 4*e)
*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5 + (5*(4*d + 7*e)*x^
12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d + 9*e)*x^16)/16 + ((d + 10*e)*x^
18)/18 + (e*x^20)/20 + (10*d + e)*Log[x]
```

Rubi [A] time = 0.135537, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {28, 446, 76}

$$\frac{1}{18}x^{18}(d+10e)+\frac{5}{16}x^{16}(2d+9e)+\frac{15}{14}x^{14}(3d+8e)+\frac{5}{2}x^{12}(4d+7e)+\frac{21}{5}x^{10}(5d+6e)+\frac{21}{4}x^{8}(6d+5e)+5x^{6}(7d+4e)+$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```



```
[Out] -d/(2*x^2) + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(7*d + 4*e)
*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5 + (5*(4*d + 7*e)*x^
12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d + 9*e)*x^16)/16 + ((d + 10*e)*x^
18)/18 + (e*x^20)/20 + (10*d + e)*Log[x]
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rubi steps

$$\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx = \int \frac{(1+x^2)^{10}(d+ex^2)}{x^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^{10}(d+ex)}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(5(9d+2e)+\frac{d}{x^2}+\frac{10d+e}{x}+15(8d+3e)x+30(7d+4e)x^2+42(6d+\right.\right.$$
$$= -\frac{d}{2x^2}+\frac{5}{2}(9d+2e)x^2+\frac{15}{4}(8d+3e)x^4+5(7d+4e)x^6+\frac{21}{4}(6d+5e)x^8+\frac{21}{5}(5d+$$

Mathematica [A] time = 0.0399448, size = 147, normalized size = 1.

$$\frac{1}{18}x^{18}(d+10e)+\frac{5}{16}x^{16}(2d+9e)+\frac{15}{14}x^{14}(3d+8e)+\frac{5}{2}x^{12}(4d+7e)+\frac{21}{5}x^{10}(5d+6e)+\frac{21}{4}x^{8}(6d+5e)+5x^6(7d+4e$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -d/(2*x^2) + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(7*d + 4*e)
*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5 + (5*(4*d + 7*e)*x^
12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d + 9*e)*x^16)/16 + ((d + 10*e)*x^
18)/18 + (e*x^20)/20 + (10*d + e)*Log[x]
```

Maple [A] time = 0.006, size = 131, normalized size = 0.9

$$\frac{ex^{20}}{20}+\frac{dx^{18}}{18}+\frac{5x^{18}e}{9}+\frac{5dx^{16}}{8}+\frac{45x^{16}e}{16}+\frac{45dx^{14}}{14}+\frac{60x^{14}e}{7}+10dx^{12}+\frac{35x^{12}e}{2}+21dx^{10}+\frac{126x^{10}e}{5}+\frac{63dx^8}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x)
```

```
[Out] 1/20*e*x^20+1/18*d*x^18+5/9*x^18*e+5/8*d*x^16+45/16*x^16*e+45/14*d*x^14+60/
7*x^14*e+10*d*x^12+35/2*x^12*e+21*d*x^10+126/5*x^10*e+63/2*d*x^8+105/4*x^8*
e+35*d*x^6+20*x^6*e+30*d*x^4+45/4*x^4*e+45/2*d*x^2+5*e*x^2+10*d*ln(x)+ln(x)
*e-1/2*d/x^2
```

Maxima [A] time = 0.946439, size = 176, normalized size = 1.2

$$\frac{1}{20}ex^{20}+\frac{1}{18}(d+10e)x^{18}+\frac{5}{16}(2d+9e)x^{16}+\frac{15}{14}(3d+8e)x^{14}+\frac{5}{2}(4d+7e)x^{12}+\frac{21}{5}(5d+6e)x^{10}+\frac{21}{4}(6d+5e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*e*x^20 + 1/18*(d + 10*e)*x^18 + 5/16*(2*d + 9*e)*x^16 + 15/14*(3*d + 8
*e)*x^14 + 5/2*(4*d + 7*e)*x^12 + 21/5*(5*d + 6*e)*x^10 + 21/4*(6*d + 5*e)*
x^8 + 5*(7*d + 4*e)*x^6 + 15/4*(8*d + 3*e)*x^4 + 5/2*(9*d + 2*e)*x^2 + 1/2*
(10*d + e)*log(x^2) - 1/2*d/x^2
```

Fricas [A] time = 1.43393, size = 378, normalized size = 2.57

$$\frac{252\,ex^{22} + 280\,(d + 10\,e)x^{20} + 1575\,(2\,d + 9\,e)x^{18} + 5400\,(3\,d + 8\,e)x^{16} + 12600\,(4\,d + 7\,e)x^{14} + 21168\,(5\,d + 6\,e)x^{12} + 26}{5040}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*e*x^22 + 280*(d + 10*e)*x^20 + 1575*(2*d + 9*e)*x^18 + 5400*(3*
d + 8*e)*x^16 + 12600*(4*d + 7*e)*x^14 + 21168*(5*d + 6*e)*x^12 + 26460*(6*
d + 5*e)*x^10 + 25200*(7*d + 4*e)*x^8 + 18900*(8*d + 3*e)*x^6 + 12600*(9*d
+ 2*e)*x^4 + 5040*(10*d + e)*x^2*log(x) - 2520*d)/x^2
```

Sympy [A] time = 0.533572, size = 131, normalized size = 0.89

$$-\frac{d}{2x^2} + \frac{ex^{20}}{20} + x^{18}\left(\frac{d}{18} + \frac{5e}{9}\right) + x^{16}\left(\frac{5d}{8} + \frac{45e}{16}\right) + x^{14}\left(\frac{45d}{14} + \frac{60e}{7}\right) + x^{12}\left(10d + \frac{35e}{2}\right) + x^{10}\left(21d + \frac{126e}{5}\right) + x^{8}\left(\frac{63d}{2} +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**3,x)
```

```
[Out] -d/(2*x**2) + e*x**20/20 + x**18*(d/18 + 5*e/9) + x**16*(5*d/8 + 45*e/16) +
 x**14*(45*d/14 + 60*e/7) + x**12*(10*d + 35*e/2) + x**10*(21*d + 126*e/5)
+ x**8*(63*d/2 + 105*e/4) + x**6*(35*d + 20*e) + x**4*(30*d + 45*e/4) + x**
2*(45*d/2 + 5*e) + (10*d + e)*log(x)
```

Giac [A] time = 1.11002, size = 211, normalized size = 1.44

$$\frac{1}{20}\,x^{20}e + \frac{1}{18}\,dx^{18} + \frac{5}{9}\,x^{18}e + \frac{5}{8}\,dx^{16} + \frac{45}{16}\,x^{16}e + \frac{45}{14}\,dx^{14} + \frac{60}{7}\,x^{14}e + 10\,dx^{12} + \frac{35}{2}\,x^{12}e + 21\,dx^{10} + \frac{126}{5}\,x^{10}e + \frac{63}{2}\,dx^{8} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*x^20*e + 1/18*d*x^18 + 5/9*x^18*e + 5/8*d*x^16 + 45/16*x^16*e + 45/14*
d*x^14 + 60/7*x^14*e + 10*d*x^12 + 35/2*x^12*e + 21*d*x^10 + 126/5*x^10*e +
 63/2*d*x^8 + 105/4*x^8*e + 35*d*x^6 + 20*x^6*e + 30*d*x^4 + 45/4*x^4*e + 4
5/2*d*x^2 + 5*x^2*e + 1/2*(10*d + e)*log(x^2) - 1/2*(10*d*x^2 + x^2*e + d)/
x^2
```

3.65 $\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=203

$$\frac{11(fx)^{m+3}}{f^3(m+3)}+\frac{55(fx)^{m+5}}{f^5(m+5)}+\frac{165(fx)^{m+7}}{f^7(m+7)}+\frac{330(fx)^{m+9}}{f^9(m+9)}+\frac{462(fx)^{m+11}}{f^{11}(m+11)}+\frac{462(fx)^{m+13}}{f^{13}(m+13)}+\frac{330(fx)^{m+15}}{f^{15}(m+15)}+\frac{165(fx)^{m+17}}{f^{17}(m+17)}$$

```
[Out] (f*x)^(1 + m)/(f*(1 + m)) + (11*(f*x)^(3 + m))/(f^3*(3 + m)) + (55*(f*x)^(5
 + m))/(f^5*(5 + m)) + (165*(f*x)^(7 + m))/(f^7*(7 + m)) + (330*(f*x)^(9 +
m))/(f^9*(9 + m)) + (462*(f*x)^(11 + m))/(f^11*(11 + m)) + (462*(f*x)^(13 +
 m))/(f^13*(13 + m)) + (330*(f*x)^(15 + m))/(f^15*(15 + m)) + (165*(f*x)^(1
7 + m))/(f^17*(17 + m)) + (55*(f*x)^(19 + m))/(f^19*(19 + m)) + (11*(f*x)^(
21 + m))/(f^21*(21 + m)) + (f*x)^(23 + m)/(f^23*(23 + m))
```

Rubi [A] time = 0.073443, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {28, 270}

$$\frac{11(fx)^{m+3}}{f^3(m+3)}+\frac{55(fx)^{m+5}}{f^5(m+5)}+\frac{165(fx)^{m+7}}{f^7(m+7)}+\frac{330(fx)^{m+9}}{f^9(m+9)}+\frac{462(fx)^{m+11}}{f^{11}(m+11)}+\frac{462(fx)^{m+13}}{f^{13}(m+13)}+\frac{330(fx)^{m+15}}{f^{15}(m+15)}+\frac{165(fx)^{m+17}}{f^{17}(m+17)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```


```
[Out] (f*x)^(1 + m)/(f*(1 + m)) + (11*(f*x)^(3 + m))/(f^3*(3 + m)) + (55*(f*x)^(5
 + m))/(f^5*(5 + m)) + (165*(f*x)^(7 + m))/(f^7*(7 + m)) + (330*(f*x)^(9 +
m))/(f^9*(9 + m)) + (462*(f*x)^(11 + m))/(f^11*(11 + m)) + (462*(f*x)^(13 +
 m))/(f^13*(13 + m)) + (330*(f*x)^(15 + m))/(f^15*(15 + m)) + (165*(f*x)^(1
7 + m))/(f^17*(17 + m)) + (55*(f*x)^(19 + m))/(f^19*(19 + m)) + (11*(f*x)^(
21 + m))/(f^21*(21 + m)) + (f*x)^(23 + m)/(f^23*(23 + m))
```


Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx &= \int (fx)^m \left(1+x^2\right)^{11} \, dx \\
&= \int \left((fx)^m + \frac{11(fx)^{2+m}}{f^2} + \frac{55(fx)^{4+m}}{f^4} + \frac{165(fx)^{6+m}}{f^6} + \frac{330(fx)^{8+m}}{f^8} + \frac{462(fx)^{\cdot}}{f^{10}} \right. \\
&= \frac{(fx)^{1+m}}{f(1+m)} + \frac{11(fx)^{3+m}}{f^3(3+m)} + \frac{55(fx)^{5+m}}{f^5(5+m)} + \frac{165(fx)^{7+m}}{f^7(7+m)} + \frac{330(fx)^{9+m}}{f^9(9+m)} + \frac{462(fx)^{1}}{f^{11}(11+}
\end{aligned}$$

Mathematica [A] time = 0.0521488, size = 122, normalized size = 0.6

$$x\left(\frac{x^{22}}{m+23}+\frac{11x^{20}}{m+21}+\frac{55x^{18}}{m+19}+\frac{165x^{16}}{m+17}+\frac{330x^{14}}{m+15}+\frac{462x^{12}}{m+13}+\frac{462x^{10}}{m+11}+\frac{330x^8}{m+9}+\frac{165x^6}{m+7}+\frac{55x^4}{m+5}+\frac{11x^2}{m+3}+\frac{1}{m+1}\right)($$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x*(f*x)^m*((1 + m)^(-1) + (11*x^2)/(3 + m) + (55*x^4)/(5 + m) + (165*x^6)/(
7 + m) + (330*x^8)/(9 + m) + (462*x^10)/(11 + m) + (462*x^12)/(13 + m) + (3
30*x^14)/(15 + m) + (165*x^16)/(17 + m) + (55*x^18)/(19 + m) + (11*x^20)/(2
1 + m) + x^22/(23 + m))
```

Maple [B] time = 0.01, size = 1121, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] (f*x)^m*(m^11*x^22+121*m^10*x^22+11*m^11*x^20+6435*m^9*x^22+1353*m^10*x^20+
197835*m^8*x^22+55*m^11*x^18+72985*m^9*x^20+3889578*m^7*x^22+6875*m^10*x^18
+2271555*m^8*x^20+51069018*m^6*x^22+165*m^11*x^16+376365*m^9*x^18+45134958*
m^7*x^20+453714470*m^5*x^22+20955*m^10*x^16+11870265*m^8*x^18+597988314*m^6
*x^20+2702025590*m^4*x^22+330*m^11*x^14+1164735*m^9*x^16+238653030*m^7*x^18
+5353566130*m^5*x^20+10431670821*m^3*x^22+42570*m^10*x^14+37263105*m^8*x^16
+3194704590*m^6*x^18+32087153670*m^4*x^20+24372200061*m^2*x^22+462*m^11*x^1
2+2403390*m^9*x^14+759091410*m^7*x^16+28857216410*m^5*x^18+124530626231*m^3
*x^20+29985521895*m*x^22+60522*m^10*x^12+78076350*m^8*x^14+10282782510*m^6*
x^16+174273100210*m^4*x^18+292163767533*m^2*x^20+13749310575*x^22+462*m^11*
x^10+3471930*m^9*x^12+1613983140*m^7*x^14+93862508190*m^5*x^16+680615362515
*m^3*x^18+360568238085*m*x^20+61446*m^10*x^10+114642990*m^8*x^12+2216492586
0*m^6*x^14+572017996770*m^4*x^16+1604842704135*m^2*x^18+165646455975*x^20+3
30*m^11*x^8+3582810*m^9*x^10+2408820876*m^7*x^12+204865733820*m^5*x^14+2251
106854425*m^3*x^16+1988025402825*m*x^18+44550*m^10*x^8+120367170*m^8*x^10+3
3609870756*m^6*x^12+1262375264700*m^4*x^14+5340787250535*m^2*x^16+915414625
125*x^18+165*m^11*x^6+2640990*m^9*x^8+2575140876*m^7*x^10+315347150580*m^5*
x^12+5015196628530*m^3*x^14+6646727085075*m*x^16+22605*m^10*x^6+90358290*m^
8*x^8+36597992508*m^6*x^10+1969992823260*m^4*x^12+11991258123570*m^2*x^14+3
069331390125*x^16+55*m^11*x^4+1362735*m^9*x^6+1971903780*m^7*x^8+3496975528
20*m^5*x^10+7921249136262*m^3*x^12+15011348834790*m*x^14+7645*m^10*x^4+4752
4455*m^8*x^6+28627538940*m^6*x^8+2222832699780*m^4*x^10+19130651800722*m^2*
x^12+6957151150950*x^14+11*m^11*x^2+468765*m^9*x^4+1059893010*m^7*x^6+27969
1771260*m^5*x^8+9079996141062*m^3*x^10+24133835554290*m*x^12+1551*m^10*x^2+
16677375*m^8*x^4+15768085410*m^6*x^6+1818135330660*m^4*x^8+22226933020446*m
^2*x^10+11238474936150*x^12+m^11+96745*m^9*x^2+380801190*m^7*x^4+1582932129
90*m^5*x^6+7587607623090*m^3*x^8+28336045738770*m*x^10+143*m^10+3514005*m^8
*x^2+5825106210*m^6*x^4+1059628145070*m^4*x^6+18930738943710*m^2*x^8+132818
34015450*x^10+9075*m^9+82295598*m^7*x^2+60431072570*m^5*x^4+4558015784025*m
^3*x^6+24503570194950*m*x^8+336765*m^8+1298935638*m^6*x^2+420404849150*m^4*
x^4+11703493287585*m^2*x^6+11595251918250*x^8+8103018*m^7+14014513810*m^5*x
^2+1889780020755*m^3*x^4+15515657331075*m*x^6+132426294*m^6+102468500970*m^
4*x^2+5087634488145*m^2*x^4+7454090518875*x^6+1495875590*m^5+490955350391*m
^3*x^2+7041864340665*m*x^4+11641582810*m^4+1434440867211*m^2*x^2+3478575575
```

```
475*x^4+60936676581*m^3+2192684754645*m*x^2+203363952363*m^2+1159525191825*
x^2+387182170935*m+316234143225)*x/(1+m)/(3+m)/(5+m)/(7+m)/(9+m)/(11+m)/(13
+m)/(15+m)/(17+m)/(19+m)/(21+m)/(23+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.63075, size = 3202, normalized size = 15.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] ((m^11 + 121*m^10 + 6435*m^9 + 197835*m^8 + 3889578*m^7 + 51069018*m^6 + 45
3714470*m^5 + 2702025590*m^4 + 10431670821*m^3 + 24372200061*m^2 + 29985521
895*m + 13749310575)*x^23 + 11*(m^11 + 123*m^10 + 6635*m^9 + 206505*m^8 + 4
103178*m^7 + 54362574*m^6 + 486687830*m^5 + 2917013970*m^4 + 11320966021*m^
3 + 26560342503*m^2 + 32778930735*m + 15058768725)*x^21 + 55*(m^11 + 125*m^
10 + 6843*m^9 + 215823*m^8 + 4339146*m^7 + 58085538*m^6 + 524676662*m^5 + 3
168601822*m^4 + 12374824773*m^3 + 29178958257*m^2 + 36145916415*m + 1664390
2275)*x^19 + 165*(m^11 + 127*m^10 + 7059*m^9 + 225837*m^8 + 4600554*m^7 + 6
2319894*m^6 + 568863686*m^5 + 3466775738*m^4 + 13643071845*m^3 + 3236840757
9*m^2 + 40283194455*m + 18602008425)*x^17 + 330*(m^11 + 129*m^10 + 7283*m^9
 + 236595*m^8 + 4890858*m^7 + 67166442*m^6 + 620805254*m^5 + 3825379590*m^4
 + 15197565541*m^3 + 36337145829*m^2 + 45488935863*m + 21082276215)*x^15 +
462*(m^11 + 131*m^10 + 7515*m^9 + 248145*m^8 + 5213898*m^7 + 72748638*m^6 +
 682569590*m^5 + 4264053730*m^4 + 17145560901*m^3 + 41408337231*m^2 + 52237
739295*m + 24325703325)*x^13 + 462*(m^11 + 133*m^10 + 7755*m^9 + 260535*m^8
 + 5573898*m^7 + 79216434*m^6 + 756921110*m^5 + 4811326190*m^4 + 1965367130
1*m^3 + 48110244633*m^2 + 61333432335*m + 28748558475)*x^11 + 330*(m^11 + 1
35*m^10 + 8003*m^9 + 273813*m^8 + 5975466*m^7 + 86750118*m^6 + 847550822*m^
5 + 5509501002*m^4 + 22992750373*m^3 + 57365875587*m^2 + 74253243015*m + 35
137127025)*x^9 + 165*(m^11 + 137*m^10 + 8259*m^9 + 288027*m^8 + 6423594*m^7
 + 95564154*m^6 + 959352806*m^5 + 6421988758*m^4 + 27624338085*m^3 + 709302
62349*m^2 + 94034286855*m + 45176306175)*x^7 + 55*(m^11 + 139*m^10 + 8523*m
^9 + 303225*m^8 + 6923658*m^7 + 105911022*m^6 + 1098746774*m^5 + 7643724530
*m^4 + 34359636741*m^3 + 92502445239*m^2 + 128033897103*m + 63246828645)*x^
5 + 11*(m^11 + 141*m^10 + 8795*m^9 + 319455*m^8 + 7481418*m^7 + 118085058*m
^6 + 1274046710*m^5 + 9315318270*m^4 + 44632304581*m^3 + 130403715201*m^2 +
 199334977695*m + 105411381075)*x^3 + (m^11 + 143*m^10 + 9075*m^9 + 336765*
m^8 + 8103018*m^7 + 132426294*m^6 + 1495875590*m^5 + 11641582810*m^4 + 6093
6676581*m^3 + 203363952363*m^2 + 387182170935*m + 316234143225)*x)*(f*x)^m/
(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529312*m^7 + 1
628301884*m^6 + 13137458400*m^5 + 72578259391*m^4 + 264300628944*m^3 + 5905
46123298*m^2 + 703416314160*m + 316234143225)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22598, size = 2495, normalized size = 12.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] ((f*x)^m*m^11*x^23 + 121*(f*x)^m*m^10*x^23 + 11*(f*x)^m*m^11*x^21 + 6435*(f
*x)^m*m^9*x^23 + 1353*(f*x)^m*m^10*x^21 + 197835*(f*x)^m*m^8*x^23 + 55*(f*x
)^m*m^11*x^19 + 72985*(f*x)^m*m^9*x^21 + 3889578*(f*x)^m*m^7*x^23 + 6875*(f
*x)^m*m^10*x^19 + 2271555*(f*x)^m*m^8*x^21 + 51069018*(f*x)^m*m^6*x^23 + 16
5*(f*x)^m*m^11*x^17 + 376365*(f*x)^m*m^9*x^19 + 45134958*(f*x)^m*m^7*x^21 +
 453714470*(f*x)^m*m^5*x^23 + 20955*(f*x)^m*m^10*x^17 + 11870265*(f*x)^m*m^
8*x^19 + 597988314*(f*x)^m*m^6*x^21 + 2702025590*(f*x)^m*m^4*x^23 + 330*(f*
x)^m*m^11*x^15 + 1164735*(f*x)^m*m^9*x^17 + 238653030*(f*x)^m*m^7*x^19 + 53
53566130*(f*x)^m*m^5*x^21 + 10431670821*(f*x)^m*m^3*x^23 + 42570*(f*x)^m*m^
10*x^15 + 37263105*(f*x)^m*m^8*x^17 + 3194704590*(f*x)^m*m^6*x^19 + 3208715
3670*(f*x)^m*m^4*x^21 + 24372200061*(f*x)^m*m^2*x^23 + 462*(f*x)^m*m^11*x^1
3 + 2403390*(f*x)^m*m^9*x^15 + 759091410*(f*x)^m*m^7*x^17 + 28857216410*(f*
x)^m*m^5*x^19 + 124530626231*(f*x)^m*m^3*x^21 + 29985521895*(f*x)^m*m*x^23
+ 60522*(f*x)^m*m^10*x^13 + 78076350*(f*x)^m*m^8*x^15 + 10282782510*(f*x)^m
*m^6*x^17 + 174273100210*(f*x)^m*m^4*x^19 + 292163767533*(f*x)^m*m^2*x^21 +
 13749310575*(f*x)^m*x^23 + 462*(f*x)^m*m^11*x^11 + 3471930*(f*x)^m*m^9*x^1
3 + 1613983140*(f*x)^m*m^7*x^15 + 93862508190*(f*x)^m*m^5*x^17 + 6806153625
15*(f*x)^m*m^3*x^19 + 360568238085*(f*x)^m*m*x^21 + 61446*(f*x)^m*m^10*x^11
 + 114642990*(f*x)^m*m^8*x^13 + 22164925860*(f*x)^m*m^6*x^15 + 572017996770
*(f*x)^m*m^4*x^17 + 1604842704135*(f*x)^m*m^2*x^19 + 165646455975*(f*x)^m*x
^21 + 330*(f*x)^m*m^11*x^9 + 3582810*(f*x)^m*m^9*x^11 + 2408820876*(f*x)^m*
m^7*x^13 + 204865733820*(f*x)^m*m^5*x^15 + 2251106854425*(f*x)^m*m^3*x^17 +
 1988025402825*(f*x)^m*m*x^19 + 44550*(f*x)^m*m^10*x^9 + 120367170*(f*x)^m*
m^8*x^11 + 33609870756*(f*x)^m*m^6*x^13 + 1262375264700*(f*x)^m*m^4*x^15 +
5340787250535*(f*x)^m*m^2*x^17 + 915414625125*(f*x)^m*x^19 + 165*(f*x)^m*m^
11*x^7 + 2640990*(f*x)^m*m^9*x^9 + 2575140876*(f*x)^m*m^7*x^11 + 3153471505
80*(f*x)^m*m^5*x^13 + 5015196628530*(f*x)^m*m^3*x^15 + 6646727085075*(f*x)^
m*m*x^17 + 22605*(f*x)^m*m^10*x^7 + 90358290*(f*x)^m*m^8*x^9 + 36597992508*
(f*x)^m*m^6*x^11 + 1969992823260*(f*x)^m*m^4*x^13 + 11991258123570*(f*x)^m*
m^2*x^15 + 3069331390125*(f*x)^m*x^17 + 55*(f*x)^m*m^11*x^5 + 1362735*(f*x)
^m*m^9*x^7 + 1971903780*(f*x)^m*m^7*x^9 + 349697552820*(f*x)^m*m^5*x^11 + 7
921249136262*(f*x)^m*m^3*x^13 + 15011348834790*(f*x)^m*m*x^15 + 7645*(f*x)^
m*m^10*x^5 + 47524455*(f*x)^m*m^8*x^7 + 28627538940*(f*x)^m*m^6*x^9 + 22228
32699780*(f*x)^m*m^4*x^11 + 19130651800722*(f*x)^m*m^2*x^13 + 6957151150950
*(f*x)^m*x^15 + 11*(f*x)^m*m^11*x^3 + 468765*(f*x)^m*m^9*x^5 + 1059893010*(
f*x)^m*m^7*x^7 + 279691771260*(f*x)^m*m^5*x^9 + 9079996141062*(f*x)^m*m^3*x
^11 + 24133835554290*(f*x)^m*m*x^13 + 1551*(f*x)^m*m^10*x^3 + 16677375*(f*x
```

)^m*m^8*x^5 + 15768085410*(f*x)^m*m^6*x^7 + 1818135330660*(f*x)^m*m^4*x^9 + 22226933020446*(f*x)^m*m^2*x^11 + 11238474936150*(f*x)^m*x^13 + (f*x)^m*m^11*x + 96745*(f*x)^m*m^9*x^3 + 380801190*(f*x)^m*m^7*x^5 + 158293212990*(f*x)^m*m^5*x^7 + 7587607623090*(f*x)^m*m^3*x^9 + 28336045738770*(f*x)^m*m*x^11 + 143*(f*x)^m*m^10*x + 3514005*(f*x)^m*m^8*x^3 + 5825106210*(f*x)^m*m^6*x^5 + 1059628145070*(f*x)^m*m^4*x^7 + 18930738943710*(f*x)^m*m^2*x^9 + 13281834015450*(f*x)^m*x^11 + 9075*(f*x)^m*m^9*x + 82295598*(f*x)^m*m^7*x^3 + 60431072570*(f*x)^m*m^5*x^5 + 4558015784025*(f*x)^m*m^3*x^7 + 24503570194950*(f*x)^m*m*x^9 + 336765*(f*x)^m*m^8*x + 1298935638*(f*x)^m*m^6*x^3 + 420404849150*(f*x)^m*m^4*x^5 + 11703493287585*(f*x)^m*m^2*x^7 + 11595251918250*(f*x)^m*x^9 + 8103018*(f*x)^m*m^7*x + 14014513810*(f*x)^m*m^5*x^3 + 1889780020755*(f*x)^m*m^3*x^5 + 15515657331075*(f*x)^m*m*x^7 + 132426294*(f*x)^m*m^6*x + 102468500970*(f*x)^m*m^4*x^3 + 5087634488145*(f*x)^m*m^2*x^5 + 7454090518875*(f*x)^m*x^7 + 1495875590*(f*x)^m*m^5*x + 490955350391*(f*x)^m*m^3*x^3 + 7041864340665*(f*x)^m*m*x^5 + 11641582810*(f*x)^m*m^4*x + 1434440867211*(f*x)^m*m^2*x^3 + 3478575575475*(f*x)^m*x^5 + 60936676581*(f*x)^m*m^3*x + 2192684754645*(f*x)^m*m*x^3 + 203363952363*(f*x)^m*m^2*x + 1159525191825*(f*x)^m*x^3 + 387182170935*(f*x)^m*m*x + 316234143225*(f*x)^m*x)/(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529312*m^7 + 1628301884*m^6 + 13137458400*m^5 + 72578259391*m^4 + 264300628944*m^3 + 590546123298*m^2 + 703416314160*m + 316234143225)

3.66 $\int x^5 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=34

$$\frac{1}{28} \left(x^2+1\right)^{14}-\frac{1}{13} \left(x^2+1\right)^{13}+\frac{1}{24} \left(x^2+1\right)^{12}$$

```
[Out] (1 + x^2)^12/24 - (1 + x^2)^13/13 + (1 + x^2)^14/28
```

Rubi [A] time = 0.0467126, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 266, 43}

$$\frac{1}{28} \left(x^2+1\right)^{14}-\frac{1}{13} \left(x^2+1\right)^{13}+\frac{1}{24} \left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]

[Out] (1 + x^2)^12/24 - (1 + x^2)^13/13 + (1 + x^2)^14/28
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx &= \int x^5 \left(1+x^2\right)^{11} \, dx\\
&= \frac{1}{2} \text{Subst}\left(\int x^2 (1+x)^{11} \, dx, x, x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left((1+x)^{11}-2(1+x)^{12}+(1+x)^{13}\right) \, dx, x, x^2\right)\\
&= \frac{1}{24} \left(1+x^2\right)^{12}-\frac{1}{13} \left(1+x^2\right)^{13}+\frac{1}{28} \left(1+x^2\right)^{14}
\end{aligned}$$

Mathematica [B] time = 0.0019485, size = 85, normalized size = 2.5

$$\frac{x^{28}}{28}+\frac{11x^{26}}{26}+\frac{55x^{24}}{24}+\frac{15x^{22}}{2}+\frac{33x^{20}}{2}+\frac{77x^{18}}{3}+\frac{231x^{16}}{8}+\frac{165x^{14}}{7}+\frac{55x^{12}}{4}+\frac{11x^{10}}{2}+\frac{11x^8}{8}+\frac{x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^6/6 + (11*x^8)/8 + (11*x^10)/2 + (55*x^12)/4 + (165*x^14)/7 + (231*x^16)/
8 + (77*x^18)/3 + (33*x^20)/2 + (15*x^22)/2 + (55*x^24)/24 + (11*x^26)/26 +
 x^28/28
```

Maple [B] time = 0.001, size = 62, normalized size = 1.8

$$\frac{x^{28}}{28}+\frac{11\,x^{26}}{26}+\frac{55\,x^{24}}{24}+\frac{15\,x^{22}}{2}+\frac{33\,x^{20}}{2}+\frac{77\,x^{18}}{3}+\frac{231\,x^{16}}{8}+\frac{165\,x^{14}}{7}+\frac{55\,x^{12}}{4}+\frac{11\,x^{10}}{2}+\frac{11\,x^{8}}{8}+\frac{x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/28*x^28+11/26*x^26+55/24*x^24+15/2*x^22+33/2*x^20+77/3*x^18+231/8*x^16+16
5/7*x^14+55/4*x^12+11/2*x^10+11/8*x^8+1/6*x^6
```

Maxima [B] time = 0.953037, size = 82, normalized size = 2.41

$$\frac{1}{28}\,x^{28}+\frac{11}{26}\,x^{26}+\frac{55}{24}\,x^{24}+\frac{15}{2}\,x^{22}+\frac{33}{2}\,x^{20}+\frac{77}{3}\,x^{18}+\frac{231}{8}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{55}{4}\,x^{12}+\frac{11}{2}\,x^{10}+\frac{11}{8}\,x^{8}+\frac{1}{6}\,x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

Fricas [B] time = 1.25211, size = 194, normalized size = 5.71

$$\frac{1}{28}x^{28}+\frac{11}{26}x^{26}+\frac{55}{24}x^{24}+\frac{15}{2}x^{22}+\frac{33}{2}x^{20}+\frac{77}{3}x^{18}+\frac{231}{8}x^{16}+\frac{165}{7}x^{14}+\frac{55}{4}x^{12}+\frac{11}{2}x^{10}+\frac{11}{8}x^{8}+\frac{1}{6}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

Sympy [B] time = 0.068427, size = 76, normalized size = 2.24

$$\frac{x^{28}}{28}+\frac{11x^{26}}{26}+\frac{55x^{24}}{24}+\frac{15x^{22}}{2}+\frac{33x^{20}}{2}+\frac{77x^{18}}{3}+\frac{231x^{16}}{8}+\frac{165x^{14}}{7}+\frac{55x^{12}}{4}+\frac{11x^{10}}{2}+\frac{11x^{8}}{8}+\frac{x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**28/28 + 11*x**26/26 + 55*x**24/24 + 15*x**22/2 + 33*x**20/2 + 77*x**18/3
 + 231*x**16/8 + 165*x**14/7 + 55*x**12/4 + 11*x**10/2 + 11*x**8/8 + x**6/6
```

Giac [B] time = 1.11737, size = 82, normalized size = 2.41

$$\frac{1}{28}\,x^{28} + \frac{11}{26}\,x^{26} + \frac{55}{24}\,x^{24} + \frac{15}{2}\,x^{22} + \frac{33}{2}\,x^{20} + \frac{77}{3}\,x^{18} + \frac{231}{8}\,x^{16} + \frac{165}{7}\,x^{14} + \frac{55}{4}\,x^{12} + \frac{11}{2}\,x^{10} + \frac{11}{8}\,x^{8} + \frac{1}{6}\,x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

3.67 $\int x^4 (1+x^2) (1+2x^2+x^4)^5 \, dx$

Optimal. Leaf size=83

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

Rubi [A] time = 0.0314287, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {28, 270}

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 (1+x^2) (1+2x^2+x^4)^5 \, dx &= \int x^4 (1+x^2)^{11} \, dx\\ &= \int (x^4+11x^6+55x^8+165x^{10}+330x^{12}+462x^{14}+462x^{16}+330x^{18}+165x^{20}+\\ &= \frac{x^5}{5}+\frac{11x^7}{7}+\frac{55x^9}{9}+15x^{11}+\frac{330x^{13}}{13}+\frac{154x^{15}}{5}+\frac{462x^{17}}{17}+\frac{330x^{19}}{19}+\frac{55x^{21}}{7}+\end{aligned}$$

Mathematica [A] time = 0.0016816, size = 83, normalized size = 1.

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

Maple [A] time = 0.001, size = 62, normalized size = 0.8

$$\frac{x^5}{5}+\frac{11\,x^7}{7}+\frac{55\,x^9}{9}+15\,x^{11}+\frac{330\,x^{13}}{13}+\frac{154\,x^{15}}{5}+\frac{462\,x^{17}}{17}+\frac{330\,x^{19}}{19}+\frac{55\,x^{21}}{7}+\frac{55\,x^{23}}{23}+\frac{11\,x^{25}}{25}+\frac{x^{27}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/5*x^5+11/7*x^7+55/9*x^9+15*x^11+330/13*x^13+154/5*x^15+462/17*x^17+330/19
*x^19+55/7*x^21+55/23*x^23+11/25*x^25+1/27*x^27
```

Maxima [A] time = 0.964315, size = 82, normalized size = 0.99

$$\frac{1}{27}\,x^{27}+\frac{11}{25}\,x^{25}+\frac{55}{23}\,x^{23}+\frac{55}{7}\,x^{21}+\frac{330}{19}\,x^{19}+\frac{462}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{330}{13}\,x^{13}+15\,x^{11}+\frac{55}{9}\,x^9+\frac{11}{7}\,x^7+\frac{1}{5}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17
 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

Fricas [A] time = 1.25451, size = 197, normalized size = 2.37

$$\frac{1}{27}x^{27}+\frac{11}{25}x^{25}+\frac{55}{23}x^{23}+\frac{55}{7}x^{21}+\frac{330}{19}x^{19}+\frac{462}{17}x^{17}+\frac{154}{5}x^{15}+\frac{330}{13}x^{13}+15x^{11}+\frac{55}{9}x^9+\frac{11}{7}x^7+\frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17
 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

Sympy [A] time = 0.069771, size = 75, normalized size = 0.9

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**27/27 + 11*x**25/25 + 55*x**23/23 + 55*x**21/7 + 330*x**19/19 + 462*x**1
7/17 + 154*x**15/5 + 330*x**13/13 + 15*x**11 + 55*x**9/9 + 11*x**7/7 + x**5
/5
```

Giac [A] time = 1.10929, size = 82, normalized size = 0.99

$$\frac{1}{27}x^{27} + \frac{11}{25}x^{25} + \frac{55}{23}x^{23} + \frac{55}{7}x^{21} + \frac{330}{19}x^{19} + \frac{462}{17}x^{17} + \frac{154}{5}x^{15} + \frac{330}{13}x^{13} + 15\,x^{11} + \frac{55}{9}x^{9} + \frac{11}{7}x^{7} + \frac{1}{5}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17
 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

3.68 $\int x^3\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=23

$$\frac{1}{26}\left(x^2+1\right)^{13}-\frac{1}{24}\left(x^2+1\right)^{12}$$

```
[Out] -(1 + x^2)^12/24 + (1 + x^2)^13/26
```

Rubi [A] time = 0.0219489, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 266, 43}

$$\frac{1}{26}\left(x^2+1\right)^{13}-\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] -(1 + x^2)^12/24 + (1 + x^2)^13/26
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x^3\left(1+x^2\right)^{11}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int x(1+x)^{11}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(-(1+x)^{11}+(1+x)^{12}\right)dx,x,x^2\right)\\ &= -\frac{1}{24}\left(1+x^2\right)^{12}+\frac{1}{26}\left(1+x^2\right)^{13}\end{aligned}$$

Mathematica [B] time = 0.0016193, size = 83, normalized size = 3.61

$$\frac{x^{26}}{26}+\frac{11x^{24}}{24}+\frac{5x^{22}}{2}+\frac{33x^{20}}{4}+\frac{55x^{18}}{3}+\frac{231x^{16}}{8}+33x^{14}+\frac{55x^{12}}{2}+\frac{33x^{10}}{2}+\frac{55x^8}{8}+\frac{11x^6}{6}+\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^4/4 + (11*x^6)/6 + (55*x^8)/8 + (33*x^10)/2 + (55*x^12)/2 + 33*x^14 + (23
1*x^16)/8 + (55*x^18)/3 + (33*x^20)/4 + (5*x^22)/2 + (11*x^24)/24 + x^26/26
```

Maple [B] time = 0.002, size = 62, normalized size = 2.7

$$\frac{x^{26}}{26}+\frac{11\,x^{24}}{24}+\frac{5\,x^{22}}{2}+\frac{33\,x^{20}}{4}+\frac{55\,x^{18}}{3}+\frac{231\,x^{16}}{8}+33\,x^{14}+\frac{55\,x^{12}}{2}+\frac{33\,x^{10}}{2}+\frac{55\,x^{8}}{8}+\frac{11\,x^{6}}{6}+\frac{x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/26*x^26+11/24*x^24+5/2*x^22+33/4*x^20+55/3*x^18+231/8*x^16+33*x^14+55/2*x
^12+33/2*x^10+55/8*x^8+11/6*x^6+1/4*x^4
```

Maxima [B] time = 0.940523, size = 82, normalized size = 3.57

$$\frac{1}{26}\,x^{26}+\frac{11}{24}\,x^{24}+\frac{5}{2}\,x^{22}+\frac{33}{4}\,x^{20}+\frac{55}{3}\,x^{18}+\frac{231}{8}\,x^{16}+33\,x^{14}+\frac{55}{2}\,x^{12}+\frac{33}{2}\,x^{10}+\frac{55}{8}\,x^{8}+\frac{11}{6}\,x^{6}+\frac{1}{4}\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

Fricas [B] time = 1.25516, size = 186, normalized size = 8.09

$$\frac{1}{26}x^{26}+\frac{11}{24}x^{24}+\frac{5}{2}x^{22}+\frac{33}{4}x^{20}+\frac{55}{3}x^{18}+\frac{231}{8}x^{16}+33x^{14}+\frac{55}{2}x^{12}+\frac{33}{2}x^{10}+\frac{55}{8}x^{8}+\frac{11}{6}x^{6}+\frac{1}{4}x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

Sympy [B] time = 0.07234, size = 75, normalized size = 3.26

$$\frac{x^{26}}{26}+\frac{11x^{24}}{24}+\frac{5x^{22}}{2}+\frac{33x^{20}}{4}+\frac{55x^{18}}{3}+\frac{231x^{16}}{8}+33x^{14}+\frac{55x^{12}}{2}+\frac{33x^{10}}{2}+\frac{55x^{8}}{8}+\frac{11x^{6}}{6}+\frac{x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**26/26 + 11*x**24/24 + 5*x**22/2 + 33*x**20/4 + 55*x**18/3 + 231*x**16/8
+ 33*x**14 + 55*x**12/2 + 33*x**10/2 + 55*x**8/8 + 11*x**6/6 + x**4/4
```

Giac [B] time = 1.11224, size = 82, normalized size = 3.57

$$\frac{1}{26}\,x^{26}+\frac{11}{24}\,x^{24}+\frac{5}{2}\,x^{22}+\frac{33}{4}\,x^{20}+\frac{55}{3}\,x^{18}+\frac{231}{8}\,x^{16}+33\,x^{14}+\frac{55}{2}\,x^{12}+\frac{33}{2}\,x^{10}+\frac{55}{8}\,x^{8}+\frac{11}{6}\,x^{6}+\frac{1}{4}\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

3.69 $\int x^2 (1+x^2) (1+2x^2+x^4)^5 \, dx$

Optimal. Leaf size=83

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

Rubi [A] time = 0.0268132, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {28, 270}

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 (1+x^2) (1+2x^2+x^4)^5 \, dx &= \int x^2 (1+x^2)^{11} \, dx \\
&= \int (x^2+11x^4+55x^6+165x^8+330x^{10}+462x^{12}+462x^{14}+330x^{16}+165x^{18}+5 \\
&= \frac{x^3}{3}+\frac{11x^5}{5}+\frac{55x^7}{7}+\frac{55x^9}{3}+30x^{11}+\frac{462x^{13}}{13}+\frac{154x^{15}}{5}+\frac{330x^{17}}{17}+\frac{165x^{19}}{19}+\frac{55}{2}
\end{aligned}$$

Mathematica [A] time = 0.0015298, size = 83, normalized size = 1.

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

Maple [A] time = 0.001, size = 62, normalized size = 0.8

$$\frac{x^3}{3}+\frac{11\,x^5}{5}+\frac{55\,x^7}{7}+\frac{55\,x^9}{3}+30\,x^{11}+\frac{462\,x^{13}}{13}+\frac{154\,x^{15}}{5}+\frac{330\,x^{17}}{17}+\frac{165\,x^{19}}{19}+\frac{55\,x^{21}}{21}+\frac{11\,x^{23}}{23}+\frac{x^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/3*x^3+11/5*x^5+55/7*x^7+55/3*x^9+30*x^11+462/13*x^13+154/5*x^15+330/17*x^
17+165/19*x^19+55/21*x^21+11/23*x^23+1/25*x^25
```

Maxima [A] time = 0.96931, size = 82, normalized size = 0.99

$$\frac{1}{25}\,x^{25}+\frac{11}{23}\,x^{23}+\frac{55}{21}\,x^{21}+\frac{165}{19}\,x^{19}+\frac{330}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{462}{13}\,x^{13}+30\,x^{11}+\frac{55}{3}\,x^9+\frac{55}{7}\,x^7+\frac{11}{5}\,x^5+\frac{1}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^1
5 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

Fricas [A] time = 1.24423, size = 196, normalized size = 2.36

$$\frac{1}{25}x^{25}+\frac{11}{23}x^{23}+\frac{55}{21}x^{21}+\frac{165}{19}x^{19}+\frac{330}{17}x^{17}+\frac{154}{5}x^{15}+\frac{462}{13}x^{13}+30x^{11}+\frac{55}{3}x^9+\frac{55}{7}x^7+\frac{11}{5}x^5+\frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^1
5 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

Sympy [A] time = 0.068047, size = 75, normalized size = 0.9

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**25/25 + 11*x**23/23 + 55*x**21/21 + 165*x**19/19 + 330*x**17/17 + 154*x*
*15/5 + 462*x**13/13 + 30*x**11 + 55*x**9/3 + 55*x**7/7 + 11*x**5/5 + x**3/
3
```

Giac [A] time = 1.13183, size = 82, normalized size = 0.99

$$\frac{1}{25}\,x^{25} + \frac{11}{23}\,x^{23} + \frac{55}{21}\,x^{21} + \frac{165}{19}\,x^{19} + \frac{330}{17}\,x^{17} + \frac{154}{5}\,x^{15} + \frac{462}{13}\,x^{13} + 30\,x^{11} + \frac{55}{3}\,x^{9} + \frac{55}{7}\,x^{7} + \frac{11}{5}\,x^{5} + \frac{1}{3}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^1
5 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

3.70 $\int x\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=11

$$\frac{1}{24}\left(x^2+1\right)^{12}$$

```
[Out] (1 + x^2)^12/24
```

Rubi [A] time = 0.0023995, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {28, 261}

$$\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x\left(1+x^2\right)^{11}\,dx\\ &= \frac{1}{24}\left(1+x^2\right)^{12}\end{aligned}$$

Mathematica [A] time = 0.0017517, size = 11, normalized size = 1.

$$\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24
```

Maple [B] time = 0.001, size = 62, normalized size = 5.6

$$\frac{x^{24}}{24}+\frac{x^{22}}{2}+\frac{11\,x^{20}}{4}+\frac{55\,x^{18}}{6}+\frac{165\,x^{16}}{8}+33\,x^{14}+\frac{77\,x^{12}}{2}+33\,x^{10}+\frac{165\,x^{8}}{8}+\frac{55\,x^{6}}{6}+\frac{11\,x^{4}}{4}+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] 1/24*x^24+1/2*x^22+11/4*x^20+55/6*x^18+165/8*x^16+33*x^14+77/2*x^12+33*x^10
+165/8*x^8+55/6*x^6+11/4*x^4+1/2*x^2
```

Maxima [B] time = 0.981964, size = 82, normalized size = 7.45

$$\frac{1}{24}\,x^{24}+\frac{1}{2}\,x^{22}+\frac{11}{4}\,x^{20}+\frac{55}{6}\,x^{18}+\frac{165}{8}\,x^{16}+33\,x^{14}+\frac{77}{2}\,x^{12}+33\,x^{10}+\frac{165}{8}\,x^{8}+\frac{55}{6}\,x^{6}+\frac{11}{4}\,x^{4}+\frac{1}{2}\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2*
x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2
```

Fricas [B] time = 1.46461, size = 182, normalized size = 16.55

$$\frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+\frac{165}{8}x^{16}+33x^{14}+\frac{77}{2}x^{12}+33x^{10}+\frac{165}{8}x^{8}+\frac{55}{6}x^{6}+\frac{11}{4}x^{4}+\frac{1}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2*
x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2
```

Sympy [B] time = 0.068124, size = 71, normalized size = 6.45

$$\frac{x^{24}}{24}+\frac{x^{22}}{2}+\frac{11x^{20}}{4}+\frac{55x^{18}}{6}+\frac{165x^{16}}{8}+33x^{14}+\frac{77x^{12}}{2}+33x^{10}+\frac{165x^{8}}{8}+\frac{55x^{6}}{6}+\frac{11x^{4}}{4}+\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**24/24 + x**22/2 + 11*x**20/4 + 55*x**18/6 + 165*x**16/8 + 33*x**14 + 77*
x**12/2 + 33*x**10 + 165*x**8/8 + 55*x**6/6 + 11*x**4/4 + x**2/2
```

Giac [B] time = 1.11146, size = 82, normalized size = 7.45

$$\frac{1}{24}\,x^{24}+\frac{1}{2}\,x^{22}+\frac{11}{4}\,x^{20}+\frac{55}{6}\,x^{18}+\frac{165}{8}\,x^{16}+33\,x^{14}+\frac{77}{2}\,x^{12}+33\,x^{10}+\frac{165}{8}\,x^{8}+\frac{55}{6}\,x^{6}+\frac{11}{4}\,x^{4}+\frac{1}{2}\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2*
x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2
```

3.71 $\int \left(1+x^2\right) \left(1+2 x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=73

$$\frac{x^{23}}{23}+\frac{11 x^{21}}{21}+\frac{55 x^{19}}{19}+\frac{165 x^{17}}{17}+22 x^{15}+\frac{462 x^{13}}{13}+42 x^{11}+\frac{110 x^9}{3}+\frac{165 x^7}{7}+11 x^5+\frac{11 x^3}{3}+x$$

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

Rubi [A] time = 0.0217871, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {28, 194}

$$\frac{x^{23}}{23}+\frac{11 x^{21}}{21}+\frac{55 x^{19}}{19}+\frac{165 x^{17}}{17}+22 x^{15}+\frac{462 x^{13}}{13}+42 x^{11}+\frac{110 x^9}{3}+\frac{165 x^7}{7}+11 x^5+\frac{11 x^3}{3}+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \left(1+x^2\right) \left(1+2 x^2+x^4\right)^5 \, dx &= \int \left(1+x^2\right)^{11} \, dx\\ &= \int \left(1+11 x^2+55 x^4+165 x^6+330 x^8+462 x^{10}+462 x^{12}+330 x^{14}+165 x^{16}+55 x^{18}\right.\\ &= x+\frac{11 x^3}{3}+11 x^5+\frac{165 x^7}{7}+\frac{110 x^9}{3}+42 x^{11}+\frac{462 x^{13}}{13}+22 x^{15}+\frac{165 x^{17}}{17}+\frac{55 x^{19}}{19}+\frac{1}{} \end{aligned}$$

Mathematica [A] time = 0.0008704, size = 73, normalized size = 1.

$$\frac{x^{23}}{23}+\frac{11 x^{21}}{21}+\frac{55 x^{19}}{19}+\frac{165 x^{17}}{17}+22 x^{15}+\frac{462 x^{13}}{13}+42 x^{11}+\frac{110 x^9}{3}+\frac{165 x^7}{7}+11 x^5+\frac{11 x^3}{3}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

Maple [A] time = 0.001, size = 58, normalized size = 0.8

$$x+\frac{11\,x^{3}}{3}+11\,x^{5}+\frac{165\,x^{7}}{7}+\frac{110\,x^{9}}{3}+42\,x^{11}+\frac{462\,x^{13}}{13}+22\,x^{15}+\frac{165\,x^{17}}{17}+\frac{55\,x^{19}}{19}+\frac{11\,x^{21}}{21}+\frac{x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5,x)
```

```
[Out] x+11/3*x^3+11*x^5+165/7*x^7+110/3*x^9+42*x^11+462/13*x^13+22*x^15+165/17*x^
17+55/19*x^19+11/21*x^21+1/23*x^23
```

Maxima [A] time = 0.965563, size = 77, normalized size = 1.05

$$\frac{1}{23}\,x^{23}+\frac{11}{21}\,x^{21}+\frac{55}{19}\,x^{19}+\frac{165}{17}\,x^{17}+22\,x^{15}+\frac{462}{13}\,x^{13}+42\,x^{11}+\frac{110}{3}\,x^{9}+\frac{165}{7}\,x^{7}+11\,x^{5}+\frac{11}{3}\,x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

Fricas [A] time = 1.52189, size = 180, normalized size = 2.47

$$\frac{1}{23}x^{23}+\frac{11}{21}x^{21}+\frac{55}{19}x^{19}+\frac{165}{17}x^{17}+22x^{15}+\frac{462}{13}x^{13}+42x^{11}+\frac{110}{3}x^{9}+\frac{165}{7}x^{7}+11x^{5}+\frac{11}{3}x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

Sympy [A] time = 0.071701, size = 68, normalized size = 0.93

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^{9}}{3}+\frac{165x^{7}}{7}+11x^{5}+\frac{11x^{3}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**23/23 + 11*x**21/21 + 55*x**19/19 + 165*x**17/17 + 22*x**15 + 462*x**13/
13 + 42*x**11 + 110*x**9/3 + 165*x**7/7 + 11*x**5 + 11*x**3/3 + x
```

Giac [A] time = 1.11735, size = 77, normalized size = 1.05

$$\frac{1}{23}\,x^{23} + \frac{11}{21}\,x^{21} + \frac{55}{19}\,x^{19} + \frac{165}{17}\,x^{17} + 22\,x^{15} + \frac{462}{13}\,x^{13} + 42\,x^{11} + \frac{110}{3}\,x^{9} + \frac{165}{7}\,x^{7} + 11\,x^{5} + \frac{11}{3}\,x^{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

3.72 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} \, dx$

Optimal. Leaf size=80

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{55x^4}{4}+\frac{11x^2}{2}+\log(x)$$

```
[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5 + (77*x^1
2)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20 + x^22/22
 + Log[x]
```

Rubi [A] time = 0.0332073, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 266, 43}

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{55x^4}{4}+\frac{11x^2}{2}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5 + (77*x^1
2)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20 + x^22/22
 + Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx = \int \frac{(1+x^2)^{11}}{x}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{(1+x)^{11}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(11+\frac{1}{x}+55x+165x^2+330x^3+462x^4+462x^5+330x^6+165x^7+5\right.\right.$$
$$= \frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11}{2}$$

Mathematica [A] time = 0.0032637, size = 80, normalized size = 1.

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{55x^4}{4}+\frac{11x^2}{2}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5 + (77*x^1
2)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20 + x^22/22
 + Log[x]
```

Maple [A] time = 0.001, size = 59, normalized size = 0.7

$$\frac{11\,x^2}{2}+\frac{55\,x^4}{4}+\frac{55\,x^6}{2}+\frac{165\,x^8}{4}+\frac{231\,x^{10}}{5}+\frac{77\,x^{12}}{2}+\frac{165\,x^{14}}{7}+\frac{165\,x^{16}}{16}+\frac{55\,x^{18}}{18}+\frac{11\,x^{20}}{20}+\frac{x^{22}}{22}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x,x)
```

```
[Out] 11/2*x^2+55/4*x^4+55/2*x^6+165/4*x^8+231/5*x^10+77/2*x^12+165/7*x^14+165/16
*x^16+55/18*x^18+11/20*x^20+1/22*x^22+ln(x)
```

Maxima [A] time = 0.973522, size = 84, normalized size = 1.05

$$\frac{1}{22}\,x^{22}+\frac{11}{20}\,x^{20}+\frac{55}{18}\,x^{18}+\frac{165}{16}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{77}{2}\,x^{12}+\frac{231}{5}\,x^{10}+\frac{165}{4}\,x^8+\frac{55}{2}\,x^6+\frac{55}{4}\,x^4+\frac{11}{2}\,x^2+\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + 1/2*log(x^2)
```

Fricas [A] time = 1.66668, size = 193, normalized size = 2.41

$$\frac{1}{22}\,x^{22}+\frac{11}{20}\,x^{20}+\frac{55}{18}\,x^{18}+\frac{165}{16}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{77}{2}\,x^{12}+\frac{231}{5}\,x^{10}+\frac{165}{4}\,x^8+\frac{55}{2}\,x^6+\frac{55}{4}\,x^4+\frac{11}{2}\,x^2+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="fricas")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + log(x)
```

Sympy [A] time = 0.10352, size = 75, normalized size = 0.94

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^{8}}{4}+\frac{55x^{6}}{2}+\frac{55x^{4}}{4}+\frac{11x^{2}}{2}+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x,x)
```

```
[Out] x**22/22 + 11*x**20/20 + 55*x**18/18 + 165*x**16/16 + 165*x**14/7 + 77*x**1
2/2 + 231*x**10/5 + 165*x**8/4 + 55*x**6/2 + 55*x**4/4 + 11*x**2/2 + log(x)
```

Giac [A] time = 1.10755, size = 84, normalized size = 1.05

$$\frac{1}{22}x^{22}+\frac{11}{20}x^{20}+\frac{55}{18}x^{18}+\frac{165}{16}x^{16}+\frac{165}{7}x^{14}+\frac{77}{2}x^{12}+\frac{231}{5}x^{10}+\frac{165}{4}x^{8}+\frac{55}{2}x^{6}+\frac{55}{4}x^{4}+\frac{11}{2}x^{2}+\frac{1}{2}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + 1/2*log(x^2)
```

3.73 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2} \, dx$

Optimal. Leaf size=73

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

```
[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11
+ (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

Rubi [A] time = 0.0247976, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {28, 270}

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11
+ (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2}\,dx &= \int \frac{(1+x^2)^{11}}{x^2}\,dx\\ &= \int \left(11+\frac{1}{x^2}+55x^2+165x^4+330x^6+462x^8+462x^{10}+330x^{12}+165x^{14}+55x^{16}+\right.\\ &= -\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}+\frac{11}{}\end{aligned}$$

Mathematica [A] time = 0.0030978, size = 73, normalized size = 1.

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11
+ (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

Maple [A] time = 0.005, size = 60, normalized size = 0.8

$$-x^{-1}+11\,x+\frac{55\,x^{3}}{3}+33\,x^{5}+\frac{330\,x^{7}}{7}+\frac{154\,x^{9}}{3}+42\,x^{11}+\frac{330\,x^{13}}{13}+11\,x^{15}+\frac{55\,x^{17}}{17}+\frac{11\,x^{19}}{19}+\frac{x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x^2,x)
```

```
[Out] -1/x+11*x+55/3*x^3+33*x^5+330/7*x^7+154/3*x^9+42*x^11+330/13*x^13+11*x^15+5
5/17*x^17+11/19*x^19+1/21*x^21
```

Maxima [A] time = 0.927436, size = 80, normalized size = 1.1

$$\frac{1}{21}\,x^{21}+\frac{11}{19}\,x^{19}+\frac{55}{17}\,x^{17}+11\,x^{15}+\frac{330}{13}\,x^{13}+42\,x^{11}+\frac{154}{3}\,x^{9}+\frac{330}{7}\,x^{7}+33\,x^{5}+\frac{55}{3}\,x^{3}+11\,x-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*x^11 + 154
/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x
```

Fricas [A] time = 1.39501, size = 232, normalized size = 3.18

$$\frac{4199\,x^{22}+51051\,x^{20}+285285\,x^{18}+969969\,x^{16}+2238390\,x^{14}+3703518\,x^{12}+4526522\,x^{10}+4157010\,x^{8}+2909907\,x}{88179\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/88179*(4199*x^22 + 51051*x^20 + 285285*x^18 + 969969*x^16 + 2238390*x^14
+ 3703518*x^12 + 4526522*x^10 + 4157010*x^8 + 2909907*x^6 + 1616615*x^4 + 9
69969*x^2 - 88179)/x
```

Sympy [A] time = 0.100858, size = 66, normalized size = 0.9

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^{9}}{3}+\frac{330x^{7}}{7}+33x^{5}+\frac{55x^{3}}{3}+11x-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x**2,x)
```

```
[Out] x**21/21 + 11*x**19/19 + 55*x**17/17 + 11*x**15 + 330*x**13/13 + 42*x**11 +
 154*x**9/3 + 330*x**7/7 + 33*x**5 + 55*x**3/3 + 11*x - 1/x
```

Giac [A] time = 1.11949, size = 80, normalized size = 1.1

$$\frac{1}{21}x^{21}+\frac{11}{19}x^{19}+\frac{55}{17}x^{17}+11\,x^{15}+\frac{330}{13}x^{13}+42\,x^{11}+\frac{154}{3}x^{9}+\frac{330}{7}x^{7}+33\,x^{5}+\frac{55}{3}x^{3}+11\,x-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*x^11 + 154
/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x
```

3.74 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} \, dx$

Optimal. Leaf size=80

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}-\frac{1}{2x^2}+11\log(x)$$

```
[Out] -1/(2*x^2) + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (231*x^10)/5
 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x^20/20 + 11
*Log[x]
```

Rubi [A] time = 0.0396765, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {28, 266, 43}

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}-\frac{1}{2x^2}+11\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/(2*x^2) + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (231*x^10)/5
 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x^20/20 + 11
*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3}\,dx = \int \frac{(1+x^2)^{11}}{x^3}\,dx$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(1+x)^{11}}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(55+\frac{1}{x^2}+\frac{11}{x}+165x+330x^2+462x^3+462x^4+330x^5+165x^6+55\right.\right.$$
$$= -\frac{1}{2x^2}+\frac{55x^2}{2}+\frac{165x^4}{4}+55x^6+\frac{231x^8}{4}+\frac{231x^{10}}{5}+\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}+\frac{11x}{18}$$

Mathematica [A] time = 0.0029021, size = 80, normalized size = 1.

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}-\frac{1}{2x^2}+11\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/(2*x^2) + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (231*x^10)/5
 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x^20/20 + 11
*Log[x]
```

Maple [A] time = 0.007, size = 61, normalized size = 0.8

$$-\frac{1}{2x^2}+\frac{55x^2}{2}+\frac{165x^4}{4}+55x^6+\frac{231x^8}{4}+\frac{231x^{10}}{5}+\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}+\frac{11x^{18}}{18}+\frac{x^{20}}{20}+11\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x^3,x)
```

```
[Out] -1/2/x^2+55/2*x^2+165/4*x^4+55*x^6+231/4*x^8+231/5*x^10+55/2*x^12+165/14*x^
14+55/16*x^16+11/18*x^18+1/20*x^20+11*ln(x)
```

Maxima [A] time = 0.964415, size = 84, normalized size = 1.05

$$\frac{1}{20}x^{20}+\frac{11}{18}x^{18}+\frac{55}{16}x^{16}+\frac{165}{14}x^{14}+\frac{55}{2}x^{12}+\frac{231}{5}x^{10}+\frac{231}{4}x^8+55x^6+\frac{165}{4}x^4+\frac{55}{2}x^2-\frac{1}{2x^2}+\frac{11}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*x^20 + 11/18*x^18 + 55/16*x^16 + 165/14*x^14 + 55/2*x^12 + 231/5*x^10
+ 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2/x^2 + 11/2*log(x^2)
```

Fricas [A] time = 1.46213, size = 227, normalized size = 2.84

$$\frac{252x^{22}+3080x^{20}+17325x^{18}+59400x^{16}+138600x^{14}+232848x^{12}+291060x^{10}+277200x^8+207900x^6+138}{5040x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*x^22 + 3080*x^20 + 17325*x^18 + 59400*x^16 + 138600*x^14 + 2328
48*x^12 + 291060*x^10 + 277200*x^8 + 207900*x^6 + 138600*x^4 + 55440*x^2*lo
g(x) - 2520)/x^2
```

Sympy [A] time = 0.117264, size = 75, normalized size = 0.94

$$\frac{x^{20}}{20} + \frac{11x^{18}}{18} + \frac{55x^{16}}{16} + \frac{165x^{14}}{14} + \frac{55x^{12}}{2} + \frac{231x^{10}}{5} + \frac{231x^{8}}{4} + 55x^{6} + \frac{165x^{4}}{4} + \frac{55x^{2}}{2} + 11\log(x) - \frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x**3,x)
```

```
[Out] x**20/20 + 11*x**18/18 + 55*x**16/16 + 165*x**14/14 + 55*x**12/2 + 231*x**1
0/5 + 231*x**8/4 + 55*x**6 + 165*x**4/4 + 55*x**2/2 + 11*log(x) - 1/(2*x**2
)
```

Giac [A] time = 1.11874, size = 93, normalized size = 1.16

$$\frac{1}{20}x^{20} + \frac{11}{18}x^{18} + \frac{55}{16}x^{16} + \frac{165}{14}x^{14} + \frac{55}{2}x^{12} + \frac{231}{5}x^{10} + \frac{231}{4}x^{8} + 55x^{6} + \frac{165}{4}x^{4} + \frac{55}{2}x^{2} - \frac{11x^{2}+1}{2x^{2}} + \frac{11}{2}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*x^20 + 11/18*x^18 + 55/16*x^16 + 165/14*x^14 + 55/2*x^12 + 231/5*x^10
+ 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2*(11*x^2 + 1)/x^2 + 11/2*l
og(x^2)
```

3.75 $\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=145

$$\frac{x\left(a+bx^2\right)(bd-ae)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((b*d - a*e)*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (e*x^3*
(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[a]*(b*d - a*e)*(
a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])
```

Rubi [A] time = 0.0901167, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1250, 459, 321, 205}

$$\frac{x\left(a+bx^2\right)(bd-ae)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((b*d - a*e)*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (e*x^3*
(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[a]*(b*d - a*e)*(
a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(d+ex^2\right)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{x^2\left(d+ex^2\right)}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(\left(-3b^2d+3abe\right)\left(ab+b^2x^2\right)\right)\int \frac{x^2}{ab+b^2x^2}\,dx}{3b^2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{(bd-ae)x\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a\left(-3b^2d+3abe\right)\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}}{3b^3\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{(bd-ae)x\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}(bd-ae)\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A] time = 0.0493694, size = 80, normalized size = 0.55

$$\frac{\left(a+bx^2\right)\left(\sqrt{b}x\left(-3ae+3bd+bex^2\right)+3\sqrt{a}(ae-bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{3b^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(Sqrt[b]*x*(3*b*d - 3*a*e + b*e*x^2) + 3*Sqrt[a]*(-(b*d) + a*e
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(3*b^(5/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.046, size = 90, normalized size = 0.6

$$\frac{bx^2+a}{3\,b^2}\left(\sqrt{ab}x^3be+3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2e-3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)abd-3\,\sqrt{ab}xae+3\,\sqrt{ab}xbd\right)\frac{1}{\sqrt{\left(bx^2+a\right)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)*((a*b)^(1/2)*x^3*b*e+3*arctan(b*x/(a*b)^(1/2))*a^2*e-3*arctan
(b*x/(a*b)^(1/2))*a*b*d-3*(a*b)^(1/2)*x*a*e+3*(a*b)^(1/2)*x*b*d)/((b*x^2+a)
^2)^(1/2)/b^2/(a*b)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55687, size = 277, normalized size = 1.91

$$\left[\frac{2\,bex^3 - 3\,(bd - ae)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right) + 6\,(bd - ae)x}{6\,b^2}, \frac{bex^3 - 3\,(bd - ae)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 3\,(bd - ae)x}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*e*x^3 - 3*(b*d - a*e)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) -
a)/(b*x^2 + a)) + 6*(b*d - a*e)*x)/b^2, 1/3*(b*e*x^3 - 3*(b*d - a*e)*sqrt(a
/b)*arctan(b*x*sqrt(a/b)/a) + 3*(b*d - a*e)*x)/b^2]
```

Sympy [A] time = 0.511312, size = 90, normalized size = 0.62

$$-\frac{\sqrt{-\frac{a}{b^5}}\,(ae - bd)\log\left(-b^2\sqrt{-\frac{a}{b^5}} + x\right)}{2} + \frac{\sqrt{-\frac{a}{b^5}}\,(ae - bd)\log\left(b^2\sqrt{-\frac{a}{b^5}} + x\right)}{2} + \frac{ex^3}{3b} - \frac{x\,(ae - bd)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-a/b**5)*(a*e - b*d)*log(-b**2*sqrt(-a/b**5) + x)/2 + sqrt(-a/b**5)*(
a*e - b*d)*log(b**2*sqrt(-a/b**5) + x)/2 + e*x**3/(3*b) - x*(a*e - b*d)/b**
2
```

Giac [A] time = 1.10465, size = 136, normalized size = 0.94

$$-\frac{\left(abd\mathrm{sgn}\left(bx^2 + a\right) - a^2e\mathrm{sgn}\left(bx^2 + a\right)\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2x^3e\mathrm{sgn}\left(bx^2 + a\right) + 3\,b^2dx\mathrm{sgn}\left(bx^2 + a\right) - 3\,abxe\mathrm{sgn}\left(bx^2 \cdot\right.}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*b*d*sgn(b*x^2 + a) - a^2*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*b^2) + 1/3*(b^2*x^3*e*sgn(b*x^2 + a) + 3*b^2*d*x*sgn(b*x^2 + a) - 3*a*
b*x*e*sgn(b*x^2 + a))/b^3
```

3.76 $\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=83

$$\frac{(a+bx^2)(bd-ae)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}$$

```
[Out] (e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*b^2) + ((b*d - a*e)*(a + b*x^2)*Log[
a + b*x^2])/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.072228, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1247, 640, 608, 31}

$$\frac{(a+bx^2)(bd-ae)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```


```
[Out] (e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*b^2) + ((b*d - a*e)*(a + b*x^2)*Log[
a + b*x^2])/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```


Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 608

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x\left(d+ex^2\right)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+ex}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^2\right)$$
$$= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} + \frac{(bd-ae)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^2\right)}{2b}$$
$$= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} + \frac{\left((bd-ae)\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^2\right)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} + \frac{(bd-ae)\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0212931, size = 51, normalized size = 0.61

$$\frac{\left(a+bx^2\right)\left((bd-ae)\log\left(a+bx^2\right)+bex^2\right)}{2b^2\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(b*e*x^2 + (b*d - a*e)*Log[a + b*x^2]))/(2*b^2*Sqrt[(a + b*x^2
)^2])
```

Maple [A] time = 0.007, size = 55, normalized size = 0.7

$$-\frac{\left(bx^2+a\right)\left(-x^2eb+\ln\left(bx^2+a\right)ae-\ln\left(bx^2+a\right)bd\right)}{2\,b^2}\frac{1}{\sqrt{\left(bx^2+a\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)*(-x^2*e*b+ln(b*x^2+a)*a*e-ln(b*x^2+a)*b*d)/((b*x^2+a)^2)^(1/
2)/b^2
```

Maxima [A] time = 0.99303, size = 92, normalized size = 1.11

$$\frac{1}{2}\sqrt{\frac{1}{b^2}}d\log\left(x^2+\frac{a}{b}\right)-\frac{1}{2}\left(\frac{a\sqrt{\frac{1}{b^2}}\log\left(x^2+\frac{a}{b}\right)}{b}-\frac{\sqrt{b^2x^4+2\,abx^2+a^2}}{b^2}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b^(-2))*d*log(x^2 + a/b) - 1/2*(a*sqrt(b^(-2))*log(x^2 + a/b)/b -
sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)/b^2)*e
```

Fricas [A] time = 1.52182, size = 65, normalized size = 0.78

$$\frac{bex^2 + (bd - ae)\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*e*x^2 + (b*d - a*e)*log(b*x^2 + a))/b^2
```

Sympy [A] time = 0.432277, size = 27, normalized size = 0.33

$$\frac{ex^2}{2b} - \frac{(ae - bd)\log\left(a + bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] e*x**2/(2*b) - (a*e - b*d)*log(a + b*x**2)/(2*b**2)
```

Giac [A] time = 1.13645, size = 57, normalized size = 0.69

$$\frac{1}{2}\left(\frac{x^2e}{b} + \frac{(bd - ae)\log\left(\left|bx^2 + a\right|\right)}{b^2}\right)\mathrm{sgn}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e/b + (b*d - a*e)*log(abs(b*x^2 + a))/b^2)*sgn(b*x^2 + a)
```

3.77 $\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=97

$$\frac{(a+bx^2)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (e*x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*(a + b
*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])
```

Rubi [A] time = 0.0470934, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1148, 388, 205}

$$\frac{(a+bx^2)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*(a + b
*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])
```

Rule 1148

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{d+ex^2}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{ex\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(\left(-b^2d+abe\right)\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{b^2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{ex\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0293192, size = 69, normalized size = 0.71

$$-\frac{\left(a+bx^2\right)\left((ae-bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{a}\sqrt{b}ex\right)}{\sqrt{a}b^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*e*x) + (-(b*d) + a*e)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]]))/(Sqrt[a]*b^(3/2)*Sqrt[(a + b*x^2)^2]))
```

Maple [A] time = 0.008, size = 62, normalized size = 0.6

$$\frac{bx^2+a}{b}\left(ex\sqrt{ab}-\arctan\left(bx\frac{1}{\sqrt{ab}}\right)ae+\arctan\left(bx\frac{1}{\sqrt{ab}}\right)bd\right)\frac{1}{\sqrt{\left(bx^2+a\right)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] (b*x^2+a)*(e*x*(a*b)^(1/2)-arctan(b*x/(a*b)^(1/2))*a*e+arctan(b*x/(a*b)^(1/
2))*b*d)/((b*x^2+a)^2)^(1/2)/b/(a*b)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51208, size = 223, normalized size = 2.3

$$\left[\frac{2\,abex+\sqrt{-ab}(bd-ae)\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab^2}, \frac{abex+\sqrt{ab}(bd-ae)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*e*x + sqrt(-a*b)*(b*d - a*e)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (a*b*e*x + sqrt(a*b)*(b*d - a*e)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

Sympy [A] time = 0.4661, size = 82, normalized size = 0.85

$$\frac{\sqrt{-\frac{1}{ab^3}}\,(ae-bd)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^3}}\,(ae-bd)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}+\frac{ex}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-1/(a*b**3))*(a*e - b*d)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 - sqrt(-1/(
a*b**3))*(a*e - b*d)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 + e*x/b
```

Giac [A] time = 1.13127, size = 80, normalized size = 0.82

$$\frac{xe\,\mathrm{sgn}\left(bx^2+a\right)}{b}+\frac{\left(bd\,\mathrm{sgn}\left(bx^2+a\right)-ae\,\mathrm{sgn}\left(bx^2+a\right)\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] x*e*sgn(b*x^2 + a)/b + (b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*b)
```

3.78 $\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}} dx$

Optimal. Leaf size=92

$$\frac{d\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (d*(a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*(
a + b*x^2)*Log[a + b*x^2])/(2*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0717011, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 446, 72}

$$\frac{d\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (d*(a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*(
a + b*x^2)*Log[a + b*x^2])/(2*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{d+ex^2}{x\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{d+ex}{x\left(ab+b^2x\right)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \left(\frac{d}{abx}+\frac{-bd+ae}{ab(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{d\left(a+bx^2\right)\log(x)}{a\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)\left(a+bx^2\right)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0209118, size = 54, normalized size = 0.59

$$\frac{\left(a+bx^2\right)\left((ae-bd)\log\left(a+bx^2\right)+2bd\log(x)\right)}{2ab\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(2*b*d*Log[x] + (-(b*d) + a*e)*Log[a + b*x^2]))/(2*a*b*Sqrt[(a
 + b*x^2)^2])
```

Maple [A] time = 0.01, size = 57, normalized size = 0.6

$$\frac{\left(bx^2+a\right)\left(2\,d\ln(x)\,b+\ln\left(bx^2+a\right)ae-\ln\left(bx^2+a\right)bd\right)}{2\,ab}\frac{1}{\sqrt{\left(bx^2+a\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)*(2*d*ln(x)*b+ln(b*x^2+a)*a*e-ln(b*x^2+a)*b*d)/((b*x^2+a)^2)^(
1/2)/a/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49825, size = 74, normalized size = 0.8

$$\frac{2\,bd\log\left(x\right) - \left(bd - ae\right)\log\left(bx^2 + a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*log(x) - (b*d - a*e)*log(b*x^2 + a))/(a*b)
```

Sympy [A] time = 0.731905, size = 26, normalized size = 0.28

$$\frac{d\log\left(x\right)}{a} + \frac{\left(ae - bd\right)\log\left(\frac{a}{b} + x^2\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] d*log(x)/a + (a*e - b*d)*log(a/b + x**2)/(2*a*b)
```

Giac [A] time = 1.11569, size = 82, normalized size = 0.89

$$\frac{d\log\left(x^2\right)\operatorname{sgn}\left(bx^2 + a\right)}{2\,a} - \frac{\left(bd\operatorname{sgn}\left(bx^2 + a\right) - ae\operatorname{sgn}\left(bx^2 + a\right)\right)\log\left(\left|bx^2 + a\right|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d*log(x^2)*sgn(b*x^2 + a)/a - 1/2*(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 +
 a))*log(abs(b*x^2 + a))/(a*b)
```

3.79 $\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=101

$$-\frac{\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{ax\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -((d*(a + b*x^2))/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - ((b*d - a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])
```

Rubi [A] time = 0.0629733, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 453, 205}

$$-\frac{\left(a+bx^2\right)(bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{ax\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((d*(a + b*x^2))/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - ((b*d - a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{d+ex^2}{x^2\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{d\left(a+bx^2\right)}{ax\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(\left(b^2d-abe\right)\left(ab+b^2x^2\right)\right)\int \frac{1}{ab+b^2x^2}\,dx}{ab\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{d\left(a+bx^2\right)}{ax\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0313869, size = 72, normalized size = 0.71

$$\frac{\left(a+bx^2\right)\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(aex-bdx)-\sqrt{a}\sqrt{b}d\right)}{a^{3/2}\sqrt{b}x\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*d) + (-(b*d*x) + a*e*x)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]]))/(a^(3/2)*Sqrt[b]*x*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.01, size = 67, normalized size = 0.7

$$-\frac{bx^2+a}{ax}\left(-\arctan\left(bx\frac{1}{\sqrt{ab}}\right)xae+\arctan\left(bx\frac{1}{\sqrt{ab}}\right)xbd+d\sqrt{ab}\right)\frac{1}{\sqrt{\left(bx^2+a\right)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -(b*x^2+a)*(-arctan(b*x/(a*b)^(1/2))*x*a*e+arctan(b*x/(a*b)^(1/2))*x*b*d+d*
(a*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a/x/(a*b)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57388, size = 230, normalized size = 2.28

$$\left[\frac{\sqrt{-ab}(bd-ae)x\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-2\,abd}{2\,a^2bx}, -\frac{\sqrt{ab}(bd-ae)x\arctan\left(\frac{\sqrt{ab}x}{a}\right)+abd}{a^2bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a*b)*(b*d - a*e)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) - 2*a*b*d)/(a^2*b*x), -(sqrt(a*b)*(b*d - a*e)*x*arctan(sqrt(a*b)*x/a) + a
*b*d)/(a^2*b*x)]
```

Sympy [A] time = 0.514622, size = 82, normalized size = 0.81

$$-\frac{\sqrt{-\frac{1}{a^3b}}\left(ae-bd\right)\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b}}\left(ae-bd\right)\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-1/(a**3*b))*(a*e - b*d)*log(-a**2*sqrt(-1/(a**3*b)) + x)/2 + sqrt(-1
/(a**3*b))*(a*e - b*d)*log(a**2*sqrt(-1/(a**3*b)) + x)/2 - d/(a*x)
```

Giac [A] time = 1.11881, size = 84, normalized size = 0.83

$$-\frac{\left(bd\mathrm{sgn}\left(bx^2+a\right)-ae\mathrm{sgn}\left(bx^2+a\right)\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}-\frac{d\mathrm{sgn}\left(bx^2+a\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a) - d*sgn(b*x^2 + a)/(a*x)
```

3.80 $\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=137

$$-\frac{\log(x)\left(a+bx^2\right)(bd-ae)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -(d*(a + b*x^2))/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*(
a + b*x^2)*Log[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*(a
+ b*x^2)*Log[a + b*x^2])/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0981535, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 446, 77}

$$-\frac{\log(x)\left(a+bx^2\right)(bd-ae)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(bd-ae)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(d*(a + b*x^2))/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*(
a + b*x^2)*Log[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*(a
+ b*x^2)*Log[a + b*x^2])/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))))
```

Rubi steps

$$\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{d+ex^2}{x^3\left(ab+b^2x^2\right)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \frac{d+ex}{x^2\left(ab+b^2x\right)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(ab+b^2x^2\right)\text{Subst}\left(\int \left(\frac{d}{abx^2}+\frac{-bd+ae}{a^2bx}+\frac{bd-ae}{a^2(a+bx)}\right)dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{d\left(a+bx^2\right)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)\left(a+bx^2\right)\log(x)}{a^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)\left(a+bx^2\right)\log\left(a+\right.}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0327839, size = 70, normalized size = 0.51

$$\frac{\left(a+bx^2\right)\left(2x^2\log(x)(ae-bd)+x^2(bd-ae)\log\left(a+bx^2\right)-ad\right)}{2a^2x^2\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(-(a*d) + 2*(-(b*d) + a*e)*x^2*Log[x] + (b*d - a*e)*x^2*Log[a
+ b*x^2]))/(2*a^2*x^2*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.014, size = 79, normalized size = 0.6

$$\frac{\left(bx^2+a\right)\left(2\ln(x)x^2ae-2\ln(x)x^2bd-\ln\left(bx^2+a\right)x^2ae+\ln\left(bx^2+a\right)x^2bd-ad\right)}{2a^2x^2}\frac{1}{\sqrt{\left(bx^2+a\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)*(2*ln(x)*x^2*a*e-2*ln(x)*x^2*b*d-ln(b*x^2+a)*x^2*a*e+ln(b*x^2
+a)*x^2*b*d-a*d)/((b*x^2+a)^2)^(1/2)/a^2/x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53572, size = 109, normalized size = 0.8

$$\frac{(bd-ae)x^2\log\left(bx^2+a\right)-2\,(bd-ae)x^2\log\left(x\right)-ad}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((b*d - a*e)*x^2*log(b*x^2 + a) - 2*(b*d - a*e)*x^2*log(x) - a*d)/(a^2*
x^2)
```

Sympy [A] time = 0.878829, size = 41, normalized size = 0.3

$$-\frac{d}{2ax^2}+\frac{(ae-bd)\log\left(x\right)}{a^2}-\frac{(ae-bd)\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -d/(2*a*x**2) + (a*e - b*d)*log(x)/a**2 - (a*e - b*d)*log(a/b + x**2)/(2*a*
*2)
```

Giac [A] time = 1.1387, size = 177, normalized size = 1.29

$$-\frac{\left(bd\mathrm{sgn}\left(bx^2+a\right)-ae\mathrm{sgn}\left(bx^2+a\right)\right)\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2d\mathrm{sgn}\left(bx^2+a\right)-abe\mathrm{sgn}\left(bx^2+a\right)\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^2b}+\frac{bdx^2\mathrm{sgn}\left(bx^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*log(x^2)/a^2 + 1/2*(b^2*d*sg
n(b*x^2 + a) - a*b*e*sgn(b*x^2 + a))*log(abs(b*x^2 + a))/(a^2*b) + 1/2*(b*d
*x^2*sgn(b*x^2 + a) - a*x^2*e*sgn(b*x^2 + a) - a*d*sgn(b*x^2 + a))/(a^2*x^2
)
```

3.81 $\int \frac{x^2\left(d+ex^2\right)}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{x(bd-5ae)}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(bd-ae)}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((b*d - 5*a*e)*x)/(8*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*
x)/(4*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d + 3*a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rubi [A] time = 0.119201, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1250, 455, 385, 205}

$$\frac{x(bd-5ae)}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(bd-ae)}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((b*d - 5*a*e)*x)/(8*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*
x)/(4*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d + 3*a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^2\right)}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{x^2\left(d+ex^2\right)}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(bd-ae)x}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(ab+b^2x^2\right)\int \frac{-b(bd-ae)-4b^2ex^2}{\left(ab+b^2x^2\right)^2}\,dx}{4b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(bd-5ae)x}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left((bd+3ae)\left(ab+b^2x^2\right)\right)}{8ab^2\sqrt{a^2+2abx^2}}$$

$$= \frac{(bd-5ae)x}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd+3ae)\left(a+bx^2\right)\tan}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2}}$$

Mathematica [A] time = 0.0630064, size = 108, normalized size = 0.71

$$\frac{\left(a+bx^2\right)^2(3ae+bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{a}\sqrt{b}x\left(3a^2e+ab\left(d+5ex^2\right)-b^2dx^2\right)}{8a^{3/2}b^{5/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[b]*x*(3*a^2*e - b^2*d*x^2 + a*b*(d + 5*e*x^2))) + (b*d + 3*
a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)*(a + b*x
^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.016, size = 188, normalized size = 1.2

$$-\frac{bx^2+a}{8\,b^2a}\left(-3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4ab^2e-\arctan\left(bx\frac{1}{\sqrt{ab}}\right)x^4b^3d+5\,\sqrt{ab}x^3abe-\sqrt{ab}x^3b^2d-6\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^2a^2be-2\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/8*(-3*arctan(b*x/(a*b)^(1/2))*x^4*a*b^2*e-arctan(b*x/(a*b)^(1/2))*x^4*b^
3*d+5*(a*b)^(1/2)*x^3*a*b*e-(a*b)^(1/2)*x^3*b^2*d-6*arctan(b*x/(a*b)^(1/2))
*x^2*a^2*b*e-2*arctan(b*x/(a*b)^(1/2))*x^2*a*b^2*d+3*(a*b)^(1/2)*x*a^2*e+(a
*b)^(1/2)*x*a*b*d-3*arctan(b*x/(a*b)^(1/2))*a^3*e-arctan(b*x/(a*b)^(1/2))*a
^2*b*d)*(b*x^2+a)/(a*b)^(1/2)/a/b^2/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56592, size = 621, normalized size = 4.06

$$\left[\frac{2\left(ab^3d-5\,a^2b^2e\right)x^3-\left(\left(b^3d+3\,ab^2e\right)x^4+a^2bd+3\,a^3e+2\left(ab^2d+3\,a^2be\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)-2\left(a^2b^2d+\right.}{16\left(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(2*(a*b^3*d - 5*a^2*b^2*e)*x^3 - ((b^3*d + 3*a*b^2*e)*x^4 + a^2*b*d +
 3*a^3*e + 2*(a*b^2*d + 3*a^2*b*e)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) - 2*(a^2*b^2*d + 3*a^3*b*e)*x)/(a^2*b^5*x^4 + 2*a^3*b
^4*x^2 + a^4*b^3), 1/8*((a*b^3*d - 5*a^2*b^2*e)*x^3 + ((b^3*d + 3*a*b^2*e)*
x^4 + a^2*b*d + 3*a^3*e + 2*(a*b^2*d + 3*a^2*b*e)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) - (a^2*b^2*d + 3*a^3*b*e)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4
*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(d+ex^2\right)}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.82 $\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{bd-ae}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -e/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*b^2*(a + b*x^2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0657618, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1247, 640, 607}

$$-\frac{bd-ae}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -e/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*b^2*(a + b*x^2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 607

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(2*(a + b*x
+ c*x^2)^(p + 1))/((2*p + 1)*(b + 2*c*x)), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)\,\text{Subst}\left(\int \frac{1}{(a^2+2abx+b^2x^2)^{3/2}}\,dx,x,x^2\right)}{2b}\\
&= -\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{bd-ae}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A] time = 0.0195536, size = 45, normalized size = 0.58

$$\frac{-ae-b\left(d+2ex^2\right)}{4b^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(a*e) - b*(d + 2*e*x^2))/(4*b^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.009, size = 38, normalized size = 0.5

$$-\frac{\left(bx^2+a\right)\left(2x^2eb+ae+bd\right)}{4b^2}\left(\left(bx^2+a\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(b*x^2+a)*(2*b*e*x^2+a*e+b*d)/b^2/((b*x^2+a)^2)^(3/2)
```

Maxima [A] time = 0.985069, size = 96, normalized size = 1.25

$$-\frac{1}{4}e\left(\frac{2}{\sqrt{b^2x^4+2abx^2+a^2}b^2}-\frac{a}{\left(b^2\right)^{\frac{3}{2}}\left(x^2+\frac{a}{b}\right)^2b}\right)-\frac{d}{4\left(b^2\right)^{\frac{3}{2}}\left(x^2+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*e*(2/(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*b^2) - a/((b^2)^(3/2)*(x^2 + a/b
)^2*b)) - 1/4*d/((b^2)^(3/2)*(x^2 + a/b)^2)
```

Fricas [A] time = 1.49238, size = 86, normalized size = 1.12

$$-\frac{2bex^2+bd+ae}{4\left(b^4x^4+2ab^3x^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*e*x^2 + b*d + a*e)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(d+ex^2)}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.83 $\int \frac{d+ex^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=156

$$\frac{x(ae+3bd)}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x(bd-ae)}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(ae+3bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((3*b*d + a*e)*x)/(8*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*
x)/(4*a*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((3*b*d + a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rubi [A] time = 0.0894809, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1148, 385, 199, 205}

$$\frac{x(ae+3bd)}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x(bd-ae)}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(ae+3bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((3*b*d + a*e)*x)/(8*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*
x)/(4*a*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((3*b*d + a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rule 1148

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{d+ex^2}{\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(bd-ae)x}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left((3bd+ae)\left(ab+b^2x^2\right)\right)\int \frac{1}{\left(ab+b^2x^2\right)^2}\,dx}{4a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(3bd+ae)x}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)x}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left((3bd+ae)\left(ab+b^2x^2\right)\right)}{8a^2b\sqrt{a^2+2abx^2}}$$

$$= \frac{(3bd+ae)x}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)x}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3bd+ae)\left(a+bx^2\right)\tan}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2}}$$

Mathematica [A] time = 0.0532503, size = 108, normalized size = 0.69

$$\frac{\sqrt{a}\sqrt{b}x\left(a^2(-e)+ab\left(5d+ex^2\right)+3b^2dx^2\right)+\left(a+bx^2\right)^2(ae+3bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-(a^2*e) + 3*b^2*d*x^2 + a*b*(5*d + e*x^2)) + (3*b*d +
a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*(a + b*x
^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.016, size = 186, normalized size = 1.2

$$\frac{bx^2+a}{8\,a^2b}\left(\arctan\left(bx\frac{1}{\sqrt{ab}}\right)x^4ab^2e+3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4b^3d+\sqrt{ab}x^3abe+3\,\sqrt{ab}x^3b^2d+2\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^2a^2be+6\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(arctan(b*x/(a*b)^(1/2))*x^4*a*b^2*e+3*arctan(b*x/(a*b)^(1/2))*x^4*b^3*
d+(a*b)^(1/2)*x^3*a*b*e+3*(a*b)^(1/2)*x^3*b^2*d+2*arctan(b*x/(a*b)^(1/2))*x
^2*a^2*b*e+6*arctan(b*x/(a*b)^(1/2))*x^2*a*b^2*d-(a*b)^(1/2)*x*a^2*e+5*(a*b
)^(1/2)*x*a*b*d+arctan(b*x/(a*b)^(1/2))*a^3*e+3*arctan(b*x/(a*b)^(1/2))*a^2
*b*d)*(b*x^2+a)/(a*b)^(1/2)/b/a^2/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52063, size = 621, normalized size = 3.98

$$\left[\frac{2\left(3\,ab^3d + a^2b^2e\right)x^3 - \left(\left(3\,b^3d + ab^2e\right)x^4 + 3\,a^2bd + a^3e + 2\left(3\,ab^2d + a^2be\right)x^2\right)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right) + 2\left(5\,a^2b^2d\right.}{16\left(a^3b^4x^4 + 2\,a^4b^3x^2 + a^5b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a*b^3*d + a^2*b^2*e)*x^3 - ((3*b^3*d + a*b^2*e)*x^4 + 3*a^2*b*d
 + a^3*e + 2*(3*a*b^2*d + a^2*b*e)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^2*d - a^3*b*e)*x)/(a^3*b^4*x^4 + 2*a^4*b
^3*x^2 + a^5*b^2), 1/8*((3*a*b^3*d + a^2*b^2*e)*x^3 + ((3*b^3*d + a*b^2*e)*
x^4 + 3*a^2*b*d + a^3*e + 2*(3*a*b^2*d + a^2*b*e)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) + (5*a^2*b^2*d - a^3*b*e)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5
*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.84 $\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=161

$$\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] d/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*d - a*e)/(4*a*b*(a + b*x^2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*(a + b*x^2)*Log[x])/(a^3*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2)*Log[a + b*x^2])/(2*a^3*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.118266, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 446, 77}

$$\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```


```
[Out] d/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*d - a*e)/(4*a*b*(a + b*x^2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*(a + b*x^2)*Log[x])/(a^3*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2)*Log[a + b*x^2])/(2*a^3*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```


Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{d+ex^2}{x\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{d+ex}{x\left(ab+b^2x\right)^3}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{d}{a^3b^3x}+\frac{-bd+ae}{ab^3(a+bx)^3}-\frac{d}{a^2b^2(a+bx)^2}-\frac{d}{a^3b^2(a+bx)}\right)dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\left(a+bx^2\right)\log(x)}{a^3\sqrt{a^2+2abx^2+b^2}}$$

Mathematica [A] time = 0.0433481, size = 92, normalized size = 0.57

$$\frac{a\left(a^2(-e)+3abd+2b^2dx^2\right)+4bd\log(x)\left(a+bx^2\right)^2-2bd\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4a^3b\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(3*a*b*d - a^2*e + 2*b^2*d*x^2) + 4*b*d*(a + b*x^2)^2*Log[x] - 2*b*d*(a
+ b*x^2)^2*Log[a + b*x^2])/(4*a^3*b*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.021, size = 133, normalized size = 0.8

$$\frac{\left(4\,\ln(x)\,x^4b^3d-2\,\ln\left(bx^2+a\right)x^4b^3d+8\,\ln(x)\,x^2ab^2d-4\,\ln\left(bx^2+a\right)x^2ab^2d+2\,b^2dx^2a+4\,\ln(x)\,a^2bd-2\,\ln\left(bx^2\right.}{4\,ba^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/4*(4*ln(x)*x^4*b^3*d-2*ln(b*x^2+a)*x^4*b^3*d+8*ln(x)*x^2*a*b^2*d-4*ln(b*x
^2+a)*x^2*a*b^2*d+2*b^2*d*x^2*a+4*ln(x)*a^2*b*d-2*ln(b*x^2+a)*a^2*b*d-a^3*e
+3*a^2*b*d)*(b*x^2+a)/b/a^3/((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56315, size = 250, normalized size = 1.55

$$\frac{2\,ab^2dx^2+3\,a^2bd-a^3e-2\left(b^3dx^4+2\,ab^2dx^2+a^2bd\right)\log\left(bx^2+a\right)+4\left(b^3dx^4+2\,ab^2dx^2+a^2bd\right)\log\left(x\right)}{4\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b^2*d*x^2 + 3*a^2*b*d - a^3*e - 2*(b^3*d*x^4 + 2*a*b^2*d*x^2 + a^2
*b*d)*log(b*x^2 + a) + 4*(b^3*d*x^4 + 2*a*b^2*d*x^2 + a^2*b*d)*log(x))/(a^3
*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{d+ex^2}{x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x*((a + b*x**2)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.85 $\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{x(7bd-3ae)}{8a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(bd-ae)}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3\left(a+bx^2\right)(5bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{a^3x\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -((7*b*d - 3*a*e)*x)/(8*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)
*x)/(4*a^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2))/(
a^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*(5*b*d - a*e)*(a + b*x^2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.185439, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {1250, 456, 453, 205}

$$-\frac{x(7bd-3ae)}{8a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(bd-ae)}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3\left(a+bx^2\right)(5bd-ae)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{a^3x\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -((7*b*d - 3*a*e)*x)/(8*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)
*x)/(4*a^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2))/(
a^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*(5*b*d - a*e)*(a + b*x^2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
```

```
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{d+ex^2}{x^2\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{-\frac{4d}{ab}+\frac{3(bd-ae)x^2}{a^2b}}{x^2\left(ab+b^2x^2\right)^2}\,dx}{4\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{8c}{a^2 b}}{8\sqrt{a^2+2abx^2}}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\left(a+bx^2\right)}{a^3x\sqrt{a^2+2abx^2+b^2}}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{d\left(a+bx^2\right)}{a^3x\sqrt{a^2+2abx^2+b^2}}$$

Mathematica [A] time = 0.0683688, size = 124, normalized size = 0.65

$$\frac{\sqrt{a}\sqrt{b}\left(a^2\left(5ex^2-8d\right)+ab\left(3ex^4-25dx^2\right)-15b^2dx^4\right)+3x\left(a+bx^2\right)^2(ae-5bd)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}x\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*(-15*b^2*d*x^4 + a^2*(-8*d + 5*e*x^2) + a*b*(-25*d*x^2 + 3
*e*x^4)) + 3*(-5*b*d + a*e)*x*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8
*a^(7/2)*Sqrt[b]*x*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.019, size = 206, normalized size = 1.1

$$\frac{bx^2+a}{8\,xa^3}\left(3\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^5ab^2e-15\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^5b^3d+3\sqrt{ab}x^4abe-15\sqrt{ab}x^4b^2d+6\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^3a^2be-30\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(3*arctan(b*x/(a*b)^(1/2))*x^5*a*b^2*e-15*arctan(b*x/(a*b)^(1/2))*x^5*b
^3*d+3*(a*b)^(1/2)*x^4*a*b*e-15*(a*b)^(1/2)*x^4*b^2*d+6*arctan(b*x/(a*b)^(1
```

```
/2))*x^3*a^2*b*e-30*arctan(b*x/(a*b)^(1/2))*x^3*a*b^2*d+5*(a*b)^(1/2)*x^2*a
^2*e-25*(a*b)^(1/2)*x^2*a*b*d+3*arctan(b*x/(a*b)^(1/2))*x*a^3*e-15*arctan(b
*x/(a*b)^(1/2))*x*a^2*b*d-8*(a*b)^(1/2)*a^2*d)*(b*x^2+a)/(a*b)^(1/2)/x/a^3/
((b*x^2+a)^2)^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59615, size = 686, normalized size = 3.61

$$\left[-\frac{16\,a^{3}bd+6\left(5\,ab^{3}d-a^{2}b^{2}e\right)x^{4}+10\left(5\,a^{2}b^{2}d-a^{3}be\right)x^{2}-3\left(\left(5\,b^{3}d-ab^{2}e\right)x^{5}+2\left(5\,ab^{2}d-a^{2}be\right)x^{3}+\left(5\,a^{2}bd-a^{3}e\right)\cdots\right.}{16\left(a^{4}b^{3}x^{5}+2\,a^{5}b^{2}x^{3}+a^{6}bx\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*(16*a^3*b*d + 6*(5*a*b^3*d - a^2*b^2*e)*x^4 + 10*(5*a^2*b^2*d - a^3*
b*e)*x^2 - 3*((5*b^3*d - a*b^2*e)*x^5 + 2*(5*a*b^2*d - a^2*b*e)*x^3 + (5*a^
2*b*d - a^3*e)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))
/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), -1/8*(8*a^3*b*d + 3*(5*a*b^3*d -
a^2*b^2*e)*x^4 + 5*(5*a^2*b^2*d - a^3*b*e)*x^2 + 3*((5*b^3*d - a*b^2*e)*x^5
 + 2*(5*a*b^2*d - a^2*b*e)*x^3 + (5*a^2*b*d - a^3*e)*x)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex^{2}}{x^{2}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x**2*((a + b*x**2)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.86 $\int \frac{d+ex^2}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{bd-ae}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\log(x)\left(a+bx^2\right)(3bd-ae)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3bd-ae)\,\mathrm{lo}}{2a^4\sqrt{a^2+2abx^2+}}$$

```
[Out] -(2*b*d - a*e)/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*a^2
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2))/(2*a^3*x^2*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((3*b*d - a*e)*(a + b*x^2)*Log[x])/(a^4*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((3*b*d - a*e)*(a + b*x^2)*Log[a + b*x^2
])/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.183783, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1250, 446, 77}

$$-\frac{bd-ae}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\log(x)\left(a+bx^2\right)(3bd-ae)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3bd-ae)\,\mathrm{lo}}{2a^4\sqrt{a^2+2abx^2+}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -(2*b*d - a*e)/(2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*a^2
*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2))/(2*a^3*x^2*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((3*b*d - a*e)*(a + b*x^2)*Log[x])/(a^4*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((3*b*d - a*e)*(a + b*x^2)*Log[a + b*x^2
])/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{d+ex^2}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{d+ex^2}{x^3\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \frac{d+ex}{x^2\left(ab+b^2x\right)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\text{Subst}\left(\int \left(\frac{d}{a^3b^3x^2}+\frac{-3bd+ae}{a^4b^3x}+\frac{bd-ae}{a^2b^2(a+bx)^3}+\frac{2bd-ae}{a^3b^2(a+bx)^2}+\frac{3bd-ae}{a^4b^2(a+bx)}\right)dx,x\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{bd-ae}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{2a^3x^2\sqrt{a^2+2abx^2+}}$$

Mathematica [A] time = 0.0731758, size = 130, normalized size = 0.58

$$\frac{a\left(a^2\left(3ex^2-2d\right)+ab\left(2ex^4-9dx^2\right)-6b^2dx^4\right)+4x^2\log(x)\left(a+bx^2\right)^2(ae-3bd)+2x^2\left(a+bx^2\right)^2(3bd-ae)\log\left(a+bx^2\right.}{4a^4x^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(-6*b^2*d*x^4 + a^2*(-2*d + 3*e*x^2) + a*b*(-9*d*x^2 + 2*e*x^4)) + 4*(-3
*b*d + a*e)*x^2*(a + b*x^2)^2*Log[x] + 2*(3*b*d - a*e)*x^2*(a + b*x^2)^2*Lo
g[a + b*x^2])/(4*a^4*x^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.021, size = 249, normalized size = 1.1

$$\frac{\left(4\ln(x)x^6ab^2e-12\ln(x)x^6b^3d-2\ln\left(bx^2+a\right)x^6ab^2e+6\ln\left(bx^2+a\right)x^6b^3d+8\ln(x)x^4a^2be-24\ln(x)x^4ab^2d-4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/4*(4*ln(x)*x^6*a*b^2*e-12*ln(x)*x^6*b^3*d-2*ln(b*x^2+a)*x^6*a*b^2*e+6*ln(
b*x^2+a)*x^6*b^3*d+8*ln(x)*x^4*a^2*b*e-24*ln(x)*x^4*a*b^2*d-4*ln(b*x^2+a)*x
^4*a^2*b*e+12*ln(b*x^2+a)*x^4*a*b^2*d+2*x^4*a^2*b*e-6*x^4*a*b^2*d+4*ln(x)*x
^2*a^3*e-12*ln(x)*x^2*a^2*b*d-2*ln(b*x^2+a)*x^2*a^3*e+6*ln(b*x^2+a)*x^2*a^2
*b*d+3*x^2*a^3*e-9*x^2*a^2*b*d-2*a^3*d)*(b*x^2+a)/x^2/a^4/((b*x^2+a)^2)^(3/
2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.56555, size = 413, normalized size = 1.85

$$-\frac{2\left(3\,ab^2d-a^2be\right)x^4+2\,a^3d+3\left(3\,a^2bd-a^3e\right)x^2-2\left(\left(3\,b^3d-ab^2e\right)x^6+2\left(3\,ab^2d-a^2be\right)x^4+\left(3\,a^2bd-a^3e\right)x^2\right)\log\left(\right.}{4\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/4*(2*(3*a*b^2*d - a^2*b*e)*x^4 + 2*a^3*d + 3*(3*a^2*b*d - a^3*e)*x^2 - 2
*((3*b^3*d - a*b^2*e)*x^6 + 2*(3*a*b^2*d - a^2*b*e)*x^4 + (3*a^2*b*d - a^3*
e)*x^2)*log(b*x^2 + a) + 4*((3*b^3*d - a*b^2*e)*x^6 + 2*(3*a*b^2*d - a^2*b*
e)*x^4 + (3*a^2*b*d - a^3*e)*x^2)*log(x))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*
x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex^2}{x^3\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x**3*((a + b*x**2)**2)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.87 $\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=400

$$\frac{a^4\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+5bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{5a^3b\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}(ae+2bd)}{f^5(m+5)\left(a+bx^2\right)}+\frac{10a^2b^2\sqrt{a^2+2abx^2+b^2x^4}(fx)}{f^7(m+7)\left(a+bx^2\right.}$$

```
[Out] (a^5*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2
)) + (a^4*(5*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3
*(3 + m)*(a + b*x^2)) + (5*a^3*b*(2*b*d + a*e)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2)) + (10*a^2*b^2*(b*d + a*e)*(f*x
)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (5*a
*b^3*(b*d + 2*a*e)*(f*x)^(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 +
 m)*(a + b*x^2)) + (b^4*(b*d + 5*a*e)*(f*x)^(11 + m)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(f^11*(11 + m)*(a + b*x^2)) + (b^5*e*(f*x)^(13 + m)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(f^13*(13 + m)*(a + b*x^2))
```

Rubi [A] time = 0.242838, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1250, 448}

$$\frac{a^4\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+5bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{5a^3b\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}(ae+2bd)}{f^5(m+5)\left(a+bx^2\right)}+\frac{10a^2b^2\sqrt{a^2+2abx^2+b^2x^4}(fx)}{f^7(m+7)\left(a+bx^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (a^5*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2
)) + (a^4*(5*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3
*(3 + m)*(a + b*x^2)) + (5*a^3*b*(2*b*d + a*e)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2)) + (10*a^2*b^2*(b*d + a*e)*(f*x
)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (5*a
*b^3*(b*d + 2*a*e)*(f*x)^(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 +
 m)*(a + b*x^2)) + (b^4*(b*d + 5*a*e)*(f*x)^(11 + m)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(f^11*(11 + m)*(a + b*x^2)) + (b^5*e*(f*x)^(13 + m)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(f^13*(13 + m)*(a + b*x^2))
```


Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (fx)^m\left(ab+b^2x^2\right)^5\left(d+ex^2\right)dx}{b^4\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int\left(a^5b^5d(fx)^m+\frac{a^4b^5(5bd+ae)(fx)^{2+m}}{f^2}+\frac{5a^3b^6(2bd+ae)(f}{f^4}\right.}{b^4}$$

$$= \frac{a^5d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)}+\frac{a^4(5bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+}}{f^3(3+m)\left(a+bx^2\right)}$$

Mathematica [A] time = 0.233154, size = 160, normalized size = 0.4

$$\frac{x\sqrt{\left(a+bx^2\right)^2}(fx)^m\left(\frac{10a^2b^2x^6(ae+bd)}{m+7}+\frac{5a^3bx^4(ae+2bd)}{m+5}+\frac{a^4x^2(ae+5bd)}{m+3}+\frac{a^5d}{m+1}+\frac{b^4x^{10}(5ae+bd)}{m+11}+\frac{5ab^3x^8(2ae+bd)}{m+9}+\frac{b^5ex^{12}}{m+13}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x*(f*x)^m*Sqrt[(a + b*x^2)^2]*((a^5*d)/(1 + m) + (a^4*(5*b*d + a*e)*x^2)/(
3 + m) + (5*a^3*b*(2*b*d + a*e)*x^4)/(5 + m) + (10*a^2*b^2*(b*d + a*e)*x^6)
/(7 + m) + (5*a*b^3*(b*d + 2*a*e)*x^8)/(9 + m) + (b^4*(b*d + 5*a*e)*x^10)/(
11 + m) + (b^5*e*x^12)/(13 + m)))/(a + b*x^2)
```

Maple [B] time = 0.009, size = 1099, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] x*(b^5*e*m^6*x^12+36*b^5*e*m^5*x^12+5*a*b^4*e*m^6*x^10+b^5*d*m^6*x^10+505*b
^5*e*m^4*x^12+190*a*b^4*e*m^5*x^10+38*b^5*d*m^5*x^10+3480*b^5*e*m^3*x^12+10
*a^2*b^3*e*m^6*x^8+5*a*b^4*d*m^6*x^8+2775*a*b^4*e*m^4*x^10+555*b^5*d*m^4*x^
10+12139*b^5*e*m^2*x^12+400*a^2*b^3*e*m^5*x^8+200*a*b^4*d*m^5*x^8+19700*a*b
^4*e*m^3*x^10+3940*b^5*d*m^3*x^10+19524*b^5*e*m*x^12+10*a^3*b^2*e*m^6*x^6+1
0*a^2*b^3*d*m^6*x^6+6130*a^2*b^3*e*m^4*x^8+3065*a*b^4*d*m^4*x^8+70195*a*b^4
*e*m^2*x^10+14039*b^5*d*m^2*x^10+10395*b^5*e*x^12+420*a^3*b^2*e*m^5*x^6+420
*a^2*b^3*d*m^5*x^6+45280*a^2*b^3*e*m^3*x^8+22640*a*b^4*d*m^3*x^8+114510*a*b
^4*e*m*x^10+22902*b^5*d*m*x^10+5*a^4*b*e*m^6*x^4+10*a^3*b^2*d*m^6*x^4+6790*
a^3*b^2*e*m^4*x^6+6790*a^2*b^3*d*m^4*x^6+166270*a^2*b^3*e*m^2*x^8+83135*a*b
^4*d*m^2*x^8+61425*a*b^4*e*x^10+12285*b^5*d*x^10+220*a^4*b*e*m^5*x^4+440*a^
3*b^2*d*m^5*x^4+52920*a^3*b^2*e*m^3*x^6+52920*a^2*b^3*d*m^3*x^6+276880*a^2*
b^3*e*m*x^8+138440*a*b^4*d*m*x^8+a^5*e*m^6*x^2+5*a^4*b*d*m^6*x^2+3765*a^4*b
*e*m^4*x^4+7530*a^3*b^2*d*m^4*x^4+203350*a^3*b^2*e*m^2*x^6+203350*a^2*b^3*d
*m^2*x^6+150150*a^2*b^3*e*x^8+75075*a*b^4*d*x^8+46*a^5*e*m^5*x^2+230*a^4*b*
d*m^5*x^2+31400*a^4*b*e*m^3*x^4+62800*a^3*b^2*d*m^3*x^4+349860*a^3*b^2*e*m*
x^6+349860*a^2*b^3*d*m*x^6+a^5*d*m^6+835*a^5*e*m^4*x^2+4175*a^4*b*d*m^4*x^2
+129895*a^4*b*e*m^2*x^4+259790*a^3*b^2*d*m^2*x^4+193050*a^3*b^2*e*x^6+19305
0*a^2*b^3*d*x^6+48*a^5*d*m^5+7540*a^5*e*m^3*x^2+37700*a^4*b*d*m^3*x^2+23718
0*a^4*b*e*m*x^4+474360*a^3*b^2*d*m*x^4+925*a^5*d*m^4+34759*a^5*e*m^2*x^2+17
3795*a^4*b*d*m^2*x^2+135135*a^4*b*e*x^4+270270*a^3*b^2*d*x^4+9120*a^5*d*m^3
```

```
+73054*a^5*e*m*x^2+365270*a^4*b*d*m*x^2+48259*a^5*d*m^2+45045*a^5*e*x^2+225
225*a^4*b*d*x^2+129072*a^5*d*m+135135*a^5*d)*(f*x)^m*((b*x^2+a)^2)^(5/2)/(1
3+m)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)/(b*x^2+a)^5
```

Maxima [A] time = 1.03974, size = 663, normalized size = 1.66

$$\frac{\left(\left(m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945\right)b^5 f^m x^{11} + 5\left(m^5 + 27\,m^4 + 262\,m^3 + 1122\,m^2 + 2041\,m + 1155\right)ab^4 f^{m}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="max
ima")
```

```
[Out] ((m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)*b^5*f^m*x^11 + 5*(m^5 +
27*m^4 + 262*m^3 + 1122*m^2 + 2041*m + 1155)*a*b^4*f^m*x^9 + 10*(m^5 + 29*m
^4 + 302*m^3 + 1366*m^2 + 2577*m + 1485)*a^2*b^3*f^m*x^7 + 10*(m^5 + 31*m^4
 + 350*m^3 + 1730*m^2 + 3489*m + 2079)*a^3*b^2*f^m*x^5 + 5*(m^5 + 33*m^4 +
406*m^3 + 2262*m^2 + 5353*m + 3465)*a^4*b*f^m*x^3 + (m^5 + 35*m^4 + 470*m^3
 + 3010*m^2 + 9129*m + 10395)*a^5*f^m*x)*d*x^m/(m^6 + 36*m^5 + 505*m^4 + 34
80*m^3 + 12139*m^2 + 19524*m + 10395) + ((m^5 + 35*m^4 + 470*m^3 + 3010*m^2
 + 9129*m + 10395)*b^5*f^m*x^13 + 5*(m^5 + 37*m^4 + 518*m^3 + 3422*m^2 + 10
617*m + 12285)*a*b^4*f^m*x^11 + 10*(m^5 + 39*m^4 + 574*m^3 + 3954*m^2 + 126
73*m + 15015)*a^2*b^3*f^m*x^9 + 10*(m^5 + 41*m^4 + 638*m^3 + 4654*m^2 + 156
81*m + 19305)*a^3*b^2*f^m*x^7 + 5*(m^5 + 43*m^4 + 710*m^3 + 5570*m^2 + 2040
9*m + 27027)*a^4*b*f^m*x^5 + (m^5 + 45*m^4 + 790*m^3 + 6750*m^2 + 28009*m +
 45045)*a^5*f^m*x^3)*e*x^m/(m^6 + 48*m^5 + 925*m^4 + 9120*m^3 + 48259*m^2 +
 129072*m + 135135)
```

Fricas [B] time = 1.63556, size = 2005, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fri
cas")
```

```
[Out] ((b^5*e*m^6 + 36*b^5*e*m^5 + 505*b^5*e*m^4 + 3480*b^5*e*m^3 + 12139*b^5*e*m
^2 + 19524*b^5*e*m + 10395*b^5*e)*x^13 + ((b^5*d + 5*a*b^4*e)*m^6 + 12285*b
^5*d + 61425*a*b^4*e + 38*(b^5*d + 5*a*b^4*e)*m^5 + 555*(b^5*d + 5*a*b^4*e)
*m^4 + 3940*(b^5*d + 5*a*b^4*e)*m^3 + 14039*(b^5*d + 5*a*b^4*e)*m^2 + 22902
*(b^5*d + 5*a*b^4*e)*m)*x^11 + 5*((a*b^4*d + 2*a^2*b^3*e)*m^6 + 15015*a*b^4
*d + 30030*a^2*b^3*e + 40*(a*b^4*d + 2*a^2*b^3*e)*m^5 + 613*(a*b^4*d + 2*a^
2*b^3*e)*m^4 + 4528*(a*b^4*d + 2*a^2*b^3*e)*m^3 + 16627*(a*b^4*d + 2*a^2*b^
3*e)*m^2 + 27688*(a*b^4*d + 2*a^2*b^3*e)*m)*x^9 + 10*((a^2*b^3*d + a^3*b^2*
e)*m^6 + 19305*a^2*b^3*d + 19305*a^3*b^2*e + 42*(a^2*b^3*d + a^3*b^2*e)*m^5
 + 679*(a^2*b^3*d + a^3*b^2*e)*m^4 + 5292*(a^2*b^3*d + a^3*b^2*e)*m^3 + 203
35*(a^2*b^3*d + a^3*b^2*e)*m^2 + 34986*(a^2*b^3*d + a^3*b^2*e)*m)*x^7 + 5*(
(2*a^3*b^2*d + a^4*b*e)*m^6 + 54054*a^3*b^2*d + 27027*a^4*b*e + 44*(2*a^3*b
^2*d + a^4*b*e)*m^5 + 753*(2*a^3*b^2*d + a^4*b*e)*m^4 + 6280*(2*a^3*b^2*d +
 a^4*b*e)*m^3 + 25979*(2*a^3*b^2*d + a^4*b*e)*m^2 + 47436*(2*a^3*b^2*d + a^
4*b*e)*m)*x^5 + ((5*a^4*b*d + a^5*e)*m^6 + 225225*a^4*b*d + 45045*a^5*e + 4
6*(5*a^4*b*d + a^5*e)*m^5 + 835*(5*a^4*b*d + a^5*e)*m^4 + 7540*(5*a^4*b*d +
 a^5*e)*m^3 + 34759*(5*a^4*b*d + a^5*e)*m^2 + 73054*(5*a^4*b*d + a^5*e)*m)*
```

```
x^3 + (a^5*d*m^6 + 48*a^5*d*m^5 + 925*a^5*d*m^4 + 9120*a^5*d*m^3 + 48259*a^
5*d*m^2 + 129072*a^5*d*m + 135135*a^5*d)*x)*(f*x)^m/(m^7 + 49*m^6 + 973*m^5
 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)

[Out] Timed out
```

Giac [B] time = 1.28371, size = 2988, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="gia
c")

[Out] ((f*x)^m*b^5*m^6*x^13*e*sgn(b*x^2 + a) + 36*(f*x)^m*b^5*m^5*x^13*e*sgn(b*x^
2 + a) + (f*x)^m*b^5*d*m^6*x^11*sgn(b*x^2 + a) + 5*(f*x)^m*a*b^4*m^6*x^11*e
*sgn(b*x^2 + a) + 505*(f*x)^m*b^5*m^4*x^13*e*sgn(b*x^2 + a) + 38*(f*x)^m*b^
5*d*m^5*x^11*sgn(b*x^2 + a) + 190*(f*x)^m*a*b^4*m^5*x^11*e*sgn(b*x^2 + a) +
 3480*(f*x)^m*b^5*m^3*x^13*e*sgn(b*x^2 + a) + 5*(f*x)^m*a*b^4*d*m^6*x^9*sgn
(b*x^2 + a) + 555*(f*x)^m*b^5*d*m^4*x^11*sgn(b*x^2 + a) + 10*(f*x)^m*a^2*b^
3*m^6*x^9*e*sgn(b*x^2 + a) + 2775*(f*x)^m*a*b^4*m^4*x^11*e*sgn(b*x^2 + a) +
 12139*(f*x)^m*b^5*m^2*x^13*e*sgn(b*x^2 + a) + 200*(f*x)^m*a*b^4*d*m^5*x^9*
sgn(b*x^2 + a) + 3940*(f*x)^m*b^5*d*m^3*x^11*sgn(b*x^2 + a) + 400*(f*x)^m*a
^2*b^3*m^5*x^9*e*sgn(b*x^2 + a) + 19700*(f*x)^m*a*b^4*m^3*x^11*e*sgn(b*x^2
+ a) + 19524*(f*x)^m*b^5*m*x^13*e*sgn(b*x^2 + a) + 10*(f*x)^m*a^2*b^3*d*m^6
*x^7*sgn(b*x^2 + a) + 3065*(f*x)^m*a*b^4*d*m^4*x^9*sgn(b*x^2 + a) + 14039*(
f*x)^m*b^5*d*m^2*x^11*sgn(b*x^2 + a) + 10*(f*x)^m*a^3*b^2*m^6*x^7*e*sgn(b*x
^2 + a) + 6130*(f*x)^m*a^2*b^3*m^4*x^9*e*sgn(b*x^2 + a) + 70195*(f*x)^m*a*b
^4*m^2*x^11*e*sgn(b*x^2 + a) + 10395*(f*x)^m*b^5*x^13*e*sgn(b*x^2 + a) + 42
0*(f*x)^m*a^2*b^3*d*m^5*x^7*sgn(b*x^2 + a) + 22640*(f*x)^m*a*b^4*d*m^3*x^9*
sgn(b*x^2 + a) + 22902*(f*x)^m*b^5*d*m*x^11*sgn(b*x^2 + a) + 420*(f*x)^m*a^
3*b^2*m^5*x^7*e*sgn(b*x^2 + a) + 45280*(f*x)^m*a^2*b^3*m^3*x^9*e*sgn(b*x^2
+ a) + 114510*(f*x)^m*a*b^4*m*x^11*e*sgn(b*x^2 + a) + 10*(f*x)^m*a^3*b^2*d*
m^6*x^5*sgn(b*x^2 + a) + 6790*(f*x)^m*a^2*b^3*d*m^4*x^7*sgn(b*x^2 + a) + 83
135*(f*x)^m*a*b^4*d*m^2*x^9*sgn(b*x^2 + a) + 12285*(f*x)^m*b^5*d*x^11*sgn(b
*x^2 + a) + 5*(f*x)^m*a^4*b*m^6*x^5*e*sgn(b*x^2 + a) + 6790*(f*x)^m*a^3*b^2
*m^4*x^7*e*sgn(b*x^2 + a) + 166270*(f*x)^m*a^2*b^3*m^2*x^9*e*sgn(b*x^2 + a)
 + 61425*(f*x)^m*a*b^4*x^11*e*sgn(b*x^2 + a) + 440*(f*x)^m*a^3*b^2*d*m^5*x^
5*sgn(b*x^2 + a) + 52920*(f*x)^m*a^2*b^3*d*m^3*x^7*sgn(b*x^2 + a) + 138440*
(f*x)^m*a*b^4*d*m*x^9*sgn(b*x^2 + a) + 220*(f*x)^m*a^4*b*m^5*x^5*e*sgn(b*x^
2 + a) + 52920*(f*x)^m*a^3*b^2*m^3*x^7*e*sgn(b*x^2 + a) + 276880*(f*x)^m*a^
2*b^3*m*x^9*e*sgn(b*x^2 + a) + 5*(f*x)^m*a^4*b*d*m^6*x^3*sgn(b*x^2 + a) + 7
530*(f*x)^m*a^3*b^2*d*m^4*x^5*sgn(b*x^2 + a) + 203350*(f*x)^m*a^2*b^3*d*m^2
*x^7*sgn(b*x^2 + a) + 75075*(f*x)^m*a*b^4*d*x^9*sgn(b*x^2 + a) + (f*x)^m*a^
5*m^6*x^3*e*sgn(b*x^2 + a) + 3765*(f*x)^m*a^4*b*m^4*x^5*e*sgn(b*x^2 + a) +
```

```
203350*(f*x)^m*a^3*b^2*m^2*x^7*e*sgn(b*x^2 + a) + 150150*(f*x)^m*a^2*b^3*x^
9*e*sgn(b*x^2 + a) + 230*(f*x)^m*a^4*b*d*m^5*x^3*sgn(b*x^2 + a) + 62800*(f*
x)^m*a^3*b^2*d*m^3*x^5*sgn(b*x^2 + a) + 349860*(f*x)^m*a^2*b^3*d*m*x^7*sgn(
b*x^2 + a) + 46*(f*x)^m*a^5*m^5*x^3*e*sgn(b*x^2 + a) + 31400*(f*x)^m*a^4*b*
m^3*x^5*e*sgn(b*x^2 + a) + 349860*(f*x)^m*a^3*b^2*m*x^7*e*sgn(b*x^2 + a) +
(f*x)^m*a^5*d*m^6*x*sgn(b*x^2 + a) + 4175*(f*x)^m*a^4*b*d*m^4*x^3*sgn(b*x^2
 + a) + 259790*(f*x)^m*a^3*b^2*d*m^2*x^5*sgn(b*x^2 + a) + 193050*(f*x)^m*a^
2*b^3*d*x^7*sgn(b*x^2 + a) + 835*(f*x)^m*a^5*m^4*x^3*e*sgn(b*x^2 + a) + 129
895*(f*x)^m*a^4*b*m^2*x^5*e*sgn(b*x^2 + a) + 193050*(f*x)^m*a^3*b^2*x^7*e*s
gn(b*x^2 + a) + 48*(f*x)^m*a^5*d*m^5*x*sgn(b*x^2 + a) + 37700*(f*x)^m*a^4*b
*d*m^3*x^3*sgn(b*x^2 + a) + 474360*(f*x)^m*a^3*b^2*d*m*x^5*sgn(b*x^2 + a) +
 7540*(f*x)^m*a^5*m^3*x^3*e*sgn(b*x^2 + a) + 237180*(f*x)^m*a^4*b*m*x^5*e*s
gn(b*x^2 + a) + 925*(f*x)^m*a^5*d*m^4*x*sgn(b*x^2 + a) + 173795*(f*x)^m*a^4
*b*d*m^2*x^3*sgn(b*x^2 + a) + 270270*(f*x)^m*a^3*b^2*d*x^5*sgn(b*x^2 + a) +
 34759*(f*x)^m*a^5*m^2*x^3*e*sgn(b*x^2 + a) + 135135*(f*x)^m*a^4*b*x^5*e*sg
n(b*x^2 + a) + 9120*(f*x)^m*a^5*d*m^3*x*sgn(b*x^2 + a) + 365270*(f*x)^m*a^4
*b*d*m*x^3*sgn(b*x^2 + a) + 73054*(f*x)^m*a^5*m*x^3*e*sgn(b*x^2 + a) + 4825
9*(f*x)^m*a^5*d*m^2*x*sgn(b*x^2 + a) + 225225*(f*x)^m*a^4*b*d*x^3*sgn(b*x^2
 + a) + 45045*(f*x)^m*a^5*x^3*e*sgn(b*x^2 + a) + 129072*(f*x)^m*a^5*d*m*x*s
gn(b*x^2 + a) + 135135*(f*x)^m*a^5*d*x*sgn(b*x^2 + a))/(m^7 + 49*m^6 + 973*
m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

3.88 $\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=276

$$\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+3bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{3ab\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}(ae+bd)}{f^5(m+5)\left(a+bx^2\right)}+\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+7}}{f^7(m+7)\left(a+bx^2\right)}$$

```
[Out] (a^3*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2
)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3
*(3 + m)*(a + b*x^2)) + (3*a*b*(b*d + a*e)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2)) + (b^2*(b*d + 3*a*e)*(f*x)^(7 + m)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (b^3*e*(f*x)^
(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 + m)*(a + b*x^2))
```

Rubi [A] time = 0.152863, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1250, 448}

$$\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+3bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{3ab\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}(ae+bd)}{f^5(m+5)\left(a+bx^2\right)}+\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+7}}{f^7(m+7)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```


```
[Out] (a^3*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2
)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3
*(3 + m)*(a + b*x^2)) + (3*a*b*(b*d + a*e)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2)) + (b^2*(b*d + 3*a*e)*(f*x)^(7 + m)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (b^3*e*(f*x)^
(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 + m)*(a + b*x^2))
```


Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (fx)^m \left(ab+b^2x^2\right)^3 \left(d+ex^2\right) dx}{b^2 \left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3d(fx)^m + \frac{a^2b^3(3bd+ae)(fx)^{2+m}}{f^2} + \frac{3ab^4(bd+ae)(fx)^{4+m}}{f^4}\right.}{b^2 \left(ab+b^2x^2\right)}$$
$$= \frac{a^3d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)} + \frac{a^2(3bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^3(3+m)\left(a+bx^2\right)}$$

Mathematica [A] time = 0.127584, size = 112, normalized size = 0.41

$$\frac{x\left(\left(a+bx^2\right)^2\right)^{3/2} (fx)^m \left(\frac{a^2x^2(ae+3bd)}{m+3} + \frac{a^3d}{m+1} + \frac{b^2x^6(3ae+bd)}{m+7} + \frac{3abx^4(ae+bd)}{m+5} + \frac{b^3ex^8}{m+9}\right)}{\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*((a + b*x^2)^2)^(3/2)*((a^3*d)/(1 + m) + (a^2*(3*b*d + a*e)*x^2)
/(3 + m) + (3*a*b*(b*d + a*e)*x^4)/(5 + m) + (b^2*(b*d + 3*a*e)*x^6)/(7 + m
) + (b^3*e*x^8)/(9 + m)))/(a + b*x^2)^3
```

Maple [B] time = 0.009, size = 495, normalized size = 1.8

$$\frac{\left(b^3em^4x^8 + 16\,b^3em^3x^8 + 3\,ab^2em^4x^6 + b^3dm^4x^6 + 86\,b^3em^2x^8 + 54\,ab^2em^3x^6 + 18\,b^3dm^3x^6 + 176\,b^3emx^8 + 3\,a^2bem^4x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] x*(b^3*e*m^4*x^8+16*b^3*e*m^3*x^8+3*a*b^2*e*m^4*x^6+b^3*d*m^4*x^6+86*b^3*e*
m^2*x^8+54*a*b^2*e*m^3*x^6+18*b^3*d*m^3*x^6+176*b^3*e*m*x^8+3*a^2*b*e*m^4*x
^4+3*a*b^2*d*m^4*x^4+312*a*b^2*e*m^2*x^6+104*b^3*d*m^2*x^6+105*b^3*e*x^8+60
*a^2*b*e*m^3*x^4+60*a*b^2*d*m^3*x^4+666*a*b^2*e*m*x^6+222*b^3*d*m*x^6+a^3*e
*m^4*x^2+3*a^2*b*d*m^4*x^2+390*a^2*b*e*m^2*x^4+390*a*b^2*d*m^2*x^4+405*a*b^
2*e*x^6+135*b^3*d*x^6+22*a^3*e*m^3*x^2+66*a^2*b*d*m^3*x^2+900*a^2*b*e*m*x^4
+900*a*b^2*d*m*x^4+a^3*d*m^4+164*a^3*e*m^2*x^2+492*a^2*b*d*m^2*x^2+567*a^2*
b*e*x^4+567*a*b^2*d*x^4+24*a^3*d*m^3+458*a^3*e*m*x^2+1374*a^2*b*d*m*x^2+206
*a^3*d*m^2+315*a^3*e*x^2+945*a^2*b*d*x^2+744*a^3*d*m+945*a^3*d)*(f*x)^m*((b
*x^2+a)^2)^(3/2)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)/(b*x^2+a)^3
```

Maxima [A] time = 0.998404, size = 328, normalized size = 1.19

$$\frac{\left(\left(m^3+9\,m^2+23\,m+15\right)b^3f^mx^7 + 3\left(m^3+11\,m^2+31\,m+21\right)ab^2f^mx^5 + 3\left(m^3+13\,m^2+47\,m+35\right)a^2bf^mx^3 + \left(m^3+\right.\right.}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="max
ima")
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*f^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a*b^2
*f^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*f^m*x^3 + (m^3 + 15*m^2 + 71*
m + 105)*a^3*f^m*x)*d*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105) + ((m^3 + 1
5*m^2 + 71*m + 105)*b^3*f^m*x^9 + 3*(m^3 + 17*m^2 + 87*m + 135)*a*b^2*f^m*x
^7 + 3*(m^3 + 19*m^2 + 111*m + 189)*a^2*b*f^m*x^5 + (m^3 + 21*m^2 + 143*m +
 315)*a^3*f^m*x^3)*e*x^m/(m^4 + 24*m^3 + 206*m^2 + 744*m + 945)
```

Fricas [A] time = 1.61066, size = 871, normalized size = 3.16

$$\frac{\left(\left(b^3em^4 + 16\,b^3em^3 + 86\,b^3em^2 + 176\,b^3em + 105\,b^3e\right)x^9 + \left(\left(b^3d + 3\,ab^2e\right)m^4 + 135\,b^3d + 405\,ab^2e + 18\,\left(b^3d + 3\,ab^2e\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fri
cas")
```

```
[Out] ((b^3*e*m^4 + 16*b^3*e*m^3 + 86*b^3*e*m^2 + 176*b^3*e*m + 105*b^3*e)*x^9 +
((b^3*d + 3*a*b^2*e)*m^4 + 135*b^3*d + 405*a*b^2*e + 18*(b^3*d + 3*a*b^2*e)
*m^3 + 104*(b^3*d + 3*a*b^2*e)*m^2 + 222*(b^3*d + 3*a*b^2*e)*m)*x^7 + 3*((a
*b^2*d + a^2*b*e)*m^4 + 189*a*b^2*d + 189*a^2*b*e + 20*(a*b^2*d + a^2*b*e)*
m^3 + 130*(a*b^2*d + a^2*b*e)*m^2 + 300*(a*b^2*d + a^2*b*e)*m)*x^5 + ((3*a^
2*b*d + a^3*e)*m^4 + 945*a^2*b*d + 315*a^3*e + 22*(3*a^2*b*d + a^3*e)*m^3 +
 164*(3*a^2*b*d + a^3*e)*m^2 + 458*(3*a^2*b*d + a^3*e)*m)*x^3 + (a^3*d*m^4
+ 24*a^3*d*m^3 + 206*a^3*d*m^2 + 744*a^3*d*m + 945*a^3*d)*x)*(f*x)^m/(m^5 +
 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^m \left(d + ex^2\right)\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*((a + b*x**2)**2)**(3/2), x)
```

Giac [B] time = 1.17867, size = 1368, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="gia
c")
```

```
[Out] ((f*x)^m*b^3*m^4*x^9*e*sgn(b*x^2 + a) + 16*(f*x)^m*b^3*m^3*x^9*e*sgn(b*x^2
+ a) + (f*x)^m*b^3*d*m^4*x^7*sgn(b*x^2 + a) + 3*(f*x)^m*a*b^2*m^4*x^7*e*sgn
```

```
(b*x^2 + a) + 86*(f*x)^m*b^3*m^2*x^9*e*sgn(b*x^2 + a) + 18*(f*x)^m*b^3*d*m^
3*x^7*sgn(b*x^2 + a) + 54*(f*x)^m*a*b^2*m^3*x^7*e*sgn(b*x^2 + a) + 176*(f*x
)^m*b^3*m*x^9*e*sgn(b*x^2 + a) + 3*(f*x)^m*a*b^2*d*m^4*x^5*sgn(b*x^2 + a) +
 104*(f*x)^m*b^3*d*m^2*x^7*sgn(b*x^2 + a) + 3*(f*x)^m*a^2*b*m^4*x^5*e*sgn(b
*x^2 + a) + 312*(f*x)^m*a*b^2*m^2*x^7*e*sgn(b*x^2 + a) + 105*(f*x)^m*b^3*x^
9*e*sgn(b*x^2 + a) + 60*(f*x)^m*a*b^2*d*m^3*x^5*sgn(b*x^2 + a) + 222*(f*x)^
m*b^3*d*m*x^7*sgn(b*x^2 + a) + 60*(f*x)^m*a^2*b*m^3*x^5*e*sgn(b*x^2 + a) +
666*(f*x)^m*a*b^2*m*x^7*e*sgn(b*x^2 + a) + 3*(f*x)^m*a^2*b*d*m^4*x^3*sgn(b*
x^2 + a) + 390*(f*x)^m*a*b^2*d*m^2*x^5*sgn(b*x^2 + a) + 135*(f*x)^m*b^3*d*x
^7*sgn(b*x^2 + a) + (f*x)^m*a^3*m^4*x^3*e*sgn(b*x^2 + a) + 390*(f*x)^m*a^2*
b*m^2*x^5*e*sgn(b*x^2 + a) + 405*(f*x)^m*a*b^2*x^7*e*sgn(b*x^2 + a) + 66*(f
*x)^m*a^2*b*d*m^3*x^3*sgn(b*x^2 + a) + 900*(f*x)^m*a*b^2*d*m*x^5*sgn(b*x^2
+ a) + 22*(f*x)^m*a^3*m^3*x^3*e*sgn(b*x^2 + a) + 900*(f*x)^m*a^2*b*m*x^5*e*
sgn(b*x^2 + a) + (f*x)^m*a^3*d*m^4*x*sgn(b*x^2 + a) + 492*(f*x)^m*a^2*b*d*m
^2*x^3*sgn(b*x^2 + a) + 567*(f*x)^m*a*b^2*d*x^5*sgn(b*x^2 + a) + 164*(f*x)^
m*a^3*m^2*x^3*e*sgn(b*x^2 + a) + 567*(f*x)^m*a^2*b*x^5*e*sgn(b*x^2 + a) + 2
4*(f*x)^m*a^3*d*m^3*x*sgn(b*x^2 + a) + 1374*(f*x)^m*a^2*b*d*m*x^3*sgn(b*x^2
 + a) + 458*(f*x)^m*a^3*m*x^3*e*sgn(b*x^2 + a) + 206*(f*x)^m*a^3*d*m^2*x*sg
n(b*x^2 + a) + 945*(f*x)^m*a^2*b*d*x^3*sgn(b*x^2 + a) + 315*(f*x)^m*a^3*x^3
*e*sgn(b*x^2 + a) + 744*(f*x)^m*a^3*d*m*x*sgn(b*x^2 + a) + 945*(f*x)^m*a^3*
d*x*sgn(b*x^2 + a))/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

3.89 $\int (fx)^m \left(d+ex^2\right)\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{ad\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+1}}{f(m+1)\left(a+bx^2\right)}+\frac{be\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}}{f^5(m+5)\left(a+bx^2\right)}$$

```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2))
 + ((b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3*(3 + m)
*(a + b*x^2)) + (b*e*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^5*(5
 + m)*(a + b*x^2))
```

Rubi [A] time = 0.0762581, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.057, Rules used = {1250, 448}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+3}(ae+bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{ad\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+1}}{f(m+1)\left(a+bx^2\right)}+\frac{be\sqrt{a^2+2abx^2+b^2x^4}(fx)^{m+5}}{f^5(m+5)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2))
 + ((b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3*(3 + m)
*(a + b*x^2)) + (b*e*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^5*(5
 + m)*(a + b*x^2))
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (fx)^m \left(d+ex^2\right)\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (fx)^m\left(ab+b^2x^2\right)\left(d+ex^2\right)dx}{ab+b^2x^2}\\ &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(abd(fx)^m+\frac{b(bd+ae)(fx)^{2+m}}{f^2}+\frac{b^2e(fx)^{4+m}}{f^4}\right)dx}{ab+b^2x^2}\\ &= \frac{ad(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)}+\frac{(bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^3(3+m)\left(a+bx^2\right)}+\end{aligned}$$

Mathematica [A] time = 0.0527489, size = 86, normalized size = 0.56

$$\frac{x\sqrt{\left(a+bx^2\right)^2}(fx)^m\left(a(m+5)\left(d(m+3)+e(m+1)x^2\right)+b(m+1)x^2\left(d(m+5)+e(m+3)x^2\right)\right)}{(m+1)(m+3)(m+5)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(f*x)^m*Sqrt[(a + b*x^2)^2]*(a*(5 + m)*(d*(3 + m) + e*(1 + m)*x^2) + b*(
1 + m)*x^2*(d*(5 + m) + e*(3 + m)*x^2)))/((1 + m)*(3 + m)*(5 + m)*(a + b*x^
2))
```

Maple [A] time = 0.004, size = 131, normalized size = 0.9

$$\frac{\left(bem^2x^4+4\,bemx^4+aem^2x^2+bdm^2x^2+3\,bex^4+6\,aemx^2+6\,bdmx^2+adm^2+5\,aex^2+5\,bdx^2+8\,adm+15\,ad\right)x\left(fx\right)}{(5+m)\left(3+m\right)\left(1+m\right)\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

```
[Out] x*(b*e*m^2*x^4+4*b*e*m*x^4+a*e*m^2*x^2+b*d*m^2*x^2+3*b*e*x^4+6*a*e*m*x^2+6*
b*d*m*x^2+a*d*m^2+5*a*e*x^2+5*b*d*x^2+8*a*d*m+15*a*d)*(f*x)^m*((b*x^2+a)^2)
^(1/2)/(5+m)/(3+m)/(1+m)/(b*x^2+a)
```

Maxima [A] time = 1.01512, size = 101, normalized size = 0.66

$$\frac{\left(bf^m(m+1)x^3+af^m(m+3)x\right)dx^m}{m^2+4\,m+3}+\frac{\left(bf^m(m+3)x^5+af^m(m+5)x^3\right)ex^m}{m^2+8\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] (b*f^m*(m + 1)*x^3 + a*f^m*(m + 3)*x)*d*x^m/(m^2 + 4*m + 3) + (b*f^m*(m + 3
)*x^5 + a*f^m*(m + 5)*x^3)*e*x^m/(m^2 + 8*m + 15)
```

Fricas [A] time = 1.58691, size = 216, normalized size = 1.41

$$\frac{\left(\left(bem^2+4\,bem+3\,be\right)x^5+\left((bd+ae)m^2+5\,bd+5\,ae+6\,(bd+ae)m\right)x^3+\left(adm^2+8\,adm+15\,ad\right)x\right)\left(fx\right)^m}{m^3+9\,m^2+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] ((b*e*m^2 + 4*b*e*m + 3*b*e)*x^5 + ((b*d + a*e)*m^2 + 5*b*d + 5*a*e + 6*(b*
d + a*e)*m)*x^3 + (a*d*m^2 + 8*a*d*m + 15*a*d)*x)*(f*x)^m/(m^3 + 9*m^2 + 23
*m + 15)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m \left(d + ex^2\right) \sqrt{\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*sqrt((a + b*x**2)**2), x)
```

Giac [B] time = 1.16549, size = 363, normalized size = 2.37

$$\frac{(fx)^m bm^2x^5 e\,\mathrm{sgn}\left(bx^2 + a\right) + 4\,(fx)^m bmx^5 e\,\mathrm{sgn}\left(bx^2 + a\right) + (fx)^m bdm^2x^3\mathrm{sgn}\left(bx^2 + a\right) + (fx)^m am^2x^3 e\,\mathrm{sgn}\left(bx^2 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] ((f*x)^m*b*m^2*x^5*e*sgn(b*x^2 + a) + 4*(f*x)^m*b*m*x^5*e*sgn(b*x^2 + a) +
(f*x)^m*b*d*m^2*x^3*sgn(b*x^2 + a) + (f*x)^m*a*m^2*x^3*e*sgn(b*x^2 + a) + 3
*(f*x)^m*b*x^5*e*sgn(b*x^2 + a) + 6*(f*x)^m*b*d*m*x^3*sgn(b*x^2 + a) + 6*(f
*x)^m*a*m*x^3*e*sgn(b*x^2 + a) + (f*x)^m*a*d*m^2*x*sgn(b*x^2 + a) + 5*(f*x)
^m*b*d*x^3*sgn(b*x^2 + a) + 5*(f*x)^m*a*x^3*e*sgn(b*x^2 + a) + 8*(f*x)^m*a*
d*m*x*sgn(b*x^2 + a) + 15*(f*x)^m*a*d*x*sgn(b*x^2 + a))/(m^3 + 9*m^2 + 23*m
 + 15)
```

3.90 $\int \frac{(fx)^m\left(d+ex^2\right)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=134

$$\frac{\left(a+bx^2\right)(fx)^{m+1}(bd-ae)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e\left(a+bx^2\right)(fx)^{m+1}}{bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (e*(f*x)^(1 + m)*(a + b*x^2))/(b*f*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 + ((b*d - a*e)*(f*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, (
3 + m)/2, -((b*x^2)/a)])/(a*b*f*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0883244, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1250, 459, 364}

$$\frac{\left(a+bx^2\right)(fx)^{m+1}(bd-ae)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e\left(a+bx^2\right)(fx)^{m+1}}{bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```


```
[Out] (e*(f*x)^(1 + m)*(a + b*x^2))/(b*f*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 + ((b*d - a*e)*(f*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, (
3 + m)/2, -((b*x^2)/a)])/(a*b*f*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```


Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] && !IntegerQ[p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(fx)^m\left(d+ex^2\right)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{\left(ab+b^2x^2\right)\int \frac{(fx)^m\left(d+ex^2\right)}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{e(fx)^{1+m}\left(a+bx^2\right)}{bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(\left(-b^2d(1+m)+abe(1+m)\right)\left(ab+b^2x^2\right)\right)\int \frac{(fx)^m}{ab+b^2x^2}\,dx}{b^2(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{e(fx)^{1+m}\left(a+bx^2\right)}{bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)(fx)^{1+m}\left(a+bx^2\right){}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{abf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0734766, size = 78, normalized size = 0.58

$$-\frac{x\left(a+bx^2\right)(fx)^m\left((ae-bd)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)-ae\right)}{ab(m+1)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((x*(f*x)^m*(a + b*x^2)*(-(a*e) + (-(b*d) + a*e)*Hypergeometric2F1[1, (1 +
 m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a*b*(1 + m)*Sqrt[(a + b*x^2)^2]))
```

Maple [F] time = 0.059, size = 0, normalized size = 0.

$$\int (fx)^m\left(ex^2+d\right)\frac{1}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)(fx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)(fx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m (d + ex^2)}{\sqrt{(a + bx^2)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/sqrt((a + b*x**2)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(fx)^m}{\sqrt{b^2x^4 + 2\,abx^2 + a^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

3.91 $\int \frac{(fx)^m \left(d+ex^2\right)}{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=154

$$\frac{\left(a+bx^2\right)(fx)^{m+1}(ae(m+1)+bd(3-m))\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{4a^3bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(fx)^{m+1}(bd-ae)}{4abf\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((b*d - a*e)*(f*x)^(1 + m))/(4*a*b*f*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) + ((b*d*(3 - m) + a*e*(1 + m))*(f*x)^(1 + m)*(a + b*x^2)*Hypergeomet
ric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(4*a^3*b*f*(1 + m)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.119163, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1250, 457, 364}

$$\frac{\left(a+bx^2\right)(fx)^{m+1}(ae(m+1)+bd(3-m))\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{4a^3bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(fx)^{m+1}(bd-ae)}{4abf\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((b*d - a*e)*(f*x)^(1 + m))/(4*a*b*f*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) + ((b*d*(3 - m) + a*e*(1 + m))*(f*x)^(1 + m)*(a + b*x^2)*Hypergeomet
ric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(4*a^3*b*f*(1 + m)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(fx)^m (d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{\left(b^2(ab+b^2x^2)\right)\int \frac{(fx)^m(d+ex^2)}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(bd-ae)(fx)^{1+m}}{4abf(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{\left((bd(3-m)+ae(1+m))(ab+b^2x^2)\right)\int \frac{(fx)^m}{(ab+b^2x^2)^2}\,d}{4a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(bd-ae)(fx)^{1+m}}{4abf(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd(3-m)+ae(1+m))(fx)^{1+m}(a+bx^2)\,{}_2F_1\left(2,\right.}{4a^3bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A] time = 0.0844478, size = 101, normalized size = 0.66

$$\frac{x(a+bx^2)(fx)^m\left((bd-ae)\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)+ae\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)\right)}{a^3b(m+1)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*(a + b*x^2)*(a*e*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b
*x^2)/a)] + (b*d - a*e)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2
)/a)]))/(a^3*b*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int (fx)^m (ex^2+d)(b^2x^4+2abx^2+a^2)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)(fx)^m}{(b^2x^4+2abx^2+a^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b^2x^4 + 2\,abx^2 + a^2}\left(ex^2 + d\right)\left(fx\right)^m}{b^4x^8 + 4\,ab^3x^6 + 6\,a^2b^2x^4 + 4\,a^3bx^2 + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(e*x^2 + d)*(f*x)^m/(b^4*x^8 + 4*a
*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^m\left(d + ex^2\right)}{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)\left(fx\right)^m}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

3.92 $\int x \left(a+bx^2\right) \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=34

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b(p+1)}$$

```
[Out] (a^2 + 2*a*b*x^2 + b^2*x^4)^(1 + p)/(4*b*(1 + p))
```

Rubi [A] time = 0.0289665, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1247, 629}

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2 + 2*a*b*x^2 + b^2*x^4)^(1 + p)/(4*b*(1 + p))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int x \left(a+bx^2\right) \left(a^2+2abx^2+b^2x^4\right)^p \, dx &= \frac{1}{2} \text{Subst}\left(\int (a+bx) \left(a^2+2abx+b^2x^2\right)^p \, dx, x, x^2\right) \\ &= \frac{\left(a^2+2abx^2+b^2x^4\right)^{1+p}}{4b(1+p)} \end{aligned}$$

Mathematica [A] time = 0.0088728, size = 25, normalized size = 0.74

$$\frac{\left(\left(a+bx^2\right)^2\right)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)^2)^(1 + p)/(4*b*(1 + p))
```

Maple [A] time = 0.003, size = 40, normalized size = 1.2

$$\frac{\left(bx^2+a\right)^2\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\,b\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/4*(b*x^2+a)^2/b/(1+p)*(b^2*x^4+2*a*b*x^2+a^2)^p
```

Maxima [B] time = 0.994282, size = 116, normalized size = 3.41

$$\frac{\left(bx^2+a\right)\left(bx^2+a\right)^{2\,p}a}{2\,b\left(2\,p+1\right)}+\frac{\left(b^2\left(2\,p+1\right)x^4+2\,abpx^2-a^2\right)\left(bx^2+a\right)^{2\,p}}{4\,\left(2\,p^2+3\,p+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^(2*p)*a/(b*(2*p + 1)) + 1/4*(b^2*(2*p + 1)*x^4
+ 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)/((2*p^2 + 3*p + 1)*b)
```

Fricas [A] time = 1.60822, size = 99, normalized size = 2.91

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\,\left(bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(b*p + b)
```

Sympy [A] time = 11.9969, size = 165, normalized size = 4.85

$$\begin{cases} \frac{x^2}{2a} & \text{for}\: b=0 \wedge p=-1 \\ \frac{ax^2\left(a^2\right)^p}{2} & \text{for}\: b=0 \\ \frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b}+\frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+x\right)}{2b} & \text{for}\: p=-1 \\ \frac{a^2\left(a^2+2abx^2+b^2x^4\right)^p}{4bp+4b}+\frac{2abx^2\left(a^2+2abx^2+b^2x^4\right)^p}{4bp+4b}+\frac{b^2x^4\left(a^2+2abx^2+b^2x^4\right)^p}{4bp+4b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**2/(2*a), Eq(b, 0) & Eq(p, -1)), (a*x**2*(a**2)**p/2, Eq(b, 0)
), (log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b) + log(I*sqrt(a)*sqrt(1/b) + x)/(2*b
), Eq(p, -1)), (a**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b) + 2*a
*b*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b) + b**2*x**4*(a**2
+ 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b), True))
```

Giac [B] time = 1.12262, size = 119, normalized size = 3.5

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p b^2x^4+2\left(b^2x^4+2\,abx^2+a^2\right)^p abx^2+\left(b^2x^4+2\,abx^2+a^2\right)^p a^2}{4\left(bp+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*((b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^2*x^4 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^
p*a*b*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2)/(b*p + b)
```

3.93 $\int x^3 \left(a+bx^2\right) \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=86

$$\frac{\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+3)}-\frac{a\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}$$

```
[Out] -(a*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^2*(1 + p)) + ((a + b*
x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(3 + 2*p))
```

Rubi [A] time = 0.0887778, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1249, 770, 21, 43}

$$\frac{\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+3)}-\frac{a\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -(a*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^2*(1 + p)) + ((a + b*
x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(3 + 2*p))
```

Rule 1249

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b
^2 - 4*a*c, 0] &&  !IntegerQ[p] && IGtQ[(m + 1)/2, 0]
```

Rule 770

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPa
rt[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(f + g*x)*(b/2 + c*x)^(
2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^3 \left(a+bx^2\right) \left(a^2+2abx^2+b^2x^4\right)^p dx = \frac{1}{2}\text{Subst}\left(\int x(a+bx)\left(a^2+2abx+b^2x^2\right)^p dx, x, x^2\right)$$
$$= \frac{1}{2}\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x(a+bx)\left(ab+b^2x\right)^{2p} dx\right.$$
$$= \frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x\left(ab+b^2x\right)^{1+2p} dx, x, x^2\right)}{2b}$$
$$= \frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^{1+2p}}{b}+\frac{\left(ab+b^2x\right)^{2+2p}}{b^2}\right.\right.}{2b}$$
$$= -\frac{a\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(1+p)} + \frac{\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(3+2p)}$$

Mathematica [A] time = 0.0248835, size = 45, normalized size = 0.52

$$\frac{\left(\left(a+bx^2\right)^2\right)^{p+1}\left(2b(p+1)x^2-a\right)}{4b^2(p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (((a + b*x^2)^2)^(1 + p)*(-a + 2*b*(1 + p)*x^2))/(4*b^2*(1 + p)*(3 + 2*p))
```

Maple [A] time = 0.005, size = 62, normalized size = 0.7

$$-\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p\left(-2\,x^2pb-2\,bx^2+a\right)\left(bx^2+a\right)^2}{4\,b^2\left(2\,p^2+5\,p+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-2*b*p*x^2-2*b*x^2+a)*(b*x^2+a)^2/b^2/(2*p^
2+5*p+3)
```

Maxima [A] time = 1.01171, size = 182, normalized size = 2.12

$$\frac{\left(b^2\left(2\,p+1\right)x^4+2\,abpx^2-a^2\right)\left(bx^2+a\right)^{2\,p}a}{4\left(2\,p^2+3\,p+1\right)b^2} + \frac{\left(\left(2\,p^2+3\,p+1\right)b^3x^6+\left(2\,p^2+p\right)ab^2x^4-2\,a^2bpx^2+a^3\right)\left(bx^2+a\right)^{2\,p}}{2\left(4\,p^3+12\,p^2+11\,p+3\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*(2*p + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)*a/((2*p^2 + 3
*p + 1)*b^2) + 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a
^2*b*p*x^2 + a^3)*(b*x^2 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^2)
```

Fricas [A] time = 1.55942, size = 185, normalized size = 2.15

$$\frac{\left(2\left(b^3p+b^3\right)x^6+2\,a^2bpx^2+\left(4\,ab^2p+3\,ab^2\right)x^4-a^3\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\left(2\,b^2p^2+5\,b^2p+3\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^3*p + b^3)*x^6 + 2*a^2*b*p*x^2 + (4*a*b^2*p + 3*a*b^2)*x^4 - a^3)
*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b^2*p^2 + 5*b^2*p + 3*b^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.14165, size = 265, normalized size = 3.08

$$\frac{2\left(b^2x^4+2\,abx^2+a^2\right)^pb^3px^6+2\left(b^2x^4+2\,abx^2+a^2\right)^pb^3x^6+4\left(b^2x^4+2\,abx^2+a^2\right)^pab^2px^4+3\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\left(2\,b^2p^2+5\,b^2p+3\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^3*p*x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a
^2)^p*b^3*x^6 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p*x^4 + 3*(b^2*x^4 +
2*a*b*x^2 + a^2)^p*a*b^2*x^4 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b*p*x^2
- (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^3)/(2*b^2*p^2 + 5*b^2*p + 3*b^2)
```

3.94 $\int x^5 \left(a+bx^2\right) \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=128

$$\frac{\left(a+bx^2\right)^4 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+2)}-\frac{a\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{b^3(2p+3)}+\frac{a^2\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+1)}$$

```
[Out] (a^2*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(1 + p)) - (a*(a +
 b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(b^3*(3 + 2*p)) + ((a + b*x^2)^4*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(2 + p))
```

Rubi [A] time = 0.128993, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1249, 770, 21, 43}

$$\frac{\left(a+bx^2\right)^4 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+2)}-\frac{a\left(a+bx^2\right)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{b^3(2p+3)}+\frac{a^2\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```


```
[Out] (a^2*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(1 + p)) - (a*(a +
 b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(b^3*(3 + 2*p)) + ((a + b*x^2)^4*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(2 + p))
```


Rule 1249

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b
^2 - 4*a*c, 0] &&  !IntegerQ[p] && IGtQ[(m + 1)/2, 0]
```

Rule 770

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPa
rt[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(f + g*x)*(b/2 + c*x)^(
2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^5\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{1}{2}\,\text{Subst}\left(\int x^2(a+bx)\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)$$
$$=\frac{1}{2}\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^2(a+bx)\left(ab+b^2x\right)^2\right.$$
$$=\frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^{1+2p}\,dx,x,x\right.}{2b}$$
$$=\frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^{1+2p}}{b^2}-\frac{2a\left(ab+b^2x\right.}{b^3}\right.\right.}{2b}$$
$$=\frac{a^2\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(1+p)}-\frac{a\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x^4\right)^p}{b^3(3+2p)}+$$

Mathematica [A] time = 0.0368035, size = 68, normalized size = 0.53

$$\frac{\left(\left(a+bx^2\right)^2\right)^{p+1}\left(a^2-2ab(p+1)x^2+b^2\left(2p^2+5p+3\right)x^4\right)}{4b^3(p+1)(p+2)(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (((a + b*x^2)^2)^(1 + p)*(a^2 - 2*a*b*(1 + p)*x^2 + b^2*(3 + 5*p + 2*p^2)*x
^4))/(4*b^3*(1 + p)*(2 + p)*(3 + 2*p))
```

Maple [A] time = 0.006, size = 99, normalized size = 0.8

$$\frac{\left(2\,b^2p^2x^4+5\,b^2px^4+3\,b^2x^4-2\,abpx^2-2\,abx^2+a^2\right)\left(bx^2+a\right)^2\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\,b^3\left(2\,p^3+9\,p^2+13\,p+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/4*(b*x^2+a)^2*(2*b^2*p^2*x^4+5*b^2*p*x^4+3*b^2*x^4-2*a*b*p*x^2-2*a*b*x^2+
a^2)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(2*p^3+9*p^2+13*p+6)
```

Maxima [A] time = 1.01826, size = 265, normalized size = 2.07

$$\frac{\left(\left(2\,p^2+3\,p+1\right)b^3x^6+\left(2\,p^2+p\right)ab^2x^4-2\,a^2bpx^2+a^3\right)\left(bx^2+a\right)^{2\,p}a}{2\left(4\,p^3+12\,p^2+11\,p+3\right)b^3}+\frac{\left(\left(4\,p^3+12\,p^2+11\,p+3\right)b^4x^8+2\left(2\,p^3+3\,p\right.\right.}{4\left(4\,p^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + a^
3)*(b*x^2 + a)^(2*p)*a/((4*p^3 + 12*p^2 + 11*p + 3)*b^3) + 1/4*((4*p^3 + 12
```

```
*p^2 + 11*p + 3)*b^4*x^8 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^6 - 3*(2*p^2 + p)*
a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 3*a^4)*(b*x^2 + a)^(2*p)/((4*p^4 + 20*p^3 + 3
5*p^2 + 25*p + 6)*b^3)
```

Fricas [A] time = 1.56729, size = 284, normalized size = 2.22

$$\frac{\left(\left(2\,b^4p^2+5\,b^4p+3\,b^4\right)x^8-2\,a^3bpx^2+4\left(ab^3p^2+2\,ab^3p+ab^3\right)x^6+\left(2\,a^2b^2p^2+a^2b^2p\right)x^4+a^4\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\left(2\,b^3p^3+9\,b^3p^2+13\,b^3p+6\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^4*p^2 + 5*b^4*p + 3*b^4)*x^8 - 2*a^3*b*p*x^2 + 4*(a*b^3*p^2 + 2*a
*b^3*p + a*b^3)*x^6 + (2*a^2*b^2*p^2 + a^2*b^2*p)*x^4 + a^4)*(b^2*x^4 + 2*a
*b*x^2 + a^2)^p/(2*b^3*p^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.13106, size = 447, normalized size = 3.49

$$\frac{2\left(b^2x^4+2\,abx^2+a^2\right)^pb^4p^2x^8+5\left(b^2x^4+2\,abx^2+a^2\right)^pb^4px^8+4\left(b^2x^4+2\,abx^2+a^2\right)^pab^3p^2x^6+3\left(b^2x^4+2\,abx^2+a^2\right)^p}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p^2*x^8 + 5*(b^2*x^4 + 2*a*b*x^2 +
 a^2)^p*b^4*p*x^8 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p^2*x^6 + 3*(b^2*
x^4 + 2*a*b*x^2 + a^2)^p*b^4*x^8 + 8*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p*
x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p^2*x^4 + 4*(b^2*x^4 + 2*a*b*
x^2 + a^2)^p*a*b^3*x^6 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p*x^4 - 2*(b
^2*x^4 + 2*a*b*x^2 + a^2)^p*a^3*b*p*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^4
)/(2*b^3*p^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)
```

3.95 $\int x^3 \left(A+Bx^2\right) \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=166

$$\frac{1}{6}a^2x^6(aB+3Ab)+\frac{1}{4}a^3Ax^4+\frac{1}{12}x^{12}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{14}cx^{14}\left(aBc+Abc+b^2B\right)+\frac{1}{10}x^{10}\left(A\left(6abc\right.\right.$$

```
[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/10 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*(b^2*B + A*b*c + a*B*c)*x^14)/
14 + (c^2*(3*b*B + A*c)*x^16)/16 + (B*c^3*x^18)/18
```

Rubi [A] time = 0.393488, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1251, 765}

$$\frac{1}{6}a^2x^6(aB+3Ab)+\frac{1}{4}a^3Ax^4+\frac{1}{12}x^{12}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{14}cx^{14}\left(aBc+Abc+b^2B\right)+\frac{1}{10}x^{10}\left(A\left(6abc\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/10 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*(b^2*B + A*b*c + a*B*c)*x^14)/
14 + (c^2*(3*b*B + A*c)*x^16)/16 + (B*c^3*x^18)/18
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 765

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int x^3\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int x(A+Bx)\left(a+bx+cx^2\right)^3\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(a^3Ax+a^2(3Ab+aB)x^2+3a\left(abB+A\left(b^2+ac\right)\right)x^3+\left(3aB\left(b^2\right.\right.\right.\right.\\ &= \frac{1}{4}a^3Ax^4+\frac{1}{6}a^2(3Ab+aB)x^6+\frac{3}{8}a\left(abB+A\left(b^2+ac\right)\right)x^8+\frac{1}{10}\left(3aB\left(b^2+ac\right)\right.\end{aligned}$$

Mathematica [A] time = 0.0507348, size = 166, normalized size = 1.

$$\frac{1}{6}a^2x^6(aB+3Ab)+\frac{1}{4}a^3Ax^4+\frac{1}{12}x^{12}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{14}cx^{14}\left(aBc+Abc+b^2B\right)+\frac{1}{10}x^{10}\left(A\left(6abc\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/10 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*(b^2*B + A*b*c + a*B*c)*x^14)/
14 + (c^2*(3*b*B + A*c)*x^16)/16 + (B*c^3*x^18)/18
```

Maple [A] time = 0.001, size = 226, normalized size = 1.4

$$\frac{Bc^3x^{18}}{18} + \frac{\left(Ac^3 + 3\,Bbc^2\right)x^{16}}{16} + \frac{\left(3\,Abc^2 + B\left(ac^2 + 2\,b^2c + c\left(2\,ac + b^2\right)\right)\right)x^{14}}{14} + \frac{\left(A\left(ac^2 + 2\,b^2c + c\left(2\,ac + b^2\right)\right) + B\left(4\,ab\right.\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/18*B*c^3*x^18+1/16*(A*c^3+3*B*b*c^2)*x^16+1/14*(3*A*b*c^2+B*(a*c^2+2*b^2*
c+c*(2*a*c+b^2)))*x^14+1/12*(A*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+B*(4*a*b*c+b*(
2*a*c+b^2)))*x^12+1/10*(A*(4*a*b*c+b*(2*a*c+b^2))+B*(a*(2*a*c+b^2)+2*b^2*a+
c*a^2))*x^10+1/8*(A*(a*(2*a*c+b^2)+2*b^2*a+c*a^2)+3*B*a^2*b)*x^8+1/6*(3*A*a
^2*b+B*a^3)*x^6+1/4*a^3*A*x^4
```

Maxima [A] time = 1.00325, size = 224, normalized size = 1.35

$$\frac{1}{18}\,Bc^3x^{18} + \frac{1}{16}\left(3\,Bbc^2 + Ac^3\right)x^{16} + \frac{3}{14}\left(Bb^2c + (Ba + Ab)c^2\right)x^{14} + \frac{1}{12}\left(Bb^3 + 3\,Aac^2 + 3\left(2\,Bab + Ab^2\right)c\right)x^{12} + \frac{1}{10}\left(3\,Ba\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*B*c^3*x^18 + 1/16*(3*B*b*c^2 + A*c^3)*x^16 + 3/14*(B*b^2*c + (B*a + A*
b)*c^2)*x^14 + 1/12*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^12 + 1/10
*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^10 + 3/8*(B*a^2*b + A*a*b^2
+ A*a^2*c)*x^8 + 1/4*A*a^3*x^4 + 1/6*(B*a^3 + 3*A*a^2*b)*x^6
```

Fricas [A] time = 1.21434, size = 501, normalized size = 3.02

$$\frac{1}{18}x^{18}c^3B + \frac{3}{16}x^{16}c^2bB + \frac{1}{16}x^{16}c^3A + \frac{3}{14}x^{14}cb^2B + \frac{3}{14}x^{14}c^2aB + \frac{3}{14}x^{14}c^2bA + \frac{1}{12}x^{12}b^3B + \frac{1}{2}x^{12}cbaB + \frac{1}{4}x^{12}cb^2A + \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*c^3*B + 3/16*x^16*c^2*b*B + 1/16*x^16*c^3*A + 3/14*x^14*c*b^2*B +
 3/14*x^14*c^2*a*B + 3/14*x^14*c^2*b*A + 1/12*x^12*b^3*B + 1/2*x^12*c*b*a*B
 + 1/4*x^12*c*b^2*A + 1/4*x^12*c^2*a*A + 3/10*x^10*b^2*a*B + 3/10*x^10*c*a^
2*B + 1/10*x^10*b^3*A + 3/5*x^10*c*b*a*A + 3/8*x^8*b*a^2*B + 3/8*x^8*b^2*a*
A + 3/8*x^8*c*a^2*A + 1/6*x^6*a^3*B + 1/2*x^6*b*a^2*A + 1/4*x^4*a^3*A
```

Sympy [A] time = 0.098254, size = 202, normalized size = 1.22

$$\frac{Aa^3x^4}{4} + \frac{Bc^3x^{18}}{18} + x^{16}\left(\frac{Ac^3}{16} + \frac{3Bbc^2}{16}\right) + x^{14}\left(\frac{3Abc^2}{14} + \frac{3Bac^2}{14} + \frac{3Bb^2c}{14}\right) + x^{12}\left(\frac{Aac^2}{4} + \frac{Ab^2c}{4} + \frac{Babc}{2} + \frac{Bb^3}{12}\right) + x^{10}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**4/4 + B*c**3*x**18/18 + x**16*(A*c**3/16 + 3*B*b*c**2/16) + x**14
*(3*A*b*c**2/14 + 3*B*a*c**2/14 + 3*B*b**2*c/14) + x**12*(A*a*c**2/4 + A*b*
*2*c/4 + B*a*b*c/2 + B*b**3/12) + x**10*(3*A*a*b*c/5 + A*b**3/10 + 3*B*a**2
*c/10 + 3*B*a*b**2/10) + x**8*(3*A*a**2*c/8 + 3*A*a*b**2/8 + 3*B*a**2*b/8)
+ x**6*(A*a**2*b/2 + B*a**3/6)
```

Giac [A] time = 1.1432, size = 261, normalized size = 1.57

$$\frac{1}{18}Bc^3x^{18} + \frac{3}{16}Bbc^2x^{16} + \frac{1}{16}Ac^3x^{16} + \frac{3}{14}Bb^2cx^{14} + \frac{3}{14}Bac^2x^{14} + \frac{3}{14}Abc^2x^{14} + \frac{1}{12}Bb^3x^{12} + \frac{1}{2}Babcx^{12} + \frac{1}{4}Ab^2cx^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/18*B*c^3*x^18 + 3/16*B*b*c^2*x^16 + 1/16*A*c^3*x^16 + 3/14*B*b^2*c*x^14 +
 3/14*B*a*c^2*x^14 + 3/14*A*b*c^2*x^14 + 1/12*B*b^3*x^12 + 1/2*B*a*b*c*x^12
 + 1/4*A*b^2*c*x^12 + 1/4*A*a*c^2*x^12 + 3/10*B*a*b^2*x^10 + 1/10*A*b^3*x^1
0 + 3/10*B*a^2*c*x^10 + 3/5*A*a*b*c*x^10 + 3/8*B*a^2*b*x^8 + 3/8*A*a*b^2*x^
8 + 3/8*A*a^2*c*x^8 + 1/6*B*a^3*x^6 + 1/2*A*a^2*b*x^6 + 1/4*A*a^3*x^4
```

3.96 $\int x^2 \left(A + Bx^2\right) \left(a + bx^2 + cx^4\right)^3 \, dx$

Optimal. Leaf size=166

$$\frac{1}{5}a^2x^5(aB + 3Ab) + \frac{1}{3}a^3Ax^3 + \frac{1}{11}x^{11}\left(3aAc^2 + 6abBc + 3Ab^2c + b^3B\right) + \frac{3}{13}cx^{13}\left(aBc + Abc + b^2B\right) + \frac{1}{9}x^9\left(A\left(6abc + b^3\right.\right.$$

```
[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9 + ((b^3*B + 3*A*b^2*
c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b^2*B + A*b*c + a*B*c)*x^13)/13
 + (c^2*(3*b*B + A*c)*x^15)/15 + (B*c^3*x^17)/17
```

Rubi [A] time = 0.153507, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1261}

$$\frac{1}{5}a^2x^5(aB + 3Ab) + \frac{1}{3}a^3Ax^3 + \frac{1}{11}x^{11}\left(3aAc^2 + 6abBc + 3Ab^2c + b^3B\right) + \frac{3}{13}cx^{13}\left(aBc + Abc + b^2B\right) + \frac{1}{9}x^9\left(A\left(6abc + b^3\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9 + ((b^3*B + 3*A*b^2*
c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b^2*B + A*b*c + a*B*c)*x^13)/13
 + (c^2*(3*b*B + A*c)*x^15)/15 + (B*c^3*x^17)/17
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int x^2 \left(A + Bx^2\right) \left(a + bx^2 + cx^4\right)^3 \, dx = \int \left(a^3Ax^2 + a^2(3Ab + aB)x^4 + 3a\left(abB + A\left(b^2 + ac\right)\right)x^6 + \left(3aB\left(b^2 + ac\right) + A\left(b\right.\right.\right.$$
$$= \frac{1}{3}a^3Ax^3 + \frac{1}{5}a^2(3Ab + aB)x^5 + \frac{3}{7}a\left(abB + A\left(b^2 + ac\right)\right)x^7 + \frac{1}{9}\left(3aB\left(b^2 + ac\right) + A\right.$$

Mathematica [A] time = 0.05703, size = 166, normalized size = 1.

$$\frac{1}{5}a^2x^5(aB + 3Ab) + \frac{1}{3}a^3Ax^3 + \frac{1}{11}x^{11}\left(3aAc^2 + 6abBc + 3Ab^2c + b^3B\right) + \frac{3}{13}cx^{13}\left(aBc + Abc + b^2B\right) + \frac{1}{9}x^9\left(A\left(6abc + b^3\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9 + ((b^3*B + 3*A*b^2*
c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b^2*B + A*b*c + a*B*c)*x^13)/13
```

```
+ (c^2*(3*b*B + A*c)*x^15)/15 + (B*c^3*x^17)/17
```

Maple [A] time = 0., size = 226, normalized size = 1.4

$$\frac{Bc^3x^{17}}{17}+\frac{\left(Ac^3+3\,Bbc^2\right)x^{15}}{15}+\frac{\left(3\,Abc^2+B\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)\right)x^{13}}{13}+\frac{\left(A\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)+B\left(4\right.\right.}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/17*B*c^3*x^17+1/15*(A*c^3+3*B*b*c^2)*x^15+1/13*(3*A*b*c^2+B*(a*c^2+2*b^2*
c+c*(2*a*c+b^2)))*x^13+1/11*(A*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+B*(4*a*b*c+b*(
2*a*c+b^2)))*x^11+1/9*(A*(4*a*b*c+b*(2*a*c+b^2))+B*(a*(2*a*c+b^2)+2*b^2*a+c
*a^2))*x^9+1/7*(A*(a*(2*a*c+b^2)+2*b^2*a+c*a^2)+3*B*a^2*b)*x^7+1/5*(3*A*a^2
*b+B*a^3)*x^5+1/3*a^3*A*x^3
```

Maxima [A] time = 0.988011, size = 224, normalized size = 1.35

$$\frac{1}{17}\,Bc^3x^{17}+\frac{1}{15}\left(3\,Bbc^2+Ac^3\right)x^{15}+\frac{3}{13}\left(Bb^2c+(Ba+Ab)c^2\right)x^{13}+\frac{1}{11}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^{11}+\frac{1}{9}\left(3\,B\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/17*B*c^3*x^17 + 1/15*(3*B*b*c^2 + A*c^3)*x^15 + 3/13*(B*b^2*c + (B*a + A*
b)*c^2)*x^13 + 1/11*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^11 + 1/9*
(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^9 + 3/7*(B*a^2*b + A*a*b^2 +
A*a^2*c)*x^7 + 1/3*A*a^3*x^3 + 1/5*(B*a^3 + 3*A*a^2*b)*x^5
```

Fricas [A] time = 1.25964, size = 494, normalized size = 2.98

$$\frac{1}{17}x^{17}c^3B+\frac{1}{5}x^{15}c^2bB+\frac{1}{15}x^{15}c^3A+\frac{3}{13}x^{13}cb^2B+\frac{3}{13}x^{13}c^2aB+\frac{3}{13}x^{13}c^2bA+\frac{1}{11}x^{11}b^3B+\frac{6}{11}x^{11}cbaB+\frac{3}{11}x^{11}cb^2A+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*c^3*B + 1/5*x^15*c^2*b*B + 1/15*x^15*c^3*A + 3/13*x^13*c*b^2*B +
3/13*x^13*c^2*a*B + 3/13*x^13*c^2*b*A + 1/11*x^11*b^3*B + 6/11*x^11*c*b*a*B
 + 3/11*x^11*c*b^2*A + 3/11*x^11*c^2*a*A + 1/3*x^9*b^2*a*B + 1/3*x^9*c*a^2*
B + 1/9*x^9*b^3*A + 2/3*x^9*c*b*a*A + 3/7*x^7*b*a^2*B + 3/7*x^7*b^2*a*A + 3
/7*x^7*c*a^2*A + 1/5*x^5*a^3*B + 3/5*x^5*b*a^2*A + 1/3*x^3*a^3*A
```

Sympy [A] time = 0.098198, size = 204, normalized size = 1.23

$$\frac{Aa^3x^3}{3}+\frac{Bc^3x^{17}}{17}+x^{15}\left(\frac{Ac^3}{15}+\frac{Bbc^2}{5}\right)+x^{13}\left(\frac{3Abc^2}{13}+\frac{3Bac^2}{13}+\frac{3Bb^2c}{13}\right)+x^{11}\left(\frac{3Aac^2}{11}+\frac{3Ab^2c}{11}+\frac{6Babc}{11}+\frac{Bb^3}{11}\right)+x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**3/3 + B*c**3*x**17/17 + x**15*(A*c**3/15 + B*b*c**2/5) + x**13*(3
*A*b*c**2/13 + 3*B*a*c**2/13 + 3*B*b**2*c/13) + x**11*(3*A*a*c**2/11 + 3*A*
b**2*c/11 + 6*B*a*b*c/11 + B*b**3/11) + x**9*(2*A*a*b*c/3 + A*b**3/9 + B*a*
*2*c/3 + B*a*b**2/3) + x**7*(3*A*a**2*c/7 + 3*A*a*b**2/7 + 3*B*a**2*b/7) +
x**5*(3*A*a**2*b/5 + B*a**3/5)
```

Giac [A] time = 1.1507, size = 261, normalized size = 1.57

$$\frac{1}{17}Bc^3x^{17} + \frac{1}{5}Bbc^2x^{15} + \frac{1}{15}Ac^3x^{15} + \frac{3}{13}Bb^2cx^{13} + \frac{3}{13}Bac^2x^{13} + \frac{3}{13}Abc^2x^{13} + \frac{1}{11}Bb^3x^{11} + \frac{6}{11}Babcx^{11} + \frac{3}{11}Ab^2cx^{11} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/17*B*c^3*x^17 + 1/5*B*b*c^2*x^15 + 1/15*A*c^3*x^15 + 3/13*B*b^2*c*x^13 +
3/13*B*a*c^2*x^13 + 3/13*A*b*c^2*x^13 + 1/11*B*b^3*x^11 + 6/11*B*a*b*c*x^11
 + 3/11*A*b^2*c*x^11 + 3/11*A*a*c^2*x^11 + 1/3*B*a*b^2*x^9 + 1/9*A*b^3*x^9
+ 1/3*B*a^2*c*x^9 + 2/3*A*a*b*c*x^9 + 3/7*B*a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 3
/7*A*a^2*c*x^7 + 1/5*B*a^3*x^5 + 3/5*A*a^2*b*x^5 + 1/3*A*a^3*x^3
```

3.97 $\int x\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=166

$$\frac{1}{4}a^2x^4(aB+3Ab)+\frac{1}{2}a^3Ax^2+\frac{1}{10}x^{10}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{4}cx^{12}\left(aBc+Abc+b^2B\right)+\frac{1}{8}x^8\left(A\left(6abc+\right.\right.$$

```
[Out] (a^3*A*x^2)/2 + (a^2*(3*A*b + a*B)*x^4)/4 + (a*(a*b*B + A*(b^2 + a*c))*x^6)
/2 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^8)/8 + ((b^3*B + 3*A*b^2*c
+ 6*a*b*B*c + 3*a*A*c^2)*x^10)/10 + (c*(b^2*B + A*b*c + a*B*c)*x^12)/4 + (c
^2*(3*b*B + A*c)*x^14)/14 + (B*c^3*x^16)/16
```

Rubi [A] time = 0.286762, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1247, 631}

$$\frac{1}{4}a^2x^4(aB+3Ab)+\frac{1}{2}a^3Ax^2+\frac{1}{10}x^{10}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{4}cx^{12}\left(aBc+Abc+b^2B\right)+\frac{1}{8}x^8\left(A\left(6abc+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^2)/2 + (a^2*(3*A*b + a*B)*x^4)/4 + (a*(a*b*B + A*(b^2 + a*c))*x^6)
/2 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^8)/8 + ((b^3*B + 3*A*b^2*c
+ 6*a*b*B*c + 3*a*A*c^2)*x^10)/10 + (c*(b^2*B + A*b*c + a*B*c)*x^12)/4 + (c
^2*(3*b*B + A*c)*x^14)/14 + (B*c^3*x^16)/16
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx&=\frac{1}{2}\text{Subst}\left(\int(A+Bx)\left(a+bx+cx^2\right)^3\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(a^3A+a^2(3Ab+aB)x+3a\left(abB+A\left(b^2+ac\right)\right)x^2+\left(3aB\left(b^2+a\right.\right.\right.\right.\\&=\frac{1}{2}a^3Ax^2+\frac{1}{4}a^2(3Ab+aB)x^4+\frac{1}{2}a\left(abB+A\left(b^2+ac\right)\right)x^6+\frac{1}{8}\left(3aB\left(b^2+ac\right)+\right.\end{aligned}$$

Mathematica [A] time = 0.0620843, size = 154, normalized size = 0.93

$$\frac{1}{560}x^2\left(140a^2x^2(aB+3Ab)+280a^3A+56x^8\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+140cx^{10}\left(aBc+Abc+b^2B\right)+70x^6\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(280*a^3*A + 140*a^2*(3*A*b + a*B)*x^2 + 280*a*(a*b*B + A*(b^2 + a*c))
*x^4 + 70*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6 + 56*(b^3*B + 3*A*b^2
*c + 6*a*b*B*c + 3*a*A*c^2)*x^8 + 140*c*(b^2*B + A*b*c + a*B*c)*x^10 + 40*c
^2*(3*b*B + A*c)*x^12 + 35*B*c^3*x^14))/560
```

Maple [A] time = 0.001, size = 226, normalized size = 1.4

$$\frac{Bc^3x^{16}}{16}+\frac{\left(Ac^3+3\,Bbc^2\right)x^{14}}{14}+\frac{\left(3\,Abc^2+B\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)\right)x^{12}}{12}+\frac{\left(A\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)+B\left(4\,ab\right.\right.}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/16*B*c^3*x^16+1/14*(A*c^3+3*B*b*c^2)*x^14+1/12*(3*A*b*c^2+B*(a*c^2+2*b^2*
c+c*(2*a*c+b^2)))*x^12+1/10*(A*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+B*(4*a*b*c+b*(
2*a*c+b^2)))*x^10+1/8*(A*(4*a*b*c+b*(2*a*c+b^2))+B*(a*(2*a*c+b^2)+2*b^2*a+c
*a^2))*x^8+1/6*(A*(a*(2*a*c+b^2)+2*b^2*a+c*a^2)+3*B*a^2*b)*x^6+1/4*(3*A*a^2
*b+B*a^3)*x^4+1/2*a^3*A*x^2
```

Maxima [A] time = 1.09262, size = 224, normalized size = 1.35

$$\frac{1}{16}\,Bc^3x^{16}+\frac{1}{14}\left(3\,Bbc^2+Ac^3\right)x^{14}+\frac{1}{4}\left(Bb^2c+(Ba+Ab)c^2\right)x^{12}+\frac{1}{10}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^{10}+\frac{1}{8}\left(3\,Bab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*B*c^3*x^16 + 1/14*(3*B*b*c^2 + A*c^3)*x^14 + 1/4*(B*b^2*c + (B*a + A*b
)*c^2)*x^12 + 1/10*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^10 + 1/8*(
3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^8 + 1/2*(B*a^2*b + A*a*b^2 + A
*a^2*c)*x^6 + 1/2*A*a^3*x^2 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4
```

Fricas [A] time = 1.27122, size = 490, normalized size = 2.95

$$\frac{1}{16}x^{16}c^3B+\frac{3}{14}x^{14}c^2bB+\frac{1}{14}x^{14}c^3A+\frac{1}{4}x^{12}cb^2B+\frac{1}{4}x^{12}c^2aB+\frac{1}{4}x^{12}c^2bA+\frac{1}{10}x^{10}b^3B+\frac{3}{5}x^{10}cbaB+\frac{3}{10}x^{10}cb^2A+\frac{3}{10}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*c^3*B + 3/14*x^14*c^2*b*B + 1/14*x^14*c^3*A + 1/4*x^12*c*b^2*B +
1/4*x^12*c^2*a*B + 1/4*x^12*c^2*b*A + 1/10*x^10*b^3*B + 3/5*x^10*c*b*a*B +
3/10*x^10*c*b^2*A + 3/10*x^10*c^2*a*A + 3/8*x^8*b^2*a*B + 3/8*x^8*c*a^2*B +
 1/8*x^8*b^3*A + 3/4*x^8*c*b*a*A + 1/2*x^6*b*a^2*B + 1/2*x^6*b^2*a*A + 1/2*
x^6*c*a^2*A + 1/4*x^4*a^3*B + 3/4*x^4*b*a^2*A + 1/2*x^2*a^3*A
```

Sympy [A] time = 0.100483, size = 199, normalized size = 1.2

$$\frac{Aa^3x^2}{2} + \frac{Bc^3x^{16}}{16} + x^{14}\left(\frac{Ac^3}{14} + \frac{3Bbc^2}{14}\right) + x^{12}\left(\frac{Abc^2}{4} + \frac{Bac^2}{4} + \frac{Bb^2c}{4}\right) + x^{10}\left(\frac{3Aac^2}{10} + \frac{3Ab^2c}{10} + \frac{3Babc}{5} + \frac{Bb^3}{10}\right) + x^8\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**2/2 + B*c**3*x**16/16 + x**14*(A*c**3/14 + 3*B*b*c**2/14) + x**12
*(A*b*c**2/4 + B*a*c**2/4 + B*b**2*c/4) + x**10*(3*A*a*c**2/10 + 3*A*b**2*c
/10 + 3*B*a*b*c/5 + B*b**3/10) + x**8*(3*A*a*b*c/4 + A*b**3/8 + 3*B*a**2*c/
8 + 3*B*a*b**2/8) + x**6*(A*a**2*c/2 + A*a*b**2/2 + B*a**2*b/2) + x**4*(3*A
*a**2*b/4 + B*a**3/4)
```

Giac [A] time = 1.11051, size = 261, normalized size = 1.57

$$\frac{1}{16}Bc^3x^{16} + \frac{3}{14}Bbc^2x^{14} + \frac{1}{14}Ac^3x^{14} + \frac{1}{4}Bb^2cx^{12} + \frac{1}{4}Bac^2x^{12} + \frac{1}{4}Abc^2x^{12} + \frac{1}{10}Bb^3x^{10} + \frac{3}{5}Babcx^{10} + \frac{3}{10}Ab^2cx^{10} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*B*c^3*x^16 + 3/14*B*b*c^2*x^14 + 1/14*A*c^3*x^14 + 1/4*B*b^2*c*x^12 +
1/4*B*a*c^2*x^12 + 1/4*A*b*c^2*x^12 + 1/10*B*b^3*x^10 + 3/5*B*a*b*c*x^10 +
3/10*A*b^2*c*x^10 + 3/10*A*a*c^2*x^10 + 3/8*B*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3
/8*B*a^2*c*x^8 + 3/4*A*a*b*c*x^8 + 1/2*B*a^2*b*x^6 + 1/2*A*a*b^2*x^6 + 1/2*
A*a^2*c*x^6 + 1/4*B*a^3*x^4 + 3/4*A*a^2*b*x^4 + 1/2*A*a^3*x^2
```

3.98 $\int \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=161

$$\frac{1}{3}a^2x^3(aB+3Ab)+a^3Ax+\frac{1}{9}x^9\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{11}cx^{11}\left(aBc+Abc+b^2B\right)+\frac{1}{7}x^7\left(A\left(6abc+b^3\right)+3\right.$$

```
[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x^5)/5 +
 ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^3*B + 3*A*b^2*c + 6*
a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A*b*c + a*B*c)*x^11)/11 + (c^2*
(3*b*B + A*c)*x^13)/13 + (B*c^3*x^15)/15
```

Rubi [A] time = 0.11778, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1153}

$$\frac{1}{3}a^2x^3(aB+3Ab)+a^3Ax+\frac{1}{9}x^9\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{11}cx^{11}\left(aBc+Abc+b^2B\right)+\frac{1}{7}x^7\left(A\left(6abc+b^3\right)+3\right.$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x^5)/5 +
 ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^3*B + 3*A*b^2*c + 6*
a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A*b*c + a*B*c)*x^11)/11 + (c^2*
(3*b*B + A*c)*x^13)/13 + (B*c^3*x^15)/15
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx=\int \left(a^3A+a^2(3Ab+aB)x^2+3a\left(abB+A\left(b^2+ac\right)\right)x^4+\left(3aB\left(b^2+ac\right)+A\left(b^3+6\right.\right.\right.$$
$$=a^3Ax+\frac{1}{3}a^2(3Ab+aB)x^3+\frac{3}{5}a\left(abB+A\left(b^2+ac\right)\right)x^5+\frac{1}{7}\left(3aB\left(b^2+ac\right)+A\left(b^3+\right.\right.$$

Mathematica [A] time = 0.0497132, size = 161, normalized size = 1.

$$\frac{1}{3}a^2x^3(aB+3Ab)+a^3Ax+\frac{1}{9}x^9\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{11}cx^{11}\left(aBc+Abc+b^2B\right)+\frac{1}{7}x^7\left(A\left(6abc+b^3\right)+3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x^5)/5 +
 ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^3*B + 3*A*b^2*c + 6*
a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A*b*c + a*B*c)*x^11)/11 + (c^2*
```

```
(3*b*B + A*c)*x^13)/13 + (B*c^3*x^15)/15
```

Maple [A] time = 0.001, size = 223, normalized size = 1.4

$$\frac{Bc^3x^{15}}{15}+\frac{\left(Ac^3+3\,Bbc^2\right)x^{13}}{13}+\frac{\left(3\,Abc^2+B\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)\right)x^{11}}{11}+\frac{\left(A\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)+B\left(4\right.\right.}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/15*B*c^3*x^15+1/13*(A*c^3+3*B*b*c^2)*x^13+1/11*(3*A*b*c^2+B*(a*c^2+2*b^2*
c+c*(2*a*c+b^2)))*x^11+1/9*(A*(a*c^2+2*b^2*c+c*(2*a*c+b^2))+B*(4*a*b*c+b*(2
*a*c+b^2)))*x^9+1/7*(A*(4*a*b*c+b*(2*a*c+b^2))+B*(a*(2*a*c+b^2)+2*b^2*a+c*a
^2))*x^7+1/5*(A*(a*(2*a*c+b^2)+2*b^2*a+c*a^2)+3*B*a^2*b)*x^5+1/3*(3*A*a^2*b
+B*a^3)*x^3+a^3*A*x
```

Maxima [A] time = 1.12351, size = 220, normalized size = 1.37

$$\frac{1}{15}\,Bc^3x^{15}+\frac{1}{13}\left(3\,Bbc^2+Ac^3\right)x^{13}+\frac{3}{11}\left(Bb^2c+(Ba+Ab)c^2\right)x^{11}+\frac{1}{9}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^9+\frac{1}{7}\left(3\,Ba\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/15*B*c^3*x^15 + 1/13*(3*B*b*c^2 + A*c^3)*x^13 + 3/11*(B*b^2*c + (B*a + A*
b)*c^2)*x^11 + 1/9*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^9 + 1/7*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^7 + 3/5*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^5 + A*a^3*x + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

Fricas [A] time = 1.23447, size = 471, normalized size = 2.93

$$\frac{1}{15}x^{15}c^3B+\frac{3}{13}x^{13}c^2bB+\frac{1}{13}x^{13}c^3A+\frac{3}{11}x^{11}cb^2B+\frac{3}{11}x^{11}c^2aB+\frac{3}{11}x^{11}c^2bA+\frac{1}{9}x^9b^3B+\frac{2}{3}x^9cbaB+\frac{1}{3}x^9cb^2A+\frac{1}{3}x^9c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*c^3*B + 3/13*x^13*c^2*b*B + 1/13*x^13*c^3*A + 3/11*x^11*c*b^2*B +
 3/11*x^11*c^2*a*B + 3/11*x^11*c^2*b*A + 1/9*x^9*b^3*B + 2/3*x^9*c*b*a*B +
1/3*x^9*c*b^2*A + 1/3*x^9*c^2*a*A + 3/7*x^7*b^2*a*B + 3/7*x^7*c*a^2*B + 1/7
*x^7*b^3*A + 6/7*x^7*c*b*a*A + 3/5*x^5*b*a^2*B + 3/5*x^5*b^2*a*A + 3/5*x^5*
c*a^2*A + 1/3*x^3*a^3*B + x^3*b*a^2*A + x*a^3*A
```

Sympy [A] time = 0.10174, size = 199, normalized size = 1.24

$$Aa^3x+\frac{Bc^3x^{15}}{15}+x^{13}\left(\frac{Ac^3}{13}+\frac{3Bbc^2}{13}\right)+x^{11}\left(\frac{3Abc^2}{11}+\frac{3Bac^2}{11}+\frac{3Bb^2c}{11}\right)+x^9\left(\frac{Aac^2}{3}+\frac{Ab^2c}{3}+\frac{2Babc}{3}+\frac{Bb^3}{9}\right)+x^7\left(\frac{6}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x + B*c**3*x**15/15 + x**13*(A*c**3/13 + 3*B*b*c**2/13) + x**11*(3*A
*b*c**2/11 + 3*B*a*c**2/11 + 3*B*b**2*c/11) + x**9*(A*a*c**2/3 + A*b**2*c/3
 + 2*B*a*b*c/3 + B*b**3/9) + x**7*(6*A*a*b*c/7 + A*b**3/7 + 3*B*a**2*c/7 +
3*B*a*b**2/7) + x**5*(3*A*a**2*c/5 + 3*A*a*b**2/5 + 3*B*a**2*b/5) + x**3*(A
*a**2*b + B*a**3/3)
```

Giac [A] time = 1.10814, size = 255, normalized size = 1.58

$$\frac{1}{15}\,Bc^3x^{15} + \frac{3}{13}\,Bbc^2x^{13} + \frac{1}{13}\,Ac^3x^{13} + \frac{3}{11}\,Bb^2cx^{11} + \frac{3}{11}\,Bac^2x^{11} + \frac{3}{11}\,Abc^2x^{11} + \frac{1}{9}\,Bb^3x^9 + \frac{2}{3}\,Babcx^9 + \frac{1}{3}\,Ab^2cx^9 + \frac{1}{3}\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/15*B*c^3*x^15 + 3/13*B*b*c^2*x^13 + 1/13*A*c^3*x^13 + 3/11*B*b^2*c*x^11 +
 3/11*B*a*c^2*x^11 + 3/11*A*b*c^2*x^11 + 1/9*B*b^3*x^9 + 2/3*B*a*b*c*x^9 +
1/3*A*b^2*c*x^9 + 1/3*A*a*c^2*x^9 + 3/7*B*a*b^2*x^7 + 1/7*A*b^3*x^7 + 3/7*B
*a^2*c*x^7 + 6/7*A*a*b*c*x^7 + 3/5*B*a^2*b*x^5 + 3/5*A*a*b^2*x^5 + 3/5*A*a^
2*c*x^5 + 1/3*B*a^3*x^3 + A*a^2*b*x^3 + A*a^3*x
```

3.99 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x} dx$

Optimal. Leaf size=162

$$\frac{1}{2}a^2x^2(aB+3Ab)+a^3A\log(x)+\frac{1}{8}x^8\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{10}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{1}{6}x^6\left(A\left(6abc\right.\right.$$

```
[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 + ((3*a*B*(
b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c +
3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/10 + (c^2*(3*b*B + A
*c)*x^12)/12 + (B*c^3*x^14)/14 + a^3*A*Log[x]
```

Rubi [A] time = 0.227069, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1251, 765}

$$\frac{1}{2}a^2x^2(aB+3Ab)+a^3A\log(x)+\frac{1}{8}x^8\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{10}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{1}{6}x^6\left(A\left(6abc\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

```
[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 + ((3*a*B*(
b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c +
3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/10 + (c^2*(3*b*B + A
*c)*x^12)/12 + (B*c^3*x^14)/14 + a^3*A*Log[x]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 765

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(a^2(3Ab+aB)+\frac{a^3A}{x}+3a\left(abB+A\left(b^2+ac\right)\right)x+\left(3aB\left(b^2+ac\right)\right.\right.\right.\\ &= \frac{1}{2}a^2(3Ab+aB)x^2+\frac{3}{4}a\left(abB+A\left(b^2+ac\right)\right)x^4+\frac{1}{6}\left(3aB\left(b^2+ac\right)+A\left(b^3+6ab\right.\right.\end{aligned}$$

Mathematica [A] time = 0.0617853, size = 162, normalized size = 1.

$$\frac{1}{2}a^2x^2(aB+3Ab)+a^3A\log(x)+\frac{1}{8}x^8\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{10}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{1}{6}x^6\left(A\left(6abc\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

```
[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 + ((3*a*B*(
b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c +
3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/10 + (c^2*(3*b*B + A
*c)*x^12)/12 + (B*c^3*x^14)/14 + a^3*A*Log[x]
```

Maple [A] time = 0.003, size = 191, normalized size = 1.2

$$\frac{Bc^3x^{14}}{14}+\frac{Ax^{12}c^3}{12}+\frac{Bx^{12}bc^2}{4}+\frac{3\,Ax^{10}bc^2}{10}+\frac{3\,Bx^{10}ac^2}{10}+\frac{3\,Bx^{10}b^2c}{10}+\frac{3\,Ax^8ac^2}{8}+\frac{3\,Ax^8b^2c}{8}+\frac{3\,Bx^8abc}{4}+\frac{Bx^8b^3}{8}+Ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x)
```

```
[Out] 1/14*B*c^3*x^14+1/12*A*x^12*c^3+1/4*B*x^12*b*c^2+3/10*A*x^10*b*c^2+3/10*B*x
^10*a*c^2+3/10*B*x^10*b^2*c+3/8*A*x^8*a*c^2+3/8*A*x^8*b^2*c+3/4*B*x^8*a*b*c
+1/8*B*x^8*b^3+A*x^6*a*b*c+1/6*A*x^6*b^3+1/2*B*x^6*a^2*c+1/2*B*x^6*a*b^2+3/
4*A*x^4*a^2*c+3/4*A*x^4*a*b^2+3/4*B*x^4*a^2*b+3/2*A*x^2*a^2*b+1/2*B*x^2*a^3
+a^3*A*ln(x)
```

Maxima [A] time = 0.989105, size = 225, normalized size = 1.39

$$\frac{1}{14}\,Bc^3x^{14}+\frac{1}{12}\left(3\,Bbc^2+Ac^3\right)x^{12}+\frac{3}{10}\left(Bb^2c+(Ba+Ab)c^2\right)x^{10}+\frac{1}{8}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^8+\frac{1}{6}\left(3\,Bab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c + (B*a + A*
b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 1/6*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/4*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^4 + 1/2*A*a^3*log(x^2) + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

Fricas [A] time = 1.45317, size = 381, normalized size = 2.35

$$\frac{1}{14}\,Bc^3x^{14}+\frac{1}{12}\left(3\,Bbc^2+Ac^3\right)x^{12}+\frac{3}{10}\left(Bb^2c+(Ba+Ab)c^2\right)x^{10}+\frac{1}{8}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^8+\frac{1}{6}\left(3\,Bab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c + (B*a + A*
b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 1/6*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/4*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^4 + A*a^3*log(x) + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

Sympy [A] time = 0.480521, size = 199, normalized size = 1.23

$$Aa^3\log(x) + \frac{Bc^3x^{14}}{14} + x^{12}\left(\frac{Ac^3}{12} + \frac{Bbc^2}{4}\right) + x^{10}\left(\frac{3Abc^2}{10} + \frac{3Bac^2}{10} + \frac{3Bb^2c}{10}\right) + x^8\left(\frac{3Aac^2}{8} + \frac{3Ab^2c}{8} + \frac{3Babc}{4} + \frac{Bb^3}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x,x)
```

```
[Out] A*a**3*log(x) + B*c**3*x**14/14 + x**12*(A*c**3/12 + B*b*c**2/4) + x**10*(3
*A*b*c**2/10 + 3*B*a*c**2/10 + 3*B*b**2*c/10) + x**8*(3*A*a*c**2/8 + 3*A*b*
*2*c/8 + 3*B*a*b*c/4 + B*b**3/8) + x**6*(A*a*b*c + A*b**3/6 + B*a**2*c/2 +
B*a*b**2/2) + x**4*(3*A*a**2*c/4 + 3*A*a*b**2/4 + 3*B*a**2*b/4) + x**2*(3*A
*a**2*b/2 + B*a**3/2)
```

Giac [A] time = 1.10362, size = 261, normalized size = 1.61

$$\frac{1}{14}Bc^3x^{14} + \frac{1}{4}Bbc^2x^{12} + \frac{1}{12}Ac^3x^{12} + \frac{3}{10}Bb^2cx^{10} + \frac{3}{10}Bac^2x^{10} + \frac{3}{10}Abc^2x^{10} + \frac{1}{8}Bb^3x^8 + \frac{3}{4}Babcx^8 + \frac{3}{8}Ab^2cx^8 + \frac{3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/14*B*c^3*x^14 + 1/4*B*b*c^2*x^12 + 1/12*A*c^3*x^12 + 3/10*B*b^2*c*x^10 +
3/10*B*a*c^2*x^10 + 3/10*A*b*c^2*x^10 + 1/8*B*b^3*x^8 + 3/4*B*a*b*c*x^8 + 3
/8*A*b^2*c*x^8 + 3/8*A*a*c^2*x^8 + 1/2*B*a*b^2*x^6 + 1/6*A*b^3*x^6 + 1/2*B*
a^2*c*x^6 + A*a*b*c*x^6 + 3/4*B*a^2*b*x^4 + 3/4*A*a*b^2*x^4 + 3/4*A*a^2*c*x
^4 + 1/2*B*a^3*x^2 + 3/2*A*a^2*b*x^2 + 1/2*A*a^3*log(x^2)
```

3.100 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx$

Optimal. Leaf size=156

$$a^2x(aB+3Ab)-\frac{a^3A}{x}+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{3}cx^9\left(aBc+Abc+b^2B\right)+\frac{1}{5}x^5\left(A\left(6abc+b^3\right)+3aB\left(a\right.\right.$$

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*a*
B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c
 + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*B*c)*x^9)/3 + (c^2*(3*b*B + A*
c)*x^11)/11 + (B*c^3*x^13)/13
```

Rubi [A] time = 0.108046, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1261}

$$a^2x(aB+3Ab)-\frac{a^3A}{x}+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{3}cx^9\left(aBc+Abc+b^2B\right)+\frac{1}{5}x^5\left(A\left(6abc+b^3\right)+3aB\left(a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*a*
B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c
 + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*B*c)*x^9)/3 + (c^2*(3*b*B + A*
c)*x^11)/11 + (B*c^3*x^13)/13
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3}{x^2}\,dx &= \int \left(a^2(3Ab+aB)+\frac{a^3A}{x^2}+3a\left(abB+A\left(b^2+ac\right)\right)x^2+\left(3aB\left(b^2+ac\right)+A\left(b^3+6a\right.\right.\right.\\ &= -\frac{a^3A}{x}+a^2(3Ab+aB)x+a\left(abB+A\left(b^2+ac\right)\right)x^3+\frac{1}{5}\left(3aB\left(b^2+ac\right)+A\left(b^3+6ab\right.\right.\end{aligned}$$

Mathematica [A] time = 0.0944426, size = 156, normalized size = 1.

$$a^2x(aB+3Ab)-\frac{a^3A}{x}+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{3}cx^9\left(aBc+Abc+b^2B\right)+\frac{1}{5}x^5\left(A\left(6abc+b^3\right)+3aB\left(a\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*a*
B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c
```

```
+ 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*B*c)*x^9)/3 + (c^2*(3*b*B + A*
c)*x^11)/11 + (B*c^3*x^13)/13
```

Maple [A] time = 0.005, size = 186, normalized size = 1.2

$$\frac{Bc^3x^{13}}{13}+\frac{Ax^{11}c^3}{11}+\frac{3\,Bx^{11}bc^2}{11}+\frac{Ax^9bc^2}{3}+\frac{Bx^9ac^2}{3}+\frac{Bx^9b^2c}{3}+\frac{3\,Ax^7ac^2}{7}+\frac{3\,Ax^7b^2c}{7}+\frac{6\,Bx^7abc}{7}+\frac{Bx^7b^3}{7}+\frac{6\,Ax^5a}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x)
```

```
[Out] 1/13*B*c^3*x^13+1/11*A*x^11*c^3+3/11*B*x^11*b*c^2+1/3*A*x^9*b*c^2+1/3*B*x^9
*a*c^2+1/3*B*x^9*b^2*c+3/7*A*x^7*a*c^2+3/7*A*x^7*b^2*c+6/7*B*x^7*a*b*c+1/7*
B*x^7*b^3+6/5*A*x^5*a*b*c+1/5*A*x^5*b^3+3/5*B*x^5*a^2*c+3/5*B*x^5*a*b^2+A*x
^3*a^2*c+A*x^3*a*b^2+B*x^3*a^2*b+3*A*a^2*b*x+B*a^3*x-a^3*A/x
```

Maxima [A] time = 0.987112, size = 219, normalized size = 1.4

$$\frac{1}{13}\,Bc^3x^{13}+\frac{1}{11}\left(3\,Bbc^2+Ac^3\right)x^{11}+\frac{1}{3}\left(Bb^2c+(Ba+Ab)c^2\right)x^9+\frac{1}{7}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^7+\frac{1}{5}\left(3\,Bab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + (B*a + A*b
)*c^2)*x^9 + 1/7*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^7 + 1/5*(3*B
*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^5 + (B*a^2*b + A*a*b^2 + A*a^2*c)
*x^3 - A*a^3/x + (B*a^3 + 3*A*a^2*b)*x
```

Fricas [A] time = 1.48085, size = 404, normalized size = 2.59

$$\frac{1155\,Bc^3x^{14}+1365\left(3\,Bbc^2+Ac^3\right)x^{12}+5005\left(Bb^2c+(Ba+Ab)c^2\right)x^{10}+2145\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^8}{150}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*B*c^3*x^14 + 1365*(3*B*b*c^2 + A*c^3)*x^12 + 5005*(B*b^2*c +
(B*a + A*b)*c^2)*x^10 + 2145*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^
8 + 3003*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 15015*(B*a^2*b +
 A*a*b^2 + A*a^2*c)*x^4 - 15015*A*a^3 + 15015*(B*a^3 + 3*A*a^2*b)*x^2)/x
```

Sympy [A] time = 0.478917, size = 185, normalized size = 1.19

$$-\frac{Aa^3}{x}+\frac{Bc^3x^{13}}{13}+x^{11}\left(\frac{Ac^3}{11}+\frac{3Bbc^2}{11}\right)+x^9\left(\frac{Abc^2}{3}+\frac{Bac^2}{3}+\frac{Bb^2c}{3}\right)+x^7\left(\frac{3Aac^2}{7}+\frac{3Ab^2c}{7}+\frac{6Babc}{7}+\frac{Bb^3}{7}\right)+x^5\left(\frac{6A}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**2,x)
```

```
[Out] -A*a**3/x + B*c**3*x**13/13 + x**11*(A*c**3/11 + 3*B*b*c**2/11) + x**9*(A*b
*c**2/3 + B*a*c**2/3 + B*b**2*c/3) + x**7*(3*A*a*c**2/7 + 3*A*b**2*c/7 + 6*
B*a*b*c/7 + B*b**3/7) + x**5*(6*A*a*b*c/5 + A*b**3/5 + 3*B*a**2*c/5 + 3*B*a
*b**2/5) + x**3*(A*a**2*c + A*a*b**2 + B*a**2*b) + x*(3*A*a**2*b + B*a**3)
```

Giac [A] time = 1.13561, size = 250, normalized size = 1.6

$$\frac{1}{13}Bc^3x^{13} + \frac{3}{11}Bbc^2x^{11} + \frac{1}{11}Ac^3x^{11} + \frac{1}{3}Bb^2cx^9 + \frac{1}{3}Bac^2x^9 + \frac{1}{3}Abc^2x^9 + \frac{1}{7}Bb^3x^7 + \frac{6}{7}Babcx^7 + \frac{3}{7}Ab^2cx^7 + \frac{3}{7}Aac^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/13*B*c^3*x^13 + 3/11*B*b*c^2*x^11 + 1/11*A*c^3*x^11 + 1/3*B*b^2*c*x^9 + 1
/3*B*a*c^2*x^9 + 1/3*A*b*c^2*x^9 + 1/7*B*b^3*x^7 + 6/7*B*a*b*c*x^7 + 3/7*A*
b^2*c*x^7 + 3/7*A*a*c^2*x^7 + 3/5*B*a*b^2*x^5 + 1/5*A*b^3*x^5 + 3/5*B*a^2*c
*x^5 + 6/5*A*a*b*c*x^5 + B*a^2*b*x^3 + A*a*b^2*x^3 + A*a^2*c*x^3 + B*a^3*x
+ 3*A*a^2*b*x - A*a^3/x
```

3.101 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3} \, dx$

Optimal. Leaf size=162

$$a^2\log(x)(aB+3Ab)-\frac{a^3A}{2x^2}+\frac{1}{6}x^6\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{8}cx^8\left(aBc+Abc+b^2B\right)+\frac{1}{4}x^4\left(A\left(6abc+b^3\right)\right.$$

```
[Out] -(a^3*A)/(2*x^2) + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 + a*c
) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2
)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8 + (c^2*(3*b*B + A*c)*x^10)/1
0 + (B*c^3*x^12)/12 + a^2*(3*A*b + a*B)*Log[x]
```

Rubi [A] time = 0.22608, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1251, 765}

$$a^2\log(x)(aB+3Ab)-\frac{a^3A}{2x^2}+\frac{1}{6}x^6\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{8}cx^8\left(aBc+Abc+b^2B\right)+\frac{1}{4}x^4\left(A\left(6abc+b^3\right)\right.$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```



```
[Out] -(a^3*A)/(2*x^2) + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 + a*c
) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2
)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8 + (c^2*(3*b*B + A*c)*x^10)/1
0 + (B*c^3*x^12)/12 + a^2*(3*A*b + a*B)*Log[x]
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 765

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(3a\left(abB+A\left(b^2+ac\right)\right)+\frac{a^3A}{x^2}+\frac{a^2(3Ab+aB)}{x}+\left(3aB\left(b^2+ac\right)+\right.\right.\right.\\ &= -\frac{a^3A}{2x^2}+\frac{3}{2}a\left(abB+A\left(b^2+ac\right)\right)x^2+\frac{1}{4}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^4+\frac{1}{6}\left(\right.\end{aligned}$$

Mathematica [A] time = 0.0745245, size = 162, normalized size = 1.

$$a^2\log(x)(aB+3Ab)-\frac{a^3A}{2x^2}+\frac{1}{6}x^6\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{3}{8}cx^8\left(aBc+Abc+b^2B\right)+\frac{1}{4}x^4\left(A\left(6abc+b^3\right)+3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] -(a^3*A)/(2*x^2) + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 + a*c
) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2
)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8 + (c^2*(3*b*B + A*c)*x^10)/1
0 + (B*c^3*x^12)/12 + a^2*(3*A*b + a*B)*Log[x]
```

Maple [A] time = 0.008, size = 190, normalized size = 1.2

$$\frac{Bc^3x^{12}}{12}+\frac{Ax^{10}c^3}{10}+\frac{3\,Bx^{10}bc^2}{10}+\frac{3\,Ax^8bc^2}{8}+\frac{3\,Bx^8ac^2}{8}+\frac{3\,Bx^8b^2c}{8}+\frac{Ax^6ac^2}{2}+\frac{Ax^6b^2c}{2}+Bx^6abc+\frac{Bx^6b^3}{6}+\frac{3\,Ax^4abc}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x)
```

```
[Out] 1/12*B*c^3*x^12+1/10*A*x^10*c^3+3/10*B*x^10*b*c^2+3/8*A*x^8*b*c^2+3/8*B*x^8
*a*c^2+3/8*B*x^8*b^2*c+1/2*A*x^6*a*c^2+1/2*A*x^6*b^2*c+B*x^6*a*b*c+1/6*B*x^
6*b^3+3/2*A*x^4*a*b*c+1/4*A*x^4*b^3+3/4*B*x^4*a^2*c+3/4*B*x^4*a*b^2+3/2*A*x
^2*a^2*c+3/2*A*x^2*a*b^2+3/2*B*x^2*a^2*b+3*A*ln(x)*a^2*b+B*ln(x)*a^3-1/2*a^
3*A/x^2
```

Maxima [A] time = 0.978896, size = 225, normalized size = 1.39

$$\frac{1}{12}Bc^3x^{12}+\frac{1}{10}\left(3\,Bbc^2+Ac^3\right)x^{10}+\frac{3}{8}\left(Bb^2c+(Ba+Ab)c^2\right)x^8+\frac{1}{6}\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^6+\frac{1}{4}\left(3\,Bab^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + (B*a + A*b
)*c^2)*x^8 + 1/6*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^6 + 1/4*(3*B
*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^4 + 3/2*(B*a^2*b + A*a*b^2 + A*a^
2*c)*x^2 - 1/2*A*a^3/x^2 + 1/2*(B*a^3 + 3*A*a^2*b)*log(x^2)
```

Fricas [A] time = 1.46054, size = 390, normalized size = 2.41

$$\frac{10\,Bc^3x^{14}+12\left(3\,Bbc^2+Ac^3\right)x^{12}+45\left(Bb^2c+(Ba+Ab)c^2\right)x^{10}+20\left(Bb^3+3\,Aac^2+3\left(2\,Bab+Ab^2\right)c\right)x^8+30\left(3\,Bab^2\right.}{120\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/120*(10*B*c^3*x^14 + 12*(3*B*b*c^2 + A*c^3)*x^12 + 45*(B*b^2*c + (B*a + A
*b)*c^2)*x^10 + 20*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 30*(3*
B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 180*(B*a^2*b + A*a*b^2 + A*a
^2*c)*x^4 - 60*A*a^3 + 120*(B*a^3 + 3*A*a^2*b)*x^2*log(x))/x^2
```

Sympy [A] time = 0.573669, size = 197, normalized size = 1.22

$$-\frac{Aa^3}{2x^2} + \frac{Bc^3x^{12}}{12} + a^2(3Ab + Ba)\log(x) + x^{10}\left(\frac{Ac^3}{10} + \frac{3Bbc^2}{10}\right) + x^8\left(\frac{3Abc^2}{8} + \frac{3Bac^2}{8} + \frac{3Bb^2c}{8}\right) + x^6\left(\frac{Aac^2}{2} + \frac{Ab^2c}{2} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**3,x)
```

```
[Out] -A*a**3/(2*x**2) + B*c**3*x**12/12 + a**2*(3*A*b + B*a)*log(x) + x**10*(A*c
**3/10 + 3*B*b*c**2/10) + x**8*(3*A*b*c**2/8 + 3*B*a*c**2/8 + 3*B*b**2*c/8)
 + x**6*(A*a*c**2/2 + A*b**2*c/2 + B*a*b*c + B*b**3/6) + x**4*(3*A*a*b*c/2
+ A*b**3/4 + 3*B*a**2*c/4 + 3*B*a*b**2/4) + x**2*(3*A*a**2*c/2 + 3*A*a*b**2
/2 + 3*B*a**2*b/2)
```

Giac [A] time = 1.11929, size = 286, normalized size = 1.77

$$\frac{1}{12}Bc^3x^{12} + \frac{3}{10}Bbc^2x^{10} + \frac{1}{10}Ac^3x^{10} + \frac{3}{8}Bb^2cx^8 + \frac{3}{8}Bac^2x^8 + \frac{3}{8}Abc^2x^8 + \frac{1}{6}Bb^3x^6 + Babcx^6 + \frac{1}{2}Ab^2cx^6 + \frac{1}{2}Aac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/12*B*c^3*x^12 + 3/10*B*b*c^2*x^10 + 1/10*A*c^3*x^10 + 3/8*B*b^2*c*x^8 + 3
/8*B*a*c^2*x^8 + 3/8*A*b*c^2*x^8 + 1/6*B*b^3*x^6 + B*a*b*c*x^6 + 1/2*A*b^2*
c*x^6 + 1/2*A*a*c^2*x^6 + 3/4*B*a*b^2*x^4 + 1/4*A*b^3*x^4 + 3/4*B*a^2*c*x^4
 + 3/2*A*a*b*c*x^4 + 3/2*B*a^2*b*x^2 + 3/2*A*a*b^2*x^2 + 3/2*A*a^2*c*x^2 +
1/2*(B*a^3 + 3*A*a^2*b)*log(x^2) - 1/2*(B*a^3*x^2 + 3*A*a^2*b*x^2 + A*a^3)/
x^2
```

3.102 $\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=133

$$\frac{(-aBc-Abc+b^2B)\log(a+bx^2+cx^4)}{4c^3}+\frac{(2aAc^2-3abBc-Ab^2c+b^3B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}-\frac{x^2(bB-Ac)}{2c^2}+\frac{Bx^4}{4c}$$

```
[Out] -((b*B - A*c)*x^2)/(2*c^2) + (B*x^4)/(4*c) + ((b^3*B - A*b^2*c - 3*a*b*B*c
+ 2*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*
a*c]) + ((b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] time = 0.206724, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 800, 634, 618, 206, 628}

$$\frac{(-aBc-Abc+b^2B)\log(a+bx^2+cx^4)}{4c^3}+\frac{(2aAc^2-3abBc-Ab^2c+b^3B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}-\frac{x^2(bB-Ac)}{2c^2}+\frac{Bx^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*B - A*c)*x^2)/(2*c^2) + (B*x^4)/(4*c) + ((b^3*B - A*b^2*c - 3*a*b*B*c
+ 2*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*
a*c]) + ((b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^5\left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{a+bx+cx^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{bB-Ac}{c^2}+\frac{Bx}{c}+\frac{a(bB-Ac)+\left(b^2B-Abc-aBc\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)$$
$$= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{\operatorname{Subst}\left(\int \frac{a(bB-Ac)+\left(b^2B-Abc-aBc\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c^2}$$
$$= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{\left(b^2B-Abc-aBc\right)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3}-\frac{\left(b^3B-Ab^2c-3abBc\right.}{}$$
$$= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{\left(b^2B-Abc-aBc\right)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{\left(b^3B-Ab^2c-3abBc+2aAc\right.}{}$$
$$= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{\left(b^3B-Ab^2c-3abBc+2aAc^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(b^2B-Abc-aBc\right)}{4}$$

Mathematica [A] time = 0.060516, size = 126, normalized size = 0.95

$$\frac{\frac{2\left(-2aAc^2+3abBc+Ab^2c+b^3(-B)\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\left(-aBc-Abc+b^2B\right)\log\left(a+bx^2+cx^4\right)+2cx^2(Ac-bB)+Bc^2x^4}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(-(b*B) + A*c)*x^2 + B*c^2*x^4 + (2*(-(b^3*B) + A*b^2*c + 3*a*b*B*c -
2*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (
b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [B] time = 0.005, size = 261, normalized size = 2.

$$\frac{Bx^4}{4c}+\frac{Ax^2}{2c}-\frac{bBx^2}{2c^2}-\frac{\ln\left(cx^4+bx^2+a\right)Ab}{4c^2}-\frac{\ln\left(cx^4+bx^2+a\right)aB}{4c^2}+\frac{\ln\left(cx^4+bx^2+a\right)b^2B}{4c^3}-\frac{aA}{c}\arctan\left((2cx^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*B*x^4/c+1/2/c*A*x^2-1/2/c^2*b*B*x^2-1/4/c^2*ln(c*x^4+b*x^2+a)*A*b-1/4/c
^2*ln(c*x^4+b*x^2+a)*a*B+1/4/c^3*ln(c*x^4+b*x^2+a)*b^2*B-1/c/(4*a*c-b^2)^(1
/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*A+3/2/c^2/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*B+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^2-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^2+b)/(4*a*c-b^2)^(1/2))*b^3*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69731, size = 896, normalized size = 6.74

$$\left[\frac{\left(Bb^2c^2 - 4\,Bac^3\right)x^4 - 2\left(Bb^3c + 4\,Aac^3 - \left(4\,Bab + Ab^2\right)c^2\right)x^2 + \left(Bb^3 + 2\,Aac^2 - \left(3\,Bab + Ab^2\right)c\right)\sqrt{b^2 - 4\,ac}\log\left(\frac{2\,c^2x^4+2}{\right.}\right.}{4\left(b^2c^3 - 4\,ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((B*b^2*c^2 - 4*B*a*c^3)*x^4 - 2*(B*b^3*c + 4*A*a*c^3 - (4*B*a*b + A*b
^2)*c^2)*x^2 + (B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*sqrt(b^2 - 4*a*c)*
log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))
/(c*x^4 + b*x^2 + a)) + (B*b^4 + 4*(B*a^2 + A*a*b)*c^2 - (5*B*a*b^2 + A*b^3
)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((B*b^2*c^2 - 4*B*a*c
^3)*x^4 - 2*(B*b^3*c + 4*A*a*c^3 - (4*B*a*b + A*b^2)*c^2)*x^2 + 2*(B*b^3 +
2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (B*b^4 + 4*(B*a^2 + A*a*b)*c^2 - (5*B*a*
b^2 + A*b^3)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] time = 8.12853, size = 619, normalized size = 4.65

$$\frac{Bx^4}{4c} + \left(-\frac{\sqrt{-4ac+b^2}\left(-2Aac^2 + Ab^2c + 3Babc - Bb^3\right)}{4c^3\left(4ac-b^2\right)} - \frac{Abc + Bac - Bb^2}{4c^3}\right)\log\left(x^2 + \frac{Aabc + 2Ba^2c - Bab^2 + 8ac^3\left(-\frac{\sqrt{-}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x**4/(4*c) + (-sqrt(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c -
B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3))*log(x*
*2 + (A*a*b*c + 2*B*a**2*c - B*a*b**2 + 8*a*c**3*(-sqrt(-4*a*c + b**2)*(-2*
```

```
A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c
+ B*a*c - B*b**2)/(4*c**3)) - 2*b**2*c**2*(-sqrt(-4*a*c + b**2)*(-2*A*a*c**
2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c
 - B*b**2)/(4*c**3)))/(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)) + (sqr
t(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)/(4*c**3*(4*a
*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3))*log(x**2 + (A*a*b*c + 2*B*
a**2*c - B*a*b**2 + 8*a*c**3*(sqrt(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c +
 3*B*a*b*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*
c**3)) - 2*b**2*c**2*(sqrt(-4*a*c + b**2)*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b
*c - B*b**3)/(4*c**3*(4*a*c - b**2)) - (A*b*c + B*a*c - B*b**2)/(4*c**3)))/
(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3)) - x**2*(-A*c + B*b)/(2*c**2)
```

Giac [A] time = 1.21063, size = 170, normalized size = 1.28

$$\frac{Bcx^4 - 2\,Bbx^2 + 2\,Acx^2}{4\,c^2} + \frac{\left(Bb^2 - Bac - Abc\right)\log\left(cx^4 + bx^2 + a\right)}{4\,c^3} - \frac{\left(Bb^3 - 3\,Babc - Ab^2c + 2\,Aac^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,a}}\right.}{2\,\sqrt{-b^2 + 4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(B*c*x^4 - 2*B*b*x^2 + 2*A*c*x^2)/c^2 + 1/4*(B*b^2 - B*a*c - A*b*c)*log
(c*x^4 + b*x^2 + a)/c^3 - 1/2*(B*b^3 - 3*B*a*b*c - A*b^2*c + 2*A*a*c^2)*arc
tan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.103 $\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4} dx$

Optimal. Leaf size=97

$$-\frac{(-2aBc-Abc+b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{Bx^2}{2c}$$

```
[Out] (B*x^2)/(2*c) - ((b^2*B - A*b*c - 2*a*B*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - ((b*B - A*c)*Log[a + b*x^2 + c*x^4])/
(4*c^2)
```

Rubi [A] time = 0.116055, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 773, 634, 618, 206, 628}

$$-\frac{(-2aBc-Abc+b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{Bx^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x^2)/(2*c) - ((b^2*B - A*b*c - 2*a*B*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - ((b*B - A*c)*Log[a + b*x^2 + c*x^4])/
(4*c^2)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3 \left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{a+bx+cx^2}\,dx,x,x^2\right)\\ &= \frac{Bx^2}{2c}+\frac{\operatorname{Subst}\left(\int \frac{-aB+(-bB+Ac)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c}\\ &= \frac{Bx^2}{2c}-\frac{(bB-Ac)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}+\frac{\left(b^2B-Abc-2aBc\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}\\ &= \frac{Bx^2}{2c}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{\left(b^2B-Abc-2aBc\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c^2}\\ &= \frac{Bx^2}{2c}-\frac{\left(b^2B-Abc-2aBc\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}\end{aligned}$$

Mathematica [A] time = 0.0680335, size = 93, normalized size = 0.96

$$\frac{\frac{2\left(-2aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+(Ac-bB)\log\left(a+bx^2+cx^4\right)+2Bcx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*B*c*x^2 + (2*(b^2*B - A*b*c - 2*a*B*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*B) + A*c)*Log[a + b*x^2 + c*x^4])/(4*c^2
)
```

Maple [A] time = 0.001, size = 175, normalized size = 1.8

$$\frac{Bx^2}{2c}+\frac{\ln\left(cx^4+bx^2+a\right)A}{4c}-\frac{\ln\left(cx^4+bx^2+a\right)bB}{4c^2}-\frac{aB}{c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{Ab}{2c}\arctan\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*B*x^2/c+1/4/c*ln(c*x^4+b*x^2+a)*A-1/4/c^2*ln(c*x^4+b*x^2+a)*b*B-1/c/(4*
a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*B-1/2/c/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b+1/2/c^2/(4*a*c-b^2)^(1/2)*a
```

```
rctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62066, size = 670, normalized size = 6.91

$$\left[\frac{2\left(Bb^2c-4\,Bac^2\right)x^2-\left(Bb^2-(2\,Ba+Ab)c\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(Bb^3+4\,Aac^2-\left(4\,Bab\right.\right.}{4\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b^2*c - 4*B*a*c^2)*x^2 - (B*b^2 - (2*B*a + A*b)*c)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) - (B*b^3 + 4*A*a*c^2 - (4*B*a*b + A*b^2)*c)*log(
c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1/4*(2*(B*b^2*c - 4*B*a*c^2)*x^2 -
 2*(B*b^2 - (2*B*a + A*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(
-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (B*b^3 + 4*A*a*c^2 - (4*B*a*b + A*b^2)*c)*lo
g(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] time = 4.97837, size = 434, normalized size = 4.47

$$\frac{Bx^2}{2c}+\left(-\frac{-Ac+Bb}{4c^2}-\frac{\sqrt{-4ac+b^2}\left(Abc+2Bac-Bb^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{2Aac-Bab-8ac^2\left(-\frac{-Ac+Bb}{4c^2}-\frac{\sqrt{-4ac+b^2}\left(Abc+2Bac-Bb^2\right)}{4c^2\left(4ac-b^2\right)}\right)}{Abc+2Bac-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x**2/(2*c) + (-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a
*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b - 8*a*c**
2*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/
(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c +
 b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*c + 2*B*a*
c - B*b**2)) + (-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a
*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b - 8*a*c**
2*(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/
(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c +
 b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*c + 2*B*a*
```

```
c - B*b**2))
```

Giac [A] time = 1.17829, size = 123, normalized size = 1.27

$$\frac{Bx^2}{2\,c} - \frac{(Bb - Ac)\log\left(cx^4 + bx^2 + a\right)}{4\,c^2} + \frac{\left(Bb^2 - 2\,Bac - Abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c - 1/4*(B*b - A*c)*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(B*b^2 - 2*B
*a*c - A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*
c^2)
```

3.104 $\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=71

$$\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*
a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rubi [A] time = 0.0702653, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1247, 634, 618, 206, 628}

$$\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*
a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{B\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c} + \frac{(-bB+2Ac)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c}$$
$$= \frac{B\log\left(a+bx^2+cx^4\right)}{4c} - \frac{(-bB+2Ac)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c}$$
$$= \frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

Mathematica [A] time = 0.0517813, size = 71, normalized size = 1.

$$\frac{B\log\left(a+bx^2+cx^4\right) - \frac{2(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-2*(b*B - 2*A*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*
a*c] + B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Maple [A] time = 0.003, size = 98, normalized size = 1.4

$$\frac{B\ln\left(cx^4+bx^2+a\right)}{4c} + A\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}} - \frac{bB}{2c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*B*ln(c*x^4+b*x^2+a)/c+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2
)^(1/2))*A-1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*b/
c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53889, size = 487, normalized size = 6.86

$$\left[-\frac{(Bb-2\,Ac)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(Bb^2-4\,Bac\right)\log\left(cx^4+bx^2+a\right)}{4\left(b^2c-4\,ac^2\right)},\frac{2\,(Bb-2\,Ac)\sqrt{-b^2}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((B*b - 2*A*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (B*b^2 - 4*B
*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*(B*b - 2*A*c)*sqrt(
-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (B*
b^2 - 4*B*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] time = 2.51237, size = 287, normalized size = 4.04

$$\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-Ab+2Ba-8ac\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}\right)+2b^2\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\left(4ac-b^2\right)}\right)}{-2Ac+Bb}\right)+\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] (B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)))*log(x**
2 + (-A*b + 2*B*a - 8*a*c*(B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*
c*(4*a*c - b**2))) + 2*b**2*(B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(
4*c*(4*a*c - b**2))))/(-2*A*c + B*b)) + (B/(4*c) + (-2*A*c + B*b)*sqrt(-4*a
*c + b**2)/(4*c*(4*a*c - b**2)))*log(x**2 + (-A*b + 2*B*a - 8*a*c*(B/(4*c)
+ (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))) + 2*b**2*(B/(4*c
) + (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))))/(-2*A*c + B*b
))
```

Giac [A] time = 1.18495, size = 90, normalized size = 1.27

$$\frac{B\log\left(cx^4+bx^2+a\right)}{4\,c}-\frac{(Bb-2\,Ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*B*log(c*x^4 + b*x^2 + a)/c - 1/2*(B*b - 2*A*c)*arctan((2*c*x^2 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

3.105 $\int \frac{A+Bx^2}{x(a+bx^2+cx^4)} dx$

Optimal. Leaf size=78

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}$$

```
[Out] ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*
a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rubi [A] time = 0.138659, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 800, 634, 618, 206, 628}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*
a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{A}{ax}+\frac{-Ab+aB-Acx}{a\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)\\
&= \frac{A\log(x)}{a}+\frac{\operatorname{Subst}\left(\int \frac{-Ab+aB-Acx}{a+bx+cx^2}\,dx, x, x^2\right)}{2a}\\
&= \frac{A\log(x)}{a}-\frac{A\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a}+\frac{(-Ab+2aB)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a}\\
&= \frac{A\log(x)}{a}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}-\frac{(-Ab+2aB)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a}\\
&= \frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{A\log(x)}{a}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.108057, size = 128, normalized size = 1.64

$$\frac{-\left(A\left(\sqrt{b^2-4ac}+b\right)-2aB\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)+\left(A\left(b-\sqrt{b^2-4ac}\right)-2aB\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)+4A}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*A*Sqrt[b^2 - 4*a*c]*Log[x] - (-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*Log[b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (-2*a*B + A*(b - Sqrt[b^2 - 4*a*c]))*Log[b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.005, size = 105, normalized size = 1.4

$$-\frac{A\ln\left(cx^4+bx^2+a\right)}{4\,a}-\frac{Ab}{2\,a}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+B\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*A*ln(c*x^4+b*x^2+a)/a-1/2/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*
c-b^2)^(1/2))*A*b+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

```
*B+A*ln(x)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73968, size = 567, normalized size = 7.27

$$\left[-\frac{(2\,Ba - Ab)\sqrt{b^2 - 4\,ac}\log\left(\frac{2\,c^2x^4 + 2\,bcx^2 + b^2 - 2\,ac + \left(2\,cx^2 + b\right)\sqrt{b^2 - 4\,ac}}{cx^4 + bx^2 + a}\right) + \left(Ab^2 - 4\,Aac\right)\log\left(cx^4 + bx^2 + a\right) - 4\left(Ab^2 - 4\,Aa\right.}{4\left(ab^2 - 4\,a^2c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*B*a - A*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (A*b^2 - 4*A
*a*c)*log(c*x^4 + b*x^2 + a) - 4*(A*b^2 - 4*A*a*c)*log(x))/(a*b^2 - 4*a^2*c
), -1/4*(2*(2*B*a - A*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2
 + 4*a*c)/(b^2 - 4*a*c)) + (A*b^2 - 4*A*a*c)*log(c*x^4 + b*x^2 + a) - 4*(A*
b^2 - 4*A*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] time = 53.6247, size = 330, normalized size = 4.23

$$\frac{A\log(x)}{a} + \left(-\frac{A}{4a} - \frac{(-Ab + 2Ba)\sqrt{-4ac + b^2}}{4a\left(4ac - b^2\right)}\right)\log\left(x^2 + \frac{2Aac - Ab^2 + Bab + 8a^2c\left(-\frac{A}{4a} - \frac{(-Ab+2Ba)\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}\right) - 2ab^2\left(\right.}{-Abc + 2Bac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] A*log(x)/a + (-A/(4*a) - (-A*b + 2*B*a)*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b
**2)))*log(x**2 + (2*A*a*c - A*b**2 + B*a*b + 8*a**2*c*(-A/(4*a) - (-A*b +
2*B*a)*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2))) - 2*a*b**2*(-A/(4*a) - (-A
*b + 2*B*a)*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2))))/(-A*b*c + 2*B*a*c))
+ (-A/(4*a) + (-A*b + 2*B*a)*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)))*log(
x**2 + (2*A*a*c - A*b**2 + B*a*b + 8*a**2*c*(-A/(4*a) + (-A*b + 2*B*a)*sqrt
(-4*a*c + b**2)/(4*a*(4*a*c - b**2))) - 2*a*b**2*(-A/(4*a) + (-A*b + 2*B*a)
*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2))))/(-A*b*c + 2*B*a*c))
```

Giac [A] time = 1.19475, size = 105, normalized size = 1.35

$$-\frac{A\log\left(cx^4+bx^2+a\right)}{4\,a}+\frac{A\log\left(x^2\right)}{2\,a}+\frac{(2\,Ba-Ab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*A*log(c*x^4 + b*x^2 + a)/a + 1/2*A*log(x^2)/a + 1/2*(2*B*a - A*b)*arct
an((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

3.106 $\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)} dx$

Optimal. Leaf size=112

$$-\frac{(-2aAc-abB+Ab^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

```
[Out] -A/(2*a*x^2) - ((A*b^2 - a*b*B - 2*a*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B
)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rubi [A] time = 0.245491, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 800, 634, 618, 206, 628}

$$-\frac{(-2aAc-abB+Ab^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -A/(2*a*x^2) - ((A*b^2 - a*b*B - 2*a*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B
)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{A}{ax^2}+\frac{-Ab+aB}{a^2x}+\frac{-abB+A\left(b^2-ac\right)+(Ab-aB)cx}{a^2\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)\\
&= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{-abB+A\left(b^2-ac\right)+(Ab-aB)cx}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2}\\
&= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2}+\frac{\left(-abB+A\left(b^2-2ac\right)\right)}{}\\
&= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\left(-abB+A\left(b^2-2ac\right)\right)\text{Subst}\left(\right.}{2a^2}\\
&= -\frac{A}{2ax^2}+\frac{\left(abB-A\left(b^2-2ac\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^2\right.}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.226514, size = 186, normalized size = 1.66

$$\frac{\frac{\left(A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)-aB\left(\sqrt{b^2-4ac}+b\right)\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(A\left(b\sqrt{b^2-4ac}+2ac-b^2\right)+aB\left(b-\sqrt{b^2-4ac}\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+4\log(x)(aB-}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^2 + 4*(-(A*b) + a*B)*Log[x] + ((-(a*B*(b + Sqrt[b^2 - 4*a*c]))
+ A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/Sqrt[b^2 - 4*a*c] + ((a*B*(b - Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 2*a*c + b
*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c
])/(4*a^2)
```

Maple [A] time = 0.01, size = 191, normalized size = 1.7

$$\frac{\ln\left(cx^4+bx^2+a\right)Ab}{4\,a^2}-\frac{\ln\left(cx^4+bx^2+a\right)B}{4\,a}-\frac{Ac}{a}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{Ab^2}{2\,a^2}\arctan\left((2\,cx^2+b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/a^2*ln(c*x^4+b*x^2+a)*A*b-1/4/a*ln(c*x^4+b*x^2+a)*B-1/a/(4*a*c-b^2)^(1/
2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arct
an((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^2-1/2/a/(4*a*c-b^2)^(1/2)*arctan((2*c
*x^2+b)/(4*a*c-b^2)^(1/2))*b*B-1/2*A/a/x^2-1/a^2*ln(x)*A*b+1/a*ln(x)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.70211, size = 834, normalized size = 7.45

$$\left[\frac{\left(Bab - Ab^2 + 2\,Aac\right)\sqrt{b^2 - 4\,ac}x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right) - 2\,Aab^2 + 8\,Aa^2c - \left(Bab^2 - Ab^3 - 4\left(Ba^2 \right.\right.}{4\left(a^2b^2 - 4\,a^3c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((B*a*b - A*b^2 + 2*A*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*
c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
 - 2*A*a*b^2 + 8*A*a^2*c - (B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(
c*x^4 + b*x^2 + a) + 4*(B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(x))/
((a^2*b^2 - 4*a^3*c)*x^2), 1/4*(2*(B*a*b - A*b^2 + 2*A*a*c)*sqrt(-b^2 + 4*a
*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*A*a*b^2
 + 8*A*a^2*c - (B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(c*x^4 + b*x^
2 + a) + 4*(B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(x))/((a^2*b^2 -
4*a^3*c)*x^2)]
```

Sympy [B] time = 170.625, size = 495, normalized size = 4.42

$$-\frac{A}{2ax^2} + \left(-\frac{-Ab + Ba}{4a^2} - \frac{\sqrt{-4ac + b^2}\left(2Aac - Ab^2 + Bab\right)}{4a^2\left(4ac - b^2\right)}\right)\log\left(x^2 + \frac{3Aabc - Ab^3 - 2Ba^2c + Bab^2 - 8a^3c\left(-\frac{-Ab+Ba}{4a^2} \right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] -A/(2*a*x**2) + (-(-A*b + B*a)/(4*a**2) - sqrt(-4*a*c + b**2)*(2*A*a*c - A*
b**2 + B*a*b)/(4*a**2*(4*a*c - b**2)))*log(x**2 + (3*A*a*b*c - A*b**3 - 2*B
*a**2*c + B*a*b**2 - 8*a**3*c*(-(-A*b + B*a)/(4*a**2) - sqrt(-4*a*c + b**2)
*(2*A*a*c - A*b**2 + B*a*b)/(4*a**2*(4*a*c - b**2))) + 2*a**2*b**2*(-(-A*b
+ B*a)/(4*a**2) - sqrt(-4*a*c + b**2)*(2*A*a*c - A*b**2 + B*a*b)/(4*a**2*(4
```

```
*a*c - b**2))))/(2*A*a*c**2 - A*b**2*c + B*a*b*c)) + (-(-A*b + B*a)/(4*a**2
) + sqrt(-4*a*c + b**2)*(2*A*a*c - A*b**2 + B*a*b)/(4*a**2*(4*a*c - b**2)))
*log(x**2 + (3*A*a*b*c - A*b**3 - 2*B*a**2*c + B*a*b**2 - 8*a**3*c*(-(-A*b
+ B*a)/(4*a**2) + sqrt(-4*a*c + b**2)*(2*A*a*c - A*b**2 + B*a*b)/(4*a**2*(4
*a*c - b**2))) + 2*a**2*b**2*(-(-A*b + B*a)/(4*a**2) + sqrt(-4*a*c + b**2)*
(2*A*a*c - A*b**2 + B*a*b)/(4*a**2*(4*a*c - b**2))))/(2*A*a*c**2 - A*b**2*c
 + B*a*b*c)) + (-A*b + B*a)*log(x)/a**2
```

Giac [A] time = 1.16742, size = 167, normalized size = 1.49

$$-\frac{(Ba - Ab)\log\left(cx^4 + bx^2 + a\right)}{4\,a^2} + \frac{(Ba - Ab)\log\left(x^2\right)}{2\,a^2} - \frac{\left(Bab - Ab^2 + 2\,Aac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}a^2} - \frac{Bax^2 - Abx^2 + Aa}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(B*a - A*b)*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*(B*a - A*b)*log(x^2)/a^2
- 1/2*(B*a*b - A*b^2 + 2*A*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(s
qrt(-b^2 + 4*a*c)*a^2) - 1/2*(B*a*x^2 - A*b*x^2 + A*a)/(a^2*x^2)
```

3.107 $\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=261

$$\frac{\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(((b*B - A*c)*x)/c^2) + (B*x^3)/(3*c) + ((b^2*B - A*b*c - a*B*c - (b^3*B -
 A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + ((b^2*B - A*b*c - a*B*c + (b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.48851, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1279, 1166, 205}

$$\frac{\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((b*B - A*c)*x)/c^2) + (B*x^3)/(3*c) + ((b^2*B - A*b*c - a*B*c - (b^3*B -
 A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + ((b^2*B - A*b*c - a*B*c + (b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{a+bx^2+cx^4}dx=\frac{Bx^3}{3c}-\frac{\int \frac{x^2\left(3aB+3(bB-Ac)x^2\right)}{a+bx^2+cx^4}dx}{3c}$$
$$=-\frac{(bB-Ac)x}{c^2}+\frac{Bx^3}{3c}+\frac{\int \frac{3a(bB-Ac)+3\left(b^2B-Abc-aBc\right)x^2}{a+bx^2+cx^4}dx}{3c^2}$$
$$=-\frac{(bB-Ac)x}{c^2}+\frac{Bx^3}{3c}+\frac{\left(b^2B-Abc-aBc-\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}dx}{2c^2}+\frac{\left(b^2B-Ab\right.}{}$$
$$=-\frac{(bB-Ac)x}{c^2}+\frac{Bx^3}{3c}+\frac{\left(b^2B-Abc-aBc-\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2B-Abc\right.}{}$$

Mathematica [A] time = 0.413887, size = 327, normalized size = 1.25

$$\frac{\left(-Abc\sqrt{b^2-4ac}-2aAc^2+b^2B\sqrt{b^2-4ac}-aBc\sqrt{b^2-4ac}+3abBc+Ab^2c+b^3(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(-Abc\sqrt{b^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-(b*B) + A*c)*x)/c^2 + (B*x^3)/(3*c) + ((-(b^3*B) + A*b^2*c + 3*a*b*B*c -
 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 - 4*a*c] - a*B*c*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B - A*
b^2*c - 3*a*b*B*c + 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 -
4*a*c] - a*B*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

Maple [B] time = 0.046, size = 825, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*B*x^3/c+1/c*A*x-1/c^2*b*B*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b+1/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*A*b^2+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B-1/2/c^2*2^(1/2)/((-b+(-4*a*c+
```

```
b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b^2*B-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*B+1/2/c^2/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*B-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b+1/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*A*b^2-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B+1/2/c^2*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B-
3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*B+1/2/c^2/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b^3*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bcx^3 - 3\,(Bb - Ac)x}{3\,c^2} - \frac{-\int \frac{Bab - Aac + \left(Bb^2 - (Ba + Ab)c\right)x^2}{cx^4 + bx^2 + a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(B*c*x^3 - 3*(B*b - A*c)*x)/c^2 - integrate(-(B*a*b - A*a*c + (B*b^2 -
(B*a + A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2
```

Fricas [B] time = 9.02119, size = 10329, normalized size = 39.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*c*x^3 + 3*sqrt(1/2)*c^2*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c
^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*
c + (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^
3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2
+ 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*
A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2
*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b
^2*c^5 - 4*a*c^6))*log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4 + (5*A^3
*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a^2*b^2 -
3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*b^4)*c)*x
 + sqrt(1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^2*b + 5*A
^3*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 + A^3*b^4)
*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^3*a*b^5 +
 3*A*B^2*b^6)*c - (B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^2 + A*b^3
)*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a
*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3
+ A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2
```

```
*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2
*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(B^2*b^5 - (4
*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*
B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6
- 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*
b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 +
 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 +
 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^
2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(B^2*b^5
 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2
- (5*B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2
*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3
*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*
b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*
b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c
)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-2*(B^4*a^2*b^4 - A*B^3*
a*b^5 - A^4*a^2*c^4 + (5*A^3*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*
a^3*b - 6*A^2*B^2*a^2*b^2 - 3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2
*b^3 - 3*A^2*B^2*a*b^4)*c)*x - sqrt(1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^
2*a^3 + 20*A^2*B*a^2*b + 5*A^3*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2
 + 17*A^2*B*a*b^3 + A^3*b^4)*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2
*B*b^5)*c^2 - (7*B^3*a*b^5 + 3*A*B^2*b^6)*c - (B*b^4*c^5 + 4*(2*B*a^2 + A*a
*b)*c^7 - (6*B*a*b^2 + A*b^3)*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2
*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B
^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^
2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*
b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a
*c^11)))*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*
B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*
sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c
^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b
^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*
b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4
*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*
sqrt(1/2)*c^2*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b +
 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c - (b^2*c^5 - 4*a*
c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b
^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 +
A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A
^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(
3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*
log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4 + (5*A^3*B*a^2*b + A^4*a*b^
2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a^2*b^2 - 3*A^3*B*a*b^3)*c^2
- (3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*b^4)*c)*x + sqrt(1/2)*(B^3*b
^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^2*b + 5*A^3*a*b^2)*c^4 - (4*
B^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 + A^3*b^4)*c^3 + (13*B^3*a^2*
b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^3*a*b^5 + 3*A*B^2*b^6)*c + (
B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^2 + A*b^3)*c^6)*sqrt((B^4*b^
8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^
4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*
(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (
11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A
*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*
b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b
^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 +
4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*
b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 +
 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6
*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11))
```

```
)/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A
^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*
A*B*b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a
^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2
*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*
b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^
5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c
^11)))/(b^2*c^5 - 4*a*c^6))*log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4
 + (5*A^3*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a
^2*b^2 - 3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*
b^4)*c)*x - sqrt(1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^
2*b + 5*A^3*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 +
 A^3*b^4)*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^
3*a*b^5 + 3*A*B^2*b^6)*c + (B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^
2 + A*b^3)*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*
b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3
*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*
a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6
)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(B^2
*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*
c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4
*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A
*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*
a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*
a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^
7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 6*(B*b - A*c)*x)/c^2
```

Sympy [B] time = 32.4061, size = 709, normalized size = 2.72

$$\frac{Bx^3}{3c} + \mathrm{RootSum}\left(t^4\left(256a^2c^7 - 128ab^2c^6 + 16b^4c^5\right) + t^2\left(48A^2a^2bc^4 - 28A^2ab^3c^3 + 4A^2b^5c^2 + 64ABa^3c^4 - 144ABa^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x**3/(3*c) + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b**4*c**
5) + _t**2*(48*A**2*a**2*b*c**4 - 28*A**2*a*b**3*c**3 + 4*A**2*b**5*c**2 +
64*A*B*a**3*c**4 - 144*A*B*a**2*b**2*c**3 + 64*A*B*a*b**4*c**2 - 8*A*B*b**6
*c - 80*B**2*a**3*b*c**3 + 100*B**2*a**2*b**3*c**2 - 36*B**2*a*b**5*c + 4*B
**2*b**7) + A**4*a**3*c**2 - 2*A**3*B*a**3*b*c + 2*A**2*B**2*a**4*c + A**2*
B**2*a**3*b**2 - 2*A*B**3*a**4*b + B**4*a**5, Lambda(_t, _t*log(x + (-32*_t
**3*A*a*b*c**7 + 8*_t**3*A*b**3*c**6 - 64*_t**3*B*a**2*c**7 + 48*_t**3*B*a*
b**2*c**6 - 8*_t**3*B*b**4*c**5 + 4*_t*A**3*a**2*c**5 - 8*_t*A**3*a*b**2*c*
*4 + 2*_t*A**3*b**4*c**3 - 30*_t*A**2*B*a**2*b*c**4 + 30*_t*A**2*B*a*b**3*c
**3 - 6*_t*A**2*B*b**5*c**2 - 12*_t*A*B**2*a**3*c**4 + 54*_t*A*B**2*a**2*b*
*2*c**3 - 36*_t*A*B**2*a*b**4*c**2 + 6*_t*A*B**2*b**6*c + 14*_t*B**3*a**3*b
*c**3 - 28*_t*B**3*a**2*b**3*c**2 + 14*_t*B**3*a*b**5*c - 2*_t*B**3*b**7)/(
-A**4*a**2*c**4 + A**4*a*b**2*c**3 + 5*A**3*B*a**2*b*c**3 - 3*A**3*B*a*b**3
*c**2 - 6*A**2*B**2*a**2*b**2*c**2 + 3*A**2*B**2*a*b**4*c + 3*A*B**3*a**3*b
*c**2 + A*B**3*a**2*b**3*c - A*B**3*a*b**5 + B**4*a**4*c**2 - 3*B**4*a**3*b
**2*c + B**4*a**2*b**4)))) - x*(-A*c + B*b)/c**2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.108 $\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4} dx$

Optimal. Leaf size=208

$$-\frac{\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}$$

```
[Out] (B*x)/c - ((b*B - A*c - (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - ((b*B - A*c + (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.527767, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1279, 1166, 205}

$$-\frac{\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((b*B - A*c - (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - ((b*B - A*c + (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2\right)}{a+bx^2+cx^4}\,dx = \frac{Bx}{c} - \frac{\int \frac{aB+(bB-Ac)x^2}{a+bx^2+cx^4}\,dx}{c}$$

$$= \frac{Bx}{c} - \frac{\left(bB-Ac-\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} - \frac{\left(bB-Ac+\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,d}{2c}$$

$$= \frac{Bx}{c} - \frac{\left(bB-Ac-\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(bB-Ac+\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right.}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.168397, size = 251, normalized size = 1.21

$$-\frac{\left(-Ac\sqrt{b^2-4ac}+bB\sqrt{b^2-4ac}+2aBc+Abc+b^2(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(-Ac\sqrt{b^2-4ac}+bB\sqrt{b^2-4ac}-2aBc\right.}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\quad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((-(b^2*B) + A*b*c + 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2*B - A*b*
c - 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [B] time = 0.023, size = 560, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] B*x/c-1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*A*b+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*B+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*a*B-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B+1/2*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*A+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b-1/2/c*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*b*B+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B-1/2/c/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
```

```
4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{Bx}{c} + \frac{-\int \frac{(Bb-Ac)x^2+Ba}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] B*x/c + integrate(-((B*b - A*c)*x^2 + B*a)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [B] time = 2.96396, size = 5264, normalized size = 25.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*
B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A
^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 +
2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2
 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b
^2)*c)*x + sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b +
A^2*B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 - 2
*(2*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)
*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3
*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*
B^2*a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A
^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2
*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
 sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^
2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B
*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*
B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A
*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*
c)*x - sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*
B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*
B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3
 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3
)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*
a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B
^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^
4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqr
t(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c
 - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*
c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*
b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3
*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x
 + sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^
2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c + (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*
```

```
b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (
B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)
/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b
+ 2*A*B*b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a
 + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*
b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/
2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c - (
b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3
+ (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)
*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3*b^3
 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x - s
qrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)*c
^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c + (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*b +
A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*
a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^
2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*
A*B*b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2
*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2
+ 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + 2*B*x)/c
```

Sympy [B] time = 11.205, size = 428, normalized size = 2.06

$$\frac{Bx}{c} + \text{RootSum}\Big(t^4\left(256a^2c^5 - 128ab^2c^4 + 16b^4c^3\right) + t^2\left(-16A^2abc^3 + 4A^2b^3c^2 - 64ABa^2c^3 + 48ABab^2c^2 - 8ABb^4c + 4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x/c + RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t
**2*(-16*A**2*a*b*c**3 + 4*A**2*b**3*c**2 - 64*A*B*a**2*c**3 + 48*A*B*a*b**
2*c**2 - 8*A*B*b**4*c + 48*B**2*a**2*b*c**2 - 28*B**2*a*b**3*c + 4*B**2*b**
5) + A**4*a*c**2 - 2*A**3*B*a*b*c + 2*A**2*B**2*a**2*c + A**2*B**2*a*b**2 -
 2*A*B**3*a**2*b + B**4*a**3, Lambda(_t, _t*log(x + (-64*_t**3*A*a*c**5 + 1
6*_t**3*A*b**2*c**4 + 32*_t**3*B*a*b*c**4 - 8*_t**3*B*b**3*c**3 + 2*_t*A**3
*b*c**3 + 12*_t*A**2*B*a*c**3 - 6*_t*A**2*B*b**2*c**2 - 18*_t*A*B**2*a*b*c*
*2 + 6*_t*A*B**2*b**3*c - 4*_t*B**3*a**2*c**2 + 8*_t*B**3*a*b**2*c - 2*_t*B
**3*b**4)/(-A**4*c**3 + 3*A**3*B*b*c**2 - 3*A**2*B**2*b**2*c + A*B**3*a*b*c
 + A*B**3*b**3 + B**4*a**2*c - B**4*a*b**2))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.109 $\int \frac{A+Bx^2}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=172

$$\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] ((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((B +
(b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.201438, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1166, 205}

$$\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((B +
(b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{a+bx^2+cx^4} \, dx=\frac{1}{2}\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} \, dx+\frac{1}{2}\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}$$

$$=\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.097955, size = 173, normalized size = 1.01

$$\frac{\frac{\left(B\sqrt{b^2-4ac}+2Ac-bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(B\sqrt{b^2-4ac}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((b*B - 2*A*c + B*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqr
t[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.018, size = 328, normalized size = 1.9

$$-c\sqrt{2}A\,\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}-\frac{\sqrt{2}B}{2}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-\right.}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A-1/2*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B+1
/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*B-c/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*A+1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 2.24043, size = 3077, normalized size = 17.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*
sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x + sqr
t(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*a^3
 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c
+ A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c
 + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*
c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(B^2*a*b -
 (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c
+ A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(B^4*a
^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x - sqrt(1/2)*(A*B^2*a*b^2 + 4*A^3*a*
c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2
 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^
3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt(
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c -
4*a^2*c^2))) + 1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*
c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^
3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A
^4*c^2)*x + sqrt(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*
c - (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 -
 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A
*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4
*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*s
qrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 -
 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)
)*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x - sqrt(1/2)*(A*B^2*a
*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c - (4*(2*B*a^3 - A*a^2*b)*c^2
 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2
*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4
*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3
)))/(a*b^2*c - 4*a^2*c^2)))
```

Sympy [A] time = 4.97496, size = 314, normalized size = 1.83

$$\mathrm{RootSum}\left(t^4\left(256a^3c^3 - 128a^2b^2c^2 + 16ab^4c\right) + t^2\left(-16A^2abc^2 + 4A^2b^3c + 64ABa^2c^2 - 16ABab^2c - 16B^2a^2bc + 4B^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**3 - 128*a**2*b**2*c**2 + 16*a*b**4*c) + _t**2*(-
16*A**2*a*b*c**2 + 4*A**2*b**3*c + 64*A*B*a**2*c**2 - 16*A*B*a*b**2*c - 16*
B**2*a**2*b*c + 4*B**2*a*b**3) + A**4*c**2 - 2*A**3*B*b*c + 2*A**2*B**2*a*c
 + A**2*B**2*b**2 - 2*A*B**3*a*b + B**4*a**2, Lambda(_t, _t*log(x + (-32*_t
**3*A*a**2*b*c**2 + 8*_t**3*A*a*b**3*c + 64*_t**3*B*a**3*c**2 - 16*_t**3*B*
a**2*b**2*c - 4*_t*A**3*a*c**2 + 2*_t*A**3*b**2*c - 6*_t*A**2*B*a*b*c + 12*
_t*A*B**2*a**2*c - 2*_t*B**3*a**2*b)/(-A**4*c**2 + A**3*B*b*c - A*B**3*a*b
+ B**4*a**2))))
```

Giac [C] time = 2.43223, size = 6435, normalized size = 37.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*
(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*cosh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
 + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/
4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*cosh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^
3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 -
 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)
*sqrt(b^2 - 4*a*c)*b)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3
/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*
c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*A*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*
c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*A*sin(5/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/
(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*c^4) + 1/2*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c
^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*cos(1/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*
sqrt(b^2 - 4*a*c)*b)*B*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9
*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)
*b)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cos
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*
c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3
```

```
)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*co
s(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*im
ag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/
4)*sqrt(b^2 - 4*a*c)*b)*B*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*si
nh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*
b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*cos(1/4*pi
 + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a
*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^3 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*s
qrt(b^2 - 4*a*c)*b*c^2)*A*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((
a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a
*c)*b*c^2)*A*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(
1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)
*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*
c^4) - 1/4*(((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b
^2 - 4*a*c)*b)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*((a*c
^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*
cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*im
ag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3
/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*cos(5/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4
*a*c)*b)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*
cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
+ (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((
a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)
*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^
(3/4)*sqrt(b^2 - 4*a*c)*b)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^3 + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*B*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(1/4)*b^2*c^2
- 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*A*cos(5/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(
1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*A*cos(5/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*c^3 - 4*a^2*c^4) - 1/4*((
(a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b
```

```
)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*((a*c^3)^(3/4)*b^2
 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*cos(1/4*pi +
1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*
c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^3
)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*co
s(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_part(arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/
4)*sqrt(b^2 - 4*a*c)*b)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*si
nh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3/4)*
b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*cos(1/4*pi
 + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^
2 - 4*a*c)*b)*B*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*((a*c^
3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*B*c
os(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(
1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*A*cos(1/4*pi + 1/2*real
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 +
(a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*A*cos(1/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a
*abs(c)))) + x^2 + sqrt(a/c))/(a*b^2*c^3 - 4*a^2*c^4)
```

3.110 $\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=189

$$-\frac{\sqrt{c}\left(\frac{Ab-2aB}{\sqrt{b^2-4ac}}+A\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}$$

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rubi [A] time = 0.400399, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1281, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{Ab-2aB}{\sqrt{b^2-4ac}}+A\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{A}{ax}-\frac{\int \frac{Ab-aB+Acx^2}{a+bx^2+cx^4}\,dx}{a}$$

$$= -\frac{A}{ax}-\frac{\left(c\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\frac{\left(c\left(A+\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}$$

$$= -\frac{A}{ax}-\frac{\sqrt{c}\left(A+\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.28944, size = 206, normalized size = 1.09

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}+b\right)-2aB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}-b\right)+2aB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2A}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((2*A)/x + (Sqrt[2]*Sqrt[c]*(-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*a*B + A*(-b + Sqrt[b^2 - 4*a*c])
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*
c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a)
```

Maple [B] time = 0.021, size = 353, normalized size = 1.9

$$\frac{c\sqrt{2}A}{2\,a}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}+\frac{c\sqrt{2}Ab}{2\,a}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*A+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*A*b-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B-1/2/a*c*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*A+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b-c/(-4*a*c+b^2)^(
1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*B-A/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((A*c*x^2 - B*a + A*b)/(c*x^4 + b*x^2 + a), x)/a - A/(a*x)
```

Fricas [B] time = 3.562, size = 5806, normalized size = 30.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 -
3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^
2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*
B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A
^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2
*a*b^2 - A^3*B*b^3)*c)*x + sqrt(1/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2
*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*
a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*a*b^3)*c - (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^
5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^
2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*
A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*
a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4
*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^
2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c))
)/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^
2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A
*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*
(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2
 - 4*a^4*c))*log(2*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*
B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3*B*b^3)*c)*x - sqrt(1/2)*(B^3*a^3*b^2 - 3*
A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (
4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*a*b^3)*c - (B*a^4*b^3
 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 -
 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2
- 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt
(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2
 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b
^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)
/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(B^2*a^2
*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c - (a^3*b^2 - 4*a^4*c
)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2
- 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*
c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3*B*b^3)*c)*x + sqrt(1
/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3
 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*
a*b^3)*c + (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^
2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A
^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*
```

```
b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*
A^2*a*b)*c - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*
a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*
a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/
2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c
 - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 -
4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^
4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A^4*a*c^3 + (
A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3
*B*b^3)*c)*x - sqrt(1/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A
^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A
^2*B*a^2*b^2 - 5*A^3*a*b^3)*c + (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2
*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b
+ A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*
b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B
^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A
^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
 4*a^4*c))) - 2*A)/(a*x)
```

Sympy [B] time = 12.052, size = 490, normalized size = 2.59

$$-\frac{A}{ax} + \operatorname{RootSum}\left(t^4\left(256a^5c^2 - 128a^4b^2c + 16a^3b^4\right) + t^2\left(48A^2a^2bc^2 - 28A^2ab^3c + 4A^2b^5 - 64ABa^3c^2 + 48ABa^2b^2c - 8\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] -A/(a*x) + RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) +
 _t**2*(48*A**2*a**2*b*c**2 - 28*A**2*a*b**3*c + 4*A**2*b**5 - 64*A*B*a**3*
c**2 + 48*A*B*a**2*b**2*c - 8*A*B*a*b**4 - 16*B**2*a**3*b*c + 4*B**2*a**2*b
**3) + A**4*c**3 - 2*A**3*B*b*c**2 + 2*A**2*B**2*a*c**2 + A**2*B**2*b**2*c
- 2*A*B**3*a*b*c + B**4*a**2*c, Lambda(_t, _t*log(x + (64*_t**3*A*a**5*c**2
 - 48*_t**3*A*a**4*b**2*c + 8*_t**3*A*a**3*b**4 + 32*_t**3*B*a**5*b*c - 8*_
t**3*B*a**4*b**3 + 10*_t*A**3*a**2*b*c**2 - 10*_t*A**3*a*b**3*c + 2*_t*A**3
*b**5 - 12*_t*A**2*B*a**3*c**2 + 24*_t*A**2*B*a**2*b**2*c - 6*_t*A**2*B*a*b
**4 - 18*_t*A*B**2*a**3*b*c + 6*_t*A*B**2*a**2*b**3 + 4*_t*B**3*a**4*c - 2*
_t*B**3*a**3*b**2)/(-A**4*a*c**3 + A**4*b**2*c**2 - A**3*B*a*b*c**2 - A**3*
B*b**3*c + 3*A**2*B**2*a*b**2*c - 3*A*B**3*a**2*b*c + B**4*a**3*c))))
```

Giac [C] time = 2.54777, size = 5257, normalized size = 27.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -2*(3*(a*c^3)^(3/4)*A*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*si
n(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(3/
4)*A*a*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi
 + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*(a*c^3)^(3/4)*A
```

```
*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*A*a*cosh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) +
 9*(a*c^3)^(3/4)*A*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*A*a*cosh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*A*a*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
3 + (a*c^3)^(3/4)*A*a*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - (a
*c^3)^(1/4)*B*a^2*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^
(1/4)*A*a*b*c*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5
/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*
B*a^2*c*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*A*a*b*
c*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*
imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan(-((a/c)^(1/4)*cos(5/
4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) - x)/((a/c)^(1/4)*sin(5/4*pi
 + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*a*b*c*abs(a
) - (b^2*c - 4*a*c^2)*a^2) - 2*(3*(a*c^3)^(3/4)*A*a*cos(1/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b
/(a*abs(c))))) - (a*c^3)^(3/4)*A*a*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^3 - 9*(a*c^3)^(3/4)*A*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^2*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*A*a*cos
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*rea
l_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3)^(3/4)*A*a*cos(1/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(
a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*
c^3)^(3/4)*A*a*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(
1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*A*a*cos(1/4*
pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*
real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3 + (a*c^3)^(3/4)*A*a*sin(1/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^3 - (a*c^3)^(1/4)*B*a^2*c*cosh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c))))) + (a*c^3)^(1/4)*A*a*b*c*cosh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c))))) + (a*c^3)^(1/4)*B*a^2*c*sin(1/4*pi + 1/2*real_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
))))) - (a*c^3)^(1/4)*A*a*b*c*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))
*arctan(-((a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
- x)/((a/c)^(1/4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(s
```

```
qrt(b^2 - 4*a*c)*a*b*c*abs(a) - (b^2*c - 4*a*c^2)*a^2) + ((a*c^3)^(3/4)*A*a
*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 3*(a*c^3)^(3/4)*A*a*cos
(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sin(5/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*(a*c^3)^(3/4)*A*a*cos(5/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a
*abs(c))))) + 9*(a*c^3)^(3/4)*A*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))^2*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin
h(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*A*a*
cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*
imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*(a*c^3)^(3/4)*A*a*cos(5/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - (a
*c^3)^(3/4)*A*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*(a*c^3
)^(3/4)*A*a*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 - (a*c^3)^(1/4)*B*a^2*c*c
os(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*ima
g_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*A*a*b*c*cos(5/4
*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*B*a^2*c*cos(5/4*pi +
1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*A*a*b*c*cos(5/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c))))))*log(-2*x*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))) + x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*a*b*c*abs(a
) - (b^2*c - 4*a*c^2)*a^2) + ((a*c^3)^(3/4)*A*a*cos(1/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^3 - 3*(a*c^3)^(3/4)*A*a*cos(1/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^3*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)
))))^2 - 3*(a*c^3)^(3/4)*A*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*(a*c^3)^(3/4
)*A*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(
1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sin(1/4*pi + 1/2*real_
part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c))))) + 3*(a*c^3)^(3/4)*A*a*cos(1/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^
2 - 9*(a*c^3)^(3/4)*A*a*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - (a*c^3)^(3/4)*A*a*cos(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*(a*c^3)^(3/4)*A*a*cos(1/4*pi + 1/2
*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c)))))^3 - (a*c^3)^(1/4)*B*a^2*c*cos(1/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c))))) + (a*c^3)^(1/4)*A*a*b*c*cos(1/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))) + (a*c^3)^(1/4)*B*a^2*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqr
```

```
t(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
))) - (a*c^3)^(1/4)*A*a*b*c*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*l
og(-2*x*(a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) +
x^2 + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*a*b*c*abs(a) - (b^2*c - 4*a*c^2)*a^2) -
 A/(a*x)
```

3.111 $\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=271

$$-\frac{\sqrt{c}\left(aB\left(\sqrt{b^2-4ac}+b\right)-A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(-b\sqrt{b^2-4ac}\right.\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b}}}$$

```
[Out] -A/(3*a*x^3) + (A*b - a*B)/(a^2*x) - (Sqrt[c]*(a*B*(b + Sqrt[b^2 - 4*a*c])
- A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[c]*(a*B*(b - Sqrt[b^2 - 4*a*c]) - A*(b^2 - 2*a*c - b*Sqrt[b
^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.652712, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1281, 1166, 205}

$$-\frac{\sqrt{c}\left(aB\left(\sqrt{b^2-4ac}+b\right)-A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(-b\sqrt{b^2-4ac}\right.\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -A/(3*a*x^3) + (A*b - a*B)/(a^2*x) - (Sqrt[c]*(a*B*(b + Sqrt[b^2 - 4*a*c])
- A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (Sqrt[c]*(a*B*(b - Sqrt[b^2 - 4*a*c]) - A*(b^2 - 2*a*c - b*Sqrt[b
^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx = -\frac{A}{3ax^3} - \frac{\int \frac{3(Ab-aB)+3Acx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{3a}$$

$$= -\frac{A}{3ax^3} + \frac{Ab-aB}{a^2x} + \frac{\int \frac{3\left(Ab^2-abB-aAc\right)+3(Ab-aB)cx^2}{a+bx^2+cx^4}\,dx}{3a^2}$$

$$= -\frac{A}{3ax^3} + \frac{Ab-aB}{a^2x} + \frac{\left(c\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(b^2-2ac-b\sqrt{b^2-4ac}\right)\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac+\cdots}}}{2a^2\sqrt{b^2-4ac}}$$

$$= -\frac{A}{3ax^3} + \frac{Ab-aB}{a^2x} - \frac{\sqrt{c}\left(aB\left(b+\sqrt{b^2-4ac}\right)-A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt{b-\sqrt{b^2\cdots}}}\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.329353, size = 267, normalized size = 0.99

$$\frac{-\frac{3\sqrt{2}\sqrt{c}\left(aB\left(\sqrt{b^2-4ac}+b\right)-A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}+2ac-b^2\right)+aB\left(b-\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{6Ab\cdots}{\cdots}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^3 + (6*A*b - 6*a*B)/x - (3*Sqrt[2]*Sqrt[c]*(a*B*(b + Sqrt[b^2 -
 4*a*c]) - A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (3*Sqrt[2]*Sqrt[c]*(a*B*(b - Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 2*a*c +
b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2)
```

Maple [B] time = 0.025, size = 611, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b+1/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*A-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2+1/2/a
*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*B+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
```

```
*b*B+1/2/a^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b+1/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*A-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2-1/2/a*c*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*B+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*B-1/3*A/
a/x^3+1/a^2/x*A*b-1/a/x*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{-\int \frac{(Ba-Ab)cx^2+Bab-Ab^2+Aac}{cx^4+bx^2+a}\,dx}{a^2} - \frac{3\,(Ba-Ab)x^2+Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(-((B*a - A*b)*c*x^2 + B*a*b - A*b^2 + A*a*c)/(c*x^4 + b*x^2 + a),
 x)/a^2 - 1/3*(3*(B*a - A*b)*x^2 + A*a)/(a^2*x^3)
```

Fricas [B] time = 10.1844, size = 10842, normalized size = 40.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*
B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c +
(a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6
 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b +
 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^
3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A
^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))
/(a^5*b^2 - 4*a^6*c))*log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4
- (B^4*a^4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3
 + (B^4*a^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x + s
qrt(1/2)*(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a
^4*c^4 + (4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b
 - 25*A*B^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b
^3 - 18*A*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6)*c - (B*a^6*b^4 - A*
a^5*b^5 + 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*sqrt
((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b
^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B
^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2
*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*
B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(B^2*a^2*b^3 - 2*
A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*
a^2*b^2 + 5*A^2*a*b^3)*c + (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*
a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^
2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b +
```

```
 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b
^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)
*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqr
t(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (
3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c + (a^5*b^2 - 4*a^6*c)*sqrt((B^
4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 +
 A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a
^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4
)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^
2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*
(A^4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4 - (B^4*a^4 - 5*A*B^3*a^3*b +
 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3 + (B^4*a^3*b^2 - 3*A*B^3*a^
2*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x - sqrt(1/2)*(B^3*a^3*b^5 - 3*A*
B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a^4*c^4 + (4*A*B^2*a^5 - 20*A
^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b - 25*A*B^2*a^4*b^2 + 41*A^2
*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b^3 - 18*A*B^2*a^3*b^4 + 21*A
^2*B*a^2*b^5 - 8*A^3*a*b^6)*c - (B*a^6*b^4 - A*a^5*b^5 + 4*(2*B*a^8 - 3*A*a
^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*
b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^
2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*
A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 -
 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/
(a^10*b^2 - 4*a^11*c)))*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B
*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c + (
a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6
- 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b +
3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3
*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^
2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/
(a^5*b^2 - 4*a^6*c))) + 3*sqrt(1/2)*a^2*x^3*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^
4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2
+ 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5
+ 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^
5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*
B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A
*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^1
0*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2*
b - A^4*a*b^2)*c^4 - (B^4*a^4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a
*b^3 - A^4*b^4)*c^3 + (B^4*a^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^
3*B*b^5)*c^2)*x + sqrt(1/2)*(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7
- A^3*b^8 - 4*A^3*a^4*c^4 + (4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)
*c^3 + (4*B^3*a^5*b - 25*A*B^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4)
*c^2 - (5*B^3*a^4*b^3 - 18*A*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6)*
c + (B*a^6*b^4 - A*a^5*b^5 + 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B*a^7*b^2 - 7
*A*a^6*b^3)*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*
A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^
4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a
^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^
2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))*sqrt
(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3
*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c)*sqrt((B^4
*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 +
A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^
6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)
*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2
*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sq
rt(1/2)*a^2*x^3*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5
*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c - (a^5*b^2
- 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*
```

```
B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^
3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b
^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^
3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2
 - 4*a^6*c))*log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4 - (B^4*a^
4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3 + (B^4*a
^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x - sqrt(1/2)*
(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a^4*c^4 +
(4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b - 25*A*B
^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b^3 - 18*A
*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6)*c + (B*a^6*b^4 - A*a^5*b^5 +
 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*sqrt((B^4*a^4
*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*
a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 -
8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2
 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5
 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4
 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 +
 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 +
 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5
 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B
^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*
B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10
*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 6*(B*a - A*b)*x^2 - 2*A*a)/(a^2*
x^3)
```

Sympy [B] time = 34.4356, size = 774, normalized size = 2.86

$$\text{RootSum}\left(t^4\left(256a^7c^2 - 128a^6b^2c + 16a^5b^4\right) + t^2\left(-80A^2a^3bc^3 + 100A^2a^2b^3c^2 - 36A^2ab^5c + 4A^2b^7 + 64ABa^4c^3 - 144A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**7*c**2 - 128*a**6*b**2*c + 16*a**5*b**4) + _t**2*(-80
*A**2*a**3*b*c**3 + 100*A**2*a**2*b**3*c**2 - 36*A**2*a*b**5*c + 4*A**2*b**
7 + 64*A*B*a**4*c**3 - 144*A*B*a**3*b**2*c**2 + 64*A*B*a**2*b**4*c - 8*A*B*
a*b**6 + 48*B**2*a**4*b*c**2 - 28*B**2*a**3*b**3*c + 4*B**2*a**2*b**5) + A*
*4*c**5 - 2*A**3*B*b*c**4 + 2*A**2*B**2*a*c**4 + A**2*B**2*b**2*c**3 - 2*A*
B**3*a*b*c**3 + B**4*a**2*c**3, Lambda(_t, _t*log(x + (96*_t**3*A*a**7*b*c*
*2 - 56*_t**3*A*a**6*b**3*c + 8*_t**3*A*a**5*b**5 - 64*_t**3*B*a**8*c**2 +
48*_t**3*B*a**7*b**2*c - 8*_t**3*B*a**6*b**4 + 4*_t*A**3*a**4*c**4 - 32*_t*
A**3*a**3*b**2*c**3 + 40*_t*A**3*a**2*b**4*c**2 - 16*_t*A**3*a*b**6*c + 2*_
t*A**3*b**8 + 42*_t*A**2*B*a**4*b*c**3 - 84*_t*A**2*B*a**3*b**3*c**2 + 42*_
t*A**2*B*a**2*b**5*c - 6*_t*A**2*B*a*b**7 - 12*_t*A*B**2*a**5*c**3 + 54*_t*
A*B**2*a**4*b**2*c**2 - 36*_t*A*B**2*a**3*b**4*c + 6*_t*A*B**2*a**2*b**6 -
10*_t*B**3*a**5*b*c**2 + 10*_t*B**3*a**4*b**3*c - 2*_t*B**3*a**3*b**5)/(-A*
*4*a**2*c**5 + 3*A**4*a*b**2*c**4 - A**4*b**4*c**3 - 3*A**3*B*a**2*b*c**4 -
 A**3*B*a*b**3*c**3 + A**3*B*b**5*c**2 + 6*A**2*B**2*a**2*b**2*c**3 - 3*A**
2*B**2*a*b**4*c**2 - 5*A*B**3*a**3*b*c**3 + 3*A*B**3*a**2*b**3*c**2 + B**4*
a**4*c**3 - B**4*a**3*b**2*c**2)))) - (A*a + x**2*(-3*A*b + 3*B*a))/(3*a**2
*x**3)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.112 $\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=212

$$-\frac{\left(12a^2Bc^2+6aAbc^2-12ab^2Bc-Ab^3c+2b^4B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(-6aBc-Abc+2b^2B\right)}{2c^2\left(b^2-4ac\right)}-\frac{x^4\left(x^2\left(-2aBc-Abc+\right.\right.}{2c\left(b^2-4ac\right)\left(a\right.}$$

```
[Out] ((2*b^2*B - A*b*c - 6*a*B*c)*x^2)/(2*c^2*(b^2 - 4*a*c)) - (x^4*(a*(b*B - 2*
A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*c^2)*ArcT
anh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(3/2)) - ((2*b*B
 - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] time = 0.381239, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1251, 818, 773, 634, 618, 206, 628}

$$-\frac{\left(12a^2Bc^2+6aAbc^2-12ab^2Bc-Ab^3c+2b^4B\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(-6aBc-Abc+2b^2B\right)}{2c^2\left(b^2-4ac\right)}-\frac{x^4\left(x^2\left(-2aBc-Abc+\right.\right.}{2c\left(b^2-4ac\right)\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*b^2*B - A*b*c - 6*a*B*c)*x^2)/(2*c^2*(b^2 - 4*a*c)) - (x^4*(a*(b*B - 2*
A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*c^2)*ArcT
anh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(3/2)) - ((2*b*B
 - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3(A+Bx)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\operatorname{Subst}\left(\int \frac{x\left(2a(bB-2Ac)+\left(2b^2B-Abc-6aBc\right)x\right)}{a+bx+cx^2}\,dx, x,\right.}{2c\left(b^2-4ac\right)}$$

$$= \frac{\left(2b^2B-Abc-6aBc\right)x^2}{2c^2\left(b^2-4ac\right)} - \frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\operatorname{Subst}\left(\int \frac{-a\left(2b^2B-A\right.}{}\right.}{}$$

$$= \frac{\left(2b^2B-Abc-6aBc\right)x^2}{2c^2\left(b^2-4ac\right)} - \frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(2bB-Ac)\operatorname{Subst}\left(\right.}{4}$$

$$= \frac{\left(2b^2B-Abc-6aBc\right)x^2}{2c^2\left(b^2-4ac\right)} - \frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(2bB-Ac)\log\left(a+\right.}{4c^3}$$

$$= \frac{\left(2b^2B-Abc-6aBc\right)x^2}{2c^2\left(b^2-4ac\right)} - \frac{x^4\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2b^4B-Ab^3c-12\right.}{}$$

Mathematica [A] time = 0.312656, size = 208, normalized size = 0.98

$$\frac{-\frac{2\left(a^2c\left(2c\left(A+Bx^2\right)-3bB\right)+ab\left(-bc\left(A+4Bx^2\right)+3Ac^2x^2+b^2B\right)+b^3x^2(bB-Ac)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2\left(12a^2Bc^2+6aAbc^2-12ab^2Bc-Ab^3c+2b^4B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+(Ac-2bB)\,\mathrm{l}}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*B*c*x^2 - (2*(b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a
*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) - (2*(2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*c^2
)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (-2*b*B
+ A*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [B] time = 0.017, size = 689, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*B*x^2/c^2+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*A*b-1/2/c^2/(c*x^4+b*
x^2+a)/(4*a*c-b^2)*x^2*A*b^3+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2*B-2/c^
2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b^2*B+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^
2)*x^2*b^4*B+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*A-1/2/c^2/(c*x^4+b*x^2+a)*
a/(4*a*c-b^2)*A*b^2-3/2/c^2/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b*B+1/2/c^3/(c*
x^4+b*x^2+a)*a/(4*a*c-b^2)*b^3*B+1/c/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*A-1/4/
c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*A*b^2-2/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)
*a*b*B+1/2/c^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^3*B-3/c/(4*a*c-b^2)^(3/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*a*b-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c
*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*B+6/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*B*a*b^2+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))*b^3*A-1/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*b^4*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.24119, size = 2789, normalized size = 13.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a*b^5 - 16*A*a^3*c^3 - 2*(B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c
^4)*x^6 - 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^4 + 12*(2*B*a^3*b
+ A*a^2*b^2)*c^2 + 2*(B*b^6 - 12*(2*B*a^3 + A*a^2*b)*c^3 + (26*B*a^2*b^2 +
7*A*a*b^3)*c^2 - (9*B*a*b^4 + A*b^5)*c)*x^2 + (2*B*a*b^4 + (2*B*b^4*c + 6*(
2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3)*c^2)*x^4 + 6*(2*B*a^3 + A*a^2*b
)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)*c)*x^
2 - (12*B*a^2*b^2 + A*a*b^3)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^
2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2
*(7*B*a^2*b^3 + A*a*b^4)*c + (2*B*a*b^5 - 16*A*a^3*c^3 + (2*B*b^5*c - 16*A*
a^2*c^4 + 8*(4*B*a^2*b + A*a*b^2)*c^3 - (16*B*a*b^3 + A*b^4)*c^2)*x^4 + 8*(
4*B*a^3*b + A*a^2*b^2)*c^2 + (2*B*b^6 - 16*A*a^2*b*c^3 + 8*(4*B*a^2*b^2 + A
*a*b^3)*c^2 - (16*B*a*b^4 + A*b^5)*c)*x^2 - (16*B*a^2*b^3 + A*a*b^4)*c)*log
(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8
*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2),
 -1/4*(2*B*a*b^5 - 16*A*a^3*c^3 - 2*(B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c
^4)*x^6 - 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^4 + 12*(2*B*a^3*b
+ A*a^2*b^2)*c^2 + 2*(B*b^6 - 12*(2*B*a^3 + A*a^2*b)*c^3 + (26*B*a^2*b^2 +
7*A*a*b^3)*c^2 - (9*B*a*b^4 + A*b^5)*c)*x^2 + 2*(2*B*a*b^4 + (2*B*b^4*c + 6
*(2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3)*c^2)*x^4 + 6*(2*B*a^3 + A*a^2
*b)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)*c)*
x^2 - (12*B*a^2*b^2 + A*a*b^3)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*
sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(7*B*a^2*b^3 + A*a*b^4)*c + (2*B*a*b^
5 - 16*A*a^3*c^3 + (2*B*b^5*c - 16*A*a^2*c^4 + 8*(4*B*a^2*b + A*a*b^2)*c^3
- (16*B*a*b^3 + A*b^4)*c^2)*x^4 + 8*(4*B*a^3*b + A*a^2*b^2)*c^2 + (2*B*b^6
- 16*A*a^2*b*c^3 + 8*(4*B*a^2*b^2 + A*a*b^3)*c^2 - (16*B*a*b^4 + A*b^5)*c)*
x^2 - (16*B*a^2*b^3 + A*a*b^4)*c)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^
2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)]
```

Sympy [B] time = 37.1602, size = 1266, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] B*x**2/(2*c**2) + (-sqrt(-(4*a*c - b**2)**3)*(6*A*a*b*c**2 - A*b**3*c + 12*
B*a**2*c**2 - 12*B*a*b**2*c + 2*B*b**4)/(4*c**3*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) - (-A*c + 2*B*b)/(4*c**3))*log(x**2 + (8*A*a*
*2*c**2 - A*a*b**2*c - 10*B*a**2*b*c + 2*B*a*b**3 - 32*a**2*c**4*(-sqrt(-(4
*a*c - b**2)**3)*(6*A*a*b*c**2 - A*b**3*c + 12*B*a**2*c**2 - 12*B*a*b**2*c
+ 2*B*b**4)/(4*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)
) - (-A*c + 2*B*b)/(4*c**3)) + 16*a*b**2*c**3*(-sqrt(-(4*a*c - b**2)**3)*(6
*A*a*b*c**2 - A*b**3*c + 12*B*a**2*c**2 - 12*B*a*b**2*c + 2*B*b**4)/(4*c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (-A*c + 2*B*b)/
(4*c**3)) - 2*b**4*c**2*(-sqrt(-(4*a*c - b**2)**3)*(6*A*a*b*c**2 - A*b**3*c
 + 12*B*a**2*c**2 - 12*B*a*b**2*c + 2*B*b**4)/(4*c**3*(64*a**3*c**3 - 48*a*
*2*b**2*c**2 + 12*a*b**4*c - b**6)) - (-A*c + 2*B*b)/(4*c**3)))/(6*A*a*b*c*
*2 - A*b**3*c + 12*B*a**2*c**2 - 12*B*a*b**2*c + 2*B*b**4)) + (sqrt(-(4*a*c
 - b**2)**3)*(6*A*a*b*c**2 - A*b**3*c + 12*B*a**2*c**2 - 12*B*a*b**2*c + 2*
B*b**4)/(4*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) -
(-A*c + 2*B*b)/(4*c**3))*log(x**2 + (8*A*a**2*c**2 - A*a*b**2*c - 10*B*a**2
```

```
*b*c + 2*B*a*b**3 - 32*a**2*c**4*(sqrt(-(4*a*c - b**2)**3)*(6*A*a*b*c**2 -
A*b**3*c + 12*B*a**2*c**2 - 12*B*a*b**2*c + 2*B*b**4)/(4*c**3*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (-A*c + 2*B*b)/(4*c**3)) + 16
*a*b**2*c**3*(sqrt(-(4*a*c - b**2)**3)*(6*A*a*b*c**2 - A*b**3*c + 12*B*a**2
*c**2 - 12*B*a*b**2*c + 2*B*b**4)/(4*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6)) - (-A*c + 2*B*b)/(4*c**3)) - 2*b**4*c**2*(sqrt(-(4*
a*c - b**2)**3)*(6*A*a*b*c**2 - A*b**3*c + 12*B*a**2*c**2 - 12*B*a*b**2*c +
 2*B*b**4)/(4*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))
 - (-A*c + 2*B*b)/(4*c**3)))/(6*A*a*b*c**2 - A*b**3*c + 12*B*a**2*c**2 - 12
*B*a*b**2*c + 2*B*b**4)) + (2*A*a**2*c**2 - A*a*b**2*c - 3*B*a**2*b*c + B*a
*b**3 + x**2*(3*A*a*b*c**2 - A*b**3*c + 2*B*a**2*c**2 - 4*B*a*b**2*c + B*b*
*4))/(8*a**2*c**4 - 2*a*b**2*c**3 + x**4*(8*a*c**5 - 2*b**2*c**4) + x**2*(8
*a*b*c**4 - 2*b**3*c**3))
```

Giac [A] time = 19.541, size = 323, normalized size = 1.52

$$\frac{Bx^2}{2\,c^2} + \frac{\left(2\,Bb^4 - 12\,Bab^2c - Ab^3c + 12\,Ba^2c^2 + 6\,Aabc^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(b^2c^3 - 4\,ac^4\right)\sqrt{-b^2+4\,ac}} + \frac{2\,Bb^3x^4 - 8\,Babcx^4 - Ab^2cx^4 + 4\,Aac^2x^4 + \cdots}{4\left(cx^4 + bx^2 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c^2 + 1/2*(2*B*b^4 - 12*B*a*b^2*c - A*b^3*c + 12*B*a^2*c^2 + 6*A*
a*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt
(-b^2 + 4*a*c)) + 1/4*(2*B*b^3*x^4 - 8*B*a*b*c*x^4 - A*b^2*c*x^4 + 4*A*a*c^
2*x^4 + A*b^3*x^2 - 4*B*a^2*c*x^2 - 2*A*a*b*c*x^2 - 2*B*a^2*b + A*a*b^2)/((
c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/4*(2*B*b - A*c)*log(c*x^4 + b*x
^2 + a)/c^3
```

3.113 $\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=147

$$\frac{(4aAc^2-6abBc+b^3B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}}-\frac{x^2\left(x^2(-2aBc-Abc+b^2B)+a(bB-2Ac)\right)}{2c(b^2-4ac)(a+bx^2+cx^4)}+\frac{B\log(a+bx^2+cx^4)}{4c^2}$$

```
[Out] -(x^2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) + ((b^3*B - 6*a*b*B*c + 4*a*A*c^2)*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + (B*Log[a + b*x^2 + c*
x^4])/(4*c^2)
```

Rubi [A] time = 0.174667, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 818, 634, 618, 206, 628}

$$\frac{(4aAc^2-6abBc+b^3B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}}-\frac{x^2\left(x^2(-2aBc-Abc+b^2B)+a(bB-2Ac)\right)}{2c(b^2-4ac)(a+bx^2+cx^4)}+\frac{B\log(a+bx^2+cx^4)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x^2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) + ((b^3*B - 6*a*b*B*c + 4*a*A*c^2)*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + (B*Log[a + b*x^2 + c*
x^4])/(4*c^2)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(A+Bx)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{x^2\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\text{Subst}\left(\int \frac{a(bB-2Ac)+B\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c\left(b^2-4ac\right)}$$

$$= -\frac{x^2\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} - \frac{\left(b^3B-6abBc+\right.}{}$$

$$= -\frac{x^2\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{\left(b^3B-6abBc+4aAc^2\right)}{2}$$

$$= -\frac{x^2\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^3B-6abBc+4aAc^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\left(b^2-4ac\right)^{3/2}} + \frac{B\,\text{l}}{}$$

Mathematica [A] time = 0.205738, size = 160, normalized size = 1.09

$$\frac{-\frac{2\left(2a^2Bc+a\left(bc\left(A+3Bx^2\right)-2Ac^2x^2+b^2(-B)\right)+b^2x^2(Ac-bB)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2\left(4aAc^2-6abBc+b^3B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}+B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*(2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*
(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(b^3*B - 6*a*b*B*
c + 4*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/
```

```
2) + B*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [B] time = 0.013, size = 286, normalized size = 2.

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(-\frac{\left(2\,aAc^2-Ab^2c-3\,abBc+b^3B\right)x^2}{c^2\left(4\,ac-b^2\right)}+\frac{a\left(Abc+2\,aBc-b^2B\right)}{c^2\left(4\,ac-b^2\right)}\right)+\frac{\ln\left(cx^4+bx^2+a\right)aB}{\left(4\,ac-b^2\right)c}-\frac{\ln\left(cx^4\right.}{\left(16\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-1/c^2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/(4*a*c-b^2)*x^2+a*(A*b*c+2*
B*a*c-B*b^2)/c^2/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)/c*ln(c*x^4+b*x^
2+a)*a*B-1/4/(4*a*c-b^2)/c^2*ln(c*x^4+b*x^2+a)*b^2*B+2/(4*a*c-b^2)^(3/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*A-3/(4*a*c-b^2)^(3/2)/c*arctan((2*c*x
^2+b)/(4*a*c-b^2)^(1/2))*a*b*B+1/2/(4*a*c-b^2)^(3/2)/c^2*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*b^3*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.8405, size = 1804, normalized size = 12.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*a*b^4 + 8*(2*B*a^3 + A*a^2*b)*c^2 + 2*(B*b^5 - 8*A*a^2*c^3 + 6*(2
*B*a^2*b + A*a*b^2)*c^2 - (7*B*a*b^3 + A*b^4)*c)*x^2 - (B*a*b^3 - 6*B*a^2*b
*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c^3)*x^4 + (B*b^4 - 6*B*a
*b^2*c + 4*A*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b
^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2*(6*B
*a^2*b^2 + A*a*b^3)*c + (B*a*b^4 - 8*B*a^2*b^2*c + 16*B*a^3*c^2 + (B*b^4*c
- 8*B*a*b^2*c^2 + 16*B*a^2*c^3)*x^4 + (B*b^5 - 8*B*a*b^3*c + 16*B*a^2*b*c^2
)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b
^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*
c^4)*x^2), 1/4*(2*B*a*b^4 + 8*(2*B*a^3 + A*a^2*b)*c^2 + 2*(B*b^5 - 8*A*a^2*
c^3 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (7*B*a*b^3 + A*b^4)*c)*x^2 + 2*(B*a*b^3
 - 6*B*a^2*b*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c^3)*x^4 + (B
*b^4 - 6*B*a*b^2*c + 4*A*a*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2
+ b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(6*B*a^2*b^2 + A*a*b^3)*c + (B*a
*b^4 - 8*B*a^2*b^2*c + 16*B*a^3*c^2 + (B*b^4*c - 8*B*a*b^2*c^2 + 16*B*a^2*c
```

```
^3)*x^4 + (B*b^5 - 8*B*a*b^3*c + 16*B*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a
))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^
2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

Sympy [B] time = 18.004, size = 916, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (B/(4*c**2) - sqrt(-(4*a*c - b**2)**3)*(-4*A*a*c**2 + 6*B*a*b*c - B*b**3)/(
4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x**2 +
 (-2*A*a*b*c + 8*B*a**2*c - B*a*b**2 - 32*a**2*c**3*(B/(4*c**2) - sqrt(-(4*
a*c - b**2)**3)*(-4*A*a*c**2 + 6*B*a*b*c - B*b**3)/(4*c**2*(64*a**3*c**3 -
48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 16*a*b**2*c**2*(B/(4*c**2) - sq
rt(-(4*a*c - b**2)**3)*(-4*A*a*c**2 + 6*B*a*b*c - B*b**3)/(4*c**2*(64*a**3*
c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 2*b**4*c*(B/(4*c**2) - s
qrt(-(4*a*c - b**2)**3)*(-4*A*a*c**2 + 6*B*a*b*c - B*b**3)/(4*c**2*(64*a**3
*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(-4*A*a*c**2 + 6*B*a*b*c
 - B*b**3)) + (B/(4*c**2) + sqrt(-(4*a*c - b**2)**3)*(-4*A*a*c**2 + 6*B*a*b
*c - B*b**3)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6
)))*log(x**2 + (-2*A*a*b*c + 8*B*a**2*c - B*a*b**2 - 32*a**2*c**3*(B/(4*c**
2) + sqrt(-(4*a*c - b**2)**3)*(-4*A*a*c**2 + 6*B*a*b*c - B*b**3)/(4*c**2*(6
4*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 16*a*b**2*c**2*(B
/(4*c**2) + sqrt(-(4*a*c - b**2)**3)*(-4*A*a*c**2 + 6*B*a*b*c - B*b**3)/(4*
c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 2*b**4*c*(
B/(4*c**2) + sqrt(-(4*a*c - b**2)**3)*(-4*A*a*c**2 + 6*B*a*b*c - B*b**3)/(4
*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(-4*A*a*c*
*2 + 6*B*a*b*c - B*b**3)) + (A*a*b*c + 2*B*a**2*c - B*a*b**2 + x**2*(-2*A*a
*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3))/(8*a**2*c**3 - 2*a*b**2*c**2 + x**4
*(8*a*c**4 - 2*b**2*c**3) + x**2*(8*a*b*c**3 - 2*b**3*c**2))
```

Giac [A] time = 19.9281, size = 262, normalized size = 1.78

$$-\frac{\left(Bb^3 - 6\,Babc + 4\,Aac^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(b^2c^2 - 4\,ac^3\right)\sqrt{-b^2+4\,ac}} + \frac{B\log\left(cx^4+bx^2+a\right)}{4\,c^2} - \frac{Bb^2cx^4 - 4\,Bac^2x^4 - Bb^3x^2 + 2\,Babcx^2 + 2\,Ab^2c}{4\,\left(cx^4+bx^2+a\right)\left(b^2c^2 -\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^3 - 6*B*a*b*c + 4*A*a*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c
))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/4*B*log(c*x^4 + b*x^2 + a)/
c^2 - 1/4*(B*b^2*c*x^4 - 4*B*a*c^2*x^4 - B*b^3*x^2 + 2*B*a*b*c*x^2 + 2*A*b^
2*c*x^2 - 4*A*a*c^2*x^2 - B*a*b^2 + 2*A*a*b*c)/((c*x^4 + b*x^2 + a)*(b^2*c^
2 - 4*a*c^3))
```

3.114 $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] -(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(2*c*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.113441, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1251, 777, 618, 206}

$$-\frac{x^2\left(-2aBc-Abc+b^2B\right)+a(bB-2Ac)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(2*c*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(b^2 - 4*a*c)^(3/2)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3 \left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{x(A+Bx)}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$

$$=-\frac{a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(Ab-2aB)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{2\left(b^2-4ac\right)}$$

$$=-\frac{a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(Ab-2aB)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{b^2-4ac}$$

$$=-\frac{a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.0869724, size = 111, normalized size = 1.04

$$\frac{-2ac\left(A+Bx^2\right)+abB+bx^2(bB-Ac)}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}-\frac{(Ab-2aB)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*
x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-
b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.01, size = 158, normalized size = 1.5

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(-\frac{\left(Abc+2\,aBc-b^2B\right)x^2}{c\left(4\,ac-b^2\right)}-\frac{a\,(2\,Ac-bB)}{c\left(4\,ac-b^2\right)}\right)-Ab\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\left(4\,ac-b^2\right)^{-\frac{3}{2}}+2\,\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-(A*b*c+2*B*a*c-B*b^2)/c/(4*a*c-b^2)*x^2-a*(2*A*c-B*b)/(4*a*c-b^2)/c)/
(c*x^4+b*x^2+a)-1/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A
*b+2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.52872, size = 1131, normalized size = 10.57

$$\left[-\frac{Bab^3 + 8\,Aa^2c^2 + \left(Bb^4 + 4\left(2\,Ba^2 + Aab\right)c^2 - \left(6\,Bab^2 + Ab^3\right)c\right)x^2 - \left(\left(2\,Ba - Ab\right)c^2x^4 + \left(2\,Bab - Ab^2\right)cx^2 + \left(2\,Ba^2\right.\right.}{2\left(ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3 + \left(b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4\right)x^4 + \left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(B*a*b^3 + 8*A*a^2*c^2 + (B*b^4 + 4*(2*B*a^2 + A*a*b)*c^2 - (6*B*a*b^
2 + A*b^3)*c)*x^2 - ((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B
*a^2 - A*a*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c
 + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2*(2*B*a^2*b + A
*a*b^2)*c)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(B*a*b^3
 + 8*A*a^2*c^2 + (B*b^4 + 4*(2*B*a^2 + A*a*b)*c^2 - (6*B*a*b^2 + A*b^3)*c)*
x^2 - 2*((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B*a^2 - A*a*b
)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)) - 2*(2*B*a^2*b + A*a*b^2)*c)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*x^2)]
```

Sympy [B] time = 6.08602, size = 394, normalized size = 3.68

$$-\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(-Ab + 2Ba\right)\log\left(x^2 + \frac{-Ab^2+2Bab-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(-Ab+2Ba)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(-Ab+2Ba)-b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}(-Ab+2Ba)}{-2Abc+4Bac}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a)*log(x**2 + (-A*b**2 + 2*B*a*b -
16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a) + 8*a*b**2*c*sqrt(-1
/(4*a*c - b**2)**3)*(-A*b + 2*B*a) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(-A*b
+ 2*B*a))/(-2*A*b*c + 4*B*a*c))/2 + sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*
a)*log(x**2 + (-A*b**2 + 2*B*a*b + 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*
(-A*b + 2*B*a) - 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a) + b**
4*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a))/(-2*A*b*c + 4*B*a*c))/2 - (2*A
*a*c - B*a*b + x**2*(A*b*c + 2*B*a*c - B*b**2))/(8*a**2*c**2 - 2*a*b**2*c +
 x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c))
```

Giac [A] time = 19.6521, size = 162, normalized size = 1.51

$$-\frac{(2\,Ba-Ab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2-4\,ac\right)\sqrt{-b^2+4\,ac}}-\frac{Bb^2x^2-2\,Bacx^2-Abcx^2+Bab-2\,Aac}{2\left(cx^4+bx^2+a\right)\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -(2*B*a - A*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt
(-b^2 + 4*a*c)) - 1/2*(B*b^2*x^2 - 2*B*a*c*x^2 - A*b*c*x^2 + B*a*b - 2*A*a*
c)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
```

3.115 $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=94

$$-\frac{-2aB+x^2(-(bB-2Ac))+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] -(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2
)
```

Rubi [A] time = 0.0876177, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1247, 638, 618, 206}

$$-\frac{-2aB+x^2(-(bB-2Ac))+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2
)
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)$$
$$=-\frac{Ab-2aB-(bB-2Ac)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(bB-2Ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{2\left(b^2-4ac\right)}$$
$$=-\frac{Ab-2aB-(bB-2Ac)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(bB-2Ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{b^2-4ac}$$
$$=-\frac{Ab-2aB-(bB-2Ac)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.0842634, size = 101, normalized size = 1.07

$$\frac{\frac{2(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{B\left(2a+bx^2\right)-A\left(b+2cx^2\right)}{a+bx^2+cx^4}}{2\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((B*(2*a + b*x^2) - A*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + (2*(b*B - 2*A*c)
*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*
a*c))
```

Maple [A] time = 0.006, size = 127, normalized size = 1.4

$$\frac{(2\,Ac-bB)\,x^2+Ab-2\,aB}{\left(8\,ac-2\,b^2\right)\left(cx^4+bx^2+a\right)}+2\,\frac{Ac}{\left(4\,ac-b^2\right)^{3/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)-bB\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\left(4\,ac-b^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*((2*A*c-B*b)*x^2+A*b-2*a*B)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)^(
3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*c-1/(4*a*c-b^2)^(3/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.60384, size = 1026, normalized size = 10.91

$$\left[\frac{2\,Bab^2 - Ab^3 + \left(Bb^3 + 8\,Aac^2 - 2\left(2\,Bab + Ab^2\right)c\right)x^2 + \left(\left(Bbc - 2\,Ac^2\right)x^4 + Bab - 2\,Aac + \left(Bb^2 - 2\,Abc\right)x^2\right)\sqrt{b^2 - 4}}{2\left(ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + \left(b^4c - 8\,ab^2c^2 + 16\,a^2c^3\right)x^4 + \left(b^5 - 8\,ab^3c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*b^2 - A*b^3 + (B*b^3 + 8*A*a*c^2 - 2*(2*B*a*b + A*b^2)*c)*x^2 +
 ((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B*b^2 - 2*A*b*c)*x^2)*sqrt(b^2
 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2
 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(2*B*a^2 - A*a*b)*c)/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*x^2), 1/2*(2*B*a*b^2 - A*b^3 + (B*b^3 + 8*A*a*c^2 - 2*(2*B
*a*b + A*b^2)*c)*x^2 - 2*((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B*b^2
- 2*A*b*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)
/(b^2 - 4*a*c)) - 4*(2*B*a^2 - A*a*b)*c)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)
*x^2)]
```

Sympy [B] time = 3.87652, size = 374, normalized size = 3.98

$$\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(-2Ac+Bb\right)\log\left(x^2+\frac{-2Abc+Bb^2-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(-2Ac+Bb\right)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(-2Ac+Bb\right)-b^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(-2Ac+Bb\right)}{-4Ac^2+2Bbc}\right)}{2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b)*log(x**2 + (-2*A*b*c + B*b**2 - 1
6*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b) + 8*a*b**2*c*sqrt(-1/
(4*a*c - b**2)**3)*(-2*A*c + B*b) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c
 + B*b))/(-4*A*c**2 + 2*B*b*c))/2 - sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*
b)*log(x**2 + (-2*A*b*c + B*b**2 + 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*
(-2*A*c + B*b) - 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b) + b**
4*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b))/(-4*A*c**2 + 2*B*b*c))/2 - (-A
*b + 2*B*a + x**2*(-2*A*c + B*b))/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2
*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

Giac [A] time = 20.2415, size = 138, normalized size = 1.47

$$\frac{(Bb - 2\,Ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2 - 4\,ac\right)\sqrt{-b^2 + 4\,ac}} + \frac{Bbx^2 - 2\,Acx^2 + 2\,Ba - Ab}{2\left(cx^4 + bx^2 + a\right)\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] (B*b - 2*A*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(
-b^2 + 4*a*c)) + 1/2*(B*b*x^2 - 2*A*c*x^2 + 2*B*a - A*b)/((c*x^4 + b*x^2 +
a)*(b^2 - 4*a*c))
```

3.116 $\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=150

$$\frac{\left(4a^2Bc+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))-}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c))*ArcTanh[(b + 2*c*x^2)/Sqrt
[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (A*Log[x])/a^2 - (A*Log[a + b
*x^2 + c*x^4])/(4*a^2)
```

Rubi [A] time = 0.33067, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{\left(4a^2Bc+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}-\frac{-A\left(b^2-2ac\right)+cx^2(-(Ab-2aB))-}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c))*ArcTanh[(b + 2*c*x^2)/Sqrt
[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (A*Log[x])/a^2 - (A*Log[a + b
*x^2 + c*x^4])/(4*a^2)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\mathrm{Subst}\left(\int \frac{-A\left(b^2-4ac\right)-(Ab-2aB)cx}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\mathrm{Subst}\left(\int \left(\frac{A\left(-b^2+4ac\right)}{ax} + \frac{2a^2Bc+A\left(b^3-5abc\right)+Ac\left(b^2-4ac\right)x}{a\left(a+bx+cx^2\right)}\right) d}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{\mathrm{Subst}\left(\int \frac{2a^2Bc+A\left(b^3-5abc\right)+Ac\left(b^2-4ac\right)x}{a+bx+cx^2}\,dx,\right.}{2a^2\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{A\,\mathrm{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} - \frac{\left(4a^2Bc\right.}{}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{\left(4a^2Bc+A\left(b^3\right.\right.}{}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(4a^2Bc+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} + \frac{A\log(}{a^2}$$

Mathematica [A] time = 0.351148, size = 243, normalized size = 1.62

$$\frac{-\frac{\left(4a^2Bc+A\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\left(4a^2Bc+A\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}-}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + 4*A*Log[x] - ((4*a^2*B*c + A*(b^3 - 6*a*b*c + b^2*Sqrt[b
^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/(b^2 - 4*a*c)^(3/2) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*
a*c] + 4*a*c*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2
 - 4*a*c)^(3/2))/(4*a^2)
```

Maple [B] time = 0.017, size = 361, normalized size = 2.4

$$-\frac{cx^2Ab}{2\,a\left(cx^4+bx^2+a\right)\left(4\,ac-b^2\right)}+\frac{cx^2B}{\left(cx^4+bx^2+a\right)\left(4\,ac-b^2\right)}+\frac{Ac}{\left(cx^4+bx^2+a\right)\left(4\,ac-b^2\right)}-\frac{Ab^2}{2\,a\left(cx^4+bx^2+a\right)\left(4\,ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*A*b+1/(c*x^4+b*x^2+a)*c/(4*a*c-b^2
)*x^2*B+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*c-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)
*A*b^2+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*B-1/a/(4*a*c-b^2)*c*ln(c*x^4+b*x^2
+a)*A+1/4/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*A*b^2-3/a/(4*a*c-b^2)^(3/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b*c+1/2/a^2/(4*a*c-b^2)^(3/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^3+2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*B*c+A*ln(x)/a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 5.75849, size = 2160, normalized size = 14.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*b^3 - 2*A*a*b^4 - 16*A*a^3*c^2 - 2*(4*(2*B*a^3 - A*a^2*b)*c^
2 - (2*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (A*a*b^3 + (A*b^3*c + 2*(2*B*a^2 - 3*A
*a*b)*c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3 - 3
*A*a^2*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (
2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(2*B*a^3*b - 3*A*a
^2*b^2)*c + (A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*b^4*c - 8*A*a*b^2*
c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)*x^2)*log(c
*x^4 + b*x^2 + a) - 4*(A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*b^4*c -
8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)*
x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c
^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), -1/4*(2
*B*a^2*b^3 - 2*A*a*b^4 - 16*A*a^3*c^2 - 2*(4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B
*a^2*b^2 - A*a*b^3)*c)*x^2 - 2*(A*a*b^3 + (A*b^3*c + 2*(2*B*a^2 - 3*A*a*b)*
c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3 - 3*A*a^2
*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)) - 4*(2*B*a^3*b - 3*A*a^2*b^2)*c + (A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^
3*c^2 + (A*b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c
 + 16*A*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) - 4*(A*a*b^4 - 8*A*a^2*b^2*c
 + 16*A*a^3*c^2 + (A*b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8
*A*a*b^3*c + 16*A*a^2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c
+ 16*a^4*b*c^2)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 19.8669, size = 271, normalized size = 1.81

$$-\frac{\left(Ab^3+4\,Ba^2c-6\,Aabc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}}-\frac{A\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{A\log\left(x^2\right)}{2\,a^2}+\frac{Ab^2cx^4-4\,Aac^2x^4+Ab^3x^2-4\,B}{4\left(cx^4+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c
))/((a^2*b^2 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/4*A*log(c*x^4 + b*x^2 + a)/
a^2 + 1/2*A*log(x^2)/a^2 + 1/4*(A*b^2*c*x^4 - 4*A*a*c^2*x^4 + A*b^3*x^2 - 4
*B*a^2*c*x^2 - 2*A*a*b*c*x^2 - 2*B*a^2*b + 3*A*a*b^2 - 8*A*a^2*c)/((c*x^4 +
 b*x^2 + a)*(a^2*b^2 - 4*a^3*c))
```

3.117 $\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=223

$$\frac{\left(abB\left(b^2-6ac\right)-2A\left(6a^2c^2-6ab^2c+b^4\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}}-\frac{-6aAc-abB+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}+\frac{(2Ab-aB)\log\left(a+bx^2+cx\right.}{4a^3}$$

```
[Out] -(2*A*b^2 - a*b*B - 6*a*A*c)/(2*a^2*(b^2 - 4*a*c)*x^2) - (a*b*B - A*(b^2 -
2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) +
 ((a*b*B*(b^2 - 6*a*c) - 2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2))*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*A*b - a*B)*Log
[x])/a^3 + ((2*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rubi [A] time = 0.418907, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{\left(abB\left(b^2-6ac\right)-2A\left(6a^2c^2-6ab^2c+b^4\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}}-\frac{-6aAc-abB+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}+\frac{(2Ab-aB)\log\left(a+bx^2+cx\right.}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(2*A*b^2 - a*b*B - 6*a*A*c)/(2*a^2*(b^2 - 4*a*c)*x^2) - (a*b*B - A*(b^2 -
2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) +
 ((a*b*B*(b^2 - 6*a*c) - 2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2))*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*A*b - a*B)*Log
[x])/a^3 + ((2*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-2Ab^2+abB+6aAc-2(Ab-2aB)cx}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{-2Ab^2+abB+6aAc}{ax^2} + \frac{(-2Ab+aB)\left(-b^2+4ac\right)}{a^2x} + \frac{abB\ldots}{\ldots}\right.\right.}{2a\left(b^2\ldots\right.}$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{(2Ab-aB)\log(x)}{a^3} - \frac{\text{Subst}\left(\ldots\right.}{\ldots}$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{(2Ab-aB)\log(x)}{a^3} + \frac{(2Ab-\ldots}{\ldots}$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{(2Ab-aB)\log(x)}{a^3} + \frac{(2Ab-\ldots}{\ldots}$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} + \frac{\left(abB\left(b^2-6ac\right)-2A\left(b^4-6\ldots\right.\right.}{2a^3\left(b\ldots\right.}$$

Mathematica [A] time = 0.551445, size = 379, normalized size = 1.7

$$\frac{\frac{\left(2A\left(6a^2c^2+b^3\sqrt{b^2-4ac}-6ab^2c-4abc\sqrt{b^2-4ac}+b^4\right)+aB\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}+6abc-b^3\right)\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\left(2A\left(-6a^2c^2+b^3\sqrt{b^2-4ac}+6ab^2c-\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*A)/x^2 - (2*a*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a*b*c + b^2
*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*(-2*A*b + a
*B)*Log[x] + ((a*B*(-b^3 + 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2
 - 4*a*c]) + 2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a
*b*c*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c
)^(3/2) + ((a*B*(b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4
*a*c]) + 2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*
c*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^3)
```

Maple [B] time = 0.023, size = 622, normalized size = 2.8

$$-\frac{c^2x^2A}{a\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)}+\frac{cx^2Ab^2}{2a^2\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)}-\frac{cx^2bB}{2a\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)}-\frac{3Abc}{2a\left(cx^4+bx^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2*A+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c
-b^2)*x^2*A*b^2-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b*B-3/2/a/(c*x^4+b*
x^2+a)/(4*a*c-b^2)*A*b*c+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A*b^3+1/(c*x^4
+b*x^2+a)/(4*a*c-b^2)*B*c-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B*b^2+2/a^2/(4*
a*c-b^2)*c*ln(c*x^4+b*x^2+a)*A*b-1/2/a^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*A*b^
3-1/a/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*B+1/4/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+
a)*b^2*B-6/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*c^2+
6/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^2*c-1/a^3
/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^4-3/a/(4*a*c-b
^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*B*c+1/2/a^2/(4*a*c-b^2)^(
3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*b^3-1/2*A/a^2/x^2-2/a^3*ln(x)*
A*b+1/a^2*ln(x)*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 11.263, size = 3426, normalized size = 15.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*A*a^2*b^4 - 16*A*a^3*b^2*c + 32*A*a^4*c^2 + 2*(24*A*a^3*c^3 + 2*(2
*B*a^3*b - 7*A*a^2*b^2)*c^2 - (B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 - 2*(B*a^2*b^4
 - 2*A*a*b^5 + 4*(2*B*a^4 - 7*A*a^3*b)*c^2 - 3*(2*B*a^3*b^2 - 5*A*a^2*b^3)*
c)*x^2 + ((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*b^3 - 2*A*b^4)
*c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(B*a^2*b^2 - 2*A*a*b^3)*c
)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3*c^2 - 6*(B*a^3*b - 2*A*a^2*b^2)*c
)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^
2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((16*(B*a^3 - 2*A*a^2*b)*c
^3 - 8*(B*a^2*b^2 - 2*A*a*b^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5
- 2*A*b^6 + 16*(B*a^3*b - 2*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x
^4 + (B*a^2*b^4 - 2*A*a*b^5 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2
*A*a^2*b^3)*c)*x^2)*log(c*x^4 + b*x^2 + a) - 4*((16*(B*a^3 - 2*A*a^2*b)*c^3
 - 8*(B*a^2*b^2 - 2*A*a*b^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 -
2*A*b^6 + 16*(B*a^3*b - 2*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4
 + (B*a^2*b^4 - 2*A*a*b^5 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A
*a^2*b^3)*c)*x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6
*c^2)*x^2), -1/4*(2*A*a^2*b^4 - 16*A*a^3*b^2*c + 32*A*a^4*c^2 + 2*(24*A*a^3
*c^3 + 2*(2*B*a^3*b - 7*A*a^2*b^2)*c^2 - (B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 - 2
*(B*a^2*b^4 - 2*A*a*b^5 + 4*(2*B*a^4 - 7*A*a^3*b)*c^2 - 3*(2*B*a^3*b^2 - 5*
A*a^2*b^3)*c)*x^2 + 2*((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*b
^3 - 2*A*b^4)*c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(B*a^2*b^2 -
 2*A*a*b^3)*c)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3*c^2 - 6*(B*a^3*b - 2
*A*a^2*b^2)*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*
a*c)/(b^2 - 4*a*c)) + ((16*(B*a^3 - 2*A*a^2*b)*c^3 - 8*(B*a^2*b^2 - 2*A*a*b
^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 - 2*A*b^6 + 16*(B*a^3*b - 2
*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 + (B*a^2*b^4 - 2*A*a*b^5
 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A*a^2*b^3)*c)*x^2)*log(c*x
^4 + b*x^2 + a) - 4*((16*(B*a^3 - 2*A*a^2*b)*c^3 - 8*(B*a^2*b^2 - 2*A*a*b^3
)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 - 2*A*b^6 + 16*(B*a^3*b - 2*A
*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 + (B*a^2*b^4 - 2*A*a*b^5 +
 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A*a^2*b^3)*c)*x^2)*log(x))/(
(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*
a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 19.6893, size = 338, normalized size = 1.52

$$-\frac{\left(Bab^3 - 2\,Ab^4 - 6\,Ba^2bc + 12\,Aab^2c - 12\,Aa^2c^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^3b^2 - 4\,a^4c\right)\sqrt{-b^2 + 4\,ac}} + \frac{Babcx^4 - 2\,Ab^2cx^4 + 6\,Aac^2x^4 + Bab^2x^2 - 2\,\ldots}{2\left(cx^6 + bx^4 + \ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^3 - 2*A*b^4 - 6*B*a^2*b*c + 12*A*a*b^2*c - 12*A*a^2*c^2)*arctan
((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c))
 + 1/2*(B*a*b*c*x^4 - 2*A*b^2*c*x^4 + 6*A*a*c^2*x^4 + B*a*b^2*x^2 - 2*A*b^3
*x^2 - 2*B*a^2*c*x^2 + 7*A*a*b*c*x^2 - A*a*b^2 + 4*A*a^2*c)/((c*x^6 + b*x^4
 + a*x^2)*(a^2*b^2 - 4*a^3*c)) - 1/4*(B*a - 2*A*b)*log(c*x^4 + b*x^2 + a)/a
^3 + 1/2*(B*a - 2*A*b)*log(x^2)/a^3
```

3.118 $\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=425

$$-\frac{\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+\cdots}{\sqrt{b^2-4ac}}\right.}{2\cdots}$$

```
[Out] ((3*b^2*B - A*b*c - 10*a*B*c)*x)/(2*c^2*(b^2 - 4*a*c)) - ((b*B - 2*A*c)*x^3
)/(2*c*(b^2 - 4*a*c)) - (x^5*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*B - A*b^2*c - 13*a*b*B*c + 6*a*A*c^2 -
 (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2
]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*B - A*b^2*c
- 13*a*b*B*c + 6*a*A*c^2 + (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2
+ 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c
]])
```

Rubi [A] time = 3.67472, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1275, 1279, 1166, 205}

$$-\frac{\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{cx}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+\cdots}{\sqrt{b^2-4ac}}\right.}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((3*b^2*B - A*b*c - 10*a*B*c)*x)/(2*c^2*(b^2 - 4*a*c)) - ((b*B - 2*A*c)*x^3
)/(2*c*(b^2 - 4*a*c)) - (x^5*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*B - A*b^2*c - 13*a*b*B*c + 6*a*A*c^2 -
 (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2
]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*B - A*b^2*c
- 13*a*b*B*c + 6*a*A*c^2 + (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2
+ 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c
]])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx=-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\int\frac{x^4\left(5(Ab-2aB)-3(bB-2Ac)x^2\right)}{a+bx^2+cx^4}dx}{2\left(b^2-4ac\right)}$$

$$=-\frac{(bB-2Ac)x^3}{2c\left(b^2-4ac\right)}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int\frac{x^2\left(-9a(bB-2Ac)-3\left(3b^2B-Abc-10aBc\right)x^2\right)}{a+bx^2+cx^4}dx}{6c\left(b^2-4ac\right)}$$

$$=\frac{\left(3b^2B-Abc-10aBc\right)x}{2c^2\left(b^2-4ac\right)}-\frac{(bB-2Ac)x^3}{2c\left(b^2-4ac\right)}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\int\frac{-3a\left(3b^2B-Abc-\right.}{}}{}$$

$$=\frac{\left(3b^2B-Abc-10aBc\right)x}{2c^2\left(b^2-4ac\right)}-\frac{(bB-2Ac)x^3}{2c\left(b^2-4ac\right)}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b^3B-Ab^2c\right.}{}$$

$$=\frac{\left(3b^2B-Abc-10aBc\right)x}{2c^2\left(b^2-4ac\right)}-\frac{(bB-2Ac)x^3}{2c\left(b^2-4ac\right)}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b^3B-Ab^2c\right.}{}$$

Mathematica [A] time = 1.31145, size = 455, normalized size = 1.07

$$\frac{\frac{2\sqrt{c}x\left(-2a^2Bc+a\left(-bc\left(A+3Bx^2\right)+2Ac^2x^2+b^2B\right)+b^2x^2(bB-Ac)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{2}\left(2ac^2\left(3A\sqrt{b^2-4ac}-10aB\right)+b^3\left(3B\sqrt{b^2-4ac}+Ac\right)+b^2c\left(19aB-A\sqrt{b^2-4ac}\right)-abc\left(13B\sqrt{b^2-}\right.\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*B*Sqrt[c]*x + (2*Sqrt[c]*x*(-2*a^2*B*c + b^2*(b*B - A*c)*x^2 + a*(b^2*B
+ 2*A*c^2*x^2 - b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
```

```
(Sqrt[2]*(-3*b^4*B + b^2*c*(19*a*B - A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(-10*a*
B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - a*b*c*(8*A
*c + 13*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3
*b^4*B - b^2*c*(19*a*B + A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(10*a*B + 3*A*Sqrt[
b^2 - 4*a*c]) + a*b*c*(8*A*c - 13*B*Sqrt[b^2 - 4*a*c]) + b^3*(-(A*c) + 3*B*
Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(5/2))
```

Maple [B] time = 0.04, size = 1507, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*A-1/2/c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2
)*x*b^2*B+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*B+5/(4
*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*B+1/4/c/(4*a*c-b^2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*A*b^2-3/4/c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*B-1/
4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2+3/4/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*b^3*B+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*b*B+1/2/c/(c*x^4+b*x^2+a
)*a/(4*a*c-b^2)*x*A*b-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*A*b^3-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^3+1
3/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*B+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*B-13/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*a*b*B+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*B+2/(4
*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A*b+2/(4*a*c-b^2)/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A*b+B/c^2*x+1/c/(c*x^4+b*x^2+a)*a^2/(4*
a*c-b^2)*x*B-3/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A+3/2/(4*a*c-b^2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*a*A-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^3*B+1/2/c/(c*x
^4+b*x^2+a)/(4*a*c-b^2)*x^3*A*b^2-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*a*b^2*B-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*a*b^2*B
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 24.2109, size = 15583, normalized size = 36.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(B*b^2*c - 4*B*a*c^2)*x^5 + 2*(3*B*b^3 + 2*A*a*c^2 - (11*B*a*b + A*b
^2)*c)*x^3 - sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (
b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4
 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 +
80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 1
2*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^
6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2
200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6
*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^
3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*
a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((189*
B^4*a^2*b^6 - 135*A*B^3*a*b^7 + 324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*
a^2*b^2)*c^4 - (2500*B^4*a^5 + 2500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 64
7*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3
 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3
*a^2*b^5 - 5*A^2*B^2*a*b^6)*c)*x + 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2
*B*a^4 + A^3*a^3*b)*c^6 - 8*(500*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*
b^2 + 11*A^3*a^2*b^3)*c^5 + (11360*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A
^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 +
 153*A^2*B*a*b^6 + A^3*b^7)*c^3 + 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2
*B*b^8)*c^2 - 27*(17*B^3*a*b^8 + A*B^2*b^9)*c - (3*B*b^9*c^5 - 768*A*a^4*c^
10 + 128*(8*B*a^4*b + 5*A*a^3*b^2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8
+ 24*(14*B*a^2*b^5 + A*a*b^6)*c^7 - (52*B*a*b^7 + A*b^8)*c^6)*sqrt((81*B^4*
b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5
 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3
 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^
4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6
)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*
c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3
 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5
- 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2
*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4
+ 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4
- 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)
*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B
^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqr
```

```
t(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*
c^3)*x^2)*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^
3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A
^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 12*a*b^4*c^6 + 48
*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B
^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b
 + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^
2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*
a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*
b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c
^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((189*B^4*a^2*b^6 - 13
5*A*B^3*a*b^7 + 324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (
2500*B^4*a^5 + 2500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3
 - 5*A^4*a*b^4)*c^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a
^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2
*B^2*a*b^6)*c)*x - 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3
*b)*c^6 - 8*(500*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2
*b^3)*c^5 + (11360*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 1
7*A^3*a*b^5)*c^4 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6
 + A^3*b^7)*c^3 + 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27
*(17*B^3*a*b^8 + A*B^2*b^9)*c - (3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^
4*b + 5*A*a^3*b^2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b
^5 + A*a*b^6)*c^7 - (52*B*a*b^7 + A*b^8)*c^6)*sqrt((81*B^4*b^8 + 81*A^4*a^2
*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4
 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4
 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)
*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B
^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2
*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4
 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 12*a*b^4*c^6 +
 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^
2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3
*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3
*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B
^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B
^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^
6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2)*(a*b^2*c^
2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-
(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2
*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*
(35*B^2*a*b^5 + 2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*
B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a
^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^
2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B
^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^1
0 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^
6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((189*B^4*a^2*b^6 - 135*A*B^3*a*b^7 +
324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (2500*B^4*a^5 + 2
500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c
^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B
*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2*B^2*a*b^6)*c)*x
 + 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3*b)*c^6 - 8*(500
*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2*b^3)*c^5 + (113
60*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4
 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6 + A^3*b^7)*c^3
+ 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27*(17*B^3*a*b^8 +
 A*B^2*b^9)*c + (3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^4*b + 5*A*a^3*b^
```

```
2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b^5 + A*a*b^6)*c^
7 - (52*B*a*b^7 + A*b^8)*c^6)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^
2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3
*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3
*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B
^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B
^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqr
t(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*
a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 -
 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
- 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A
^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^
2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3
*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*
A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4
*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 +
(b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-(9*B^2*b^7 + 60*
(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3
)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 +
2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt(
(81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*
b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3
*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*
B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2
*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^1
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c
^7 - 64*a^3*c^8))*log((189*B^4*a^2*b^6 - 135*A*B^3*a*b^7 + 324*A^4*a^3*c^5
- 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (2500*B^4*a^5 + 2500*A*B^3*a^4*b
- 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c^3 + 9*(625*B^4*
a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27
*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2*B^2*a*b^6)*c)*x - 1/2*sqrt(1/2)
*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3*b)*c^6 - 8*(500*B^3*a^5 + 930*A
*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2*b^3)*c^5 + (11360*B^3*a^4*b^2 +
 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4 - (8818*B^3*a^3
*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6 + A^3*b^7)*c^3 + 9*(329*B^3*a^2
*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27*(17*B^3*a*b^8 + A*B^2*b^9)*c +
(3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^4*b + 5*A*a^3*b^2)*c^9 - 192*(5*
B*a^3*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b^5 + A*a*b^6)*c^7 - (52*B*a*b^7
+ A*b^8)*c^6)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A
^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^
2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3
*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*
A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*B^2*b^7 +
60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*
b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5
 + 2*A*B*b^6)*c - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sq
rt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4
*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*
A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A
^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*
A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*
c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^
2*c^7 - 64*a^3*c^8))) + 2*(3*B*a*b^2 - (10*B*a^2 + A*a*b)*c)*x)/(a*b^2*c^2
- 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)
```

Sympy [B] time = 168.3, size = 1448, normalized size = 3.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] B*x/c**2 + (x**3*(-2*A*a*c**2 + A*b**2*c + 3*B*a*b*c - B*b**3) + x*(A*a*b*c
 + 2*B*a**2*c - B*a*b**2))/(8*a**2*c**3 - 2*a*b**2*c**2 + x**4*(8*a*c**4 -
2*b**2*c**3) + x**2*(8*a*b*c**3 - 2*b**3*c**2)) + RootSum(_t**4*(1048576*a*
*6*c**11 - 1572864*a**5*b**2*c**10 + 983040*a**4*b**4*c**9 - 327680*a**3*b*
*6*c**8 + 61440*a**2*b**8*c**7 - 6144*a*b**10*c**6 + 256*b**12*c**5) + _t**
2*(-61440*A**2*a**5*b*c**7 + 61440*A**2*a**4*b**3*c**6 - 24064*A**2*a**3*b*
*5*c**5 + 4608*A**2*a**2*b**7*c**4 - 432*A**2*a*b**9*c**3 + 16*A**2*b**11*c
**2 - 245760*A*B*a**6*c**7 + 491520*A*B*a**5*b**2*c**6 - 358400*A*B*a**4*b*
*4*c**5 + 129024*A*B*a**3*b**6*c**4 - 24768*A*B*a**2*b**8*c**3 + 2432*A*B*a
*b**10*c**2 - 96*A*B*b**12*c + 430080*B**2*a**6*b*c**6 - 716800*B**2*a**5*b
**3*c**5 + 483840*B**2*a**4*b**5*c**4 - 170496*B**2*a**3*b**7*c**3 + 33232*
B**2*a**2*b**9*c**2 - 3408*B**2*a*b**11*c + 144*B**2*b**13) + 1296*A**4*a**
5*c**4 - 360*A**4*a**4*b**2*c**3 + 25*A**4*a**3*b**4*c**2 - 5472*A**3*B*a**
5*b*c**3 + 1840*A**3*B*a**4*b**3*c**2 - 150*A**3*B*a**3*b**5*c + 7200*A**2*
B**2*a**6*c**3 + 3264*A**2*B**2*a**5*b**2*c**2 - 2070*A**2*B**2*a**4*b**4*c
 + 225*A**2*B**2*a**3*b**6 - 15200*A*B**3*a**6*b*c**2 + 6192*A*B**3*a**5*b*
*3*c - 630*A*B**3*a**4*b**5 + 10000*B**4*a**7*c**2 - 4200*B**4*a**6*b**2*c
+ 441*B**4*a**5*b**4, Lambda(_t, _t*log(x + (-49152*_t**3*A*a**4*c**10 + 40
960*_t**3*A*a**3*b**2*c**9 - 12288*_t**3*A*a**2*b**4*c**8 + 1536*_t**3*A*a*
b**6*c**7 - 64*_t**3*A*b**8*c**6 + 65536*_t**3*B*a**4*b*c**9 - 61440*_t**3*
B*a**3*b**3*c**8 + 21504*_t**3*B*a**2*b**5*c**7 - 3328*_t**3*B*a*b**7*c**6
+ 192*_t**3*B*b**9*c**5 + 1728*_t*A**3*a**3*b*c**6 - 656*_t*A**3*a**2*b**3*
c**5 + 88*_t*A**3*a*b**5*c**4 - 4*_t*A**3*b**7*c**3 + 8640*_t*A**2*B*a**4*c
**6 - 13632*_t*A**2*B*a**3*b**2*c**5 + 5124*_t*A**2*B*a**2*b**4*c**4 - 732*
_t*A**2*B*a*b**6*c**3 + 36*_t*A**2*B*b**8*c**2 - 32640*_t*A*B**2*a**4*b*c**
5 + 36336*_t*A*B**2*a**3*b**3*c**4 - 13332*_t*A*B**2*a**2*b**5*c**3 + 2016*
_t*A*B**2*a*b**7*c**2 - 108*_t*A*B**2*b**9*c - 8000*_t*B**3*a**5*c**5 + 361
60*_t*B**3*a**4*b**2*c**4 - 32476*_t*B**3*a**3*b**4*c**3 + 11592*_t*B**3*a*
*2*b**6*c**2 - 1836*_t*B**3*a*b**8*c + 108*_t*B**3*b**10)/(-324*A**4*a**3*c
**5 + 81*A**4*a**2*b**2*c**4 - 5*A**4*a*b**4*c**3 + 2268*A**3*B*a**3*b*c**4
 - 647*A**3*B*a**2*b**3*c**3 + 45*A**3*B*a*b**5*c**2 - 5016*A**2*B**2*a**3*
b**2*c**3 + 1674*A**2*B**2*a**2*b**4*c**2 - 135*A**2*B**2*a*b**6*c + 2500*A
*B**3*a**4*b*c**3 + 2727*A*B**3*a**3*b**3*c**2 - 1323*A*B**3*a**2*b**5*c +
135*A*B**3*a*b**7 + 2500*B**4*a**5*c**3 - 5625*B**4*a**4*b**2*c**2 + 1971*B
**4*a**3*b**4*c - 189*B**4*a**2*b**6))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.119 $\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=336

$$\frac{\left(-\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\tan^{-1}\left(\right.}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -((b*B - 2*A*c)*x)/(2*c*(b^2 - 4*a*c)) - (x^3*(A*b - 2*a*B - (b*B - 2*A*c)*
x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2*B + A*b*c - 6*a*B*c - (
b^3*B + A*b^2*c - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*B + A*b*c - 6*a*B*c + (b^3*B + A*b^2*c
 - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b
^2 - 4*a*c]])
```

Rubi [A] time = 1.71697, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1275, 1279, 1166, 205}

$$\frac{\left(-\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\tan^{-1}\left(\right.}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*B - 2*A*c)*x)/(2*c*(b^2 - 4*a*c)) - (x^3*(A*b - 2*a*B - (b*B - 2*A*c)*
x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2*B + A*b*c - 6*a*B*c - (
b^3*B + A*b^2*c - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*B + A*b*c - 6*a*B*c + (b^3*B + A*b^2*c
 - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b
^2 - 4*a*c]])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
```

```
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\int \frac{x^2\left(3(Ab-2aB)+(-bB+2Ac)x^2\right)}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{(bB-2Ac)x}{2c\left(b^2-4ac\right)}-\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-a(bB-2Ac)+\left(-b^2B-Abc+6aBc\right)x^2}{a+bx^2+cx^4}\,dx}{2c\left(b^2-4ac\right)}$$

$$= -\frac{(bB-2Ac)x}{2c\left(b^2-4ac\right)}-\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b^2B+Abc-6aBc-\frac{b^3B+Ab^2c-8abBc+4aAc^2}{\sqrt{b^2-4ac}}\right)\cdot}{4c\left(b^2-4ac\right)}$$

$$= -\frac{(bB-2Ac)x}{2c\left(b^2-4ac\right)}-\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b^2B+Abc-6aBc-\frac{b^3B+Ab^2c-8abBc+4aAc^2}{\sqrt{b^2-4ac}}\right)\mathrm{t}}{2\sqrt{2}c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-}}}$$

Mathematica [A] time = 0.939042, size = 362, normalized size = 1.08

$$\frac{\frac{2\sqrt{c}\left(2acx\left(A+Bx^2\right)-abBx+bx^3(Ac-bB)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b^2\left(B\sqrt{b^2-4ac}-Ac\right)+bc\left(A\sqrt{b^2-4ac}+8aB\right)-2ac\left(3B\sqrt{b^2-4ac}+2Ac\right)+b^3(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2\left(B\sqrt{b^2\right.\right.}{}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*(-(a*b*B*x) + b*(-(b*B) + A*c)*x^3 + 2*a*c*x*(A + B*x^2)))/((b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^3*B) + b*c*(8*a*B + A*Sqrt[
b^2 - 4*a*c]) + b^2*(-(A*c) + B*Sqrt[b^2 - 4*a*c]) - 2*a*c*(2*A*c + 3*B*Sqr
t[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*B + 2*a*c*
(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^2*(A*c + B*Sqrt[b^2 - 4*a*c]) + b*(-8*a
*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(3/2))
```

Maple [B] time = 0.033, size = 1030, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*(A*b*c+2*B*a*c-B*b^2)/c/(4*a*c-b^2)*x^3-1/2*a*(2*A*c-B*b)/(4*a*c-b^2)
/c*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b-1/(4*a*c-b^
2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c
*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A-1/4/(4*a*c-b^2)/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2-3/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*a*B+1/4/(4*a*c-b^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*
x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B+2/(4*a*c-b^2)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*B-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*b^3*B-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b-1/(4*a*c-
b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A-1/4/(4*a*c-b^2)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2+3/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B-1/
4/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*a*b*B-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*b^3*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 8.30557, size = 9642, normalized size = 28.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(B*b^2 - (2*B*a + A*b)*c)*x^3 + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4
+ a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*A*B
*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2
*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c
^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a
^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(
b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4
*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4*A^
4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^2*b
 + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^3 +
 9*A^2*B^2*b^4)*c)*x + 1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2
*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b
+ 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A
*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7 -
64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4*(6
*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b
)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2
- 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*
sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^
2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*
A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4
*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)
*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*
sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^
2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*
A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4
*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*
a*b^4 - 3*A*B^3*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108
*B^4*a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*
a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x - 1/2*sqrt(1/2)*(B^3*b^7 - 1
7*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b
^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^
3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256*
(3*B*a^4 - A*a^3*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2
*b^4 - A*a*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 -
2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*
b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^
2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (
60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c +
 (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4
*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A
^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7
+ 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 -
 64*a^3*c^6))) + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 +
 (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (
60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c -
 (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4
*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A
^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7
+ 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 -
 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b
 - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3
*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x +
1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3
 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 12
```

```
*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b^4 - 3*A^2*B*b^
5)*c^2 + (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A*
a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*
sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 -
 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5 - 12*(4*
A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*
B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^
3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^
4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*
b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x
^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*
A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*
B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^
3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^
4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c
)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*
b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4
*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^
2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^
3 + 9*A^2*B^2*b^4)*c)*x - 1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*
a^2*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3
*b + 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 1
8*A*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 + (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7
 - 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4
*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*
B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b
^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)
))*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a
*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a -
 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*
B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a
^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + 2*(B*a
*b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c -
4*a*b*c^2)*x^2)
```

Sympy [B] time = 68.2125, size = 1129, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -(x**3*(A*b*c + 2*B*a*c - B*b**2) + x*(2*A*a*c - B*a*b))/(8*a**2*c**2 - 2*a
*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c)) + R
ootSum(_t**4*(1048576*a**6*c**9 - 1572864*a**5*b**2*c**8 + 983040*a**4*b**4
*c**7 - 327680*a**3*b**6*c**6 + 61440*a**2*b**8*c**5 - 6144*a*b**10*c**4 +
256*b**12*c**3) + _t**2*(-12288*A**2*a**4*b*c**6 + 8192*A**2*a**3*b**3*c**5
 - 1536*A**2*a**2*b**5*c**4 + 16*A**2*b**9*c**2 + 49152*A*B*a**5*c**6 - 245
76*A*B*a**4*b**2*c**5 - 2048*A*B*a**3*b**4*c**4 + 3072*A*B*a**2*b**6*c**3 -
 576*A*B*a*b**8*c**2 + 32*A*B*b**10*c - 61440*B**2*a**5*b*c**5 + 61440*B**2
*a**4*b**3*c**4 - 24064*B**2*a**3*b**5*c**3 + 4608*B**2*a**2*b**7*c**2 - 43
2*B**2*a*b**9*c + 16*B**2*b**11) + 16*A**4*a**3*c**4 + 24*A**4*a**2*b**2*c*
*3 + 9*A**4*a*b**4*c**2 - 224*A**3*B*a**3*b*c**3 - 144*A**3*B*a**2*b**3*c**
```

```
2 + 18*A**3*B*a*b**5*c + 288*A**2*B**2*a**4*c**3 + 960*A**2*B**2*a**3*b**2*
c**2 - 198*A**2*B**2*a**2*b**4*c + 9*A**2*B**2*a*b**6 - 2016*A*B**3*a**4*b*
c**2 + 496*A*B**3*a**3*b**3*c - 30*A*B**3*a**2*b**5 + 1296*B**4*a**5*c**2 -
 360*B**4*a**4*b**2*c + 25*B**4*a**3*b**4, Lambda(_t, _t*log(x + (-16384*_t
**3*A*a**3*b*c**7 + 12288*_t**3*A*a**2*b**3*c**6 - 3072*_t**3*A*a*b**5*c**5
 + 256*_t**3*A*b**7*c**4 + 49152*_t**3*B*a**4*c**7 - 40960*_t**3*B*a**3*b**
2*c**6 + 12288*_t**3*B*a**2*b**4*c**5 - 1536*_t**3*B*a*b**6*c**4 + 64*_t**3
*B*b**8*c**3 - 64*_t*A**3*a**2*c**5 + 128*_t*A**3*a*b**2*c**4 + 4*_t*A**3*b
**4*c**3 - 768*_t*A**2*B*a**2*b*c**4 - 48*_t*A**2*B*a*b**3*c**3 + 12*_t*A**
2*B*b**5*c**2 + 1728*_t*A*B**2*a**3*c**4 + 384*_t*A*B**2*a**2*b**2*c**3 - 1
56*_t*A*B**2*a*b**4*c**2 + 12*_t*A*B**2*b**6*c - 1728*_t*B**3*a**3*b*c**3 +
 656*_t*B**3*a**2*b**3*c**2 - 88*_t*B**3*a*b**5*c + 4*_t*B**3*b**7)/(-4*A**
4*a*c**4 - 3*A**4*b**2*c**3 + 20*A**3*B*a*b*c**3 - 9*A**3*B*b**3*c**2 + 84*
A**2*B**2*a*b**2*c**2 - 9*A**2*B**2*b**4*c - 324*A*B**3*a**2*b*c**2 + 65*A*
B**3*a*b**3*c - 3*A*B**3*b**5 + 324*B**4*a**3*c**2 - 81*B**4*a**2*b**2*c +
5*B**4*a*b**4))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.120 $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=276

$$-\frac{x\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(-\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{}}$$

```
[Out] -(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + ((b*B - 2*A*c - (b^2*B - 4*A*b*c + 4*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 -
4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*B - 2*A*c + (b^2*B - 4*A*b*c + 4*
a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.553189, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1275, 1166, 205}

$$-\frac{x\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(-\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] -(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + ((b*B - 2*A*c - (b^2*B - 4*A*b*c + 4*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 -
4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*B - 2*A*c + (b^2*B - 4*A*b*c + 4*
a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx = -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\int \frac{Ab-2aB+(bB-2Ac)x^2}{a+bx^2+cx^4}dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bB-2Ac-\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}dx}{4\left(b^2-4ac\right)} + \frac{\left(bB-2Ac\right.}{}$$

$$= -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bB-2Ac-\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(bB-2Ac\right.}{2\sqrt{2}}$$

Mathematica [A] time = 0.706679, size = 298, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2x\left(B\left(2a+bx^2\right)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(-2Ac\sqrt{b^2-4ac}+bB\sqrt{b^2-4ac}-4aBc+4Abc+b^2(-B)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (Sqrt[2]*(-(b^2*B) + 4*A*b*c - 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*
c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^
2*B - 4*A*b*c + 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*c*Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*
a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/4
```

Maple [B] time = 0.03, size = 733, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (1/2*(2*A*c-B*b)/(4*a*c-b^2)*x^3+1/2*(A*b-2*B*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^
2+a)-1/2/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*
x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A+1/(4*a*c-b^2)*c/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b+1/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*B-
1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
```

```
)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B-1/4/(4*a*c-b^2
)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B+1/2/(4*a*c-b^2)*c*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*A+1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b-1/4/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*B-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*a*B-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
*B
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(Bb - 2\,Ac)x^3 + (2\,Ba - Ab)x}{2\left(\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2\right)} - \frac{-\int \frac{(Bb-2\,Ac)x^2-2\,Ba+Ab}{cx^4+bx^2+a}\,dx}{2\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 2*A*c)*x^3 + (2*B*a - A*b)*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 -
4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-((B*b - 2*A*c)*x^2 - 2*B*a
+ A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [B] time = 4.66533, size = 7073, normalized size = 25.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(B*b - 2*A*c)*x^3 - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2
*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2
+ (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 4
8*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b
^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*
b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*a*b^3 -
 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3*a^2*b
+ A^3*B*b^3)*c)*x + 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16*(2*A^2*
B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^2*b^2 - A
^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4 - A^3*b^5
)*c + (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^2)*c^4
- 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*sqrt((B^4*a^
2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4
 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2
*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2
*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 -
12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 +
 48*a^3*b^2*c^3 - 64*a^4*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*
b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*
```

```
c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)
/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c -
12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*
a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3
*a^2*b + A^3*B*b^3)*c)*x - 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16*
(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^2*
b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4 -
A^3*b^5)*c + (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^
2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*sqrt(
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*
b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 +
(12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 48*
a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6
*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^
4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 +
 a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3
*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a
^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A
^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b
^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 -
 A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 1
2*A*B^3*a^2*b + A^3*B*b^3)*c)*x + 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^
5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2
*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a
*b^4 - A^3*b^5)*c - (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A
*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c
)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 +
48*a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)
*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2*b^4*c^
2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(
a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12
*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2
)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*
a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c
 - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*
a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)
))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^
2*b^2 - A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*
a^3 - 12*A*B^3*a^2*b + A^3*B*b^3)*c)*x - 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B
^2*a*b^5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b
+ 2*A^2*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*
A^2*B*a*b^4 - A^3*b^5)*c - (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^
5*b + A*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a
*b^8)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4
*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A
^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2
*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4
*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6
*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) + 2*(2*B*a - A*b)*x)/(
(b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [B] time = 31.8216, size = 923, normalized size = 3.34

$$-\frac{x^3(-2Ac+Bb)+x(-Ab+2Ba)}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}+\mathrm{RootSum}\Big(t^4\left(1048576a^7c^7-1572864a^6b^2c^6+983040a^5b^4c^5-32\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -(x**3*(-2*A*c + B*b) + x*(-A*b + 2*B*a))/(8*a**2*c - 2*a*b**2 + x**4*(8*a*
c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**7*c
**7 - 1572864*a**6*b**2*c**6 + 983040*a**5*b**4*c**5 - 327680*a**4*b**6*c**
4 + 61440*a**3*b**8*c**3 - 6144*a**2*b**10*c**2 + 256*a*b**12*c) + _t**2*(-
12288*A**2*a**4*b*c**5 + 8192*A**2*a**3*b**3*c**4 - 1536*A**2*a**2*b**5*c**
3 + 16*A**2*b**9*c + 16384*A*B*a**5*c**5 - 6144*A*B*a**3*b**4*c**3 + 2048*A
*B*a**2*b**6*c**2 - 192*A*B*a*b**8*c - 12288*B**2*a**5*b*c**4 + 8192*B**2*a
**4*b**3*c**3 - 1536*B**2*a**3*b**5*c**2 + 16*B**2*a*b**9) + 16*A**4*a**2*c
**4 + 24*A**4*a*b**2*c**3 + 9*A**4*b**4*c**2 - 96*A**3*B*a**2*b*c**3 - 80*A
**3*B*a*b**3*c**2 - 6*A**3*B*b**5*c + 32*A**2*B**2*a**3*c**3 + 192*A**2*B**
2*a**2*b**2*c**2 + 42*A**2*B**2*a*b**4*c + A**2*B**2*b**6 - 96*A*B**3*a**3*
b*c**2 - 80*A*B**3*a**2*b**3*c - 6*A*B**3*a*b**5 + 16*B**4*a**4*c**2 + 24*B
**4*a**3*b**2*c + 9*B**4*a**2*b**4, Lambda(_t, _t*log(x + (-16384*_t**3*A*a
**5*c**5 + 8192*_t**3*A*a**4*b**2*c**4 - 512*_t**3*A*a**2*b**6*c**2 + 64*_t
**3*A*a*b**8*c + 16384*_t**3*B*a**5*b*c**4 - 12288*_t**3*B*a**4*b**3*c**3 +
 3072*_t**3*B*a**3*b**5*c**2 - 256*_t**3*B*a**2*b**7*c + 128*_t*A**3*a**2*b
*c**3 + 16*_t*A**3*a*b**3*c**2 + 4*_t*A**3*b**5*c - 192*_t*A**2*B*a**3*c**3
 - 192*_t*A**2*B*a**2*b**2*c**2 - 36*_t*A**2*B*a*b**4*c + 192*_t*A*B**2*a**
3*b*c**2 + 144*_t*A*B**2*a**2*b**3*c + 64*_t*B**3*a**4*c**2 - 128*_t*B**3*a
**3*b**2*c - 4*_t*B**3*a**2*b**4)/(-4*A**4*a*c**3 - 3*A**4*b**2*c**2 + 12*A
**3*B*a*b*c**2 + A**3*B*b**3*c - 12*A*B**3*a**2*b*c - A*B**3*a*b**3 + 4*B**
4*a**3*c + 3*B**4*a**2*b**2))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.121 $\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=293

$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abB}{\sqrt{b^2-4ac}}-2aB+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abB+Ab^2}{\sqrt{b^2-4ac}}\cdots\right.}{2\sqrt{2}a\left(b^2-\cdots\right.}$$

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(2*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[c]*(A*b - 2*a*B + (4*a*b*B + A*(b^2 - 12*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a
*B - (A*b^2 + 4*a*b*B - 12*a*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rubi [A] time = 0.846382, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1178, 1166, 205}

$$\frac{x\left(cx^2(Ab-2aB)-2aAc-abB+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abB}{\sqrt{b^2-4ac}}-2aB+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abB+Ab^2}{\sqrt{b^2-4ac}}\cdots\right.}{2\sqrt{2}a\left(b^2-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(2*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[c]*(A*b - 2*a*B + (4*a*b*B + A*(b^2 - 12*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a
*B - (A*b^2 + 4*a*b*B - 12*a*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^2}dx = \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-Ab^2-abB+6aAc-(Ab-2aB)cx^2}{a+bx^2+cx^4}dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(Ab-2aB-\frac{Ab^2+4abB-12aAc}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}}{4a\left(b^2-4ac\right)}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(2aB\left(2b-\sqrt{b^2-4ac}\right)+A\left(b^2-12ac+b\sqrt{b^2}\right.\right.}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b}}}$$

Mathematica [A] time = 0.844297, size = 304, normalized size = 1.04

$$\frac{\frac{2x\left(A\left(-2ac+b^2+bcx^2\right)-aB\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-2aB\left(\sqrt{b^2-4ac}-2b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}+12ac-b^2\right)-2aB\left(\sqrt{\right.\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{}}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-2*a*B*(-2*b + Sqrt[b^2 - 4*a*c]) +
A*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[2]*Sqrt[c]*(-2*a*B*(2*b + Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 12*a*c + b*Sq
rt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [B] time = 0.084, size = 1761, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/4/(4*a*c-b^2)*(-4*a*c+b^2)^(1/2)/a*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/
c)*A-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*A*b+1/2/(4*
a*c-b^2)*x/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*B-12*c^3/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a-8*c^2/(4*a*c-b^2)/(-4*a
```

```
*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2+3/4*c/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)/a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^4-c^2/(4*a*c-b^2)/(4*
a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b-3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*A*b^3+2*c^2/(4*a*c-b^2)*a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B+3/
2*c/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^2+4*c^2/(4*a*c-b^2)/(-
4*a*c+b^2)^(1/2)*a/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*B+3*c/(4*a*c-b^2)/(-4
*a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^3-1/4/(4*a*c-b^2)*(-4*
a*c+b^2)^(1/2)/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*A-1/4/(4*a*c-b^2)
/a*x/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*A*b+1/2/(4*a*c-b^2)*x/(x^2+1/2*
b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*B-12*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/(4*a*
c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a-8*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/(4*
a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2+3/4*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)
/a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^4+c^2/(4*a*c-b^2)/(4*a*c+3*b^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*A*b+3/4*c/(4*a*c-b^2)/a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*A*b^3-2*c^2/(4*a*c-b^2)*a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B-3/2*
c/(4*a*c-b^2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^2+4*c^2/(4*a*c-b^2)/(
-4*a*c+b^2)^(1/2)*a/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*B+3*c/(4*a*c-b^2)
/(-4*a*c+b^2)^(1/2)/(4*a*c+3*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(2\,Ba-Ab)cx^3+\left(Bab-Ab^2+2\,Aac\right)x}{2\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^2\right)}+\frac{-\int\frac{(2\,Ba-Ab)cx^2-Bab-Ab^2+6\,Aac}{cx^4+bx^2+a}\,dx}{2\left(ab^2-4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*B*a - A*b)*c*x^3 + (B*a*b - A*b^2 + 2*A*a*c)*x)/((a*b^2*c - 4*a^2*
c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate(-((
2*B*a - A*b)*c*x^2 - B*a*b - A*b^2 + 6*A*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^
2 - 4*a^2*c)
```

Fricas [B] time = 11.507, size = 10055, normalized size = 34.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*B*a - A*b)*c*x^3 - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^
2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A
^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2
- 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*s
qrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4
+ 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^
4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*
a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4
 + A^3*B*b^5)*c)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B
*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A
^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 9
5*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 +
 23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8
- 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*
A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a
*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b
^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(B^2*
a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B
^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 +
 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b
 + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B
^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(
4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^
2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3))
)/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*
c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84
*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B
^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^5
 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^
2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4
*b^2 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*
a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144
*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^
3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6
*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a
^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3
 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^
3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3
*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*
(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^
6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^
3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*
a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c -
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*
B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 -
18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 4
8*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64
*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B
```

```
^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*
c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x
+ 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 +
864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2
*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2
- (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c +
(B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 +
64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*sqrt(
(B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81
*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b
^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^
2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^
4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a
^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^
4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4
 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*
a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B
*a^2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a
^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 +
 A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)
/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a
^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 -
65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*
B^2*a*b^4 + A^3*B*b^5)*c)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6
+ 3*A^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4
*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^
3*b^3 + 95*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*
a^2*b^5 + 23*A^3*a*b^6)*c + (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 25
6*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5
*b^6 + 5*A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 +
4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b
+ A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sq
rt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2
 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c - (a^3*b^6 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*
a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^
3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9
*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + 2*(B*a*b
 - A*b^2 + 2*A*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*
b^3 - 4*a^2*b*c)*x^2)
```

Sympy [B] time = 61.4496, size = 1180, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (x**3*(-A*b*c + 2*B*a*c) + x*(2*A*a*c - A*b**2 + B*a*b))/(8*a**3*c - 2*a**2
*b**2 + x**4*(8*a**2*c**2 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + R
ootSum(_t**4*(1048576*a**9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7*b**4
*c**4 - 327680*a**6*b**6*c**3 + 61440*a**5*b**8*c**2 - 6144*a**4*b**10*c +
256*a**3*b**12) + _t**2*(-61440*A**2*a**5*b*c**5 + 61440*A**2*a**4*b**3*c**
```

```
4 - 24064*A**2*a**3*b**5*c**3 + 4608*A**2*a**2*b**7*c**2 - 432*A**2*a*b**9*
c + 16*A**2*b**11 + 49152*A*B*a**6*c**5 - 24576*A*B*a**5*b**2*c**4 - 2048*A
*B*a**4*b**4*c**3 + 3072*A*B*a**3*b**6*c**2 - 576*A*B*a**2*b**8*c + 32*A*B*
a*b**10 - 12288*B**2*a**6*b*c**4 + 8192*B**2*a**5*b**3*c**3 - 1536*B**2*a**
4*b**5*c**2 + 16*B**2*a**2*b**9) + 1296*A**4*a**2*c**5 - 360*A**4*a*b**2*c*
*4 + 25*A**4*b**4*c**3 - 2016*A**3*B*a**2*b*c**4 + 496*A**3*B*a*b**3*c**3 -
 30*A**3*B*b**5*c**2 + 288*A**2*B**2*a**3*c**4 + 960*A**2*B**2*a**2*b**2*c*
*3 - 198*A**2*B**2*a*b**4*c**2 + 9*A**2*B**2*b**6*c - 224*A*B**3*a**3*b*c**
3 - 144*A*B**3*a**2*b**3*c**2 + 18*A*B**3*a*b**5*c + 16*B**4*a**4*c**3 + 24
*B**4*a**3*b**2*c**2 + 9*B**4*a**2*b**4*c, Lambda(_t, _t*log(x + (-32768*_t
**3*A*a**7*b*c**4 + 28672*_t**3*A*a**6*b**3*c**3 - 9216*_t**3*A*a**5*b**5*c
**2 + 1280*_t**3*A*a**4*b**7*c - 64*_t**3*A*a**3*b**9 + 16384*_t**3*B*a**8*
c**4 - 8192*_t**3*B*a**7*b**2*c**3 + 512*_t**3*B*a**5*b**6*c - 64*_t**3*B*a
**4*b**8 - 1728*_t*A**3*a**4*c**4 + 2304*_t*A**3*a**3*b**2*c**3 - 740*_t*A*
*3*a**2*b**4*c**2 + 92*_t*A**3*a*b**6*c - 4*_t*A**3*b**8 - 576*_t*A**2*B*a*
*4*b*c**3 - 528*_t*A**2*B*a**3*b**3*c**2 + 168*_t*A**2*B*a**2*b**5*c - 12*_
t*A**2*B*a*b**7 + 576*_t*A*B**2*a**5*c**3 + 192*_t*A*B**2*a**4*b**2*c**2 +
60*_t*A*B**2*a**3*b**4*c - 12*_t*A*B**2*a**2*b**6 - 128*_t*B**3*a**5*b*c**2
 - 16*_t*B**3*a**4*b**3*c - 4*_t*B**3*a**3*b**5)/(-324*A**4*a**2*c**4 + 81*
A**4*a*b**2*c**3 - 5*A**4*b**4*c**2 + 324*A**3*B*a**2*b*c**3 - 65*A**3*B*a*
b**3*c**2 + 3*A**3*B*b**5*c - 84*A**2*B**2*a**2*b**2*c**2 + 9*A**2*B**2*a*b
**4*c - 20*A*B**3*a**3*b*c**2 + 9*A*B**3*a**2*b**3*c + 4*B**4*a**4*c**2 + 3
*B**4*a**3*b**2*c))))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.122 $\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=389

$$-\frac{-10aAc-abB+3Ab^2}{2a^2x\left(b^2-4ac\right)}+\frac{\sqrt{c}\left(aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\right)\tan^{-1}}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -(3*A*b^2 - a*b*B - 10*a*A*c)/(2*a^2*(b^2 - 4*a*c)*x) - (a*b*B - A*(b^2 - 2
*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) + (S
qrt[c]*(a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) - A*(3*b^3 - 16*a*b*c + 3*
b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - a*b*B - 10*a*A*c + (a*B*(b^2 -
12*a*c) - A*(3*b^3 - 16*a*b*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rubi [A] time = 1.21992, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1277, 1281, 1166, 205}

$$-\frac{-10aAc-abB+3Ab^2}{2a^2x\left(b^2-4ac\right)}+\frac{\sqrt{c}\left(aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\right)\tan^{-1}}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```


```
[Out] -(3*A*b^2 - a*b*B - 10*a*A*c)/(2*a^2*(b^2 - 4*a*c)*x) - (a*b*B - A*(b^2 - 2
*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) + (S
qrt[c]*(a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) - A*(3*b^3 - 16*a*b*c + 3*
b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - a*b*B - 10*a*A*c + (a*B*(b^2 -
12*a*c) - A*(3*b^3 - 16*a*b*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```


Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-3Ab^2+abB+10aAc-3(Ab-2aB)cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\int \frac{aB\left(b^2-6ac\right)-A\left(3b^3-13abc\right)-\cdots}{a+bx^2+\cdots}}{2a^2\left(b^2-\cdots\right.}$$

$$= -\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(aB\left(b^2-12ac+b\sqrt{b\cdots}\right.\right.\right.}{\cdots}$$

$$= -\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(aB\left(b^2-12ac+b\sqrt{\cdots}\right.\right.}{\cdots}$$

Mathematica [A] time = 1.17728, size = 382, normalized size = 0.98

$$\frac{\frac{2x\left(aB\left(-2ac+b^2+bcx^2\right)-A\left(-3abc-2ac^2x^2+b^2cx^2+b^3\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(A\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)+aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4a\cdots}}}\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\cdots}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*A)/x + (2*x*(a*B*(b^2 - 2*a*c + b*c*x^2) - A*(b^3 - 3*a*b*c + b^2*c*x^
2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(
a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*(-3*b^3 + 16*a*b*c - 3*b^2*Sqr
t[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + (Sqrt[2]*Sqrt[c]*(a*B*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*(3*b^3 -
 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sq
```

```
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Maple [B] time = 0.039, size = 1252, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*A+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c
-b^2)*x^3*A*b^2-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b*B-3/2/a/(c*x^4+b*
x^2+a)/(4*a*c-b^2)*x*A*b*c+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*b^3+1/(c
*x^4+b*x^2+a)/(4*a*c-b^2)*x*B*c-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*B*b^2+5
/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A-3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*A*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*A*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*
b^3+1/4/a*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c
*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*B-3*c^2/(4*a*c-b^2)/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*B*b^2-5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A+3/4/
a^2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*A*b-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*A*b^3-1/4/a*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*B-3*c^2/(4*a*
c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B+1/4/a*c/(4*a*c-b^2)/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^2-A/a^2/x
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 25.8835, size = 16292, normalized size = 41.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^4 - 4*A*a*b^2 + 16*A*a^2*c + 2*
(B*a*b^2 - 3*A*b^3 - (2*B*a^2 - 11*A*a*b)*c)*x^2 - sqrt(1/2)*((a^2*b^2*c -
4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(
B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 +
5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 -
 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^
2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 -
 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^
3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2
*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^
2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*
b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log((2500*A^4*a^3*c^6 + 6
25*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^4*b +
 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^
4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - 189*A
^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*
B*b^7)*c^2)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b
^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 -
 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^
6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*c^3 +
 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^
7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*
a*b^9)*c - (B*a^6*b^9 - 3*A*a^5*b^10 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b -
17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 4
9*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 - 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12
*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^
4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(2
7*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 10
17*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3
*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48
*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7
 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77
*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 1
2*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A
^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(
27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1
017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^
3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 4
8*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b
*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b
^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 +
77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c +
 (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 -
 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625
*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3
*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 +
 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*
a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c +
 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
- 64*a^8*c^3))*log((2500*A^4*a^3*c^6 + 625*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*
c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^4*b + 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a
^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^
```

```
2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - 189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*
B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*B*b^7)*c^2)*x - 1/2*sqrt(1/2)*(B^3*
a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^
6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b
^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*
A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*
b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^
2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*a*b^9)*c - (B*a^6*b^9 - 3*A*a^5*b^1
0 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3
 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 49*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 -
 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6
 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*
A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A
^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*
b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*
a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt
(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3
 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^
3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^
6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44
*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*
A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5
*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4
*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^
5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/2)*((a^2*b^2
*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sq
rt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c
^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*
b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b
^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*
b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 -
44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 96
8*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a
^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A
^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log((2500*A^4*a^3*c^
6 + 625*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^
4*b + 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B
^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 -
189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27
*A^3*B*b^7)*c^2)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*
B*a*b^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*
a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B
^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*
c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a
^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486
*A^3*a*b^9)*c + (B*a^6*b^9 - 3*A*a^5*b^10 + 1280*A*a^10*c^5 + 128*(4*B*a^10
*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^
5 - 49*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 - 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4
 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 6
25*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 +
 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3
 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^
2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c
 + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^
2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2
 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*
c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^
```

```
4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 +
625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3
+ 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^
3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B
^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*
c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c
^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*
a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*
A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b
^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5
)*c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*
b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8
+ 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^
3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*
b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2
*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))*log((2500*A^4*a^3*c^6 + 625*(4*A^3*B*a^3*b - 9*A^4*a^2*
b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^4*b + 1672*A^2*B^2*a^3*b^2 - 909*A^
3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 16
74*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - 189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 -
 9*A*B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*B*b^7)*c^2)*x - 1/2*sqrt(1/2)*
(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b^10 - 27*A^3*b^11 - 400*(6*A^2
*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*
a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 1
2414*A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2
*a^4*b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192
*A*B^2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*a*b^9)*c + (B*a^6*b^9 - 3*A*a^
5*b^10 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9
*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 49*A*a^7*b^6)*c^2 - 5*(4*B*a^7*
b^7 - 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^
2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5
- 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b +
968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4
*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459
*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))
*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b
)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a
^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c - (a^5*b^6 - 12*a^6*b^4*c + 48*a^
7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a
^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5
 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b +
 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^
4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 45
9*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))
)/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))))/((a^2*b^2*c - 4
*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.123 $\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=522

$$-\frac{\sqrt{c}\left(aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2+5b^3\sqrt{b^2-4ac}-29ab^2c-19abc\sqrt{b^2-4ac}+5\right.\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -(5*A*b^2 - 3*a*b*B - 14*a*A*c)/(6*a^2*(b^2 - 4*a*c)*x^3) - (a*B*(3*b^2 - 1
0*a*c) - A*(5*b^3 - 19*a*b*c))/(2*a^3*(b^2 - 4*a*c)*x) - (a*b*B - A*(b^2 -
2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^3*(a + b*x^2 + c*x^4)) -
 (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^
2 - 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c]
- 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]
) + (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt
[b^2 - 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*a*
c] + 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

Rubi [A] time = 1.36462, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1277, 1281, 1166, 205}

$$-\frac{\sqrt{c}\left(aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2+5b^3\sqrt{b^2-4ac}-29ab^2c-19abc\sqrt{b^2-4ac}+5\right.\right.}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```


```
[Out] -(5*A*b^2 - 3*a*b*B - 14*a*A*c)/(6*a^2*(b^2 - 4*a*c)*x^3) - (a*B*(3*b^2 - 1
0*a*c) - A*(5*b^3 - 19*a*b*c))/(2*a^3*(b^2 - 4*a*c)*x) - (a*b*B - A*(b^2 -
2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^3*(a + b*x^2 + c*x^4)) -
 (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^
2 - 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c]
- 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]
) + (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt
[b^2 - 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*a*
c] + 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```


Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
```

```
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-5Ab^2+3abB+14aAc-5(Ab-2aB)cx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} + \frac{\int \frac{-3\left(5Ab^3-3ab^2B-19aAbc+10a^2\ldots\right.}{x^2\left(a+b\ldots\right.}}{6a^2\left(b^2\ldots\right.}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{2a^3\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)-(Ab-\ldots}{2a\left(b^2-4ac\right)x^3\left(a+bx^2\ldots\right.}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{2a^3\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)-(Ab-\ldots}{2a\left(b^2-4ac\right)x^3\left(a+bx^2\ldots\right.}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{aB\left(3b^2-10ac\right)-A\left(5b^3-19abc\right)}{2a^3\left(b^2-4ac\right)x} - \frac{abB-A\left(b^2-2ac\right)-(Ab-\ldots}{2a\left(b^2-4ac\right)x^3\left(a+bx^2\ldots\right.}$$

Mathematica [A] time = 1.31993, size = 487, normalized size = 0.93

$$\frac{\frac{6x\left(A\left(2a^2c^2-4ab^2c-3abc^2x^2+b^3cx^2+b^4\right)+aB\left(3abc+2ac^2x^2-b^2cx^2-b^3\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{3\sqrt{2}\sqrt{c}\left(A\left(28a^2c^2+5b^3\sqrt{b^2-4ac}-29ab^2c-19abc\sqrt{b^2-4ac}+5b^4\right)+aB\left(-3b^2\sqrt{b^2-4ac}+10\ldots\right.\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*A)/x^3 + (24*A*b - 12*a*B)/x + (6*x*(a*B*(-b^3 + 3*a*b*c - b^2*c*x^2
 + 2*a*c^2*x^2) + A*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*a*b*c^2*x^
2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(a*B*(-3*b^3
+ 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + A*(5*b^4
 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 -
4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(a*B*(-3*b^3
+ 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]) + A*(5*b^4
 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*a*c] + 19*a*b*c*Sqrt[b^2 -
4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

Maple [B] time = 0.046, size = 1653, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -5/4/a^3*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^3-19/4/a^2*c^2/(4*a*c-b^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*A*b+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*A-3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^2+19/4/a^2*c^2/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*A+3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^2+5/4/a^3*
c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^3-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^
2)*x*A*b^4-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*B-1/a/(c*x^4+b*x^2+a)/(4
*a*c-b^2)*x*A*c^2+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*B*b^3+2/a^3/x*A*b-1
/2/a^3/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*A*b^3+3/2/a^2/(c*x^4+b*x^2+a)*c^2/
(4*a*c-b^2)*x^3*A*b+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*B*b^2+2/a^2/(
c*x^4+b*x^2+a)/(4*a*c-b^2)*x*A*b^2*c-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b*
B*c+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B-5/2/a*c^2/(4*a*c-b^2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*B-1/3*A/a^2/x^3-1/a^2/x*B-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^3-29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(
1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*A*b^2+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b*B-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*B*b^3+5/4/a^3*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^
4+5/4/a^3*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^4-29/4/a^
2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
```

```
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2+4/a*c^2/(4
*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*B
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 54.5065, size = 23162, normalized size = 44.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*((10*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^6 - 4*A*a^2
*b^2 + 16*A*a^3*c - 2*(9*B*a*b^3 - 15*A*b^4 - 14*A*a^2*c^2 - (33*B*a^2*b -
62*A*a*b^2)*c)*x^4 - 4*(3*B*a^2*b^2 - 5*A*a*b^3 - 4*(3*B*a^3 - 5*A*a^2*b)*c
)*x^2 - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^
5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b
^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2
+ 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b
^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 -
 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*
B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 24
01*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^
5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*
a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^
3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 +
3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^
3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*
b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14
*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b
^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((9604*A^4*a^4*c^8 + 7203*(4*A^3*B
*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 22500*A*B^3*a^5*b + 43524*A^2
*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)*c^6 + (5625*B^4*a^5*
b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 20190*A^3*B*a^2*b^5 - 1
2325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a^3*b^5 + 5560*A^2*B^
2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27*B^4*a^3*b^6 - 135*A
*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)*x + 1/2*sqrt(1/2)*(2
7*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2*B*a*b^13 - 125*A^3*b^14 + 109
76*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b + 709*A^3*a^6*b^2)*c^6
 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2*B*a^6*b^3 - 86495*A^
3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6*b^4 + 224705*A^2*B*a
^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5 - 8820*A*B^2*a^5*b^6
 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330*B^3*a^5*b^7 - 17889
*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^10)*c^2 - (486*B^3*a
^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2*b^11 - 2425*A^3*a*b^12)*c - (
```

3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 - 13*A*a^12*b)*c^5 + 64*(34*B*a ^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11*b^4 - 23*A*a^10*b^5)*c^3 + 28 *(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 94*A*a^8*b^9)*c)*sqrt ((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a *b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^ 6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^ 2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7 *b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A ^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4* a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c ^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9 *A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3 *b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9* b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2* B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25 *A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300 *A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4 *b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b ^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7 872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4 *a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3* b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a ^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A* B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198 *A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^ 8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4 *b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a ^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109 544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A* B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b ^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186* A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((9604*A^4*a^4*c^8 + 720 3*(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 22500*A*B^3*a^5*b + 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)*c^6 + (562 5*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 20190*A^3*B*a ^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a^3*b^5 + 5 560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27*B^4*a^3*b ^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)*x - 1/2*sq rt(1/2)*(27*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2*B*a*b^13 - 125*A^3* b^14 + 10976*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b + 709*A^3*a^ 6*b^2)*c^6 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2*B*a^6*b^3 - 86495*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6*b^4 + 2247 05*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5 - 8820*A*B ^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330*B^3*a^5*b ^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^10)*c^2 - (486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2*b^11 - 2425*A^3*a*b ^12)*c - (3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 - 13*A*a^12*b)*c^5 + 64*(34*B*a^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11*b^4 - 23*A*a^10*b^5)*c^3 + 28*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 94*A*a^8*b^

```
9)*c)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 15
00*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 18
6*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51
894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(12
75*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*
a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5
 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2
*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*
a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 -
 64*a^17*c^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A
*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*
b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(
15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 - 12*a^8*b^4*c
 + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 +
 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^
6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*
a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 766
86*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2
*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a
^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 +
 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^
2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^1
5*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9
*b^2*c^2 - 64*a^10*c^3))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3
*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*
A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5
*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3
*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a
*b^7)*c - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*
B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11
 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b +
 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6
*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2
- 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815
*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^
2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b
^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*
A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))
/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((9604*A^4*a^4
*c^8 + 7203*(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 22500*A*B
^3*a^5*b + 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)*
c^6 + (5625*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 201
90*A^3*B*a^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a
^3*b^5 + 5560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27
*B^4*a^3*b^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)*
x + 1/2*sqrt(1/2)*(27*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2*B*a*b^13
- 125*A^3*b^14 + 10976*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b +
709*A^3*a^6*b^2)*c^6 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2*
B*a^6*b^3 - 86495*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6*
b^4 + 224705*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5
- 8820*A*B^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330
*B^3*a^5*b^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^
10)*c^2 - (486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2*b^11 - 24
25*A^3*a*b^12)*c + (3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 - 13*A*a^12
*b)*c^5 + 64*(34*B*a^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11*b^4 - 23*
A*a^10*b^5)*c^3 + 28*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 9
4*A*a^8*b^9)*c)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2
*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^
2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*
```

a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c
^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77
424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B
^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^
8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9
550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16
*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9
- 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 2
3*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)
c^2 - 7(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c - (a^7*b^6 - 12
*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3
*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*
A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 +
(625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5
*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 +
51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(
1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B
*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7
+ 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^
6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*
c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*
x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2
*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*
(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 2
10*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6
+ 45*A^2*a*b^7)*c - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)
*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^
3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3
*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A
^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^
4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b
^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22
508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459
*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b
^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a
^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((96
04*A^4*a^4*c^8 + 7203*(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 -
22500*A*B^3*a^5*b + 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4
*a^2*b^4)*c^6 + (5625*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3
*b^4 - 20190*A^3*B*a^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 33
51*A*B^3*a^3*b^5 + 5560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c
^4 + 7*(27*B^4*a^3*b^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*
b^9)*c^3)*x - 1/2*sqrt(1/2)*(27*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2
*B*a*b^13 - 125*A^3*b^14 + 10976*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*
B*a^7*b + 709*A^3*a^6*b^2)*c^6 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 +
96044*A^2*B*a^6*b^3 - 86495*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*
A*B^2*a^6*b^4 + 224705*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^
3*a^6*b^5 - 8820*A*B^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c
^3 + (3330*B^3*a^5*b^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*
A^3*a^2*b^10)*c^2 - (486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2
*b^11 - 2425*A^3*a*b^12)*c + (3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 -
13*A*a^12*b)*c^5 + 64*(34*B*a^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11
*b^4 - 23*A*a^10*b^5)*c^3 + 28*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a
^9*b^8 - 94*A*a^8*b^9)*c)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A
^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*
(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9
300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*
a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^
5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4

```
- 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*
A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a
^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c
 + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 2
5*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a
^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^
2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c - (a^
7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 -
 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^
12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*
b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544
*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3
*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)
*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 2
8260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B
^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c
)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 1
2*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))))/((a^3*b^2*c - 4*a^4*c^2)*x^7
 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.124 $\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=365

$$-\frac{x^4\left(x^2\left(20a^2Bc^2+10aAbc^2-20ab^2Bc-Ab^3c+3b^4B\right)+a\left(16aAc^2-18abBc-Ab^2c+3b^3B\right)\right)}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^2\left(30a^2Bc^2+7aA\right.}{2}$$

```
[Out] ((3*b^4*B - A*b^3*c - 21*a*b^2*B*c + 7*a*A*b*c^2 + 30*a^2*B*c^2)*x^2)/(2*c^
3*(b^2 - 4*a*c)^2) - (x^8*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)
)/(4*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^4*(a*(3*b^3*B - A*b^2*c -
18*a*b*B*c + 16*a*A*c^2) + (3*b^4*B - A*b^3*c - 20*a*b^2*B*c + 10*a*A*b*c^2
 + 20*a^2*B*c^2)*x^2))/(4*c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((3*b^
6*B - A*b^5*c - 30*a*b^4*B*c + 10*a*A*b^3*c^2 + 90*a^2*b^2*B*c^2 - 30*a^2*A
*b*c^3 - 60*a^3*B*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^
2 - 4*a*c)^(5/2)) - ((3*b*B - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rubi [A] time = 1.45459, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1251, 818, 773, 634, 618, 206, 628}

$$-\frac{x^4\left(x^2\left(20a^2Bc^2+10aAbc^2-20ab^2Bc-Ab^3c+3b^4B\right)+a\left(16aAc^2-18abBc-Ab^2c+3b^3B\right)\right)}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^2\left(30a^2Bc^2+7aA\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((3*b^4*B - A*b^3*c - 21*a*b^2*B*c + 7*a*A*b*c^2 + 30*a^2*B*c^2)*x^2)/(2*c^
3*(b^2 - 4*a*c)^2) - (x^8*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)
)/(4*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^4*(a*(3*b^3*B - A*b^2*c -
18*a*b*B*c + 16*a*A*c^2) + (3*b^4*B - A*b^3*c - 20*a*b^2*B*c + 10*a*A*b*c^2
 + 20*a^2*B*c^2)*x^2))/(4*c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((3*b^
6*B - A*b^5*c - 30*a*b^4*B*c + 10*a*A*b^3*c^2 + 90*a^2*b^2*B*c^2 - 30*a^2*A
*b*c^3 - 60*a^3*B*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^
2 - 4*a*c)^(5/2)) - ((3*b*B - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
```

```
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{11}\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^5(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx,x,x^2\right)$$

$$= -\frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\text{Subst}\left(\int \frac{x^3\left(4a(bB-2Ac)+\left(3b^2B-Abc-10aBc\right)x\right)}{\left(a+bx+cx^2\right)^2}\,dx,x\right.}{4c\left(b^2-4ac\right)}$$

$$= -\frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{x^4\left(a\left(3b^3B-Ab^2c-18abBc+16aAc^2\right)+(3\right.}{4c^2\left(b^2-4ac\right.}$$

$$= \frac{\left(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)\right.}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$= \frac{\left(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)\right.}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$= \frac{\left(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)\right.}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

$$= \frac{\left(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2\right)x^2}{2c^3\left(b^2-4ac\right)^2} - \frac{x^8\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)\right.}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Mathematica [A] time = 0.758272, size = 435, normalized size = 1.19

$$\frac{-3a^2b^2c^3\left(13A+34Bx^2\right)+2a^2bc^3\left(25Acx^2-39aB\right)+4a^3c^4\left(8A+9Bx^2\right)+ab^4c^2\left(11A+48Bx^2\right)+ab^3c^2\left(61aB-30Acx^2\right)+2b^5c\left(2Acx^2-7aB\right)-b^6c\left(A+6Bx^2\right)+b^7B}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*B*c^2*x^2 + (b^7*B - b^6*c*(A + 6*B*x^2) + 4*a^3*c^4*(8*A + 9*B*x^2) - 3
*a^2*b^2*c^3*(13*A + 34*B*x^2) + a*b^4*c^2*(11*A + 48*B*x^2) + a*b^3*c^2*(6
1*a*B - 30*A*c*x^2) + 2*b^5*c*(-7*a*B + 2*A*c*x^2) + 2*a^2*b*c^3*(-39*a*B +
 25*A*c*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^5*(-(b*B) + A*c)*x
^2 + a^3*c^2*(-5*b*B + 2*c*(A + B*x^2)) + a*b^3*(-(b^2*B) - 5*A*c^2*x^2 + b
*c*(A + 6*B*x^2)) + a^2*b*c*(5*b^2*B + 5*A*c^2*x^2 - b*c*(4*A + 9*B*x^2)))/
((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*c*(-3*b^6*B + A*b^5*c + 30*a*b^4
*B*c - 10*a*A*b^3*c^2 - 90*a^2*b^2*B*c^2 + 30*a^2*A*b*c^3 + 60*a^3*B*c^3)*A
rcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*(-3*b*B +
 A*c)*Log[a + b*x^2 + c*x^4])/(4*c^5)
```

Maple [B] time = 0.026, size = 2054, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*B*x^2/c^3-12/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*B*a^2*b-3
0/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-
b^2)^(1/2))*a^3*B+3/2/c^4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arct
an((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^6*B-21/4/c^2/(c*x^4+b*x^2+a)^2*a^3/(16*
a^2*c^2-8*a*b^2*c+b^4)*A*b^2-29/2/c^2/(c*x^4+b*x^2+a)^2*a^4/(16*a^2*c^2-8*a
*b^2*c+b^4)*B*b-5/4/c^4/(c*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*
b^5+1/c^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^5-5/4/c^4/(c
*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*b^7+7/c/(c*x^4+b*x^2+a)^2*
a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B+3/4/c^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^4*A*b^6+3/4/c^3/(c*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b^2*
c+b^4)*A*b^4+9/c^3/(c*x^4+b*x^2+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*B*b^3-3
/2/c^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*b^6+6/c^3/(16*a^2
*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*B*a*b^3-1/2/c^3/(16*a^2*c^2-8*a*b^2*c
+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5*A-2/c^2/(
16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*A*a*b^2+25/2/(c*x^4+b*x^2+a)^2/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*a^2*b+6/c/(c*x^4+b*x^2+a)^2*a^4/(16*a^2*c^
2-8*a*b^2*c+b^4)*A-3/4/c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*B*b
^5+4/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*A*a^2+1/4/c^3/(16*a^2*c
^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*A*b^4+9/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^6*B*a^3+8/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*
A*a^3-51/2/c/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*a^2*b^2+12/
c^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*a*b^4+11/4/c/(c*x^4+
b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*a^2*b^2-19/4/c^2/(c*x^4+b*x^2+a
)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*a*b^4-21/2/c/(c*x^4+b*x^2+a)^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^4*B*a^3*b-41/4/c^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^4*B*a^2*b^3+31/2/c/(c*x^4+b*x^2+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c
+b^4)*x^2*A*b-15/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x^2+b)/(4*a*c-b^2)^(1/2))*A*a^2*b+5/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-
b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*a*b^3+45/c^2/(16*a^2*c^2
-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*a
^2*b^2-15/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+
b)/(4*a*c-b^2)^(1/2))*B*a*b^4+17/2/c^3/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^4*B*a*b^5+3/2/c^3/(c*x^4+b*x^2+a)^2*a/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^2*A*b^5+19/c^3/(c*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^
4-11/c^2/(c*x^4+b*x^2+a)^2*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^3-71/2/c^
2/(c*x^4+b*x^2+a)^2*a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^2-5/2/c^4/(c*x^4
+b*x^2+a)^2*a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^6-15/2/c/(c*x^4+b*x^2+a)^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*a*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.3471, size = 6745, normalized size = 18.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b^6*c^3 - 12*B*a*b^4*c^4 + 48*B*a^2*b^2*c^5 - 64*B*a^3*c^6)*x^10
 - 5*B*a^2*b^7 - 96*A*a^5*c^4 + 4*(B*b^7*c^2 - 12*B*a*b^5*c^3 + 48*B*a^2*b^
3*c^4 - 64*B*a^3*b*c^5)*x^8 - 2*(2*B*b^8*c + 100*(2*B*a^4 + A*a^3*b)*c^5 -
(254*B*a^3*b^2 + 85*A*a^2*b^3)*c^4 + (123*B*a^2*b^4 + 23*A*a*b^5)*c^3 - 2*(
13*B*a*b^6 + A*b^7)*c^2)*x^6 - (5*B*b^9 + 128*A*a^4*c^5 + 4*(22*B*a^4*b + 3
*A*a^3*b^2)*c^4 - (314*B*a^3*b^3 + 87*A*a^2*b^4)*c^3 + (225*B*a^2*b^5 + 31*
A*a*b^6)*c^2 - (58*B*a*b^7 + 3*A*b^8)*c)*x^4 + 4*(58*B*a^5*b + 27*A*a^4*b^2
)*c^3 - (202*B*a^4*b^3 + 33*A*a^3*b^4)*c^2 - 2*(5*B*a*b^8 + 4*(30*B*a^5 + 3
1*A*a^4*b)*c^4 - (346*B*a^4*b^2 + 119*A*a^3*b^3)*c^3 + (235*B*a^3*b^4 + 34*
A*a^2*b^5)*c^2 - (59*B*a^2*b^6 + 3*A*a*b^7)*c)*x^2 - (3*B*a^2*b^6 + (3*B*b^
6*c^2 - 30*(2*B*a^3 + A*a^2*b)*c^5 + 10*(9*B*a^2*b^2 + A*a*b^3)*c^4 - (30*B
*a*b^4 + A*b^5)*c^3)*x^8 + 2*(3*B*b^7*c - 30*(2*B*a^3*b + A*a^2*b^2)*c^4 +
10*(9*B*a^2*b^3 + A*a*b^4)*c^3 - (30*B*a*b^5 + A*b^6)*c^2)*x^6 + (3*B*b^8 -
 60*(2*B*a^4 + A*a^3*b)*c^4 + 10*(12*B*a^3*b^2 - A*a^2*b^3)*c^3 + 2*(15*B*a
^2*b^4 + 4*A*a*b^5)*c^2 - (24*B*a*b^6 + A*b^7)*c)*x^4 - 30*(2*B*a^5 + A*a^4
*b)*c^3 + 10*(9*B*a^4*b^2 + A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 - 30*(2*B*a^4*b +
 A*a^3*b^2)*c^3 + 10*(9*B*a^3*b^3 + A*a^2*b^4)*c^2 - (30*B*a^2*b^5 + A*a*b^
6)*c)*x^2 - (30*B*a^3*b^4 + A*a^2*b^5)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4
+ 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
 + a)) + (56*B*a^3*b^5 + 3*A*a^2*b^6)*c - (3*B*a^2*b^7 + 64*A*a^5*c^4 + (3*
B*b^7*c^2 + 64*A*a^3*c^6 - 48*(4*B*a^3*b + A*a^2*b^2)*c^5 + 12*(12*B*a^2*b^
3 + A*a*b^4)*c^4 - (36*B*a*b^5 + A*b^6)*c^3)*x^8 + 2*(3*B*b^8*c + 64*A*a^3*
b*c^5 - 48*(4*B*a^3*b^2 + A*a^2*b^3)*c^4 + 12*(12*B*a^2*b^4 + A*a*b^5)*c^3
- (36*B*a*b^6 + A*b^7)*c^2)*x^6 + (3*B*b^9 + 128*A*a^4*c^5 - 32*(12*B*a^4*b
 + A*a^3*b^2)*c^4 + 24*(4*B*a^3*b^3 - A*a^2*b^4)*c^3 + 2*(36*B*a^2*b^5 + 5*
A*a*b^6)*c^2 - (30*B*a*b^7 + A*b^8)*c)*x^4 - 48*(4*B*a^5*b + A*a^4*b^2)*c^3
 + 12*(12*B*a^4*b^3 + A*a^3*b^4)*c^2 + 2*(3*B*a*b^8 + 64*A*a^4*b*c^4 - 48*(
4*B*a^4*b^2 + A*a^3*b^3)*c^3 + 12*(12*B*a^3*b^4 + A*a^2*b^5)*c^2 - (36*B*a^
2*b^6 + A*a*b^7)*c)*x^2 - (36*B*a^3*b^5 + A*a^2*b^6)*c)*log(c*x^4 + b*x^2 +
 a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6*c^6
 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^8 + 2*(b^7*c^5 - 12*a*b^5*
c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^6 + (b^8*c^4 - 10*a*b^6*c^5 + 24*a^2
*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^4 + 2*(a*b^7*c^4 - 12*a^2*b^5*c^
5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x^2), 1/4*(2*(B*b^6*c^3 - 12*B*a*b^4*c^4
 + 48*B*a^2*b^2*c^5 - 64*B*a^3*c^6)*x^10 - 5*B*a^2*b^7 - 96*A*a^5*c^4 + 4*(
B*b^7*c^2 - 12*B*a*b^5*c^3 + 48*B*a^2*b^3*c^4 - 64*B*a^3*b*c^5)*x^8 - 2*(2*
B*b^8*c + 100*(2*B*a^4 + A*a^3*b)*c^5 - (254*B*a^3*b^2 + 85*A*a^2*b^3)*c^4
+ (123*B*a^2*b^4 + 23*A*a*b^5)*c^3 - 2*(13*B*a*b^6 + A*b^7)*c^2)*x^6 - (5*B
*b^9 + 128*A*a^4*c^5 + 4*(22*B*a^4*b + 3*A*a^3*b^2)*c^4 - (314*B*a^3*b^3 +
87*A*a^2*b^4)*c^3 + (225*B*a^2*b^5 + 31*A*a*b^6)*c^2 - (58*B*a*b^7 + 3*A*b^
8)*c)*x^4 + 4*(58*B*a^5*b + 27*A*a^4*b^2)*c^3 - (202*B*a^4*b^3 + 33*A*a^3*b
^4)*c^2 - 2*(5*B*a*b^8 + 4*(30*B*a^5 + 31*A*a^4*b)*c^4 - (346*B*a^4*b^2 + 1
19*A*a^3*b^3)*c^3 + (235*B*a^3*b^4 + 34*A*a^2*b^5)*c^2 - (59*B*a^2*b^6 + 3*
A*a*b^7)*c)*x^2 - 2*(3*B*a^2*b^6 + (3*B*b^6*c^2 - 30*(2*B*a^3 + A*a^2*b)*c^
5 + 10*(9*B*a^2*b^2 + A*a*b^3)*c^4 - (30*B*a*b^4 + A*b^5)*c^3)*x^8 + 2*(3*B
*b^7*c - 30*(2*B*a^3*b + A*a^2*b^2)*c^4 + 10*(9*B*a^2*b^3 + A*a*b^4)*c^3 -
(30*B*a*b^5 + A*b^6)*c^2)*x^6 + (3*B*b^8 - 60*(2*B*a^4 + A*a^3*b)*c^4 + 10*
(12*B*a^3*b^2 - A*a^2*b^3)*c^3 + 2*(15*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (24*B*a
*b^6 + A*b^7)*c)*x^4 - 30*(2*B*a^5 + A*a^4*b)*c^3 + 10*(9*B*a^4*b^2 + A*a^3
*b^3)*c^2 + 2*(3*B*a*b^7 - 30*(2*B*a^4*b + A*a^3*b^2)*c^3 + 10*(9*B*a^3*b^3
 + A*a^2*b^4)*c^2 - (30*B*a^2*b^5 + A*a*b^6)*c)*x^2 - (30*B*a^3*b^4 + A*a^2
*b^5)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
 4*a*c)) + (56*B*a^3*b^5 + 3*A*a^2*b^6)*c - (3*B*a^2*b^7 + 64*A*a^5*c^4 + (
3*B*b^7*c^2 + 64*A*a^3*c^6 - 48*(4*B*a^3*b + A*a^2*b^2)*c^5 + 12*(12*B*a^2*
b^3 + A*a*b^4)*c^4 - (36*B*a*b^5 + A*b^6)*c^3)*x^8 + 2*(3*B*b^8*c + 64*A*a^
3*b*c^5 - 48*(4*B*a^3*b^2 + A*a^2*b^3)*c^4 + 12*(12*B*a^2*b^4 + A*a*b^5)*c^
3 - (36*B*a*b^6 + A*b^7)*c^2)*x^6 + (3*B*b^9 + 128*A*a^4*c^5 - 32*(12*B*a^4
```

```
*b + A*a^3*b^2)*c^4 + 24*(4*B*a^3*b^3 - A*a^2*b^4)*c^3 + 2*(36*B*a^2*b^5 +
5*A*a*b^6)*c^2 - (30*B*a*b^7 + A*b^8)*c)*x^4 - 48*(4*B*a^5*b + A*a^4*b^2)*c
^3 + 12*(12*B*a^4*b^3 + A*a^3*b^4)*c^2 + 2*(3*B*a*b^8 + 64*A*a^4*b*c^4 - 48
*(4*B*a^4*b^2 + A*a^3*b^3)*c^3 + 12*(12*B*a^3*b^4 + A*a^2*b^5)*c^2 - (36*B*
a^2*b^6 + A*a*b^7)*c)*x^2 - (36*B*a^3*b^5 + A*a^2*b^6)*c)*log(c*x^4 + b*x^2
 + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^8 + 2*(b^7*c^5 - 12*a*b^
5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^6 + (b^8*c^4 - 10*a*b^6*c^5 + 24*a
^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^4 + 2*(a*b^7*c^4 - 12*a^2*b^5*
c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 38.2789, size = 807, normalized size = 2.21

$$\frac{\left(3\,Bb^6-30\,Bab^4c-Ab^5c+90\,Ba^2b^2c^2+10\,Aab^3c^2-60\,Ba^3c^3-30\,Aa^2bc^3\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(b^4c^4-8\,ab^2c^5+16\,a^2c^6\right)\sqrt{-b^2+4\,ac}}+\frac{Bx^2}{2\,c^3}+\frac{9\,Bb^5c^2x^8-72}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*b^6 - 30*B*a*b^4*c - A*b^5*c + 90*B*a^2*b^2*c^2 + 10*A*a*b^3*c^2 -
 60*B*a^3*c^3 - 30*A*a^2*b*c^3)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((
b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c)) + 1/2*B*x^2/c^3 + 1
/8*(9*B*b^5*c^2*x^8 - 72*B*a*b^3*c^3*x^8 - 3*A*b^4*c^3*x^8 + 144*B*a^2*b*c^
4*x^8 + 24*A*a*b^2*c^4*x^8 - 48*A*a^2*c^5*x^8 + 6*B*b^6*c*x^6 - 48*B*a*b^4*
c^2*x^6 + 2*A*b^5*c^2*x^6 + 84*B*a^2*b^2*c^3*x^6 - 12*A*a*b^3*c^3*x^6 + 72*
B*a^3*c^4*x^6 + 4*A*a^2*b*c^4*x^6 - B*b^7*x^4 + 14*B*a*b^5*c*x^4 + 3*A*b^6*
c*x^4 - 82*B*a^2*b^3*c^2*x^4 - 20*A*a*b^4*c^2*x^4 + 204*B*a^3*b*c^3*x^4 + 2
2*A*a^2*b^2*c^3*x^4 - 32*A*a^3*c^4*x^4 - 2*B*a*b^6*x^2 + 8*B*a^2*b^4*c*x^2
+ 6*A*a*b^5*c*x^2 + 4*B*a^3*b^2*c^2*x^2 - 40*A*a^2*b^3*c^2*x^2 + 56*B*a^4*c
^3*x^2 + 28*A*a^3*b*c^3*x^2 - B*a^2*b^5 + 3*A*a^2*b^4*c + 28*B*a^4*b*c^2 -
18*A*a^3*b^2*c^2)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*(c*x^4 + b*x^2 + a)
^2) - 1/4*(3*B*b - A*c)*log(c*x^4 + b*x^2 + a)/c^4
```

3.125 $\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=254

$$-\frac{x^2\left(x^2\left(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4B\right)+2a\left(6aAc^2-7abBc+b^3B\right)\right)}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\left(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+\ldots\right.}{2c^3\left(b^2-4ac\right)^{5/2}}$$

```
[Out] -(x^6*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(4*c*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)^2) - (x^2*(2*a*(b^3*B - 7*a*b*B*c + 6*a*A*c^2) + (2*b^
4*B - 15*a*b^2*B*c + 6*a*A*b*c^2 + 16*a^2*B*c^2)*x^2))/(4*c^2*(b^2 - 4*a*c)
^2*(a + b*x^2 + c*x^4)) + ((b^5*B - 10*a*b^3*B*c + 30*a^2*b*B*c^2 - 12*a^2*
A*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(5/2)
) + (B*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] time = 0.404194, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 818, 634, 618, 206, 628}

$$-\frac{x^2\left(x^2\left(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4B\right)+2a\left(6aAc^2-7abBc+b^3B\right)\right)}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\left(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+\ldots\right.}{2c^3\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```


```
[Out] -(x^6*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(4*c*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)^2) - (x^2*(2*a*(b^3*B - 7*a*b*B*c + 6*a*A*c^2) + (2*b^
4*B - 15*a*b^2*B*c + 6*a*A*b*c^2 + 16*a^2*B*c^2)*x^2))/(4*c^2*(b^2 - 4*a*c)
^2*(a + b*x^2 + c*x^4)) + ((b^5*B - 10*a*b^3*B*c + 30*a^2*b*B*c^2 - 12*a^2*
A*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(5/2)
) + (B*Log[a + b*x^2 + c*x^4])/(4*c^3)
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
```

```
0])
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^9\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{x^4(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx,x,x^2\right)$$

$$=-\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\text{Subst}\left(\int \frac{x^2\left(3a(bB-2Ac)+2B\left(b^2-4ac\right)x\right)}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)}{4c\left(b^2-4ac\right)}$$

$$=-\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^2\left(2a\left(b^3B-7abBc+6aAc^2\right)+\left(2b^4B-15ab^2\right.\right.}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+\right.}$$

$$=-\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^2\left(2a\left(b^3B-7abBc+6aAc^2\right)+\left(2b^4B-15ab^2\right.\right.}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+\right.}$$

$$=-\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^2\left(2a\left(b^3B-7abBc+6aAc^2\right)+\left(2b^4B-15ab^2\right.\right.}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+\right.}$$

$$=-\frac{x^6\left(a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2\right)}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^2\left(2a\left(b^3B-7abBc+6aAc^2\right)+\left(2b^4B-15ab^2\right.\right.}{4c^2\left(b^2-4ac\right)^2\left(a+bx^2+\right.}$$

Mathematica [A] time = 0.525902, size = 354, normalized size = 1.39

$$\frac{\frac{2a^2bc^3\left(11A+25Bx^2\right)+4a^2c^3\left(8aB-5Acx^2\right)-2ab^3c^2\left(4A+15Bx^2\right)+ab^2c^2\left(16Acx^2-39aB\right)+b^4c\left(11aB-2Acx^2\right)+b^5c\left(A+4Bx^2\right)+b^6(-B)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{a^2c\left(bc\left(3A+5Bx^2\right)-2Ac^2x^2\right.}{}}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-(b^6*B) + b^5*c*(A + 4*B*x^2) - 2*a*b^3*c^2*(4*A + 15*B*x^2) + 2*a^2*b*c
^3*(11*A + 25*B*x^2) + 4*a^2*c^3*(8*a*B - 5*A*c*x^2) + b^4*c*(11*a*B - 2*A*
c*x^2) + a*b^2*c^2*(-39*a*B + 16*A*c*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) + (2*a^3*B*c^2 + b^4*(b*B - A*c)*x^2 + a*b^2*(b^2*B + 4*A*c^2*x^2 - b
*c*(A + 5*B*x^2)) + a^2*c*(-4*b^2*B - 2*A*c^2*x^2 + b*c*(3*A + 5*B*x^2)))/(
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*c*(b^5*B - 10*a*b^3*B*c + 30*a^2*
b*B*c^2 - 12*a^2*A*c^3)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4
*a*c)^(5/2) + B*c*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Maple [B] time = 0.02, size = 723, normalized size = 2.9

$$\frac{1}{2\left(cx^4+bx^2+a\right)^2}\left(-\frac{\left(10\,Aa^2c^3-8\,Aab^2c^2+Ab^4c-25\,Ba^2bc^2+15\,Bab^3c-2\,Bb^5\right)x^6}{c^2\left(16\,a^2c^2-8\,ab^2c+b^4\right)}+\frac{\left(2\,Aa^2bc^3+8\,Aab^3c^2-Ab\right.}{2\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-1/c^2*(10*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c-25*B*a^2*b*c^2+15*B*a*b^3*c
-2*B*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*(2*A*a^2*b*c^3+8*A*a*b^3*c^2-A
*b^5*c+32*B*a^3*c^3+11*B*a^2*b^2*c^2-19*B*a*b^4*c+3*B*b^6)/c^3/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^4-a*(6*A*a^2*c^3-10*A*a*b^2*c^2+A*b^4*c-31*B*a^2*b*c^2+22*
B*a*b^3*c-3*B*b^5)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/2*a^2*(10*A*a*b*c^2
-A*b^3*c+24*B*a^2*c^2-21*B*a*b^2*c+3*B*b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))
/(c*x^4+b*x^2+a)^2+4/c/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*a^2*B-2
/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*a*b^2*B+1/4/c^3/(16*a^2*c
^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*b^4*B+6/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a
*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*a^2-15/c/(16*a^2*c^2-
8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*
b*B+5/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(
4*a*c-b^2)^(1/2))*B*a*b^3-1/2/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1
/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.04094, size = 4617, normalized size = 18.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*B*a^2*b^6 + 2*(2*B*b^7*c + 40*A*a^3*c^5 - 2*(50*B*a^3*b + 21*A*a^2*
b^2)*c^4 + (85*B*a^2*b^3 + 12*A*a*b^4)*c^3 - (23*B*a*b^5 + A*b^6)*c^2)*x^6
+ (3*B*b^8 - 8*(16*B*a^4 + A*a^3*b)*c^4 - 6*(2*B*a^3*b^2 + 5*A*a^2*b^3)*c^3
 + 3*(29*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (31*B*a*b^6 + A*b^7)*c)*x^4 - 8*(12*B
*a^5 + 5*A*a^4*b)*c^3 + 2*(54*B*a^4*b^2 + 7*A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 +
 24*A*a^4*c^4 - 2*(62*B*a^4*b + 23*A*a^3*b^2)*c^3 + 7*(17*B*a^3*b^3 + 2*A*a
^2*b^4)*c^2 - (34*B*a^2*b^5 + A*a*b^6)*c)*x^2 - ((B*b^5*c^2 - 10*B*a*b^3*c^
3 + 30*B*a^2*b*c^4 - 12*A*a^2*c^5)*x^8 + B*a^2*b^5 - 10*B*a^3*b^3*c + 30*B*
a^4*b*c^2 - 12*A*a^4*c^3 + 2*(B*b^6*c - 10*B*a*b^4*c^2 + 30*B*a^2*b^2*c^3 -
 12*A*a^2*b*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*c + 10*B*a^2*b^3*c^2 - 24*A*a^3*c
^4 + 12*(5*B*a^3*b - A*a^2*b^2)*c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2*b^4*c + 30
*B*a^3*b^2*c^2 - 12*A*a^3*b*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*
b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a
)) - (33*B*a^3*b^4 + A*a^2*b^5)*c + (B*a^2*b^6 - 12*B*a^3*b^4*c + 48*B*a^4*
b^2*c^2 - 64*B*a^5*c^3 + (B*b^6*c^2 - 12*B*a*b^4*c^3 + 48*B*a^2*b^2*c^4 - 6
4*B*a^3*c^5)*x^8 + 2*(B*b^7*c - 12*B*a*b^5*c^2 + 48*B*a^2*b^3*c^3 - 64*B*a^
3*b*c^4)*x^6 + (B*b^8 - 10*B*a*b^6*c + 24*B*a^2*b^4*c^2 + 32*B*a^3*b^2*c^3
- 128*B*a^4*c^4)*x^4 + 2*(B*a*b^7 - 12*B*a^2*b^5*c + 48*B*a^3*b^3*c^2 - 64*
B*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 4
8*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*
a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x
^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^
7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x^2
), 1/4*(3*B*a^2*b^6 + 2*(2*B*b^7*c + 40*A*a^3*c^5 - 2*(50*B*a^3*b + 21*A*a^
2*b^2)*c^4 + (85*B*a^2*b^3 + 12*A*a*b^4)*c^3 - (23*B*a*b^5 + A*b^6)*c^2)*x^
6 + (3*B*b^8 - 8*(16*B*a^4 + A*a^3*b)*c^4 - 6*(2*B*a^3*b^2 + 5*A*a^2*b^3)*c
^3 + 3*(29*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (31*B*a*b^6 + A*b^7)*c)*x^4 - 8*(12
*B*a^5 + 5*A*a^4*b)*c^3 + 2*(54*B*a^4*b^2 + 7*A*a^3*b^3)*c^2 + 2*(3*B*a*b^7
 + 24*A*a^4*c^4 - 2*(62*B*a^4*b + 23*A*a^3*b^2)*c^3 + 7*(17*B*a^3*b^3 + 2*A
*a^2*b^4)*c^2 - (34*B*a^2*b^5 + A*a*b^6)*c)*x^2 + 2*((B*b^5*c^2 - 10*B*a*b^
3*c^3 + 30*B*a^2*b*c^4 - 12*A*a^2*c^5)*x^8 + B*a^2*b^5 - 10*B*a^3*b^3*c + 3
0*B*a^4*b*c^2 - 12*A*a^4*c^3 + 2*(B*b^6*c - 10*B*a*b^4*c^2 + 30*B*a^2*b^2*c
^3 - 12*A*a^2*b*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*c + 10*B*a^2*b^3*c^2 - 24*A*a
^3*c^4 + 12*(5*B*a^3*b - A*a^2*b^2)*c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2*b^4*c
+ 30*B*a^3*b^2*c^2 - 12*A*a^3*b*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x
^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (33*B*a^3*b^4 + A*a^2*b^5)*c +
(B*a^2*b^6 - 12*B*a^3*b^4*c + 48*B*a^4*b^2*c^2 - 64*B*a^5*c^3 + (B*b^6*c^2
- 12*B*a*b^4*c^3 + 48*B*a^2*b^2*c^4 - 64*B*a^3*c^5)*x^8 + 2*(B*b^7*c - 12*B
*a*b^5*c^2 + 48*B*a^2*b^3*c^3 - 64*B*a^3*b*c^4)*x^6 + (B*b^8 - 10*B*a*b^6*c
 + 24*B*a^2*b^4*c^2 + 32*B*a^3*b^2*c^3 - 128*B*a^4*c^4)*x^4 + 2*(B*a*b^7 -
12*B*a^2*b^5*c + 48*B*a^3*b^3*c^2 - 64*B*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2
+ a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^
5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5
*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^
2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c
^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

[Out] Timed out

Giac [A] time = 39.023, size = 629, normalized size = 2.48

$$-\frac{\left(Bb^5-10\,Bab^3c+30\,Ba^2bc^2-12\,Aa^2c^3\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(b^4c^3-8\,ab^2c^4+16\,a^2c^5\right)\sqrt{-b^2+4\,ac}}+\frac{B\log\left(cx^4+bx^2+a\right)}{4\,c^3}-\frac{3\,Bb^4c^2x^8-24\,Bab^2c^3x^8+4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^5 - 10*B*a*b^3*c + 30*B*a^2*b*c^2 - 12*A*a^2*c^3)*arctan((2*c*x^2
 + b)/sqrt(-b^2 + 4*a*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 +
 4*a*c)) + 1/4*B*log(c*x^4 + b*x^2 + a)/c^3 - 1/8*(3*B*b^4*c^2*x^8 - 24*B*a
*b^2*c^3*x^8 + 48*B*a^2*c^4*x^8 - 2*B*b^5*c*x^6 + 12*B*a*b^3*c^2*x^6 + 4*A*
b^4*c^2*x^6 - 4*B*a^2*b*c^3*x^6 - 32*A*a*b^2*c^3*x^6 + 40*A*a^2*c^4*x^6 - 3
*B*b^6*x^4 + 20*B*a*b^4*c*x^4 + 2*A*b^5*c*x^4 - 22*B*a^2*b^2*c^2*x^4 - 16*A
*a*b^3*c^2*x^4 + 32*B*a^3*c^3*x^4 - 4*A*a^2*b*c^3*x^4 - 6*B*a*b^5*x^2 + 40*
B*a^2*b^3*c*x^2 + 4*A*a*b^4*c*x^2 - 28*B*a^3*b*c^2*x^2 - 40*A*a^2*b^2*c^2*x
^2 + 24*A*a^3*c^3*x^2 - 3*B*a^2*b^4 + 18*B*a^3*b^2*c + 2*A*a^2*b^3*c - 20*A
*a^3*b*c^2)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*(c*x^4 + b*x^2 + a)^2)
```

3.126 $\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=146

$$-\frac{x^6\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x^2\left(2a+bx^2\right)(Ab-2aB)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3a(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] -(x^6*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (3*(A*b - 2*a*B)*x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 +
 c*x^4)) + (3*a*(A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^
2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.138687, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1251, 804, 722, 618, 206}

$$-\frac{x^6\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x^2\left(2a+bx^2\right)(Ab-2aB)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3a(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```


```
[Out] -(x^6*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (3*(A*b - 2*a*B)*x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 +
 c*x^4)) + (3*a*(A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^
2 - 4*a*c)^(5/2)
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 804

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(b*f - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(m*(b*(
e*f + d*g) - 2*(c*d*f + a*e*g)))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m
- 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0] && LtQ[p, -1]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
```

```
0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^3(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^6\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{(3(Ab-2aB))\,\mathrm{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^6\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3(Ab-2aB)x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{(3a(Ab-2aB))\,\mathrm{Subst}\left(\int \right.}{2\left(b^2-4a\right.}$$

$$= -\frac{x^6\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3(Ab-2aB)x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{(3a(Ab-2aB))\,\mathrm{Subst}\left(\right.}{\left(b^2-\right.}$$

$$= -\frac{x^6\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{3(Ab-2aB)x^2\left(2a+bx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3a(Ab-2aB)\tanh^{-1}\left(\right.}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.294007, size = 261, normalized size = 1.79

$$\frac{1}{4}\left(\frac{-4a^2c^3\left(4A+5Bx^2\right)+ab^2c^2\left(5A+16Bx^2\right)+2abc^2\left(11aB-3Acx^2\right)-8ab^3Bc-b^4c\left(A+2Bx^2\right)+b^5B}{c^3\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{a^2c\left(2c\left(\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b^5*B - 8*a*b^3*B*c - b^4*c*(A + 2*B*x^2) - 4*a^2*c^3*(4*A + 5*B*x^2) + a
*b^2*c^2*(5*A + 16*B*x^2) + 2*a*b*c^2*(11*a*B - 3*A*c*x^2))/(c^3*(b^2 - 4*a
*c)^2*(a + b*x^2 + c*x^4)) + (b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A
+ B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2)))/(c^3*(-b^2 + 4*a
*c)*(a + b*x^2 + c*x^4)^2) - (12*a*(A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqrt[
-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

Maple [B] time = 0.017, size = 398, normalized size = 2.7

$$\frac{1}{2\left(cx^4+bx^2+a\right)^2}\left(-\frac{\left(3\,aAbc^2+10\,a^2Bc^2-8\,ab^2Bc+b^4B\right)x^6}{c\left(16\,a^2c^2-8\,ab^2c+b^4\right)} - \frac{\left(16\,Aa^2c^3+Aab^2c^2+Ab^4c-2\,Ba^2bc^2-8\,Bab^3c+\right.}{\left(32\,a^2c^2-16\,ab^2c+2\,b^4\right)c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-(3*A*a*b*c^2+10*B*a^2*c^2-8*B*a*b^2*c+B*b^4)/c/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^6-1/2*(16*A*a^2*c^3+A*a*b^2*c^2+A*b^4*c-2*B*a^2*b*c^2-8*B*a*b^3*c+B*
b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^4-a*(5*A*a*b*c^2+A*b^3*c+6*B*a^2*c^2-
10*B*a*b^2*c+B*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^2-1/2*a^2/c^2*(8*A*a*c
^2+A*b^2*c-10*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-
3*a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-
b^2)^(1/2))*A*b+6*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.45698, size = 2839, normalized size = 19.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(B*a^2*b^5 - 32*A*a^4*c^3 + 2*(B*b^6*c - 12*B*a*b^4*c^2 - 4*(10*B*a^3
 + 3*A*a^2*b)*c^4 + 3*(14*B*a^2*b^2 + A*a*b^3)*c^3)*x^6 + (B*b^7 - 64*A*a^3
*c^4 + 4*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 + 3*(10*B*a^2*b^3 - A*a*b^4)*c^2 - (
12*B*a*b^5 - A*b^6)*c)*x^4 + 4*(10*B*a^4*b + A*a^3*b^2)*c^2 + 2*(B*a*b^6 -
4*(6*B*a^4 + 5*A*a^3*b)*c^3 + (46*B*a^3*b^2 + A*a^2*b^3)*c^2 - (14*B*a^2*b^
4 - A*a*b^5)*c)*x^2 + 6*((2*B*a^2 - A*a*b)*c^4*x^8 + 2*(2*B*a^2*b - A*a*b^2
)*c^3*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^2 + (2*(2*B*a^3 - A*a^2*b)*c^3
+ (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a^4 - A*a^3*b)*c^2)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) - (14*B*a^3*b^3 - A*a^2*b^4)*c)/(a^2*b^6*c^2 - 1
2*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*
a^2*b^2*c^6 - 64*a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5
- 64*a^3*b*c^6)*x^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2
*c^5 - 128*a^4*c^6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 -
64*a^4*b*c^5)*x^2), -1/4*(B*a^2*b^5 - 32*A*a^4*c^3 + 2*(B*b^6*c - 12*B*a*b^
4*c^2 - 4*(10*B*a^3 + 3*A*a^2*b)*c^4 + 3*(14*B*a^2*b^2 + A*a*b^3)*c^3)*x^6
+ (B*b^7 - 64*A*a^3*c^4 + 4*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 + 3*(10*B*a^2*b^3
 - A*a*b^4)*c^2 - (12*B*a*b^5 - A*b^6)*c)*x^4 + 4*(10*B*a^4*b + A*a^3*b^2)*
c^2 + 2*(B*a*b^6 - 4*(6*B*a^4 + 5*A*a^3*b)*c^3 + (46*B*a^3*b^2 + A*a^2*b^3)
*c^2 - (14*B*a^2*b^4 - A*a*b^5)*c)*x^2 + 12*((2*B*a^2 - A*a*b)*c^4*x^8 + 2*
(2*B*a^2*b - A*a*b^2)*c^3*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^2 + (2*(2*B
*a^3 - A*a^2*b)*c^3 + (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a^4 - A*a^3*b
)*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
```

```
*a*c)) - (14*B*a^3*b^3 - A*a^2*b^4)*c)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a
^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3
*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^6
+ (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*
x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x^2)]
```

Sympy [B] time = 149.303, size = 775, normalized size = 5.31

$$-\frac{3a\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-Ab+2Ba\right)\log\left(x^2+\frac{-3Aab^2+6Ba^2b-192a^4c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-Ab+2Ba)+144a^3b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-Ab+2Ba)-36a^2b^4c\sqrt{-\frac{1}{(4ac}}}{-6Aabc+12Ba^2c}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*a*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a)*log(x**2 + (-3*A*a*b**2 + 6*
B*a**2*b - 192*a**4*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) + 144*a*
*3*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) - 36*a**2*b**4*c*sqr
t(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) + 3*a*b**6*sqrt(-1/(4*a*c - b**2)**5
)*(-A*b + 2*B*a))/(-6*A*a*b*c + 12*B*a**2*c))/2 + 3*a*sqrt(-1/(4*a*c - b**2
)**5)*(-A*b + 2*B*a)*log(x**2 + (-3*A*a*b**2 + 6*B*a**2*b + 192*a**4*c**3*s
qrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a) - 144*a**3*b**2*c**2*sqrt(-1/(4*a*
c - b**2)**5)*(-A*b + 2*B*a) + 36*a**2*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-
A*b + 2*B*a) - 3*a*b**6*sqrt(-1/(4*a*c - b**2)**5)*(-A*b + 2*B*a))/(-6*A*a*
b*c + 12*B*a**2*c))/2 - (8*A*a**3*c**2 + A*a**2*b**2*c - 10*B*a**3*b*c + B*
a**2*b**3 + x**6*(6*A*a*b*c**3 + 20*B*a**2*c**3 - 16*B*a*b**2*c**2 + 2*B*b*
*4*c) + x**4*(16*A*a**2*c**3 + A*a*b**2*c**2 + A*b**4*c - 2*B*a**2*b*c**2 -
 8*B*a*b**3*c + B*b**5) + x**2*(10*A*a**2*b*c**2 + 2*A*a*b**3*c + 12*B*a**3
*c**2 - 20*B*a**2*b**2*c + 2*B*a*b**4))/(64*a**4*c**4 - 32*a**3*b**2*c**3 +
 4*a**2*b**4*c**2 + x**8*(64*a**2*c**6 - 32*a*b**2*c**5 + 4*b**4*c**4) + x*
*6*(128*a**2*b*c**5 - 64*a*b**3*c**4 + 8*b**5*c**3) + x**4*(128*a**3*c**5 -
 24*a*b**4*c**3 + 4*b**6*c**2) + x**2*(128*a**3*b*c**4 - 64*a**2*b**3*c**3
+ 8*a*b**5*c**2))
```

Giac [B] time = 35.4841, size = 429, normalized size = 2.94

$$\frac{3\left(2Ba^2-Aab\right)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{\left(b^4-8ab^2c+16a^2c^2\right)\sqrt{-b^2+4ac}}-\frac{2Bb^4cx^6-16Bab^2c^2x^6+20Ba^2c^3x^6+6Aabc^3x^6+Bb^5x^4-8Bab^3cx^4+Ab^4cx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3*(2*B*a^2 - A*a*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^
2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(2*B*b^4*c*x^6 - 16*B*a*b^2*c^2
*x^6 + 20*B*a^2*c^3*x^6 + 6*A*a*b*c^3*x^6 + B*b^5*x^4 - 8*B*a*b^3*c*x^4 + A
*b^4*c*x^4 - 2*B*a^2*b*c^2*x^4 + A*a*b^2*c^2*x^4 + 16*A*a^2*c^3*x^4 + 2*B*a
*b^4*x^2 - 20*B*a^2*b^2*c*x^2 + 2*A*a*b^3*c*x^2 + 12*B*a^3*c^2*x^2 + 10*A*a
^2*b*c^2*x^2 + B*a^2*b^3 - 10*B*a^3*b*c + A*a^2*b^2*c + 8*A*a^3*c^2)/((b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
```

3.127 $\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=185

$$-\frac{x^2\left(4aAc^2+2abBc-4Ab^2c+b^3B\right)+a\left(8aBc-6Abc+b^2B\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^4\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{(3abB-A(2ac$$

```
[Out] -(x^4*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) - (a*(b^2*B - 6*A*b*c + 8*a*B*c) + (b^3*B - 4*A*b^2*c + 2*a*b*B*c + 4
*a*A*c^2)*x^2)/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*a*b*B - A*(b
^2 + 2*a*c))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.261968, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1251, 820, 777, 618, 206}

$$-\frac{x^2\left(4aAc^2+2abBc-4Ab^2c+b^3B\right)+a\left(8aBc-6Abc+b^2B\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^4\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{(3abB-A(2ac$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^4*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) - (a*(b^2*B - 6*A*b*c + 8*a*B*c) + (b^3*B - 4*A*b^2*c + 2*a*b*B*c + 4
*a*A*c^2)*x^2)/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*a*b*B - A*(b
^2 + 2*a*c))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*
(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p +
 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*
(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(
m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (I
ntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
```

```
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^4\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x(-2(Ab-2aB)-(bB-2Ac)x)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^4\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{a\left(b^2B-6Abc+8aBc\right)+\left(b^3B-4Ab^2c+2abBc+4aAc^2 \cdots\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x^4\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{a\left(b^2B-6Abc+8aBc\right)+\left(b^3B-4Ab^2c+2abBc+4aAc^2 \cdots\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x^4\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{a\left(b^2B-6Abc+8aBc\right)+\left(b^3B-4Ab^2c+2abBc+4aAc^2 \cdots\right.}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 0.271144, size = 233, normalized size = 1.26

$$\frac{1}{4}\left(\frac{2a^2Bc+a\left(bc\left(A+3Bx^2\right)-2Ac^2x^2+b^2(-B)\right)+b^2x^2(Ac-bB)}{c^2\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{b^2c\left(5aB+2Acx^2\right)+2abc^2\left(A-3Bx^2\right)+4ac^2\left(\cdots\right.}{c^2\left(b^2-4ac\right)^2\left(a+bx^2 \cdots\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-(b^4*B) + A*b^3*c + 2*a*b*c^2*(A - 3*B*x^2) + 4*a*c^2*(-4*a*B + A*c*x^2)
 + b^2*c*(5*a*B + 2*A*c*x^2))/(c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*(A + 3
*B*x^2)))/(c^2*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (4*(-3*a*b*B + A*(b^
2 + 2*a*c))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))
/4
```

Maple [B] time = 0.014, size = 411, normalized size = 2.2

$$\frac{1}{2\left(cx^{4}+bx^{2}+a\right)^{2}}\left(\frac{c\left(2\,aAc+Ab^{2}-3\,abB\right)x^{6}}{16\,a^{2}c^{2}-8\,ab^{2}c+b^{4}}+\frac{\left(6\,aAbc^{2}+3\,Ab^{3}c-16\,a^{2}Bc^{2}-ab^{2}Bc-b^{4}B\right)x^{4}}{2\,c\left(16\,a^{2}c^{2}-8\,ab^{2}c+b^{4}\right)}-\frac{a\left(2\,aAc^{2}-5\,Ab^{2}c\right.}{c\left(16\,a^{2}c^{2}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(c*(2*A*a*c+A*b^2-3*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*(6*A*a*b*
c^2+3*A*b^3*c-16*B*a^2*c^2-B*a*b^2*c-B*b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
4-1/c*a*(2*A*a*c^2-5*A*b^2*c+5*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
2+1/2*a^2*(6*A*b*c-8*B*a*c-B*b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^
2+a)^2+2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))*a*A*c+1/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^2-3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-
b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.46039, size = 2824, normalized size = 15.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(B*a^2*b^4 + 2*(8*A*a^2*c^4 - 2*(6*B*a^2*b - A*a*b^2)*c^3 + (3*B*a*b^
3 - A*b^4)*c^2)*x^6 + (B*b^6 - 8*(8*B*a^3 - 3*A*a^2*b)*c^3 + 6*(2*B*a^2*b^2
 + A*a*b^3)*c^2 - 3*(B*a*b^4 + A*b^5)*c)*x^4 - 8*(4*B*a^4 - 3*A*a^3*b)*c^2
+ 2*(B*a*b^5 - 8*A*a^3*c^3 - 2*(10*B*a^3*b - 11*A*a^2*b^2)*c^2 + (B*a^2*b^3
 - 5*A*a*b^4)*c)*x^2 - 2*((2*A*a*c^4 - (3*B*a*b - A*b^2)*c^3)*x^8 + 2*(2*A*
a*b*c^3 - (3*B*a*b^2 - A*b^3)*c^2)*x^6 + 2*A*a^3*c^2 + (4*A*a^2*c^3 - 2*(3*
B*a^2*b - 2*A*a*b^2)*c^2 - (3*B*a*b^3 - A*b^4)*c)*x^4 + 2*(2*A*a^2*b*c^2 -
(3*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (3*B*a^3*b - A*a^2*b^2)*c)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*
c))/(c*x^4 + b*x^2 + a)) + 2*(2*B*a^3*b^2 - 3*A*a^2*b^3)*c)/(a^2*b^6*c - 12
*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a
^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 -
 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2*c^
4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^
4*b*c^4)*x^2), -1/4*(B*a^2*b^4 + 2*(8*A*a^2*c^4 - 2*(6*B*a^2*b - A*a*b^2)*c
^3 + (3*B*a*b^3 - A*b^4)*c^2)*x^6 + (B*b^6 - 8*(8*B*a^3 - 3*A*a^2*b)*c^3 +
6*(2*B*a^2*b^2 + A*a*b^3)*c^2 - 3*(B*a*b^4 + A*b^5)*c)*x^4 - 8*(4*B*a^4 - 3
```

```
*A*a^3*b)*c^2 + 2*(B*a*b^5 - 8*A*a^3*c^3 - 2*(10*B*a^3*b - 11*A*a^2*b^2)*c^
2 + (B*a^2*b^3 - 5*A*a*b^4)*c)*x^2 + 4*((2*A*a*c^4 - (3*B*a*b - A*b^2)*c^3)
*x^8 + 2*(2*A*a*b*c^3 - (3*B*a*b^2 - A*b^3)*c^2)*x^6 + 2*A*a^3*c^2 + (4*A*a
^2*c^3 - 2*(3*B*a^2*b - 2*A*a*b^2)*c^2 - (3*B*a*b^3 - A*b^4)*c)*x^4 + 2*(2*
A*a^2*b*c^2 - (3*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (3*B*a^3*b - A*a^2*b^2)*c)*s
qrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
 2*(2*B*a^3*b^2 - 3*A*a^2*b^3)*c)/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*
c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x
^8 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*
c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(
a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2)]
```

Sympy [B] time = 63.2358, size = 833, normalized size = 4.5

$$\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-2Aac-Ab^2+3Bab\right)\log\left(x^2+\frac{-2Aabc-Ab^3+3Bab^2-64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-2Aac-Ab^2+3Bab\right)+48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-2Aac\right.}{-4Aac^2-2A}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b)*log(x**2 + (-2*A*a
*b*c - A*b**3 + 3*B*a*b**2 - 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*
a*c - A*b**2 + 3*B*a*b) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*
A*a*c - A*b**2 + 3*B*a*b) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*
c - A*b**2 + 3*B*a*b) + b**6*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2
+ 3*B*a*b))/(-4*A*a*c**2 - 2*A*b**2*c + 6*B*a*b*c))/2 - sqrt(-1/(4*a*c - b*
*2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b)*log(x**2 + (-2*A*a*b*c - A*b**3 + 3*B
*a*b**2 + 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*
a*b) - 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*
B*a*b) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*
b) - b**6*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*a*c - A*b**2 + 3*B*a*b))/(-4*A*a
*c**2 - 2*A*b**2*c + 6*B*a*b*c))/2 - (-6*A*a**2*b*c + 8*B*a**3*c + B*a**2*b
**2 + x**6*(-4*A*a*c**3 - 2*A*b**2*c**2 + 6*B*a*b*c**2) + x**4*(-6*A*a*b*c*
*2 - 3*A*b**3*c + 16*B*a**2*c**2 + B*a*b**2*c + B*b**4) + x**2*(4*A*a**2*c*
*2 - 10*A*a*b**2*c + 10*B*a**2*b*c + 2*B*a*b**3))/(64*a**4*c**3 - 32*a**3*b
**2*c**2 + 4*a**2*b**4*c + x**8*(64*a**2*c**5 - 32*a*b**2*c**4 + 4*b**4*c**
3) + x**6*(128*a**2*b*c**4 - 64*a*b**3*c**3 + 8*b**5*c**2) + x**4*(128*a**3
*c**4 - 24*a*b**4*c**2 + 4*b**6*c) + x**2*(128*a**3*b*c**3 - 64*a**2*b**3*c
**2 + 8*a*b**5*c))
```

Giac [A] time = 31.8293, size = 362, normalized size = 1.96

$$-\frac{\left(3\,Bab-Ab^2-2\,Aac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,Babc^2x^6-2\,Ab^2c^2x^6-4\,Aac^3x^6+Bb^4x^4+Bab^2cx^4-3\,Ab^3cx^4+16\,}{4\left(b^4c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -(3*B*a*b - A*b^2 - 2*A*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4
 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*B*a*b*c^2*x^6 - 2*A
*b^2*c^2*x^6 - 4*A*a*c^3*x^6 + B*b^4*x^4 + B*a*b^2*c*x^4 - 3*A*b^3*c*x^4 +
16*B*a^2*c^2*x^4 - 6*A*a*b*c^2*x^4 + 2*B*a*b^3*x^2 + 10*B*a^2*b*c*x^2 - 10*
A*a*b^2*c*x^2 + 4*A*a^2*c^2*x^2 + B*a^2*b^2 + 8*B*a^3*c - 6*A*a^2*b*c)/((b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

3.128 $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=170

$$\frac{(b+2cx^2)(2aBc-3Abc+b^2B)}{4c(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x^2(-2aBc-Abc+b^2B)+a(bB-2Ac)}{4c(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{(2aBc-3Abc+b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] -(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(4*c*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)^2) + ((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(4*c*(b^2 -
 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((b^2*B - 3*A*b*c + 2*a*B*c)*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.163101, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1251, 777, 614, 618, 206}

$$\frac{(b+2cx^2)(2aBc-3Abc+b^2B)}{4c(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x^2(-2aBc-Abc+b^2B)+a(bB-2Ac)}{4c(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{(2aBc-3Abc+b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```



```
[Out] -(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(4*c*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)^2) + ((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(4*c*(b^2 -
 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((b^2*B - 3*A*b*c + 2*a*B*c)*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x(A+Bx)}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\left(b^2B-3Abc+2aBc\right)\mathrm{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4c\left(b^2-4ac\right)}$$

$$= -\frac{a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\left(b^2B-3Abc+2aBc\right)\left(b+2cx^2\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\left(b^2B-3Abc+2\right.}{}$$

$$= -\frac{a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\left(b^2B-3Abc+2aBc\right)\left(b+2cx^2\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(b^2B-3Abc+2\right.}{}$$

$$= -\frac{a(bB-2Ac)+\left(b^2B-Abc-2aBc\right)x^2}{4c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{\left(b^2B-3Abc+2aBc\right)\left(b+2cx^2\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{\left(b^2B-3Abc+2\right.}{\left(b^2\right.}$$

Mathematica [A] time = 0.237169, size = 172, normalized size = 1.01

$$\frac{1}{4}\left(\frac{\left(b+2cx^2\right)\left(2aBc-3Abc+b^2B\right)}{c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{-2ac\left(A+Bx^2\right)+abB+bx^2(bB-Ac)}{c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2} + \frac{4\left(2aBc-3Abc+b^2B\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(c*(b^2 - 4*a*c)^2*(a + b*x^2
+ c*x^4)) + (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(c*(-b^2 + 4*a*
c)*(a + b*x^2 + c*x^4)^2) + (4*(b^2*B - 3*A*b*c + 2*a*B*c)*ArcTan[(b + 2*c*
x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

Maple [B] time = 0.013, size = 379, normalized size = 2.2

$$\frac{1}{2\left(cx^4+bx^2+a\right)^2}\left(-\frac{c\left(3\,Abc-2\,aBc-b^2B\right)x^6}{16\,a^2c^2-8\,ab^2c+b^4} - \frac{3\,b\left(3\,Abc-2\,aBc-b^2B\right)x^4}{32\,a^2c^2-16\,ab^2c+2\,b^4} - \frac{\left(5\,Aabc+Ab^3+2\,a^2Bc-5\,Bab^2\right)x^2}{16\,a^2c^2-8\,ab^2c+b^4} - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-c*(3*A*b*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-3/2*b*(3*A*b
*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-(5*A*a*b*c+A*b^3+2*B*a^2*c
-5*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/2*a*(8*A*a*c+A*b^2-6*B*a*b)/(1
6*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b*c+2/(16*a^2*c^2-8
*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*B*c
+1/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b
^2)^(1/2))*b^2*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.57635, size = 2595, normalized size = 15.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b^4*c - 4*(2*B*a^2 - 3*A*a*b)*c^3 - (2*B*a*b^2 + 3*A*b^3)*c^2)*x
^6 + 6*B*a^2*b^3 - A*a*b^4 + 32*A*a^3*c^2 + 3*(B*b^5 - 4*(2*B*a^2*b - 3*A*a
*b^2)*c^2 - (2*B*a*b^3 + 3*A*b^4)*c)*x^4 + 2*(5*B*a*b^4 - A*b^5 + 4*(2*B*a^
3 + 5*A*a^2*b)*c^2 - (22*B*a^2*b^2 + A*a*b^3)*c)*x^2 - 2*((B*b^2*c^2 + (2*B
*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a^2*b^
2 + (B*b^4 + 2*(2*B*a^2 - 3*A*a*b)*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4 + 2*(
B*a*b^3 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^
2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(6*B*a^3*b + A*a^2*b^2)*c)/((b^6*c^2 -
 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 12
8*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x
^2), 1/4*(2*(B*b^4*c - 4*(2*B*a^2 - 3*A*a*b)*c^3 - (2*B*a*b^2 + 3*A*b^3)*c^
2)*x^6 + 6*B*a^2*b^3 - A*a*b^4 + 32*A*a^3*c^2 + 3*(B*b^5 - 4*(2*B*a^2*b - 3
*A*a*b^2)*c^2 - (2*B*a*b^3 + 3*A*b^4)*c)*x^4 + 2*(5*B*a*b^4 - A*b^5 + 4*(2*
B*a^3 + 5*A*a^2*b)*c^2 - (22*B*a^2*b^2 + A*a*b^3)*c)*x^2 - 4*((B*b^2*c^2 +
(2*B*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a^
2*b^2 + (B*b^4 + 2*(2*B*a^2 - 3*A*a*b)*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4 +
 2*(B*a*b^3 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*sqr
t(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 4
*(6*B*a^3*b + A*a^2*b^2)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b
```

```
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

Sympy [B] time = 27.4522, size = 789, normalized size = 4.64

$$-\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}\left(-3Abc+2Bac+Bb^2\right)\log\left(x^2+\frac{-3Ab^2c+2Babc+Bb^3-64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}\left(-3Abc+2Bac+Bb^2\right)+48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\left(-3Abc+2B\cdots\right.}{-6Abc^2+4Bac^2+\cdots}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2)*log(x**2 + (-3*A*
b**2*c + 2*B*a*b*c + B*b**3 - 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-3*A
*b*c + 2*B*a*c + B*b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-3
*A*b*c + 2*B*a*c + B*b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b
*c + 2*B*a*c + B*b**2) + b**6*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*
c + B*b**2))/(-6*A*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c))/2 + sqrt(-1/(4*a*c -
b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2)*log(x**2 + (-3*A*b**2*c + 2*B*a*b*c
 + B*b**3 + 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B
*b**2) - 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c +
 B*b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b
**2) - b**6*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2))/(-6*A
*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c))/2 + (-8*A*a**2*c - A*a*b**2 + 6*B*a**2*
b + x**6*(-6*A*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c) + x**4*(-9*A*b**2*c + 6*B*
a*b*c + 3*B*b**3) + x**2*(-10*A*a*b*c - 2*A*b**3 - 4*B*a**2*c + 10*B*a*b**2
))/(64*a**4*c**2 - 32*a**3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a
*b**2*c**3 + 4*b**4*c**2) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5
*c) + x**4*(128*a**3*c**3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 -
 64*a**2*b**3*c + 8*a*b**5))
```

Giac [A] time = 27.6781, size = 308, normalized size = 1.81

$$\frac{\left(Bb^2+2\,Bac-3\,Abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{2\,Bb^2cx^6+4\,Bac^2x^6-6\,Abc^2x^6+3\,Bb^3x^4+6\,Babcx^4-9\,Ab^2cx^4+10\,Bab^2\cdots}{4\left(cx^4+bx^2+a\right)^2\left(b^4-8\,a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (B*b^2 + 2*B*a*c - 3*A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4
- 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*B*b^2*c*x^6 + 4*B*a*
c^2*x^6 - 6*A*b*c^2*x^6 + 3*B*b^3*x^4 + 6*B*a*b*c*x^4 - 9*A*b^2*c*x^4 + 10*
B*a*b^2*x^2 - 2*A*b^3*x^2 - 4*B*a^2*c*x^2 - 10*A*a*b*c*x^2 + 6*B*a^2*b - A*
a*b^2 - 8*A*a^2*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

3.129 $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=139

$$-\frac{3\left(b+2cx^2\right)(bB-2Ac)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{-2aB+x^2(-(bB-2Ac))+Ab}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3c(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] -(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
- (3*(b*B - 2*A*c)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
 (3*c*(b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)
^(5/2)
```

Rubi [A] time = 0.123591, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1247, 638, 614, 618, 206}

$$-\frac{3\left(b+2cx^2\right)(bB-2Ac)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{-2aB+x^2(-(bB-2Ac))+Ab}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3c(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
- (3*(b*B - 2*A*c)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
 (3*c*(b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)
^(5/2)
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$
$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{(3(bB-2Ac))\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$
$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3(bB-2Ac)\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{(3c(bB-2Ac))\,\text{Subst}\left(\int \frac{1}{a+bx}\right.}{2\left(b^2-4ac\right)^2}$$
$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3(bB-2Ac)\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{(3c(bB-2Ac))\,\text{Subst}\left(\int \frac{}{b^2-4}\right.}{\left(b^2-4ac\right)}$$
$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3(bB-2Ac)\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3c(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right.}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A] time = 0.142282, size = 142, normalized size = 1.02

$$\frac{\frac{\left(b^2-4ac\right)\left(B\left(2a+bx^2\right)-A\left(b+2cx^2\right)\right)}{\left(a+bx^2+cx^4\right)^2}-\frac{12c(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{3\left(b+2cx^2\right)(bB-2Ac)}{a+bx^2+cx^4}}{4\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-3*(b*B - 2*A*c)*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + ((b^2 - 4*a*c)*(B*(
2*a + b*x^2) - A*(b + 2*c*x^2)))/(a + b*x^2 + c*x^4)^2 - (12*c*(b*B - 2*A*c
)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(4*(b^2 - 4
*a*c)^2)
```

Maple [A] time = 0.009, size = 262, normalized size = 1.9

$$\frac{(2\,Ac-bB)\,x^2+Ab-2\,aB}{\left(16\,ac-4\,b^2\right)\left(cx^4+bx^2+a\right)^2}+3\,\frac{c^2x^2A}{\left(4\,ac-b^2\right)^2\left(cx^4+bx^2+a\right)}-\frac{3\,cx^2bB}{2\left(4\,ac-b^2\right)^2\left(cx^4+bx^2+a\right)}+\frac{3\,Abc}{2\left(4\,ac-b^2\right)^2\left(cx^4+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/4*((2*A*c-B*b)*x^2+A*b-2*a*B)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2+3/(4*a*c-b^2)
^2/(c*x^4+b*x^2+a)*c^2*x^2*A-3/2/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*c*x^2*b*B+3/
2/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*b*A*c-3/4/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*b^2
*B+6/(4*a*c-b^2)^(5/2)*c^2*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A-3/(4*a*c
-b^2)^(5/2)*c*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.40962, size = 2367, normalized size = 17.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(B*b^3*c^2 + 8*A*a*c^4 - 2*(2*B*a*b + A*b^2)*c^3)*x^6 + B*a*b^4 +
A*b^5 + 9*(B*b^4*c + 8*A*a*b*c^3 - 2*(2*B*a*b^2 + A*b^3)*c^2)*x^4 - 8*(4*B*
a^3 - 5*A*a^2*b)*c^2 + 2*(B*b^5 + 40*A*a^2*c^3 - 2*(10*B*a^2*b + A*a*b^2)*c
^2 + (B*a*b^3 - 2*A*b^4)*c)*x^2 + 6*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2
 - 2*A*b*c^3)*x^6 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2*(B*a
*b - A*b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*l
og((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/
(c*x^4 + b*x^2 + a)) + 2*(2*B*a^2*b^2 - 7*A*a*b^3)*c)/((b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^
4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)
*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), -1/4*
(6*(B*b^3*c^2 + 8*A*a*c^4 - 2*(2*B*a*b + A*b^2)*c^3)*x^6 + B*a*b^4 + A*b^5
+ 9*(B*b^4*c + 8*A*a*b*c^3 - 2*(2*B*a*b^2 + A*b^3)*c^2)*x^4 - 8*(4*B*a^3 -
5*A*a^2*b)*c^2 + 2*(B*b^5 + 40*A*a^2*c^3 - 2*(10*B*a^2*b + A*a*b^2)*c^2 + (
B*a*b^3 - 2*A*b^4)*c)*x^2 - 12*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*
A*b*c^3)*x^6 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2*(B*a*b -
A*b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arcta
n(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 2*(2*B*a^2*b^2 - 7*A*a
*b^3)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2
+ 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 +
 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c
^2 - 64*a^4*b*c^3)*x^2)]
```

Sympy [B] time = 15.4864, size = 661, normalized size = 4.76

$$\frac{3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(-2Ac+Bb\right)\log\left(x^2+\frac{-6Abc^2+3Bb^2c-192a^3c^4\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-2Ac+Bb)+144a^2b^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-2Ac+Bb)-36ab^4c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}(-}{-12Ac^3+6Bbc^2}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*c**2 + 3*B
*b**2*c - 192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 144*a**
2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 36*a*b**4*c**2*sqrt
(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)
*(-2*A*c + B*b))/(-12*A*c**3 + 6*B*b*c**2))/2 - 3*c*sqrt(-1/(4*a*c - b**2)*
*5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*c**2 + 3*B*b**2*c + 192*a**3*c**4*sqr
t(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 144*a**2*b**2*c**3*sqrt(-1/(4*a*c
- b**2)**5)*(-2*A*c + B*b) + 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*
A*c + B*b) - 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b))/(-12*A*c**
3 + 6*B*b*c**2))/2 - (-10*A*a*b*c + A*b**3 + 8*B*a**2*c + B*a*b**2 + x**6*(
-12*A*c**3 + 6*B*b*c**2) + x**4*(-18*A*b*c**2 + 9*B*b**2*c) + x**2*(-20*A*a
*c**2 - 4*A*b**2*c + 10*B*a*b*c + 2*B*b**3))/(64*a**4*c**2 - 32*a**3*b**2*c
 + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x**6*
(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c**3 - 24*a*
b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

Giac [A] time = 26.2094, size = 281, normalized size = 2.02

$$-\frac{3\left(Bbc-2\,Ac^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}-\frac{6\,Bbc^2x^6-12\,Ac^3x^6+9\,Bb^2cx^4-18\,Abc^2x^4+2\,Bb^3x^2+10\,Babcx^2-4\,Ab^2cx^2-}{4\left(cx^4+bx^2+a\right)^2\left(b^4-8\,ab^2c+16\,a^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*(B*b*c - 2*A*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b
^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*B*b*c^2*x^6 - 12*A*c^3*x^6
+ 9*B*b^2*c*x^4 - 18*A*b*c^2*x^4 + 2*B*b^3*x^2 + 10*B*a*b*c*x^2 - 4*A*b^2*c
*x^2 - 20*A*a*c^2*x^2 + B*a*b^2 + A*b^3 + 8*B*a^2*c - 10*A*a*b*c)/((c*x^4 +
 b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

3.130 $\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=252

$$\frac{2cx^2\left(6a^2Bc+A\left(b^3-7abc\right)\right)+A\left(16a^2c^2-15ab^2c+2b^4\right)+6a^2bBc}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(12a^3Bc^2-A\left(30a^2bc^2-10ab^3c+b^5\right)\right)\tanh^{-}}{2a^3\left(b^2-4ac\right)^{5/2}}$$

```
[Out] -(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(4*a*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)^2) + (6*a^2*b*B*c + A*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2) + 2*c*
(6*a^2*B*c + A*(b^3 - 7*a*b*c))*x^2)/(4*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) - ((12*a^3*B*c^2 - A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2))*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)) + (A*Log[x])/a^3 -
 (A*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rubi [A] time = 0.543441, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{2cx^2\left(6a^2Bc+A\left(b^3-7abc\right)\right)+A\left(16a^2c^2-15ab^2c+2b^4\right)+6a^2bBc}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(12a^3Bc^2-A\left(30a^2bc^2-10ab^3c+b^5\right)\right)\tanh^{-}}{2a^3\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] -(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(4*a*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)^2) + (6*a^2*b*B*c + A*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2) + 2*c*
(6*a^2*B*c + A*(b^3 - 7*a*b*c))*x^2)/(4*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) - ((12*a^3*B*c^2 - A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2))*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)) + (A*Log[x])/a^3 -
 (A*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-2A\left(b^2-4ac\right)-3(Ab-2aB)cx}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c\left(6a^2Bc\right.}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c\left(6a^2Bc\right.}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c\left(6a^2Bc\right.}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c\left(6a^2Bc\right.}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c\left(6a^2Bc\right.}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A\left(2b^4-15ab^2c+16a^2c^2\right)+2c\left(6a^2Bc\right.}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right.}$$

Mathematica [A] time = 0.677008, size = 396, normalized size = 1.57

$$-\frac{\frac{\left(A\left(16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2+b^4\sqrt{b^2-4ac}-10ab^3c-8ab^2c\sqrt{b^2-4ac}+b^5\right)-12a^3Bc^2\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\left(b^2-4ac\right)^{5/2}} - \frac{\left(A\left(16a^2c^2\sqrt{b^2-4ac}-30a^2bc^2+b^4\sqrt{b^2-4ac}+\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((a^2*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)^2) + (a*(2*A*b^3*(b + c*x^2) - a*A*b*c*(15*b + 14*c*x^2)
+ 2*a^2*c*(3*b*B + 8*A*c + 6*B*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4
)) + 4*A*Log[x] - ((-12*a^3*B*c^2 + A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^
4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4
*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) - ((12*a^
3*B*c^2 + A*(-b^5 + 10*a*b^3*c - 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a
*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2
- 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3)
```

Maple [B] time = 0.025, size = 1161, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^4+b*x^2+a)*A*b^2-1/2/a^3/(16*a^2*
c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*
A*b^5+1/2/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^5+5*a/(c
*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*c^2+5/2*a/(c*x^4+b*x^2+a)^
2/(16*a^2*c^2-8*a*b^2*c+b^4)*b*B*c+9/2/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^4*b*B-1/2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A
*b*c^2+1/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b^2*c+6/(16*a^2
*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
*B*c^2+6*a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*c^2+3/4/a/(c*x^4+
b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^4+3/(c*x^4+b*x^2+a)^2*c^3/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^6*B+4/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4
)*x^4*A-21/4/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^2*c-4/a/(16*a
^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^4+b*x^2+a)*A-1/4/a^3/(16*a^2*c^2-8*a*b^2*c
+b^4)*ln(c*x^4+b*x^2+a)*A*b^4+A*ln(x)/a^3-1/4/(c*x^4+b*x^2+a)^2/(16*a^2*c^2
-8*a*b^2*c+b^4)*B*b^3+1/a^2/(c*x^4+b*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^4*A*b^4-3/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^3*c-15/a
/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2
)^(1/2))*A*b*c^2+5/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^3*c-7/2/a/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^6*A*b+1/2/a^2/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^
2*c+b^4)*x^6*A*b^3-29/4/a/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^4*A*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 15.5241, size = 5311, normalized size = 21.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(B*a^3*b^5 - 3*A*a^2*b^6 + 96*A*a^5*c^3 - 2*(A*a*b^5*c^2 - 4*(6*B*a^4
 - 7*A*a^3*b)*c^4 + (6*B*a^3*b^2 - 11*A*a^2*b^3)*c^3)*x^6 - (4*A*a*b^6*c -
64*A*a^4*c^4 - 12*(6*B*a^4*b - 11*A*a^3*b^2)*c^3 + 9*(2*B*a^3*b^3 - 5*A*a^2
*b^4)*c^2)*x^4 + 4*(10*B*a^5*b - 27*A*a^4*b^2)*c^2 - 2*(A*a*b^7 - 4*(10*B*a
^5 - A*a^4*b)*c^3 + (2*B*a^4*b^2 + 23*A*a^3*b^3)*c^2 + 2*(B*a^3*b^4 - 5*A*a
^2*b^5)*c)*x^2 - ((A*b^5*c^2 - 10*A*a*b^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*b)*c^4
)*x^8 + A*a^2*b^5 - 10*A*a^3*b^3*c + 2*(A*b^6*c - 10*A*a*b^4*c^2 - 6*(2*B*a
^3*b - 5*A*a^2*b^2)*c^3)*x^6 + (A*b^7 - 8*A*a*b^5*c - 12*(2*B*a^4 - 5*A*a^3
*b)*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3)*c^2)*x^4 - 6*(2*B*a^5 - 5*A*a^4*b)*
```

```
c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c - 6*(2*B*a^4*b - 5*A*a^3*b^2)*c^2)*x^2)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (14*B*a^4*b^3 - 33*A*a^3*b^4)*c +
(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6*c^2
- 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c - 12*A
*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*b^6*c
 + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b^7 -
12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2
+ a) - 4*(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A
*b^6*c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7
*c - 12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*
A*a*b^6*c + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A
*a*b^7 - 12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(x))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^
4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^
2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b
^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48
*a^6*b^3*c^2 - 64*a^7*b*c^3)*x^2), -1/4*(B*a^3*b^5 - 3*A*a^2*b^6 + 96*A*a^5
*c^3 - 2*(A*a*b^5*c^2 - 4*(6*B*a^4 - 7*A*a^3*b)*c^4 + (6*B*a^3*b^2 - 11*A*a
^2*b^3)*c^3)*x^6 - (4*A*a*b^6*c - 64*A*a^4*c^4 - 12*(6*B*a^4*b - 11*A*a^3*b
^2)*c^3 + 9*(2*B*a^3*b^3 - 5*A*a^2*b^4)*c^2)*x^4 + 4*(10*B*a^5*b - 27*A*a^4
*b^2)*c^2 - 2*(A*a*b^7 - 4*(10*B*a^5 - A*a^4*b)*c^3 + (2*B*a^4*b^2 + 23*A*a
^3*b^3)*c^2 + 2*(B*a^3*b^4 - 5*A*a^2*b^5)*c)*x^2 - 2*((A*b^5*c^2 - 10*A*a*b
^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*b)*c^4)*x^8 + A*a^2*b^5 - 10*A*a^3*b^3*c + 2*
(A*b^6*c - 10*A*a*b^4*c^2 - 6*(2*B*a^3*b - 5*A*a^2*b^2)*c^3)*x^6 + (A*b^7 -
 8*A*a*b^5*c - 12*(2*B*a^4 - 5*A*a^3*b)*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3)
*c^2)*x^4 - 6*(2*B*a^5 - 5*A*a^4*b)*c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c - 6*(
2*B*a^4*b - 5*A*a^3*b^2)*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)
*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (14*B*a^4*b^3 - 33*A*a^3*b^4)*c + (A*a
^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6*c^2 - 12
*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c - 12*A*a*b
^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*b^6*c + 2
4*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b^7 - 12*A
*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a)
 - 4*(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6
*c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c -
 12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*
b^6*c + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b
^7 - 12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(x))/(a^5*
b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^
4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 +
48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c
^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6
*b^3*c^2 - 64*a^7*b*c^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 26.115, size = 568, normalized size = 2.25

$$-\frac{\left(Ab^5 - 10\,Aab^3c - 12\,Ba^3c^2 + 30\,Aa^2bc^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(a^3b^4 - 8\,a^4b^2c + 16\,a^5c^2\right)\sqrt{-b^2+4\,ac}} - \frac{A\log\left(cx^4 + bx^2 + a\right)}{4\,a^3} + \frac{A\log\left(x^2\right)}{2\,a^3} + \frac{3\,Ab^4c^2x^8 - 24\,A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(A*b^5 - 10*A*a*b^3*c - 12*B*a^3*c^2 + 30*A*a^2*b*c^2)*arctan((2*c*x^2
 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 +
 4*a*c)) - 1/4*A*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*A*log(x^2)/a^3 + 1/8*(3*A
*b^4*c^2*x^8 - 24*A*a*b^2*c^3*x^8 + 48*A*a^2*c^4*x^8 + 6*A*b^5*c*x^6 - 44*A
*a*b^3*c^2*x^6 + 24*B*a^3*c^3*x^6 + 68*A*a^2*b*c^3*x^6 + 3*A*b^6*x^4 - 10*A
*a*b^4*c*x^4 + 36*B*a^3*b*c^2*x^4 - 58*A*a^2*b^2*c^2*x^4 + 128*A*a^3*c^3*x^
4 + 10*A*a*b^5*x^2 + 8*B*a^3*b^2*c*x^2 - 72*A*a^2*b^3*c*x^2 + 40*B*a^4*c^2*
x^2 + 92*A*a^3*b*c^2*x^2 - 2*B*a^3*b^3 + 9*A*a^2*b^4 + 20*B*a^4*b*c - 66*A*
a^3*b^2*c + 96*A*a^4*c^2)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c*x^4 + b*
x^2 + a)^2)
```

3.131 $\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=363

$$\frac{abB\left(b^2-7ac\right)-3A\left(10a^2c^2-7ab^2c+b^4\right)}{2a^3x^2\left(b^2-4ac\right)^2}-\frac{-A\left(20a^2c^2-20ab^2c+3b^4\right)+cx^2\left(aB\left(b^2-16ac\right)-3A\left(b^3-6abc\right)\right)+a}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

```
[Out] (a*b*B*(b^2 - 7*a*c) - 3*A*(b^4 - 7*a*b^2*c + 10*a^2*c^2))/(2*a^3*(b^2 - 4*
a*c)^2*x^2) - (a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(4*a*(b^2 - 4
*a*c)*x^2*(a + b*x^2 + c*x^4)^2) - (a*b*B*(b^2 - 10*a*c) - A*(3*b^4 - 20*a*
b^2*c + 20*a^2*c^2) + c*(a*B*(b^2 - 16*a*c) - 3*A*(b^3 - 6*a*b*c))*x^2)/(4*
a^2*(b^2 - 4*a*c)^2*x^2*(a + b*x^2 + c*x^4)) + ((a*b*B*(b^4 - 10*a*b^2*c +
30*a^2*c^2) - 3*A*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3))*ArcTanh
[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - ((3*A*b -
a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rubi [A] time = 0.771047, antiderivative size = 363, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1251, 822, 800, 634, 618, 206, 628}

$$\frac{abB\left(b^2-7ac\right)-3A\left(10a^2c^2-7ab^2c+b^4\right)}{2a^3x^2\left(b^2-4ac\right)^2}-\frac{-A\left(20a^2c^2-20ab^2c+3b^4\right)+cx^2\left(aB\left(b^2-16ac\right)-3A\left(b^3-6abc\right)\right)+a}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```


```
[Out] (a*b*B*(b^2 - 7*a*c) - 3*A*(b^4 - 7*a*b^2*c + 10*a^2*c^2))/(2*a^3*(b^2 - 4*
a*c)^2*x^2) - (a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(4*a*(b^2 - 4
*a*c)*x^2*(a + b*x^2 + c*x^4)^2) - (a*b*B*(b^2 - 10*a*c) - A*(3*b^4 - 20*a*
b^2*c + 20*a^2*c^2) + c*(a*B*(b^2 - 16*a*c) - 3*A*(b^3 - 6*a*b*c))*x^2)/(4*
a^2*(b^2 - 4*a*c)^2*x^2*(a + b*x^2 + c*x^4)) + ((a*b*B*(b^4 - 10*a*b^2*c +
30*a^2*c^2) - 3*A*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3))*ArcTanh
[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - ((3*A*b -
a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^4)
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
```

```
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{-3Ab^2+abB+10aAc-4(Ab-2aB)cx}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)}{4a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB\left(b^2-10ac\right)-A\left(3b^4-20ab^2c+20a^2c^2\right)+ \cdots}{4a^2\left(b^2-4ac\right)^2x^2\left(a+\cdots\right.}$$

$$= -\frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB\left(b^2-10ac\right)-A\left(3b^4-20ab^2c+20a^2c^2\right)+ \cdots}{4a^2\left(b^2-4ac\right)^2x^2\left(a+\cdots\right.}$$

$$= \frac{abB\left(b^2-7ac\right)-3A\left(b^4-7ab^2c+10a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB \cdots}{\cdots}$$

$$= \frac{abB\left(b^2-7ac\right)-3A\left(b^4-7ab^2c+10a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB \cdots}{\cdots}$$

$$= \frac{abB\left(b^2-7ac\right)-3A\left(b^4-7ab^2c+10a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB \cdots}{\cdots}$$

$$= \frac{abB\left(b^2-7ac\right)-3A\left(b^4-7ab^2c+10a^2c^2\right)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A\left(b^2-2ac\right)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB \cdots}{\cdots}$$

Mathematica [A] time = 1.49994, size = 642, normalized size = 1.77

$$\frac{-\frac{a^2\left(A\left(-3abc-2ac^2x^2+b^2cx^2+b^3\right)+aB\left(2ac-b^2-bcx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{a\left(aB\left(16a^2c^2-15ab^2c-14abc^2x^2+2b^3cx^2+2b^4\right)-A\left(46a^2bc^2+28a^2c^3x^2-26ab^2c^2x^2-29ab^3c+4b^4cx^2 \cdots\right.\right.}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-2*a*A)/x^2 - (a^2*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a*b*c + b^2
*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (a*(a*B*(2*
b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x^2 - 14*a*b*c^2*x^2) - A*(4*b^5 -
29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x^2 - 26*a*b^2*c^2*x^2 + 28*a^2*c^3*x^2
)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + 4*(-3*A*b + a*B)*Log[x] + ((-(a
*B*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqr
t[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c])) + 3*A*(b^6 - 10*a*b^4*c + 3
0*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4
*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/(b^2 - 4*a*c)^(5/2) + ((a*B*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt
[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])
 + 3*A*(-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*
a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c]))*Log[b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^4)
```

Maple [B] time = 0.033, size = 1862, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -5/4/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b^5+6*a/(c*x^4+b*x^
2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*B*c^2+3/4/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)*
ln(c*x^4+b*x^2+a)*A*b^5+4/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^4*B-9/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*c^3-29/2/(c*x^4+
b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*A*b*c^2-21/4/(c*x^4+b*x^2+a)^2/(16*a^
2*c^2-8*a*b^2*c+b^4)*B*b^2*c+3/4/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+
b^4)*B*b^4-4/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln(c*x^4+b*x^2+a)*B-1/4/a^3/(
16*a^2*c^2-8*a*b^2*c+b^4)*ln(c*x^4+b*x^2+a)*B*b^4+5/a^2/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*b^3*c+45/
a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-
b^2)^(1/2))*A*b^2*c^2-37/2/a/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^4*A*b+55/4/a^2/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*
b^3-2/a^3/(c*x^4+b*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*A*b^5-29/4/a/(
c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*b^2+1/a^2/(c*x^4+b*x^
2+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*B*b^4-7/2/a/(c*x^4+b*x^2+a)^2/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^2*c^2+6/a^2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^2*A*b^4*c-3/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
2*B*b^3*c+13/2/a^2/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b
^2-1/a^3/(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A*b^4-7/2/a/(
c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*b*B+1/2/a^2/(c*x^4+b*x^
2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*B*b^3-15/a^3/(16*a^2*c^2-8*a*b^2*
c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^4*c-15/a
/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2
)^(1/2))*B*b*c^2-1/2/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*B*b*c
^2-30/a/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*
a*c-b^2)^(1/2))*A*c^3-1/2/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*B*b^5+12/a^2/(16*a^2*c^2-8*a*b^2*c+b^
4)*c^2*ln(c*x^4+b*x^2+a)*A*b-6/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^4+b*
x^2+a)*A*b^3+2/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln(c*x^4+b*x^2+a)*B*b^2-7/a
/(c*x^4+b*x^2+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*A-1/a^3/(c*x^4+b*x^2+
a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*A*b^6+1/2/a^2/(c*x^4+b*x^2+a)^2/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^2*B*b^5+9/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b
^4)*A*b^3*c+3/2/a^4/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*
c*x^2+b)/(4*a*c-b^2)^(1/2))*A*b^6-3/a^4*ln(x)*A*b-1/2*A/a^3/x^2+1/a^3*ln(x)
*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 34.5224, size = 8294, normalized size = 22.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*A*a^3*b^6 - 24*A*a^4*b^4*c + 96*A*a^5*b^2*c^2 - 128*A*a^6*c^3 - 2*
(120*A*a^4*c^5 + 2*(14*B*a^4*b - 57*A*a^3*b^2)*c^4 - 11*(B*a^3*b^3 - 3*A*a^
2*b^4)*c^3 + (B*a^2*b^5 - 3*A*a*b^6)*c^2)*x^8 + (8*(8*B*a^5 - 69*A*a^4*b)*c
^4 - 6*(22*B*a^4*b^2 - 81*A*a^3*b^3)*c^3 + 45*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2
 - 4*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 + 200*A*a^5*
c^4 + 2*(2*B*a^5*b - 11*A*a^4*b^2)*c^3 + (23*B*a^4*b^3 - 79*A*a^3*b^4)*c^2
- 10*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (3*B*a^3*b^6 - 9*A*a^2*b^7 - 8*(12*
B*a^6 - 61*A*a^5*b)*c^3 + 2*(54*B*a^5*b^2 - 197*A*a^4*b^3)*c^2 - (33*B*a^4*
b^4 - 104*A*a^3*b^5)*c)*x^2 - ((60*A*a^3*c^5 + 30*(B*a^3*b - 3*A*a^2*b^2)*c
^4 - 10*(B*a^2*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c^2)*x^10 + 2*(60
*A*a^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(B*a^2*b^4 - 3*A*a*b^5
)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^7 - 3*A*b^8 + 120*A*a^4*c^4 + 6
0*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10*(B*a^3*b^3 - 3*A*a^2*b^4)*c^2 - 8*(B*a^2
*b^5 - 3*A*a*b^6)*c)*x^6 + 2*(B*a^2*b^6 - 3*A*a*b^7 + 60*A*a^4*b*c^3 + 30*(
B*a^4*b^2 - 3*A*a^3*b^3)*c^2 - 10*(B*a^3*b^4 - 3*A*a^2*b^5)*c)*x^4 + (B*a^3
*b^5 - 3*A*a^2*b^6 + 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b^2)*c^2 - 10*(B*
a^4*b^3 - 3*A*a^3*b^4)*c)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2
 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((
64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b
^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*
a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6
)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*
a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5
)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(
B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b
^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b
)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*
x^2)*log(c*x^4 + b*x^2 + a) + 4*((64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^
2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)
*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^
4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a
*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)
*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6
 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^
3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3
*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2
- 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*x^2)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c
^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48
*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2
 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b
^3*c^2 - 64*a^8*b*c^3)*x^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)*x^2), -1/4*(2*A*a^3*b^6 - 24*A*a^4*b^4*c + 96*A*a^5*b^2*c^2 - 128*
A*a^6*c^3 - 2*(120*A*a^4*c^5 + 2*(14*B*a^4*b - 57*A*a^3*b^2)*c^4 - 11*(B*a^
3*b^3 - 3*A*a^2*b^4)*c^3 + (B*a^2*b^5 - 3*A*a*b^6)*c^2)*x^8 + (8*(8*B*a^5 -
 69*A*a^4*b)*c^4 - 6*(22*B*a^4*b^2 - 81*A*a^3*b^3)*c^3 + 45*(B*a^3*b^4 - 3*
A*a^2*b^5)*c^2 - 4*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^
8 + 200*A*a^5*c^4 + 2*(2*B*a^5*b - 11*A*a^4*b^2)*c^3 + (23*B*a^4*b^3 - 79*A
*a^3*b^4)*c^2 - 10*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (3*B*a^3*b^6 - 9*A*a^
2*b^7 - 8*(12*B*a^6 - 61*A*a^5*b)*c^3 + 2*(54*B*a^5*b^2 - 197*A*a^4*b^3)*c^
2 - (33*B*a^4*b^4 - 104*A*a^3*b^5)*c)*x^2 - 2*((60*A*a^3*c^5 + 30*(B*a^3*b
- 3*A*a^2*b^2)*c^4 - 10*(B*a^2*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c
```

```
^2)*x^10 + 2*(60*A*a^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(B*a^2
*b^4 - 3*A*a*b^5)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^7 - 3*A*b^8 + 1
20*A*a^4*c^4 + 60*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10*(B*a^3*b^3 - 3*A*a^2*b^4
)*c^2 - 8*(B*a^2*b^5 - 3*A*a*b^6)*c)*x^6 + 2*(B*a^2*b^6 - 3*A*a*b^7 + 60*A*
a^4*b*c^3 + 30*(B*a^4*b^2 - 3*A*a^3*b^3)*c^2 - 10*(B*a^3*b^4 - 3*A*a^2*b^5)
*c)*x^4 + (B*a^3*b^5 - 3*A*a^2*b^6 + 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b
^2)*c^2 - 10*(B*a^4*b^3 - 3*A*a^3*b^4)*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((64*(B*a^4 - 3*A*a^3*b)*c
^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B
*a*b^6 - 3*A*b^7)*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3
*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b
^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b
^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 -
3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c
^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4
 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 -
3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*x^2)*log(c*x^4 + b*x^2 +
 a) + 4*((64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 1
2*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)*c^2)*x^10 + 2*(64*(B*a^
4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 -
 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*
a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3
*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*
b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 1
2*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 -
 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^
3*b^5)*c)*x^2)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64
*a^7*c^5)*x^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*
c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*
a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x
^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 27.415, size = 875, normalized size = 2.41

$$-\frac{\left(Bab^5 - 3\,Ab^6 - 10\,Ba^2b^3c + 30\,Aab^4c + 30\,Ba^3bc^2 - 90\,Aa^2b^2c^2 + 60\,Aa^3c^3\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^4b^4 - 8\,a^5b^2c + 16\,a^6c^2\right)\sqrt{-b^2+4\,ac}} + \frac{3\,Bab^4c^2x^8 - 9\,Ab^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^5 - 3*A*b^6 - 10*B*a^2*b^3*c + 30*A*a*b^4*c + 30*B*a^3*b*c^2 -
90*A*a^2*b^2*c^2 + 60*A*a^3*c^3)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(
```

```
(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)) + 1/8*(3*B*a*b^4*c
^2*x^8 - 9*A*b^5*c^2*x^8 - 24*B*a^2*b^2*c^3*x^8 + 72*A*a*b^3*c^3*x^8 + 48*B
*a^3*c^4*x^8 - 144*A*a^2*b*c^4*x^8 + 6*B*a*b^5*c*x^6 - 18*A*b^6*c*x^6 - 44*
B*a^2*b^3*c^2*x^6 + 136*A*a*b^4*c^2*x^6 + 68*B*a^3*b*c^3*x^6 - 236*A*a^2*b^
2*c^3*x^6 - 56*A*a^3*c^4*x^6 + 3*B*a*b^6*x^4 - 9*A*b^7*x^4 - 10*B*a^2*b^4*c
*x^4 + 38*A*a*b^5*c*x^4 - 58*B*a^3*b^2*c^2*x^4 + 110*A*a^2*b^3*c^2*x^4 + 12
8*B*a^4*c^3*x^4 - 436*A*a^3*b*c^3*x^4 + 10*B*a^2*b^5*x^2 - 26*A*a*b^6*x^2 -
 72*B*a^3*b^3*c*x^2 + 192*A*a^2*b^4*c*x^2 + 92*B*a^4*b*c^2*x^2 - 316*A*a^3*
b^2*c^2*x^2 - 72*A*a^4*c^3*x^2 + 9*B*a^3*b^4 - 19*A*a^2*b^5 - 66*B*a^4*b^2*
c + 144*A*a^3*b^3*c + 96*B*a^5*c^2 - 260*A*a^4*b*c^2)/((a^4*b^4 - 8*a^5*b^2
*c + 16*a^6*c^2)*(c*x^4 + b*x^2 + a)^2) - 1/4*(B*a - 3*A*b)*log(c*x^4 + b*x
^2 + a)/a^4 + 1/2*(B*a - 3*A*b)*log(x^2)/a^4 - 1/2*(B*a*x^2 - 3*A*b*x^2 + A
*a)/(a^4*x^2)
```

3.132 $\int \frac{x^8\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx$

Optimal. Leaf size=554

$$\frac{\left(-\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\ldots$$

```
[Out] -((3*b^3*B + A*b^2*c - 24*a*b*B*c + 20*a*A*c^2)*x)/(8*c^2*(b^2 - 4*a*c)^2)
+ ((b^2*B + 12*A*b*c - 28*a*B*c)*x^3)/(8*c*(b^2 - 4*a*c)^2) - (x^7*(A*b - 2
*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^5*(
7*A*b^2 - 12*a*b*B - 4*a*A*c + (b^2*B + 12*A*b*c - 28*a*B*c)*x^2))/(8*(b^2
- 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a
*A*b*c^2 + 84*a^2*B*c^2 - (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^
2 + 132*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a*A*b*c
^2 + 84*a^2*B*c^2 + (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^2 + 13
2*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b +
Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 11.1945, antiderivative size = 554, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1275, 1279, 1166, 205}

$$\frac{\left(-\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -((3*b^3*B + A*b^2*c - 24*a*b*B*c + 20*a*A*c^2)*x)/(8*c^2*(b^2 - 4*a*c)^2)
+ ((b^2*B + 12*A*b*c - 28*a*B*c)*x^3)/(8*c*(b^2 - 4*a*c)^2) - (x^7*(A*b - 2
*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^5*(
7*A*b^2 - 12*a*b*B - 4*a*A*c + (b^2*B + 12*A*b*c - 28*a*B*c)*x^2))/(8*(b^2
- 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a
*A*b*c^2 + 84*a^2*B*c^2 - (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^
2 + 132*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a*A*b*c
^2 + 84*a^2*B*c^2 + (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^2 + 13
2*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b +
Sqrt[b^2 - 4*a*c]])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
```

```
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^8\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{x^7\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\int \frac{x^6\left(7(Ab-2aB)+(-bB+2Ac)x^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^7\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^5\left(7Ab^2-12abB-4aAc+\left(b^2B+12Abc-28aBc\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= \frac{\left(b^2B+12Abc-28aBc\right)x^3}{8c\left(b^2-4ac\right)^2}-\frac{x^7\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^5\left(7Ab^2-12abB-4aAc+\cdots\right.}{8\left(b^2-4ac\right)^2\left(a\cdots\right.}$$

$$= -\frac{\left(3b^3B+Ab^2c-24abBc+20aAc^2\right)x}{8c^2\left(b^2-4ac\right)^2}+\frac{\left(b^2B+12Abc-28aBc\right)x^3}{8c\left(b^2-4ac\right)^2}-\frac{x^7\left(Ab-2aB-(bB\cdots\right.}{4\left(b^2-4ac\right)\left(a+bx\cdots\right.}$$

$$= -\frac{\left(3b^3B+Ab^2c-24abBc+20aAc^2\right)x}{8c^2\left(b^2-4ac\right)^2}+\frac{\left(b^2B+12Abc-28aBc\right)x^3}{8c\left(b^2-4ac\right)^2}-\frac{x^7\left(Ab-2aB-(bB\cdots\right.}{4\left(b^2-4ac\right)\left(a+bx\cdots\right.}$$

$$= -\frac{\left(3b^3B+Ab^2c-24abBc+20aAc^2\right)x}{8c^2\left(b^2-4ac\right)^2}+\frac{\left(b^2B+12Abc-28aBc\right)x^3}{8c\left(b^2-4ac\right)^2}-\frac{x^7\left(Ab-2aB-(bB\cdots\right.}{4\left(b^2-4ac\right)\left(a+bx\cdots\right.}$$

Mathematica [A] time = 2.56918, size = 644, normalized size = 1.16

$$\frac{\frac{2x\left(-4a^2c^3\left(9A+11Bx^2\right)+ab^2c^2\left(11A+37Bx^2\right)+b^3c\left(Acx^2-17aB\right)+16abc^2\left(3aB-Acx^2\right)-b^4c\left(2A+5Bx^2\right)+2b^5B\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{4x\left(a^2c\left(2c\left(A+Bx^2\right)-3bB\right)+ab\left(-bc\left(A+4Bx^2\right)+3A\right.\right.}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(2*b^5*B - b^4*c*(2*A + 5*B*x^2) - 4*a^2*c^3*(9*A + 11*B*x^2) + a*b^2
*c^2*(11*A + 37*B*x^2) + 16*a*b*c^2*(3*a*B - A*c*x^2) + b^3*c*(-17*a*B + A*
c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (4*x*(b^3*(b*B - A*c)*x^2
+ a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*
B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(-3*b^5*
B + b^3*c*(33*a*B + A*Sqrt[b^2 - 4*a*c]) - 4*a*b*c^2*(33*a*B + 4*A*Sqrt[b^2
 - 4*a*c]) + 9*a*b^2*c*(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^4*(-(A*c) + 3*B*
Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(10*A*c + 21*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^5*B + 4*a*b*c^2*(33*a*B - 4*
A*Sqrt[b^2 - 4*a*c]) + b^4*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - 9*a*b^2*c*(2*A*c
 + 3*B*Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(-10*A*c + 21*B*Sqrt[b^2 - 4*a*c]) +
b^3*(-33*a*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(
16*c^3)
```

Maple [B] time = 0.051, size = 2015, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-1/8*(16*A*a*b*c^2-A*b^3*c+44*B*a^2*c^2-37*B*a*b^2*c+5*B*b^4)/(16*a^2*c^2-
8*a*b^2*c+b^4)/c*x^7-1/8*(36*A*a^2*c^3+5*A*a*b^2*c^2+A*b^4*c-4*B*a^2*b*c^2-
20*B*a*b^3*c+3*B*b^5)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*a/c^2*(28*A*a*
b*c^2+2*A*b^3*c+28*B*a^2*c^2-49*B*a*b^2*c+6*B*b^4)/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3-1/8*a^2*(20*A*a*c^2+A*b^2*c-24*B*a*b*c+3*B*b^3)/c^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*a*A*b-1/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^3
-5/2*c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
A*a^2-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*A*a*b^2+1/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*A*b^4-21/4/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*
B+27/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*B-3/16/c^2
/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*B+33/4/(16*a^2*c^2-8*
```

```
a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*a^2*b-33/16/c/(16*a
^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*a*b^3+3/16
/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*
b^5-1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A*b+1/16/c/(16*a^2*c^
2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^3-5/2*c/(16*a^2*c^2-8*a*b^2*c+b^4)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a^2-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*a*b^2+1/16/c/(16*a^2*c^2-8*a*b^2*c+
b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^4+21/4/(16*a^2*c^2-8*a*b^2*c
+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*a^2*B-27/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*a*b^2*B+3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^
4*B+33/4/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B
*a^2*b-33/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*B*a*b^3+3/16/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*B*b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(5\,Bb^4c+4\left(11\,Ba^2+4\,Aab\right)c^3-\left(37\,Bab^2+Ab^3\right)c^2\right)x^7+\left(3\,Bb^5+36\,Aa^2c^3-\left(4\,Ba^2b-5\,Aab^2\right)c^2-\left(20\,Bab^3-Ab\right.\right.}{8\left(\left(b^4c^4-8\,ab^2c^5+16\,a^2c^6\right)x^8+a^2b^4c^2-8\,a^3b^2c^3+16\,a^4c^4+2\left(b^5c^3-8\,ab^3c^4+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*B*b^4*c + 4*(11*B*a^2 + 4*A*a*b)*c^3 - (37*B*a*b^2 + A*b^3)*c^2)*x
^7 + (3*B*b^5 + 36*A*a^2*c^3 - (4*B*a^2*b - 5*A*a*b^2)*c^2 - (20*B*a*b^3 -
A*b^4)*c)*x^5 + (6*B*a*b^4 + 28*(B*a^3 + A*a^2*b)*c^2 - (49*B*a^2*b^2 - 2*A
*a*b^3)*c)*x^3 + (3*B*a^2*b^3 + 20*A*a^3*c^2 - (24*B*a^3*b - A*a^2*b^2)*c)*
x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3
+ 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*
a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*
x^2) - 1/8*integrate(-(3*B*a*b^3 + 20*A*a^2*c^2 + (3*B*b^4 + 4*(21*B*a^2 -
4*A*a*b)*c^2 - (27*B*a*b^2 - A*b^3)*c)*x^2 - (24*B*a^2*b - A*a*b^2)*c)/(c*x
^4 + b*x^2 + a), x)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
```

Fricas [B] time = 49.2462, size = 21535, normalized size = 38.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(5*B*b^4*c + 4*(11*B*a^2 + 4*A*a*b)*c^3 - (37*B*a*b^2 + A*b^3)*c^2
)*x^7 + 2*(3*B*b^5 + 36*A*a^2*c^3 - (4*B*a^2*b - 5*A*a*b^2)*c^2 - (20*B*a*b
^3 - A*b^4)*c)*x^5 + 2*(6*B*a*b^4 + 28*(B*a^3 + A*a^2*b)*c^2 - (49*B*a^2*b^
2 - 2*A*a*b^3)*c)*x^3 - sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8
 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16
*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 -
 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^
2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(
216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168
*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c + (b^10*c^5 - 20
*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^
5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*
B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*
B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*
A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4
- 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)
/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^
4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 -
640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(-(1701*B^4*a^2*b^8
 - 945*A*B^3*a*b^9 - 10000*A^4*a^4*c^6 + 15000*(6*A^3*B*a^4*b - A^4*a^3*b^2
)*c^5 + 3*(1037232*B^4*a^6 - 1037232*A*B^3*a^5*b + 287712*A^2*B^2*a^4*b^2 -
 32952*A^3*B*a^3*b^3 + 497*A^4*a^2*b^4)*c^4 - (1555848*B^4*a^5*b^2 - 129837
6*A*B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4 - 11277*A^3*B*a^2*b^5 + 35*A^4*a*b
^6)*c^3 + 9*(37701*B^4*a^4*b^4 - 26973*A*B^3*a^3*b^5 + 3066*A^2*B^2*a^2*b^6
 - 35*A^3*B*a*b^7)*c^2 - 27*(1341*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7 + 35*A^2*
B^2*a*b^8)*c)*x + 1/2*sqrt(1/2)*(27*B^3*b^13 + 32000*A^3*a^5*c^8 - 640*(882
*A*B^2*a^6 - 156*A^2*B*a^5*b + 37*A^3*a^4*b^2)*c^7 + 64*(10584*B^3*a^6*b +
5562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^4)*c^6 - 8*(93096*B^
3*a^5*b^3 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a^3*b^5 + 49*A^3*a^2*b^6)*c^5 +
 (337392*B^3*a^4*b^5 - 24120*A*B^2*a^3*b^6 - 84*A^2*B*a^2*b^7 - 17*A^3*a*b^
8)*c^4 - (81324*B^3*a^3*b^7 - 6993*A*B^2*a^2*b^8 + 195*A^2*B*a*b^9 - A^3*b^
10)*c^3 + 9*(1239*B^3*a^2*b^9 - 79*A*B^2*a*b^10 + A^2*B*b^11)*c^2 - 27*(31*
B^3*a*b^11 - A*B^2*b^12)*c - (3*B*b^14*c^5 - 4096*(42*B*a^7 - 13*A*a^6*b)*c
^12 + 6144*(40*B*a^6*b^2 - 11*A*a^5*b^3)*c^11 - 768*(194*B*a^5*b^4 - 45*A*a
^4*b^5)*c^10 + 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7)*c^9 - 240*(42*B*a^3*b^8 -
5*A*a^2*b^9)*c^8 + 24*(52*B*a^2*b^10 - 3*A*a*b^11)*c^7 - (90*B*a*b^12 - A*b
^13)*c^6)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^
3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^
2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 444
6*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^
4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*
c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*
a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^
3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*
B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B
*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c + (b^10*c^5 - 20*a*b
^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^
10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^
2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*
a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^
3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 11
6*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^
10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^
2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*
a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + sqrt(1/2)*((b^4*c^4 - 8
*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2
*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a
```

^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^
2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b
^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c
^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2
*A*B*b^8)*c + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8
+ 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50
*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95
256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 -
6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5
)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33
*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12
- 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*
b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c
^10))*log(-(1701*B^4*a^2*b^8 - 945*A*B^3*a*b^9 - 10000*A^4*a^4*c^6 + 15000*
(6*A^3*B*a^4*b - A^4*a^3*b^2)*c^5 + 3*(1037232*B^4*a^6 - 1037232*A*B^3*a^5*
b + 287712*A^2*B^2*a^4*b^2 - 32952*A^3*B*a^3*b^3 + 497*A^4*a^2*b^4)*c^4 - (
1555848*B^4*a^5*b^2 - 1298376*A*B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4 - 1127
7*A^3*B*a^2*b^5 + 35*A^4*a*b^6)*c^3 + 9*(37701*B^4*a^4*b^4 - 26973*A*B^3*a^
3*b^5 + 3066*A^2*B^2*a^2*b^6 - 35*A^3*B*a*b^7)*c^2 - 27*(1341*B^4*a^3*b^6 -
819*A*B^3*a^2*b^7 + 35*A^2*B^2*a*b^8)*c)*x - 1/2*sqrt(1/2)*(27*B^3*b^13 +
32000*A^3*a^5*c^8 - 640*(882*A*B^2*a^6 - 156*A^2*B*a^5*b + 37*A^3*a^4*b^2)*
c^7 + 64*(10584*B^3*a^6*b + 5562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b^3 + 89*A^
3*a^3*b^4)*c^6 - 8*(93096*B^3*a^5*b^3 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a^3
*b^5 + 49*A^3*a^2*b^6)*c^5 + (337392*B^3*a^4*b^5 - 24120*A*B^2*a^3*b^6 - 84
*A^2*B*a^2*b^7 - 17*A^3*a*b^8)*c^4 - (81324*B^3*a^3*b^7 - 6993*A*B^2*a^2*b^
8 + 195*A^2*B*a*b^9 - A^3*b^10)*c^3 + 9*(1239*B^3*a^2*b^9 - 79*A*B^2*a*b^10
+ A^2*B*b^11)*c^2 - 27*(31*B^3*a*b^11 - A*B^2*b^12)*c - (3*B*b^14*c^5 - 40
96*(42*B*a^7 - 13*A*a^6*b)*c^12 + 6144*(40*B*a^6*b^2 - 11*A*a^5*b^3)*c^11 -
768*(194*B*a^5*b^4 - 45*A*a^4*b^5)*c^10 + 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7
)*c^9 - 240*(42*B*a^3*b^8 - 5*A*a^2*b^9)*c^8 + 24*(52*B*a^2*b^10 - 3*A*a*b^
11)*c^7 - (90*B*a*b^12 - A*b^13)*c^6)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 -
50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 -
95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4
- 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b
^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(
33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^1
2 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(9*B^2*b^
9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 +
A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 +
(1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B
*b^8)*c + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 12
80*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(44
1*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*
A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(1
4553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^
3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4
*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 64
0*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*
c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)
)) - sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (
b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16
*a^3*b*c^4)*x^2)*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*
(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 3
6*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^
7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^
2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^
4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2
)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A

```
^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324
*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 +
 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*
b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a
^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1
280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(-(1701*B^4*a^2*b^8 - 945*A*B^3*a*b^9
- 10000*A^4*a^4*c^6 + 15000*(6*A^3*B*a^4*b - A^4*a^3*b^2)*c^5 + 3*(1037232*
B^4*a^6 - 1037232*A*B^3*a^5*b + 287712*A^2*B^2*a^4*b^2 - 32952*A^3*B*a^3*b^
3 + 497*A^4*a^2*b^4)*c^4 - (1555848*B^4*a^5*b^2 - 1298376*A*B^3*a^4*b^3 + 2
38464*A^2*B^2*a^3*b^4 - 11277*A^3*B*a^2*b^5 + 35*A^4*a*b^6)*c^3 + 9*(37701*
B^4*a^4*b^4 - 26973*A*B^3*a^3*b^5 + 3066*A^2*B^2*a^2*b^6 - 35*A^3*B*a*b^7)*
c^2 - 27*(1341*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7 + 35*A^2*B^2*a*b^8)*c)*x + 1
/2*sqrt(1/2)*(27*B^3*b^13 + 32000*A^3*a^5*c^8 - 640*(882*A*B^2*a^6 - 156*A^
2*B*a^5*b + 37*A^3*a^4*b^2)*c^7 + 64*(10584*B^3*a^6*b + 5562*A*B^2*a^5*b^2
- 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^4)*c^6 - 8*(93096*B^3*a^5*b^3 + 3816*A*
B^2*a^4*b^4 - 1746*A^2*B*a^3*b^5 + 49*A^3*a^2*b^6)*c^5 + (337392*B^3*a^4*b^
5 - 24120*A*B^2*a^3*b^6 - 84*A^2*B*a^2*b^7 - 17*A^3*a*b^8)*c^4 - (81324*B^3
*a^3*b^7 - 6993*A*B^2*a^2*b^8 + 195*A^2*B*a*b^9 - A^3*b^10)*c^3 + 9*(1239*B
^3*a^2*b^9 - 79*A*B^2*a*b^10 + A^2*B*b^11)*c^2 - 27*(31*B^3*a*b^11 - A*B^2*
b^12)*c + (3*B*b^14*c^5 - 4096*(42*B*a^7 - 13*A*a^6*b)*c^12 + 6144*(40*B*a^
6*b^2 - 11*A*a^5*b^3)*c^11 - 768*(194*B*a^5*b^4 - 45*A*a^4*b^5)*c^10 + 1280
*(39*B*a^4*b^6 - 7*A*a^3*b^7)*c^9 - 240*(42*B*a^3*b^8 - 5*A*a^2*b^9)*c^8 +
24*(52*B*a^2*b^10 - 3*A*a*b^11)*c^7 - (90*B*a*b^12 - A*b^13)*c^6)*sqrt((81*
B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b
^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516
*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 3
24*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5
 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*
a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024
*a^5*c^15)))*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*
B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*
B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c
^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^
6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^
8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^
5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B
*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2
*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A
^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*
c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c
^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*
a^4*b^2*c^9 - 1024*a^5*c^10))) + sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2
*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3
*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*
b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^2*b^9 - 1680*(4*A*B
*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c
^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*
b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^10
*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9
- 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 -
 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 1
7496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^
2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4
*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^
3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13
+ 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b
^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(-(1701*B^
4*a^2*b^8 - 945*A*B^3*a*b^9 - 10000*A^4*a^4*c^6 + 15000*(6*A^3*B*a^4*b - A^
4*a^3*b^2)*c^5 + 3*(1037232*B^4*a^6 - 1037232*A*B^3*a^5*b + 287712*A^2*B^2*
```

```
a^4*b^2 - 32952*A^3*B*a^3*b^3 + 497*A^4*a^2*b^4)*c^4 - (1555848*B^4*a^5*b^2
 - 1298376*A*B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4 - 11277*A^3*B*a^2*b^5 + 3
5*A^4*a*b^6)*c^3 + 9*(37701*B^4*a^4*b^4 - 26973*A*B^3*a^3*b^5 + 3066*A^2*B^
2*a^2*b^6 - 35*A^3*B*a*b^7)*c^2 - 27*(1341*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7
+ 35*A^2*B^2*a*b^8)*c)*x - 1/2*sqrt(1/2)*(27*B^3*b^13 + 32000*A^3*a^5*c^8 -
 640*(882*A*B^2*a^6 - 156*A^2*B*a^5*b + 37*A^3*a^4*b^2)*c^7 + 64*(10584*B^3
*a^6*b + 5562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^4)*c^6 - 8*
(93096*B^3*a^5*b^3 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a^3*b^5 + 49*A^3*a^2*b
^6)*c^5 + (337392*B^3*a^4*b^5 - 24120*A*B^2*a^3*b^6 - 84*A^2*B*a^2*b^7 - 17
*A^3*a*b^8)*c^4 - (81324*B^3*a^3*b^7 - 6993*A*B^2*a^2*b^8 + 195*A^2*B*a*b^9
 - A^3*b^10)*c^3 + 9*(1239*B^3*a^2*b^9 - 79*A*B^2*a*b^10 + A^2*B*b^11)*c^2
- 27*(31*B^3*a*b^11 - A*B^2*b^12)*c + (3*B*b^14*c^5 - 4096*(42*B*a^7 - 13*A
*a^6*b)*c^12 + 6144*(40*B*a^6*b^2 - 11*A*a^5*b^3)*c^11 - 768*(194*B*a^5*b^4
 - 45*A*a^4*b^5)*c^10 + 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7)*c^9 - 240*(42*B*a
^3*b^8 - 5*A*a^2*b^9)*c^8 + 24*(52*B*a^2*b^10 - 3*A*a*b^11)*c^7 - (90*B*a*b
^12 - A*b^13)*c^6)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3
 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b +
 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*
b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B
^4*a^2*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*
B^3*b^7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^1
3 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4
 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 -
 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5
- 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c - (b^10*c^5
 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 10
24*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108
*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496
*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 -
4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2
*b^4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^
7)*c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 12
80*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c
^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + 2*(3*B*a^2*b^3
 + 20*A*a^3*c^2 - (24*B*a^3*b - A*a^2*b^2)*c)*x)/((b^4*c^4 - 8*a*b^2*c^5 +
16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8
*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 +
 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.133 $\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=461

$$\frac{\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Bc^2+36aAbc^2-18ab^2B}{\sqrt{b^2-4ac}}\right.}{}$$

```
[Out] -((b^2*B - 12*A*b*c + 20*a*B*c)*x)/(8*c*(b^2 - 4*a*c)^2) - (x^5*(A*b - 2*a*
B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^3*(5*A
*b^2 - 12*a*b*B + 4*a*A*c - (b^2*B - 12*A*b*c + 20*a*B*c)*x^2))/(8*(b^2 - 4
*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^3*B + 3*A*b^2*c - 16*a*b*B*c + 12*a*A*c^
2 - (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B + 3*A*
b^2*c - 16*a*b*B*c + 12*a*A*c^2 + (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*
A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b
^2 - 4*a*c]])
```

Rubi [A] time = 4.62158, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1275, 1279, 1166, 205}

$$\frac{\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Bc^2+36aAbc^2-18ab^2B}{\sqrt{b^2-4ac}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```


```
[Out] -((b^2*B - 12*A*b*c + 20*a*B*c)*x)/(8*c*(b^2 - 4*a*c)^2) - (x^5*(A*b - 2*a*
B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^3*(5*A
*b^2 - 12*a*b*B + 4*a*A*c - (b^2*B - 12*A*b*c + 20*a*B*c)*x^2))/(8*(b^2 - 4
*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^3*B + 3*A*b^2*c - 16*a*b*B*c + 12*a*A*c^
2 - (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B + 3*A*
b^2*c - 16*a*b*B*c + 12*a*A*c^2 + (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*
A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b
^2 - 4*a*c]])
```


Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
```

```
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\int \frac{x^4\left(5(Ab-2aB)+(bB-2Ac)x^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^3\left(5Ab^2-12abB+4aAc-\left(b^2B-12Abc+20aBc\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+$$

$$= -\frac{\left(b^2B-12Abc+20aBc\right)x}{8c\left(b^2-4ac\right)^2}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^3\left(5Ab^2-12abB+4aAc-\left(b^2\right.\right.}{8\left(b^2-4ac\right)^2\left(a+\right.}$$

$$= -\frac{\left(b^2B-12Abc+20aBc\right)x}{8c\left(b^2-4ac\right)^2}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^3\left(5Ab^2-12abB+4aAc-\left(b^2\right.\right.}{8\left(b^2-4ac\right)^2\left(a+\right.}$$

$$= -\frac{\left(b^2B-12Abc+20aBc\right)x}{8c\left(b^2-4ac\right)^2}-\frac{x^5\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x^3\left(5Ab^2-12abB+4aAc-\left(b^2\right.\right.}{8\left(b^2-4ac\right)^2\left(a+\right.}$$

Mathematica [A] time = 2.21358, size = 543, normalized size = 1.18

$$\frac{-\frac{4x\left(2a^2Bc+a\left(bc\left(A+3Bx^2\right)-2Ac^2x^2+b^2(-B)\right)+b^2x^2(Ac-bB)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{2x\left(b^2c\left(11aB+3Acx^2\right)+4abc^2\left(A-4Bx^2\right)+12ac^2\left(Acx^2-3aB\right)+b^3c\left(2A+Bx^2\right)-2b^4B\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(4a\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(-2*b^4*B + 4*a*b*c^2*(A - 4*B*x^2) + b^3*c*(2*A + B*x^2) + 12*a*c^2*
(-3*a*B + A*c*x^2) + b^2*c*(11*a*B + 3*A*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x
^2 + c*x^4)) - (4*x*(2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A
*c^2*x^2 + b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sq
rt[2]*Sqrt[c]*(-(b^4*B) + 3*b^2*c*(6*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a*c^2*(
10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(-3*A*c + B*Sqrt[b^2 - 4*a*c]) - 4*a*
b*c*(9*A*c + 4*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqr
t[2]*Sqrt[c]*(b^4*B + 3*b^2*c*(-6*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a*c^2*(-10
*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + 4*a*b*c*(9*A*c - 4*B*Sqrt[b^2 - 4*a*c]) + b
^3*(3*A*c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^2)
```

Maple [B] time = 0.045, size = 1631, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (1/8*(12*A*a*c^2+3*A*b^2*c-16*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7
+1/8*(16*A*a*b*c^2+5*A*b^3*c-36*B*a^2*c^2-5*B*a*b^2*c-B*b^4)/c/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^5-1/8/c*a*(4*A*a*c^2-19*A*b^2*c+28*B*a*b*c+2*B*b^3)/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^3+1/8*a^2*(12*A*b*c-20*B*a*c-B*b^2)/c/(16*a^2*c^2-8*
a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2-3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*a*A-3/16/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b
^2+9/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*a*A*b+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*A*b^3+1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*B-1/16/(
16*a^2*c^2-8*a*b^2*c+b^4)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*B-5/2/(16*a^2*c^2-8*a
*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*B-9/8/(16*a^2*c^
2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*B+1/16/(16*
a^2*c^2-8*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*B+3/4
/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A+3/16/(16*a^2*c^2-8*a*b
^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2+9/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A*b+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^3-1/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*a*b*B+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/c*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*B
-5/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2
*B-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
```

```
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b
^2*B+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b^4*B
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 19.8498, size = 15486, normalized size = 33.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(B*b^3*c + 12*A*a*c^3 - (16*B*a*b - 3*A*b^2)*c^2)*x^7 - 2*(B*b^4 +
4*(9*B*a^2 - 4*A*a*b)*c^2 + 5*(B*a*b^2 - A*b^3)*c)*x^5 - 2*(2*B*a*b^3 + 4*A
*a^2*c^2 + (28*B*a^2*b - 19*A*a*b^2)*c)*x^3 - sqrt(1/2)*((b^4*c^3 - 8*a*b^2
*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^
2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^
4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(B^2*b^7 - 240*(4
*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*
b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5
 - 6*A*B*b^6)*c + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*
c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*
A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)
*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2
*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3
 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 10
24*a^5*c^8))*log(-(35*B^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 648*(14
*A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b + 9936
*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*a^3*b^
2 - 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*(497*B
^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^2*B^2*b^6)*c)*x + 1/2*sqrt(1/2)*(B^3*b^
10 - 2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B^2*a^4
*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 - 1284*
A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a^3*b^4
 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*a^2*b^
6 - 492*A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9)*c -
(B*b^13*c^3 - 24576*A*a^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536
*(44*B*a^5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 -
160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^10)*c^5
- 18*(4*B*a*b^11 - A*b^12)*c^4)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2
*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 -
2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^
8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(B^2*b^7 -
 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3
```

```
*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^
2*a*b^5 - 6*A*B*b^6)*c + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a
^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 -
18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B
^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 +
160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b
^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c
^7 - 1024*a^5*c^8))) + sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8
+ a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^
2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*
b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)
*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2
*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c + (b^10
*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7
- 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*
c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2
- 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*
c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a
^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log(-(35*B
^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 648*(14*A^3*B*a^2*b - 5*A^4*a*
b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b + 9936*A^2*B^2*a^2*b^2 + 1080
*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*a^3*b^2 - 3864*A*B^3*a^2*b^3
+ 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*(497*B^4*a^2*b^4 - 315*A*B^3*
a*b^5 + 45*A^2*B^2*b^6)*c)*x - 1/2*sqrt(1/2)*(B^3*b^10 - 2304*(5*A^2*B*a^4
- 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B^2*a^4*b + 198*A^2*B*a^3*b^2
- 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 - 1284*A*B^2*a^3*b^3 + 324*A^2
*B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a^3*b^4 - 1332*A*B^2*a^2*b^5 +
 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*a^2*b^6 - 492*A*B^2*a*b^7 + 6
3*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9)*c - (B*b^13*c^3 - 24576*A*a
^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536*(44*B*a^5*b^3 - 5*A*a^
4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 - 160*(56*B*a^3*b^7 - 15*
A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^10)*c^5 - 18*(4*B*a*b^11 - A*b^
12)*c^4)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (
625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B
^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1
280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2
*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*
B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c
+ (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b
^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^
3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4
*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a
^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4
+ 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) -
sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2
*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c -
 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)
*x^2)*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*
b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9
*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*c^4 + 1
60*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B
^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 30
0*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10
*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10
 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^
4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log(-(35*B^4*a*b^6 - 15*A*B^3*b^7
 - 1296*A^4*a^2*c^5 + 648*(14*A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*a
^4 - 30000*A*B^3*a^3*b + 9936*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3 - 405*A^4*
b^4)*c^3 + 3*(5000*B^4*a^3*b^2 - 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 -
```

135*A^3*B*b^5)*c^2 - 3*(497*B^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^2*B^2*b^6)
*c)*x + 1/2*sqrt(1/2)*(B^3*b^10 - 2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64
*(500*B^3*a^5 - 420*A*B^2*a^4*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 -
16*(1480*B^3*a^4*b^2 - 1284*A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b
^5)*c^4 + 4*(1424*B^3*a^3*b^4 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6 - 27*A
^3*b^7)*c^3 - (392*B^3*a^2*b^6 - 492*A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*
B^3*a*b^8 + 6*A*B^2*b^9)*c + (B*b^13*c^3 - 24576*A*a^6*c^10 + 4096*(13*B*a^
6*b + 3*A*a^5*b^2)*c^9 - 1536*(44*B*a^5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*
a^4*b^5 - 2*A*a^3*b^6)*c^7 - 160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25
*B*a^2*b^9 - 7*A*a*b^10)*c^5 - 18*(4*B*a*b^11 - A*b^12)*c^4)*sqrt((B^4*b^4
+ 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3
*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 -
20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024
*a^5*c^11)))*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B
^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*
b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*
c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*
sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a
^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c
)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b
^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640
*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) + sqrt(1/2)*((b^4*c^3 - 8
*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(
b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c
^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(B^2*b^7 -
240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*
A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2
*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^
3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 1
8*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^
2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 1
60*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^
10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^
7 - 1024*a^5*c^8))*log(-(35*B^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 6
48*(14*A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b
+ 9936*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*
a^3*b^2 - 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*
(497*B^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^2*B^2*b^6)*c)*x - 1/2*sqrt(1/2)*(
B^3*b^10 - 2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B
^2*a^4*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 -
1284*A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a
^3*b^4 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*
a^2*b^6 - 492*A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9
)*c + (B*b^13*c^3 - 24576*A*a^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9
- 1536*(44*B*a^5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*
c^7 - 160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^10
)*c^5 - 18*(4*B*a*b^11 - A*b^12)*c^4)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A
^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*
c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*
b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(B^2
b^7 - 240(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b
^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 -
(35*B^2*a*b^5 - 6*A*B*b^6)*c - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 -
640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*
c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54
*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*
c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11
)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4
*b^2*c^7 - 1024*a^5*c^8))) - 2*(B*a^2*b^2 + 4*(5*B*a^3 - 3*A*a^2*b)*c)*x)/(

```
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a
^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^
2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.134 $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=380

$$\frac{3\left(-\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right.}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}}}$$

```
[Out] -(x^3*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (3*x*(4*a*b*B - A*(b^2 + 4*a*c) + (b^2*B - 4*A*b*c + 4*a*B*c)*x^2))
/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*(b^2*B - 4*A*b*c + 4*a*B*c -
(b^3*B - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a
*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b^2*B - 4*A*b*c + 4*a*B*c + (b^3*B
 - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^2*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.41478, antiderivative size = 380, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1275, 1166, 205}

$$\frac{3\left(-\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right.}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```


```
[Out] -(x^3*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (3*x*(4*a*b*B - A*(b^2 + 4*a*c) + (b^2*B - 4*A*b*c + 4*a*B*c)*x^2))
/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*(b^2*B - 4*A*b*c + 4*a*B*c -
(b^3*B - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a
*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b^2*B - 4*A*b*c + 4*a*B*c + (b^3*B
 - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^2*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}dx = -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\int\frac{x^2\left(3(Ab-2aB)+3(bB-2Ac)x^2\right)}{\left(a+bx^2+cx^4\right)^2}dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x\left(4abB-A\left(b^2+4ac\right)+\left(b^2B-4Abc+4aBc\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\ldots$$

$$= -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x\left(4abB-A\left(b^2+4ac\right)+\left(b^2B-4Abc+4aBc\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\ldots$$

$$= -\frac{x^3\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x\left(4abB-A\left(b^2+4ac\right)+\left(b^2B-4Abc+4aBc\right)x^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\ldots$$

Mathematica [A] time = 1.83921, size = 447, normalized size = 1.18

$$\frac{\frac{2x\left(4bc\left(aB-3Acx^2\right)+4ac^2\left(A+3Bx^2\right)+b^2\left(3Bcx^2-7Ac\right)+2b^3B\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{8acx\left(A+Bx^2\right)-4abBx+4bx^3(Ac-bB)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\sqrt{2}\sqrt{c}\left(b^2\left(B\sqrt{b^2-4ac}+6Ac\right)-4bc\left(A\sqrt{b^2-4ac}+3aB\ldots\right)\ldots\right)}{\left(b^2-4ac\right)^{5/2}\ldots}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*a*b*B*x + 4*b*(-(b*B) + A*c)*x^3 + 8*a*c*x*(A + B*x^2))/((b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)^2) + (2*x*(2*b^3*B + 4*a*c^2*(A + 3*B*x^2) + 4*b*c*(a*
B - 3*A*c*x^2) + b^2*(-7*A*c + 3*B*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + (3*Sqrt[2]*Sqrt[c]*(-(b^3*B) - 4*b*c*(3*a*B + A*Sqrt[b^2 - 4*a*c])
 + 4*a*c*(2*A*c + B*Sqrt[b^2 - 4*a*c]) + b^2*(6*A*c + B*Sqrt[b^2 - 4*a*c]))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^3*B + 4*b*c*(3*a*B
 - A*Sqrt[b^2 - 4*a*c]) + b^2*(-6*A*c + B*Sqrt[b^2 - 4*a*c]) + 4*a*c*(-2*A*
c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c)
```

Maple [B] time = 0.043, size = 1283, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-3/8*c*(4*A*b*c-4*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*(4*A*a*c
^2-19*A*b^2*c+16*B*a*b*c+5*B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(16*A*
a*b*c+5*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-3/8*a*(4
*A*a*c+A*b^2-4*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3/4/(
16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b-3/2/(16*a^2*c^2-8*a*b
^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*A-9/8/(16*a^2*c^2-
8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2-3/4/(16*a^2
*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*B-3/16/(16*a^2*c^2-8*a*b^2*c+
b^4)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B+9/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*B+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*B-3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))*A*b-3/2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*a*A-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*A*b^2+3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))*a*B+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*B+9/4/(16*a^
2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*B+3/16/(16
*a^2*c^2-8*a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*B
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: AttributeError
```

Fricas [B] time = 14.6925, size = 11958, normalized size = 31.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(B*b^2*c + 4*(B*a - A*b)*c^2)*x^7 + 2*(5*B*b^3 + 4*A*a*c^2 + (16*B*
a*b - 19*A*b^2)*c)*x^5 + 2*(19*B*a*b^2 - 5*A*b^3 - 4*(B*a^2 + 4*A*a*b)*c)*x
^3 - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 -
6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*s
qrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*
B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c +
(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b
^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c
^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6
- 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^
4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*log(-27*(5*B^4*a^2*b^4 - A*B^3*a*
b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*a*b^2)*c^3 + (16*B^4*a^4 - 8
0*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 + (40*B^4*a^3*b^2 - 40*A*B^
3*a^2*b^3 + A^3*B*b^5)*c)*x + 27/2*sqrt(1/2)*(4*B^3*a^2*b^7 - A*B^2*a*b^8 -
 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a^4*b + A^3*a^3*b^2)*c^4 - 64
*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3)*c^3 + 8*(24*B^3*a^4*b^3
+ 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^3*b^5 - 8*A*B^2*a^2*b^6 + 4*
A^2*B*a*b^7 - A^3*b^8)*c - (4096*(2*B*a^8 - 3*A*a^7*b)*c^7 - 2048*(2*B*a^7*
b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*A*a^5*b^5)*c^5 + 1280*(2*B*a
^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*a^3*b^9)*c^3 + 8*(14*B*a^3*b
^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^13)*c)*sqrt((B^4*a^2 - 2*A^2*
B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a
^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))*sqrt(-(B^2*a*b^5 - 16*(4*A*
B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2
 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c + (a*b^10*c - 20*a^2*b^8*c^2
 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqr
t((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*
a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*
c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 -
 1024*a^6*c^6))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 +
2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4
*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3
*b*c^2)*x^2)*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^
2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 +
 A^2*b^5)*c + (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^
4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^
2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 128
0*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^
3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*log(-27*(5*B^4*a^2*
b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*a*b^2)*c^3 + (
16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 + (40*B^4*a^3
*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x - 27/2*sqrt(1/2)*(4*B^3*a^2*b^7 -
 A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a^4*b + A^3*a^3
*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3)*c^3 + 8*(2
4*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^3*b^5 - 8*A*B^
2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c - (4096*(2*B*a^8 - 3*A*a^7*b)*c^7 -
2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*A*a^5*b^5)*c^5
 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*a^3*b^9)*c^3 +
 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^13)*c)*sqrt((B^
4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b
^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))*sqrt(-(B^2*a*
b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 +
A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c + (a*b^10*c -
20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 102
4*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*
b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c
^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280
*a^5*b^2*c^5 - 1024*a^6*c^6))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a
```

```
^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*
b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*
b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*
c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 -
20*A*B*a*b^4 + A^2*b^5)*c - (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 -
640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^
2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5
*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 +
160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*log(-
27*(5*B^4*a^2*b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*
a*b^2)*c^3 + (16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2
 + (40*B^4*a^3*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x + 27/2*sqrt(1/2)*(4
*B^3*a^2*b^7 - A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a
^4*b + A^3*a^3*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b
^3)*c^3 + 8*(24*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^
3*b^5 - 8*A*B^2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c + (4096*(2*B*a^8 - 3*A
*a^7*b)*c^7 - 2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*
A*a^5*b^5)*c^5 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*
a^3*b^9)*c^3 + 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^1
3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c
^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))
*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*
A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c
 - (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5
*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10
*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^
6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*
b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*
b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^
2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(
a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 -
 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*
B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c - (a*b^10*c - 20*a^2*b^8*c^2 + 160*
a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*
a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6
*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*
a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a
^6*c^6))*log(-27*(5*B^4*a^2*b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*
B*a^2*b - A^4*a*b^2)*c^3 + (16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 -
5*A^4*b^4)*c^2 + (40*B^4*a^3*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x - 27/
2*sqrt(1/2)*(4*B^3*a^2*b^7 - A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a
^5 + 2*A^2*B*a^4*b + A^3*a^3*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 +
 3*A^2*B*a^3*b^3)*c^3 + 8*(24*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^
2 - (48*B^3*a^3*b^5 - 8*A*B^2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c + (4096*
(2*B*a^8 - 3*A*a^7*b)*c^7 - 2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*
B*a^6*b^4 + 5*A*a^5*b^5)*c^5 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*
B*a^4*b^8 + A*a^3*b^9)*c^3 + 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2*
b^12 - A*a*b^13)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2
 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1
024*a^7*c^7)))*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*
B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4
 + A^2*b^5)*c - (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*
c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*
c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1
280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*
c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))) + 6*(4*B*a^2*b -
 A*a*b^2 - 4*A*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 +
 (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2
```

```
)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.135 $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=438

$$-\frac{x\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(cx^2\left(12abB-A\left(20ac+b^2\right)\right)-A\left(8abc+b^3\right)+aB\left(7b^2-4ac\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(A\left(b^2\sqrt{b\right.\right.}}{}$$

```
[Out] -(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
^2) - (x*(a*B*(7*b^2 - 4*a*c) - A*(b^3 + 8*a*b*c) + c*(12*a*b*B - A*(b^2 +
20*a*c))*x^2))/(8*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(6*a*B*
(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c + b^2*Sqrt[b^2
- 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 -
 52*a*b*c - b^2*Sqrt[b^2 - 4*a*c] - 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.09024, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1275, 1178, 1166, 205}

$$-\frac{x\left(-2aB+x^2(-(bB-2Ac))+Ab\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(cx^2\left(12abB-A\left(20ac+b^2\right)\right)-A\left(8abc+b^3\right)+aB\left(7b^2-4ac\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(A\left(b^2\sqrt{b\right.\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
^2) - (x*(a*B*(7*b^2 - 4*a*c) - A*(b^3 + 8*a*b*c) + c*(12*a*b*B - A*(b^2 +
20*a*c))*x^2))/(8*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(6*a*B*
(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c + b^2*Sqrt[b^2
- 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 -
 52*a*b*c - b^2*Sqrt[b^2 - 4*a*c] - 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2\right)}{\left(a+bx^2+cx^4\right)^3}\,dx = -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\int \frac{Ab-2aB+5(bB-2Ac)x^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4\left(b^2-4ac\right)}$$

$$= -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(aB\left(7b^2-4ac\right)-A\left(b^3+8abc\right)+c\left(12abB-A\left(b^2+2\right.\right.\right.}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(aB\left(7b^2-4ac\right)-A\left(b^3+8abc\right)+c\left(12abB-A\left(b^2+2\right.\right.\right.}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x\left(Ab-2aB-(bB-2Ac)x^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(aB\left(7b^2-4ac\right)-A\left(b^3+8abc\right)+c\left(12abB-A\left(b^2+2\right.\right.\right.}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 1.66572, size = 436, normalized size = 1.

$$\frac{1}{16}\left(\frac{2x\left(A\left(8abc+20ac^2x^2+b^2cx^2+b^3\right)+aB\left(4ac-7b^2-12bcx^2\right)\right)}{a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{4x\left(B\left(2a+bx^2\right)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{2}\sqrt{c}\left(A\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (2*x*(a*B*(-7*b^2 + 4*a*c - 12*b*c*x^2) + A*(b^3 + 8*a*b*c + b^2*c*
x^2 + 20*a*c^2*x^2)))/(a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sq
```

```
rt[c]*(6*a*B*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c +
b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]
) + A*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^
(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/16
```

Maple [B] time = 0.043, size = 1335, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (1/8*c^2*(20*A*a*c+A*b^2-12*B*a*b)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8/a*c
*(28*A*a*b*c+2*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+1
/8*(36*A*a^2*c^2+5*A*a*b^2*c+A*b^4-16*B*a^2*b*c-5*B*a*b^3)/a/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^3+1/8*(16*A*a*b*c-A*b^3-12*B*a^2*c-3*B*a*b^2)/(16*a^2*c^2-8*
a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2-5/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*A-1/16/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*A*b^2+13/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*A*b-1/16/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*A*b^3+3/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b*B-3/2*a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*B-9/8/(16*a^2*c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*B*b^2+5/4/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A+1/16
/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^2+13/4/(16*a^2*c^2-8
*a*b^2*c+b^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b-1/16/a/(16*a^2*
c^2-8*a*b^2*c+b^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*A*b^3-3/4/(16*a^
2*c^2-8*a*b^2*c+b^4)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*B-3/2*a/(16*a^2*c^2-8*a*b^2*c+b
^4)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B-9/8/(16*a^2*c^2-8*a*b^2*c+b
^4)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*B*b^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 26.7878, size = 15784, normalized size = 36.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(20*A*a*c^3 - (12*B*a*b - A*b^2)*c^2)*x^7 + 2*(4*(B*a^2 + 7*A*a*b)*
c^2 - (19*B*a*b^2 - 2*A*b^3)*c)*x^5 - 2*(5*B*a*b^3 - A*b^4 - 36*A*a^2*c^2 +
 (16*B*a^2*b - 5*A*a*b^2)*c)*x^3 + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b
^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6
 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*
a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a
*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 12
80*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*
B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^
3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^
2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a
^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*
c^5))*log((10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*
(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3
 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3*a^3*b^3 - 648*A^2*B^
2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*B^4*a^3*b^4 + 27*A*B^
3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x + 1/2*sqrt(1/2)*(27*B^3*a^3*b
^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 6400*(3*A^2*B*a^6 - 4*A
^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*A^2*B*a^5*b^2 - 341*A
^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b
^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 102*A^2*B*a^3*b^6 + 47*
A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8
 + 53*A^3*a*b^9)*c - (3*B*a^4*b^13 + A*a^3*b^14 + 40960*A*a^10*c^7 - 4096*(
9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A*a^8*b^4)*c^5 - 6400*
(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17*A*a^6*b^8)*c^3 - 240
*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 19*A*a^4*b^12)*c)*sqrt
((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*
b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(
a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2
*c^4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*
(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a
^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^
10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 -
1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*
A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b
 + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c
^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^
5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/2
)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 1
6*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2
*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sq
rt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*
c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^
```

```
3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^
5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4
*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 62
5*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10
 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1
024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^
3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((10000*A^4*a^3*c^5 - 15000*(2*A^3
*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*
B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2
 - 216*A*B^3*a^3*b^3 - 648*A^2*B^2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^
2 - 15*(27*B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)
*x - 1/2*sqrt(1/2)*(27*B^3*a^3*b^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^
3*b^11 + 6400*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*
a^6*b + 66*A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324
*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a
^4*b^5 + 102*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A
*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8 + 53*A^3*a*b^9)*c - (3*B*a^4*b^13 + A*a^3*
b^14 + 40960*A*a^10*c^7 - 4096*(9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*
a^9*b^3 + A*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a
^7*b^7 - 17*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a
^5*b^11 + 19*A*a^4*b^12)*c)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2
*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 +
 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 -
 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^
5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*
a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^
2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a
^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*
a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 5
0*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c +
 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(
a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*
c^4 - 1024*a^8*c^5))) + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)
*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16
*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a
^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7
 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7
*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (
a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*
c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2
 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B
*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9
*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c +
160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((
10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^
5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3 + 497*A^4*
a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3*a^3*b^3 - 648*A^2*B^2*a^2*b^4 -
 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 +
 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x + 1/2*sqrt(1/2)*(27*B^3*a^3*b^8 + 27*A*B
^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 6400*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c
^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)
*c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b^4 - 427*A^
3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 102*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7
)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8 + 53*A^3*a
*b^9)*c + (3*B*a^4*b^13 + A*a^3*b^14 + 40960*A*a^10*c^7 - 4096*(9*B*a^10*b
+ 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^
5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9
 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 19*A*a^4*b^12)*c)*sqrt((81*B^4*a^
4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A
```

```
^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 -
20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024
*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4
- 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2
 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (a^3*b^10 - 20*a^4
*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^
5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3
 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^
2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a
^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 -
 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/2)*((a*b^4*c
^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 +
 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32
*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*
a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(1
8*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*
A*B*a^2*b^4 - 7*A^2*a*b^5)*c - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 -
 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*
A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c
^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b
^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^
5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^
7*b^2*c^4 - 1024*a^8*c^5))*log((10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b -
A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2
 + 3864*A^3*B*a^2*b^3 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3
*a^3*b^3 - 648*A^2*B^2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*
B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x - 1/2*sq
rt(1/2)*(27*B^3*a^3*b^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 64
00*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*
A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*
b^3 - 288*A^2*B*a^4*b^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 10
2*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^
7 + 267*A^2*B*a^2*b^8 + 53*A^3*a*b^9)*c + (3*B*a^4*b^13 + A*a^3*b^14 + 4096
0*A*a^10*c^7 - 4096*(9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A
*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17
*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 1
9*A*a^4*b^12)*c)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 +
12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^
2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^
4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a
*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*
A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A
^2*a*b^5)*c - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3
+ 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*
A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^
2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^
6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 -
20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*
a^8*c^5))) - 2*(3*B*a^2*b^2 + A*a*b^3 + 4*(3*B*a^3 - 4*A*a^2*b)*c)*x)/((a*b
^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c
+ 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.136 $\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=460

$$\frac{x\left(A\left(28a^2c^2-25ab^2c+3b^4\right)+cx^2\left(3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)+abB\left(8ac+b^2\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+ab}{\sqrt{b^2-4ac}}\right.}{}$$

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(4*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)^2) + (x*(a*b*B*(b^2 + 8*a*c) + A*(3*b^4 - 25*a*b^2*c + 28*a
^2*c^2) + c*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c))*x^2))/(8*a^2*(b^2 -
4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8
*a*b*c) + (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2))/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*
Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a*B*(b
^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c) - (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*
a*b^2*c + 56*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 -
4*a*c]])
```

Rubi [A] time = 1.35336, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1178, 1166, 205}

$$\frac{x\left(A\left(28a^2c^2-25ab^2c+3b^4\right)+cx^2\left(3A\left(b^3-8abc\right)+aB\left(20ac+b^2\right)\right)+abB\left(8ac+b^2\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{3A\left(56a^2c^2-10ab^2c+b^4\right)+ab}{\sqrt{b^2-4ac}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(4*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)^2) + (x*(a*b*B*(b^2 + 8*a*c) + A*(3*b^4 - 25*a*b^2*c + 28*a
^2*c^2) + c*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c))*x^2))/(8*a^2*(b^2 -
4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8
*a*b*c) + (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2))/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*
Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a*B*(b
^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c) - (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*
a*b^2*c + 56*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 -
4*a*c]])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^3}\,dx = \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\int \frac{-3Ab^2-abB+14aAc-5(Ab-2aB)cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx}{4a\left(b^2-4ac\right)}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(abB\left(b^2+8ac\right)+A\left(3b^4-25ab^2c+28a^2c^2\right)+c\left(\right.\right.}{8a^2\left(b^2-4ac\right)^2\left(a+bx\right.}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(abB\left(b^2+8ac\right)+A\left(3b^4-25ab^2c+28a^2c^2\right)+c\left(\right.\right.}{8a^2\left(b^2-4ac\right)^2\left(a+bx\right.}$$

$$= \frac{x\left(Ab^2-abB-2aAc+(Ab-2aB)cx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{x\left(abB\left(b^2+8ac\right)+A\left(3b^4-25ab^2c+28a^2c^2\right)+c\left(\right.\right.}{8a^2\left(b^2-4ac\right)^2\left(a+bx\right.}$$

Mathematica [A] time = 2.40205, size = 516, normalized size = 1.12

$$\frac{\frac{2x\left(A\left(28a^2c^2-25ab^2c-24abc^2x^2+3b^3cx^2+3b^4\right)+aB\left(8abc+20ac^2x^2+b^2cx^2+b^3\right)\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(3A\left(56a^2c^2+b^3\sqrt{b^2-4ac}-10ab^2c-8abc\sqrt{b^2-4ac}+b^4\right)+aB\left(b^2\sqrt{b^2-4ac}+2\right.\right.}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*a*x*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)^2) + (2*x*(a*B*(b^3 + 8*a*b*c + b^2*c*x^2 + 20*a*c^2*x^2)
 + A*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3*c*x^2 - 24*a*b*c^2*x^2)))/((b
^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(a*B*(b^3 - 52*a*b*c
+ b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(b^4 - 10*a*b^2*c
 + 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(a*B*(-b^3 + 52*a*b*c + b^2*Sqr
t[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(-b^4 + 10*a*b^2*c - 56*a^
2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqr
```

```
t[b^2 - 4*a*c]]))/(16*a^2)
```

Maple [B] time = 0.194, size = 11936, normalized size = 26.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(4(5Ba^2-6Aab)c^3+(Bab^2+3Ab^3)c^2)x^7+(28Aa^2c^3+7(4Ba^2b-7Aab^2)c^2+2(Bab^3+3Ab^4)c)x^5+(Bab^4+3}{8((a^2b^4c^2-8a^3b^2c^3+16a^4c^4)x^8+a^4b^4-8a^5b^2c+16a^6c^2+2(a^2b^5c-8a^3b^3c^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((4*(5*B*a^2 - 6*A*a*b)*c^3 + (B*a*b^2 + 3*A*b^3)*c^2)*x^7 + (28*A*a^2*
c^3 + 7*(4*B*a^2*b - 7*A*a*b^2)*c^2 + 2*(B*a*b^3 + 3*A*b^4)*c)*x^5 + (B*a*b
^4 + 3*A*b^5 + 4*(9*B*a^3 - A*a^2*b)*c^2 + 5*(B*a^2*b^2 - 4*A*a*b^3)*c)*x^3
 - (B*a^2*b^3 - 5*A*a*b^4 - 44*A*a^3*c^2 - (16*B*a^3*b - 37*A*a^2*b^2)*c)*x
)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c +
 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 -
 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x
^2) - 1/8*integrate(-(B*a*b^3 + 3*A*b^4 + 84*A*a^2*c^2 + (4*(5*B*a^2 - 6*A*
a*b)*c^2 + (B*a*b^2 + 3*A*b^3)*c)*x^2 - (16*B*a^2*b + 27*A*a*b^2)*c)/(c*x^4
 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Fricas [B] time = 67.1856, size = 22209, normalized size = 48.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(4*(5*B*a^2 - 6*A*a*b)*c^3 + (B*a*b^2 + 3*A*b^3)*c^2)*x^7 + 2*(28*A
*a^2*c^3 + 7*(4*B*a^2*b - 7*A*a*b^2)*c^2 + 2*(B*a*b^3 + 3*A*b^4)*c)*x^5 + 2
*(B*a*b^4 + 3*A*b^5 + 4*(9*B*a^3 - A*a^2*b)*c^2 + 5*(B*a^2*b^2 - 4*A*a*b^3)
*c)*x^3 - sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b
^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c
 + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(
4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3
*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*
(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b^8*
c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*s
```

qrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((3111696*A^4*a^4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6 - 90000*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339309*A^4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^2*B^2*a^3*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4*b^4 + 537*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^8)*c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B*b^9)*c^2)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*b^13 + 27*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a^7*b + 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 17541*A^2*B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*a^6*b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5 + 6744*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (940*B^3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*b^10)*c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 864*A^3*a*b^12)*c - (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*b)*c^7 - 2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169*A*a^10*b^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^8 + 123*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^12 + 93*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1

```
280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*
c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((3111696*A^4
*a^4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6 - 900
00*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339309*A^
4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^2*B^2*
a^3*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4*b^4 +
 537*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^8)*
c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B*b^9)*
c^2)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*b^13 +
 27*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a^7*b +
 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 17541*A^2
*B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*a^6
*b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5 +
6744*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (940*B^
3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*b^10)*
c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 864*A^3*
a*b^12)*c - (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*b)*c^7
 - 2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169*A*a^1
0*b^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^8 + 12
3*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^12
+ 93*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b
^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^
5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b
 + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2
*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2
*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 64
0*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*b^7 +
6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a
^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3
*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*
b^7)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 128
0*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^
2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25
*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*
A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b
^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 156
6*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b
^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10
- 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 102
4*a^10*c^5))) - sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 +
 a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4
*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4
*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 -
1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A
^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^
2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c - (a^5*b^10 - 20*a^
6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*
c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*
a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a
^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^
2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^
2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*
a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3
+ 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b
^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((3111696*
A^4*a^4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6 -
90000*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339309
*A^4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^2*B
^2*a^3*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4*b^
```

4 + 537*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^
8)*c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B*b^
9)*c^2)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*b^1
3 + 27*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a^7*
b + 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 17541*
A^2*B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*
a^6*b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5
 + 6744*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (940
*B^3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*b^1
0)*c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 864*A
^3*a*b^12)*c + (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*b)*
c^7 - 2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169*A*
a^10*b^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^8 +
 123*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^
12 + 93*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^
2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2
*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^
5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2
- 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*
a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 -
 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*b^7
 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^
2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*
a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2
*a*b^7)*c - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 +
1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54
*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*
(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 54
00*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^
2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 +
1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^1
2*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^
10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 -
1024*a^10*c^5))) + sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^
8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*
a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*
a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9
 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 -
9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)
c^2 - 7(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c - (a^5*b^10 - 20
*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^
10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3
*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*
B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2
*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5
*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A
^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c
^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^
7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((31116
96*A^4*a^4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6
 - 90000*A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339
309*A^4*a^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^
2*B^2*a^3*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4
*b^4 + 537*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4
*b^8)*c^3 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B
*b^9)*c^2)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*
b^13 + 27*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a
^7*b + 1143*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 175
41*A^2*B*a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B

```
^2*a^6*b^4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*
b^5 + 6744*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (
940*B^3*a^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*
b^10)*c^2 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 86
4*A^3*a*b^12)*c + (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*
b)*c^7 - 2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169
*A*a^10*b^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^
8 + 123*A*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7
*b^12 + 93*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2
*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*
B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3
*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c
^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3
*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^
2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*
b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2
*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A
*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*
A^2*a*b^7)*c - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3
 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 +
 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 8
82*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 +
 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4
*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4
 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*
a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5
*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4
 - 1024*a^10*c^5))) - 2*(B*a^2*b^3 - 5*A*a*b^4 - 44*A*a^3*c^2 - (16*B*a^3*b
 - 37*A*a^2*b^2)*c)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^
4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^
3*c + 16*a^5*b*c^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.137 $\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rubi [A] time = 0.0183348, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1247, 632, 31}

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(-7+4x^2\right)}{4-5x^2+x^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-7+4x}{4-5x+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1+x}\,dx,x,x^2\right)+\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{-4+x}\,dx,x,x^2\right)\\
&= \frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0063553, size = 25, normalized size = 1.

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Maple [A] time = 0.006, size = 18, normalized size = 0.7

$$\frac{\ln\left(x^2-1\right)}{2}+\frac{3\ln\left(x^2-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2-7)/(x^4-5*x^2+4),x)
```

```
[Out] 1/2*ln(x^2-1)+3/2*ln(x^2-4)
```

Maxima [A] time = 0.952929, size = 23, normalized size = 0.92

$$\frac{1}{2}\log\left(x^2-1\right)+\frac{3}{2}\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

Fricas [A] time = 1.6338, size = 50, normalized size = 2.

$$\frac{1}{2}\log\left(x^2-1\right)+\frac{3}{2}\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

Sympy [A] time = 0.114931, size = 17, normalized size = 0.68

$$\frac{3\log\left(x^2-4\right)}{2}+\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2-7)/(x**4-5*x**2+4),x)
```

```
[Out] 3*log(x**2 - 4)/2 + log(x**2 - 1)/2
```

Giac [A] time = 1.08686, size = 26, normalized size = 1.04

$$\frac{1}{2}\log\left(\left|x^2-1\right|\right)+\frac{3}{2}\log\left(\left|x^2-4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x^2 - 1)) + 3/2*log(abs(x^2 - 4))
```

3.138 $\int \frac{-7x+4x^3}{4-5x^2+x^4} dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rubi [A] time = 0.0293958, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 1247, 632, 31}

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]

[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx &= \int \frac{x\left(-7+4x^2\right)}{4-5x^2+x^4}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{-7+4x}{4-5x+x^2}\,dx,x,x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,x^2\right)+\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-4+x}\,dx,x,x^2\right) \\
&= \frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0054111, size = 25, normalized size = 1.

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Maple [A] time = 0.005, size = 18, normalized size = 0.7

$$\frac{\ln\left(x^2-1\right)}{2}+\frac{3\ln\left(x^2-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-7*x)/(x^4-5*x^2+4),x)
```

```
[Out] 1/2*ln(x^2-1)+3/2*ln(x^2-4)
```

Maxima [A] time = 0.945906, size = 34, normalized size = 1.36

$$\frac{3}{2}\log\left(x+2\right)+\frac{1}{2}\log\left(x+1\right)+\frac{1}{2}\log\left(x-1\right)+\frac{3}{2}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] 3/2*log(x + 2) + 1/2*log(x + 1) + 1/2*log(x - 1) + 3/2*log(x - 2)
```

Fricas [A] time = 1.47928, size = 50, normalized size = 2.

$$\frac{1}{2}\log\left(x^2-1\right)+\frac{3}{2}\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

Sympy [A] time = 0.110134, size = 17, normalized size = 0.68

$$\frac{3\log\left(x^2-4\right)}{2}+\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-7*x)/(x**4-5*x**2+4),x)
```

```
[Out] 3*log(x**2 - 4)/2 + log(x**2 - 1)/2
```

Giac [A] time = 1.08159, size = 26, normalized size = 1.04

$$\frac{1}{2}\log\left(\left|x^2-1\right|\right)+\frac{3}{2}\log\left(\left|x^2-4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x^2 - 1)) + 3/2*log(abs(x^2 - 4))
```

3.139 $\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx$

Optimal. Leaf size=37

$$\frac{1}{4}\log\left(x^4+x^2+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rubi [A] time = 0.0350982, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1247, 634, 618, 204, 628}

$$\frac{1}{4}\log\left(x^4+x^2+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + x^2))/(1 + x^2 + x^4),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(2+x^2\right)}{1+x^2+x^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx,x,x^2\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^2\right) + \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^2\right)$$
$$= \frac{1}{4}\log\left(1+x^2+x^4\right) - \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^2\right)$$
$$= \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right) + \frac{1}{4}\log\left(1+x^2+x^4\right)$$

Mathematica [A] time = 0.0105759, size = 37, normalized size = 1.

$$\frac{1}{4}\log\left(x^4+x^2+1\right) + \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + x^2))/(1 + x^2 + x^4),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Maple [A] time = 0.005, size = 31, normalized size = 0.8

$$\frac{\ln\left(x^4+x^2+1\right)}{4} + \frac{\sqrt{3}}{2}\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+2)/(x^4+x^2+1),x)
```

```
[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.45906, size = 41, normalized size = 1.11

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Fricas [A] time = 1.48884, size = 95, normalized size = 2.57

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right) + \frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Sympy [A] time = 0.11599, size = 37, normalized size = 1.

$$\frac{\log\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+2)/(x**4+x**2+1),x)
```

```
[Out] log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/2
```

Giac [A] time = 1.09156, size = 41, normalized size = 1.11

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

3.140 $\int \frac{2x+x^3}{1+x^2+x^4} dx$

Optimal. Leaf size=37

$$\frac{1}{4}\log\left(x^4+x^2+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rubi [A] time = 0.0417176, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 1247, 634, 618, 204, 628}

$$\frac{1}{4}\log\left(x^4+x^2+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^3)/(1 + x^2 + x^4),x]

[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2x+x^3}{1+x^2+x^4}\,dx &= \int \frac{x\left(2+x^2\right)}{1+x^2+x^4}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx,x,x^2\right) \\
&= \frac{1}{4}\,\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^2\right)+\frac{3}{4}\,\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^2\right) \\
&= \frac{1}{4}\log\left(1+x^2+x^4\right)-\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^2\right) \\
&= \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)+\frac{1}{4}\log\left(1+x^2+x^4\right)
\end{aligned}
$$

Mathematica [A] time = 0.0059184, size = 37, normalized size = 1.

$$\frac{1}{4}\log\left(x^4+x^2+1\right)+\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^3)/(1 + x^2 + x^4),x]

[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Maple [A] time = 0.002, size = 31, normalized size = 0.8

$$\frac{\ln\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}}{2}\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2*x)/(x^4+x^2+1),x)

[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.48006, size = 72, normalized size = 1.95

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="maxima")

[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x + 1)
```

Fricas [A] time = 1.52006, size = 95, normalized size = 2.57

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Sympy [A] time = 0.114476, size = 37, normalized size = 1.

$$\frac{\log\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2*x)/(x**4+x**2+1),x)
```

```
[Out] log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/2
```

Giac [A] time = 1.09119, size = 41, normalized size = 1.11

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

3.141 $\int \frac{11x+2x^3}{(3+2x^2+x^4)^2} \, dx$

Optimal. Leaf size=45

$$\frac{9x^2+5}{8(x^4+2x^2+3)}+\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Rubi [A] time = 0.0485355, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1593, 1247, 638, 618, 204}

$$\frac{9x^2+5}{8(x^4+2x^2+3)}+\frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{11x+2x^3}{\left(3+2x^2+x^4\right)^2}\,dx &= \int \frac{x\left(11+2x^2\right)}{\left(3+2x^2+x^4\right)^2}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{11+2x}{\left(3+2x+x^2\right)^2}\,dx, x, x^2\right)\\
&= \frac{5+9x^2}{8\left(3+2x^2+x^4\right)} + \frac{9}{8}\operatorname{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right)\\
&= \frac{5+9x^2}{8\left(3+2x^2+x^4\right)} - \frac{9}{4}\operatorname{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(1+x^2\right)\right)\\
&= \frac{5+9x^2}{8\left(3+2x^2+x^4\right)} + \frac{9\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0246261, size = 45, normalized size = 1.

$$\frac{9x^2+5}{8\left(x^4+2x^2+3\right)} + \frac{9\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Maple [A] time = 0.006, size = 41, normalized size = 0.9

$$\frac{18\,x^2+10}{16\,x^4+32\,x^2+48} + \frac{9\,\sqrt{2}}{16}\arctan\left(\frac{\left(2\,x^2+2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+11*x)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 1/16*(18*x^2+10)/(x^4+2*x^2+3)+9/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{9\,x^2+5}{8\left(x^4+2\,x^2+3\right)} + \frac{9}{4}\int \frac{x}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(9*x^2 + 5)/(x^4 + 2*x^2 + 3) + 9/4*integrate(x/(x^4 + 2*x^2 + 3), x)
```

Fricas [A] time = 1.47967, size = 132, normalized size = 2.93

$$\frac{9\sqrt{2}\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+18x^2+10}{16\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 18*x^2 +
10)/(x^4 + 2*x^2 + 3)
```

Sympy [A] time = 0.152507, size = 44, normalized size = 0.98

$$\frac{9x^2+5}{8x^4+16x^2+24}+\frac{9\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+11*x)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (9*x**2 + 5)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt
(2)/2)/16
```

Giac [A] time = 1.12985, size = 51, normalized size = 1.13

$$\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{9x^2+5}{8\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/8*(9*x^2 + 5)/(x^4 + 2*x^2 +
 3)
```

3.142 $\int x^5 \left(2+3x^2\right) \sqrt{3+5x^2+x^4} \, dx$

Optimal. Leaf size=102

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{3/2}x^4+\frac{1}{480}\left(1837-510x^2\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1633}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{21229}{512}\tanh^{-1}\Big(\frac{}{2\sqrt{}}$$

```
[Out] (-1633*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 + (3*x^4*(3 + 5*x^2 + x^4)^(3
/2))/10 + ((1837 - 510*x^2)*(3 + 5*x^2 + x^4)^(3/2))/480 + (21229*ArcTanh[(
5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rubi [A] time = 0.079444, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 832, 779, 612, 621, 206}

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{3/2}x^4+\frac{1}{480}\left(1837-510x^2\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1633}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{21229}{512}\tanh^{-1}\Big(\frac{}{2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-1633*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 + (3*x^4*(3 + 5*x^2 + x^4)^(3
/2))/10 + ((1837 - 510*x^2)*(3 + 5*x^2 + x^4)^(3/2))/480 + (21229*ArcTanh[(
5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
```

```
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^5 \left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int x^2(2+3x)\sqrt{3+5x+x^2}\,dx,x,x^2\right)$$
$$= \frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{10}\operatorname{Subst}\left(\int\left(-18-\frac{85x}{2}\right)x\sqrt{3+5x+x^2}\,dx,x,x^2\right)$$
$$= \frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}\left(1837-510x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{1633}{64}\operatorname{Subst}\left(\int\sqrt{3\right.$$
$$= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}\left(1837-510x^2\right)\left(3+\right.$$
$$= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}\left(1837-510x^2\right)\left(3+\right.$$
$$= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}\left(1837-510x^2\right)\left(3+\right.$$

Mathematica [A] time = 0.0306944, size = 71, normalized size = 0.7

$$\frac{2\sqrt{x^4+5x^2+3}\left(1152x^8+1680x^6-2248x^4+12250x^2-78387\right)+318435\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{7680}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-78387 + 12250*x^2 - 2248*x^4 + 1680*x^6 + 1152*x
^8) + 318435*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/7680
```

Maple [A] time = 0.023, size = 91, normalized size = 0.9

$$\frac{3\,x^4}{10}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{17\,x^2}{16}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}+\frac{1837}{480}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{3266\,x^2+8165}{256}\sqrt{x^4+5\,x^2+3}+\frac{21229}{512}\ln\left(\frac{5}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/10*x^4*(x^4+5*x^2+3)^(3/2)-17/16*x^2*(x^4+5*x^2+3)^(3/2)+1837/480*(x^4+5*
x^2+3)^(3/2)-1633/256*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+21229/512*ln(5/2+x^2+(x
```

```
^4+5*x^2+3)^(1/2))
```

Maxima [A] time = 0.982898, size = 140, normalized size = 1.37

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^4-\frac{17}{16}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2-\frac{1633}{128}\sqrt{x^4+5x^2+3}x^2+\frac{1837}{480}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{8165}{256}\sqrt{x^4+5x^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5*x^2 + 3)^(3/2)*x^4 - 17/16*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 1633
/128*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1837/480*(x^4 + 5*x^2 + 3)^(3/2) - 8165/25
6*sqrt(x^4 + 5*x^2 + 3) + 21229/512*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5
)
```

Fricas [A] time = 1.5956, size = 185, normalized size = 1.81

$$\frac{1}{3840}\left(1152x^8+1680x^6-2248x^4+12250x^2-78387\right)\sqrt{x^4+5x^2+3}-\frac{21229}{512}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3840*(1152*x^8 + 1680*x^6 - 2248*x^4 + 12250*x^2 - 78387)*sqrt(x^4 + 5*x^
2 + 3) - 21229/512*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [A] time = 1.11535, size = 90, normalized size = 0.88

$$\frac{1}{3840}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6\left(24x^2+35\right)x^2-281\right)x^2+6125\right)x^2-78387\right)-\frac{21229}{512}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(24*x^2 + 35)*x^2 - 281)*x^2 + 6125)*
x^2 - 78387) - 21229/512*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.143 $\int x^3 \left(2+3x^2\right) \sqrt{3+5x^2+x^4}\, dx$

Optimal. Leaf size=81

$$-\frac{1}{48}\left(59-18x^2\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{259}{128}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{3367}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] (259*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 - ((59 - 18*x^2)*(3 + 5*x^2 + x
^4)^(3/2))/48 - (3367*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256
```

Rubi [A] time = 0.0562794, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1251, 779, 612, 621, 206}

$$-\frac{1}{48}\left(59-18x^2\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{259}{128}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{3367}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (259*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 - ((59 - 18*x^2)*(3 + 5*x^2 + x
^4)^(3/2))/48 - (3367*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int x(2+3x)\sqrt{3+5x+x^2}\,dx,x,x^2\right)\\ &= -\frac{1}{48}\left(59-18x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{259}{32}\text{Subst}\left(\int\sqrt{3+5x+x^2}\,dx,x,x^2\right)\\ &= \frac{259}{128}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{48}\left(59-18x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{3367}{256}\text{Subst}\Bigg(\\ &= \frac{259}{128}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{48}\left(59-18x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{3367}{128}\text{Subst}\Bigg(\\ &= \frac{259}{128}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{48}\left(59-18x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{3367}{256}\tanh^{-1}\Bigg(\end{aligned}$$

Mathematica [A] time = 0.0207215, size = 66, normalized size = 0.81

$$\frac{1}{768}\left(2\sqrt{x^4+5x^2+3}\left(144x^6+248x^4-374x^2+2469\right)-10101\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(2469 - 374*x^2 + 248*x^4 + 144*x^6) - 10101*ArcTa
nh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/768
```

Maple [A] time = 0.014, size = 74, normalized size = 0.9

$$\frac{3x^2}{8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{59}{48}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{518x^2+1295}{128}\sqrt{x^4+5x^2+3}-\frac{3367}{256}\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/8*x^2*(x^4+5*x^2+3)^(3/2)-59/48*(x^4+5*x^2+3)^(3/2)+259/128*(2*x^2+5)*(x^
4+5*x^2+3)^(1/2)-3367/256*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A] time = 0.977678, size = 117, normalized size = 1.44

$$\frac{3}{8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2+\frac{259}{64}\sqrt{x^4+5x^2+3}x^2-\frac{59}{48}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{1295}{128}\sqrt{x^4+5x^2+3}-\frac{3367}{256}\log\Big(2x^2+2\sqrt{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 259/64*sqrt(x^4 + 5*x^2 + 3)*x^2 - 59/48*
(x^4 + 5*x^2 + 3)^(3/2) + 1295/128*sqrt(x^4 + 5*x^2 + 3) - 3367/256*log(2*x
```

```
^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.63903, size = 161, normalized size = 1.99

$$\frac{1}{384}\left(144\,x^6+248\,x^4-374\,x^2+2469\right)\sqrt{x^4+5\,x^2+3}+\frac{3367}{256}\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(144*x^6 + 248*x^4 - 374*x^2 + 2469)*sqrt(x^4 + 5*x^2 + 3) + 3367/256
*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [A] time = 1.08371, size = 81, normalized size = 1.

$$\frac{1}{384}\,\sqrt{x^4+5\,x^2+3}\left(2\left(4\left(18\,x^2+31\right)x^2-187\right)x^2+2469\right)+\frac{3367}{256}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 + 31)*x^2 - 187)*x^2 + 2469) + 33
67/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.144 $\int x\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx$

Optimal. Leaf size=74

$$\frac{1}{2}\left(x^4+5x^2+3\right)^{3/2}-\frac{11}{16}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{143}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] (-11*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 + (3 + 5*x^2 + x^4)^(3/2)/2 + (1
43*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Rubi [A] time = 0.0435766, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1247, 640, 612, 621, 206}

$$\frac{1}{2}\left(x^4+5x^2+3\right)^{3/2}-\frac{11}{16}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{143}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-11*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 + (3 + 5*x^2 + x^4)^(3/2)/2 + (1
43*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int(2+3x)\sqrt{3+5x+x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\left(3+5x^2+x^4\right)^{3/2}-\frac{11}{4}\,\text{Subst}\left(\int\sqrt{3+5x+x^2}\,dx,x,x^2\right)\\ &= -\frac{11}{16}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{2}\left(3+5x^2+x^4\right)^{3/2}+\frac{143}{32}\,\text{Subst}\left(\int\frac{1}{\sqrt{3+5x+x^2}}\,dx\right.\\ &= -\frac{11}{16}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{2}\left(3+5x^2+x^4\right)^{3/2}+\frac{143}{16}\,\text{Subst}\left(\int\frac{1}{4-x^2}\,dx,x,\frac{}{\sqrt{3}}\right.\\ &= -\frac{11}{16}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{2}\left(3+5x^2+x^4\right)^{3/2}+\frac{143}{32}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.0169141, size = 61, normalized size = 0.82

$$\frac{1}{32}\left(2\sqrt{x^4+5x^2+3}\left(8x^4+18x^2-31\right)+143\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-31 + 18*x^2 + 8*x^4) + 143*ArcTanh[(5 + 2*x^2)/(
2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Maple [A] time = 0.012, size = 57, normalized size = 0.8

$$\frac{1}{2}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{22x^2+55}{16}\sqrt{x^4+5x^2+3}+\frac{143}{32}\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/2*(x^4+5*x^2+3)^(3/2)-11/16*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+143/32*ln(5/2+x
^2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A] time = 1.22665, size = 95, normalized size = 1.28

$$-\frac{11}{8}\sqrt{x^4+5x^2+3}x^2+\frac{1}{2}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{55}{16}\sqrt{x^4+5x^2+3}+\frac{143}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -11/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 55/16*sqrt(
x^4 + 5*x^2 + 3) + 143/32*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.69985, size = 136, normalized size = 1.84

$$\frac{1}{16}\left(8\,x^4+18\,x^2-31\right)\sqrt{x^4+5\,x^2+3}-\frac{143}{32}\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*x^4 + 18*x^2 - 31)*sqrt(x^4 + 5*x^2 + 3) - 143/32*log(-2*x^2 + 2*sq
rt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [A] time = 1.11622, size = 72, normalized size = 0.97

$$\frac{1}{16}\,\sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2+9\right)x^2-31\right)-\frac{143}{32}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 9)*x^2 - 31) - 143/32*log(2*x^2 - 2*
sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.145 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx$

Optimal. Leaf size=94

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2+23\right)+\frac{1}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] ((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*
x^2 + x^4])]/16 - Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x
^4])]
```

Rubi [A] time = 0.08329, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 814, 843, 621, 206, 724}

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2+23\right)+\frac{1}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

```
[Out] ((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*
x^2 + x^4])]/16 - Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x
^4])]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(2+3x^2\right)\sqrt{3+5x^2+x^4}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x}\,dx,x,x^2\right)\\ &= \frac{1}{8}\left(23+6x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{8}\,\text{Subst}\left(\int \frac{-24-\frac{x}{2}}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{8}\left(23+6x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{16}\,\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)+3\,\text{Subst}\left(\int\right.\\ &= \frac{1}{8}\left(23+6x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)-6\,\text{Subst}\\ &= \frac{1}{8}\left(23+6x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{16}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-\sqrt{3}\tanh^{-1}\left(\frac{6+}{2\sqrt{3}\sqrt{3}}\right.\end{aligned}$$

Mathematica [A] time = 0.0377268, size = 92, normalized size = 0.98

$$\frac{1}{16}\left(2\sqrt{x^4+5x^2+3}\left(6x^2+23\right)+\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-16\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

```
[Out] (2*(23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4] + ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x
^2 + x^4])] - 16*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^
4])])/16
```

Maple [A] time = 0.011, size = 85, normalized size = 0.9

$$\frac{6x^2+15}{8}\sqrt{x^4+5x^2+3}+\frac{1}{16}\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)+\sqrt{x^4+5x^2+3}-\text{Artanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x)
```

```
[Out] 3/8*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+1/16*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))+(x^4
+5*x^2+3)^(1/2)-arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Maxima [A] time = 1.43885, size = 120, normalized size = 1.28

$$\frac{3}{4}\sqrt{x^4+5x^2+3}x^2-\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{23}{8}\sqrt{x^4+5x^2+3}+\frac{1}{16}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)
/x^2 + 6/x^2 + 5) + 23/8*sqrt(x^4 + 5*x^2 + 3) + 1/16*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.56655, size = 254, normalized size = 2.7

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2+23\right)+\sqrt{3}\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-\frac{1}{16}\log\left(-2x^2+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 23) + sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5
*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 1/16*log(-
2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+5x^2+3}\left(3x^2+2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x, x)
```

3.146 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{x^4+5x^2+3}\,(2-3x^2)}{2x^2}+\frac{19}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{7\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

```
[Out] -((2 - 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (19*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])])/4 - (7*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^
2 + x^4])])/Sqrt[3]
```

Rubi [A] time = 0.0830542, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 812, 843, 621, 206, 724}

$$-\frac{\sqrt{x^4+5x^2+3}\,(2-3x^2)}{2x^2}+\frac{19}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{7\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

```
[Out] -((2 - 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (19*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])])/4 - (7*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^
2 + x^4])])/Sqrt[3]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\sqrt{3+5x^2+x^4}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^2}\,dx,x,x^2\right)$$
$$= -\frac{\left(2-3x^2\right)\sqrt{3+5x^2+x^4}}{2x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{-28-19x}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$
$$= -\frac{\left(2-3x^2\right)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{19}{4}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right) + 7\,\text{Subst}\left(\int \frac{}{x}\right.$$
$$= -\frac{\left(2-3x^2\right)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{19}{2}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right) - 14\,\text{Subst}$$
$$= -\frac{\left(2-3x^2\right)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{19}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{7\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x}}\right.}{\sqrt{3}}$$

Mathematica [A] time = 0.0401172, size = 97, normalized size = 1.

$$\frac{\sqrt{x^4+5x^2+3}\left(3x^2-2\right)}{2x^2} + \frac{19}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - \frac{7\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

```
[Out] ((-2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (19*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])])/4 - (7*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^
2 + x^4])])/Sqrt[3]
```

Maple [A] time = 0.014, size = 104, normalized size = 1.1

$$\frac{7}{3}\sqrt{x^4+5x^2+3} + \frac{19}{4}\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right) - \frac{7\sqrt{3}}{3}\text{Artanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right) - \frac{1}{3x^2}\left(x^4+5x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x)
```

```
[Out] 7/3*(x^4+5*x^2+3)^(1/2)+19/4*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))-7/3*arctanh(1/
6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3/x^2*(x^4+5*x^2+3)^(3/2
)+1/6*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 1.46671, size = 120, normalized size = 1.24

$$-\frac{7}{3}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{\sqrt{x^4+5x^2+3}}{x^2}+\frac{19}{4}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -7/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 3/2*sqr
t(x^4 + 5*x^2 + 3) - sqrt(x^4 + 5*x^2 + 3)/x^2 + 19/4*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.52972, size = 292, normalized size = 3.01

$$\frac{56\sqrt{3}x^2\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right)-114x^2\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)+21x^2+12\sqrt{x^4+5x^2}}{24x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(56*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x
^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 114*x^2*log(-2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) - 5) + 21*x^2 + 12*sqrt(x^4 + 5*x^2 + 3)*(3*x^2 - 2))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**3,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+5x^2+3}(3x^2+2)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^3, x)
```

3.147 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx$

Optimal. Leaf size=99

$$-\frac{\sqrt{x^4+5x^2+3}\left(23x^2+6\right)}{12x^4}+\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{77\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

```
[Out] -((6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^4) + (3*ArcTanh[(5 + 2*x^2)/(2*
Sqrt[3 + 5*x^2 + x^4])])/2 - (77*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*
x^2 + x^4])])/(24*Sqrt[3])
```

Rubi [A] time = 0.0834549, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 810, 843, 621, 206, 724}

$$-\frac{\sqrt{x^4+5x^2+3}\left(23x^2+6\right)}{12x^4}+\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{77\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
```

```
[Out] -((6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^4) + (3*ArcTanh[(5 + 2*x^2)/(2*
Sqrt[3 + 5*x^2 + x^4])])/2 - (77*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*
x^2 + x^4])])/(24*Sqrt[3])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^3}\,dx, x, x^2\right)\\ &= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} - \frac{1}{24}\operatorname{Subst}\left(\int \frac{-77-36x}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\ &= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right) + \frac{77}{24}\operatorname{Subst}\Bigg(\int\\ &= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + 3\operatorname{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right) - \frac{77}{12}\operatorname{Subst}\\ &= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + \frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{77\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+}}\right.}{24\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0397514, size = 97, normalized size = 0.98

$$\frac{1}{72}\left(-\frac{6\sqrt{x^4+5x^2+3}\,(23x^2+6)}{x^4}+108\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-77\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
```

```
[Out] ((-6*(6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 + 108*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])] - 77*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 +
 5*x^2 + x^4])])/72
```

Maple [A] time = 0.014, size = 121, normalized size = 1.2

$$-\frac{1}{6x^4}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{13}{36x^2}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{77}{72}\sqrt{x^4+5x^2+3}-\frac{77\sqrt{3}}{72}\operatorname{Artanh}\left(\frac{(5x^2+6)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x)
```

```
[Out] -1/6/x^4*(x^4+5*x^2+3)^(3/2)-13/36/x^2*(x^4+5*x^2+3)^(3/2)+77/72*(x^4+5*x^2
+3)^(1/2)-77/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+
13/72*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+3/2*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A] time = 1.47378, size = 143, normalized size = 1.44

$$-\frac{77}{72}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{1}{6}\sqrt{x^4+5x^2+3}-\frac{13\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{6x^4}+\frac{3}{2}\log\left(2x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -77/72*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1/6*s
qrt(x^4 + 5*x^2 + 3) - 13/12*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/6*(x^4 + 5*x^2 +
 3)^(3/2)/x^4 + 3/2*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.63955, size = 293, normalized size = 2.96

$$\frac{77\sqrt{3}x^4\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-108x^4\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)-138x^4-6\sqrt{x^4+5x^2}}{72x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/72*(77*sqrt(3)*x^4*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x
^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 108*x^4*log(-2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) - 5) - 138*x^4 - 6*sqrt(x^4 + 5*x^2 + 3)*(23*x^2 + 6))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**5,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+5x^2+3}\left(3x^2+2\right)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^5, x)
```

3.148 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx$

Optimal. Leaf size=90

$$-\frac{(x^4+5x^2+3)^{3/2}}{9x^6}-\frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{18x^4}+\frac{13\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{36\sqrt{3}}$$

```
[Out] -((6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(18*x^4) - (3 + 5*x^2 + x^4)^(3/2)/(9*
x^6) + (13*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(36*Sqrt
[3])
```

Rubi [A] time = 0.0681416, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1251, 806, 720, 724, 206}

$$-\frac{(x^4+5x^2+3)^{3/2}}{9x^6}-\frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{18x^4}+\frac{13\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{36\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
```

```
[Out] -((6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(18*x^4) - (3 + 5*x^2 + x^4)^(3/2)/(9*
x^6) + (13*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(36*Sqrt
[3])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{2}{3}\,\text{Subst}\left(\int \frac{\sqrt{3+5x+x^2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} - \frac{13}{36}\,\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x\right.\\
&= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{13}{18}\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+}{\sqrt{3+}}\right.\\
&= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{13\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{36\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0262554, size = 74, normalized size = 0.82

$$\frac{1}{108}\left(13\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)-\frac{6\sqrt{x^4+5x^2+3}\,(7x^4+16x^2+6)}{x^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
```

```
[Out] ((-6*Sqrt[3 + 5*x^2 + x^4]*(6 + 16*x^2 + 7*x^4))/x^6 + 13*Sqrt[3]*ArcTanh[(
6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/108
```

Maple [A] time = 0.013, size = 118, normalized size = 1.3

$$-\frac{1}{9\,x^4}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}+\frac{5}{54\,x^2}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{13}{108}\sqrt{x^4+5\,x^2+3}+\frac{13\,\sqrt{3}}{108}\text{Artanh}\left(\frac{(5\,x^2+6)\,\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5\,x^2+3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x)
```

```
[Out] -1/9/x^4*(x^4+5*x^2+3)^(3/2)+5/54/x^2*(x^4+5*x^2+3)^(3/2)-13/108*(x^4+5*x^2
+3)^(1/2)+13/108*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
-5/108*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)-1/9*(x^4+5*x^2+3)^(3/2)/x^6
```

Maxima [A] time = 1.43131, size = 134, normalized size = 1.49

$$\frac{13}{108}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{1}{9}\sqrt{x^4+5x^2+3}+\frac{5\sqrt{x^4+5x^2+3}}{18x^2}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{9x^4}-\frac{\left(x^4+5x^2+3\right)}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 13/108*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1/9*s
qrt(x^4 + 5*x^2 + 3) + 5/18*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/9*(x^4 + 5*x^2 +
3)^(3/2)/x^4 - 1/9*(x^4 + 5*x^2 + 3)^(3/2)/x^6
```

Fricas [A] time = 1.391, size = 234, normalized size = 2.6

$$\frac{13\sqrt{3}x^6\log\left(\frac{25x^2+2\sqrt{3}\left(5x^2+6\right)+2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}+6\right)+30}{x^2}\right)-42x^6-6\left(7x^4+16x^2+6\right)\sqrt{x^4+5x^2+3}}{108x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/108*(13*sqrt(3)*x^6*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 42*x^6 - 6*(7*x^4 + 16*x^2 + 6)*sqrt(
x^4 + 5*x^2 + 3))/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**7,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**7, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^4+5x^2+3}\left(3x^2+2\right)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^7, x)
```

3.149 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx$

Optimal. Leaf size=111

$$-\frac{11\left(x^4+5x^2+3\right)^{3/2}}{216x^6}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{12x^8}+\frac{67\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{1728x^4}-\frac{871\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3456\sqrt{3}}$$

```
[Out] (67*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(1728*x^4) - (3 + 5*x^2 + x^4)^(3/2)
/(12*x^8) - (11*(3 + 5*x^2 + x^4)^(3/2))/(216*x^6) - (871*ArcTanh[(6 + 5*x^
2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(3456*Sqrt[3])
```

Rubi [A] time = 0.0861399, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 834, 806, 720, 724, 206}

$$-\frac{11\left(x^4+5x^2+3\right)^{3/2}}{216x^6}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{12x^8}+\frac{67\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{1728x^4}-\frac{871\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3456\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
```

```
[Out] (67*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(1728*x^4) - (3 + 5*x^2 + x^4)^(3/2)
/(12*x^8) - (11*(3 + 5*x^2 + x^4)^(3/2))/(216*x^6) - (871*ArcTanh[(6 + 5*x^
2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(3456*Sqrt[3])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(2+3x^2\right)\sqrt{3+5x^2+x^4}}{x^9}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^5}\,dx, x, x^2\right)\\
&= -\frac{\left(3+5x^2+x^4\right)^{3/2}}{12x^8}-\frac{1}{24}\,\text{Subst}\left(\int \frac{(-11+2x)\sqrt{3+5x+x^2}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{\left(3+5x^2+x^4\right)^{3/2}}{12x^8}-\frac{11\left(3+5x^2+x^4\right)^{3/2}}{216x^6}-\frac{67}{144}\,\text{Subst}\left(\int \frac{\sqrt{3+5x+x^2}}{x^3}\,dx, x, x^2\right)\\
&= \frac{67\left(6+5x^2\right)\sqrt{3+5x^2+x^4}}{1728x^4}-\frac{\left(3+5x^2+x^4\right)^{3/2}}{12x^8}-\frac{11\left(3+5x^2+x^4\right)^{3/2}}{216x^6}+\frac{871\,\text{Subst}\left(\int\right.}{}\\
&= \frac{67\left(6+5x^2\right)\sqrt{3+5x^2+x^4}}{1728x^4}-\frac{\left(3+5x^2+x^4\right)^{3/2}}{12x^8}-\frac{11\left(3+5x^2+x^4\right)^{3/2}}{216x^6}-\frac{871\,\text{Subst}\left(\int\right.}{}\\
&= \frac{67\left(6+5x^2\right)\sqrt{3+5x^2+x^4}}{1728x^4}-\frac{\left(3+5x^2+x^4\right)^{3/2}}{12x^8}-\frac{11\left(3+5x^2+x^4\right)^{3/2}}{216x^6}-\frac{871\tanh^{-1}\left(\right.}{345}
\end{aligned}
$$

Mathematica [A] time = 0.0282441, size = 82, normalized size = 0.74

$$
\frac{6\sqrt{x^4+5x^2+3}\left(247x^6-182x^4-984x^2-432\right)-871\sqrt{3}x^8\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{10368x^8}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
```

```
[Out] (6*Sqrt[3 + 5*x^2 + x^4]*(-432 - 984*x^2 - 182*x^4 + 247*x^6) - 871*Sqrt[3]
*x^8*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(10368*x^8)
```

Maple [A] time = 0.016, size = 135, normalized size = 1.2

$$-\frac{1}{12x^8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{11}{216x^6}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{67}{864x^4}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{335}{5184x^2}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{871}{10368}\sqrt{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x)
```

```
[Out] -1/12*(x^4+5*x^2+3)^(3/2)/x^8-11/216*(x^4+5*x^2+3)^(3/2)/x^6+67/864/x^4*(x^
4+5*x^2+3)^(3/2)-335/5184/x^2*(x^4+5*x^2+3)^(3/2)+871/10368*(x^4+5*x^2+3)^(
1/2)-871/10368*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+3
35/10368*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 1.46421, size = 157, normalized size = 1.41

$$-\frac{871}{10368}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{67}{864}\sqrt{x^4+5x^2+3}-\frac{335\sqrt{x^4+5x^2+3}}{1728x^2}+\frac{67\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{864x^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -871/10368*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 6
7/864*sqrt(x^4 + 5*x^2 + 3) - 335/1728*sqrt(x^4 + 5*x^2 + 3)/x^2 + 67/864*(
x^4 + 5*x^2 + 3)^(3/2)/x^4 - 11/216*(x^4 + 5*x^2 + 3)^(3/2)/x^6 - 1/12*(x^4
 + 5*x^2 + 3)^(3/2)/x^8
```

Fricas [A] time = 1.41226, size = 261, normalized size = 2.35

$$\frac{871\sqrt{3}x^8\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right)+1482x^8+6\left(247x^6-182x^4-984x^2-432\right)\sqrt{x^4+5x^2+3}}{10368x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/10368*(871*sqrt(3)*x^8*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 +
 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 1482*x^8 + 6*(247*x^6 - 182*x^4 -
984*x^2 - 432)*sqrt(x^4 + 5*x^2 + 3))/x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**9,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**9, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5x^2+3}\,(3x^2+2)}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^9, x)
```

3.150 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx$

Optimal. Leaf size=132

$$\frac{173\left(x^4+5x^2+3\right)^{3/2}}{3240x^6}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{36x^8}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{15x^{10}}-\frac{161\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{5184x^4}+\frac{2093\tanh^{-1}\left(\frac{5x^2\ldots}{2\sqrt{3}\sqrt{x^4\ldots}}\right.}{10368\sqrt{3}}$$

```
[Out] (-161*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(5184*x^4) - (3 + 5*x^2 + x^4)^(3/
2)/(15*x^10) - (3 + 5*x^2 + x^4)^(3/2)/(36*x^8) + (173*(3 + 5*x^2 + x^4)^(3
/2))/(3240*x^6) + (2093*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4
])])/(10368*Sqrt[3])
```

Rubi [A] time = 0.108624, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 834, 806, 720, 724, 206}

$$\frac{173\left(x^4+5x^2+3\right)^{3/2}}{3240x^6}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{36x^8}-\frac{\left(x^4+5x^2+3\right)^{3/2}}{15x^{10}}-\frac{161\left(5x^2+6\right)\sqrt{x^4+5x^2+3}}{5184x^4}+\frac{2093\tanh^{-1}\left(\frac{5x^2\ldots}{2\sqrt{3}\sqrt{x^4\ldots}}\right.}{10368\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

```
[Out] (-161*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(5184*x^4) - (3 + 5*x^2 + x^4)^(3/
2)/(15*x^10) - (3 + 5*x^2 + x^4)^(3/2)/(36*x^8) + (173*(3 + 5*x^2 + x^4)^(3
/2))/(3240*x^6) + (2093*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4
])])/(10368*Sqrt[3])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 720

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> -Simp[((d + e*x)^(m + 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
 + c*x^2)^p)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[(p*(b^2 - 4*a*c
))/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*
x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2, 0
] && GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2+3x^2\right)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^6}\,dx, x, x^2\right)\\
&= -\frac{\left(3+5x^2+x^4\right)^{3/2}}{15x^{10}} - \frac{1}{30}\operatorname{Subst}\left(\int \frac{(-10+4x)\sqrt{3+5x+x^2}}{x^5}\,dx, x, x^2\right)\\
&= -\frac{\left(3+5x^2+x^4\right)^{3/2}}{15x^{10}} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{36x^8} + \frac{1}{360}\operatorname{Subst}\left(\int \frac{(-173-10x)\sqrt{3+5x+x^2}}{x^4}\,dx\right.\\
&= -\frac{\left(3+5x^2+x^4\right)^{3/2}}{15x^{10}} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{36x^8} + \frac{173\left(3+5x^2+x^4\right)^{3/2}}{3240x^6} + \frac{161}{432}\operatorname{Subst}\left(\int \frac{\sqrt{3+}}{}\right.\\
&= -\frac{161\left(6+5x^2\right)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{15x^{10}} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{36x^8} + \frac{173\left(3+5x^2\right.}{3240x^6}\\
&= -\frac{161\left(6+5x^2\right)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{15x^{10}} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{36x^8} + \frac{173\left(3+5x^2\right.}{3240x^6}\\
&= -\frac{161\left(6+5x^2\right)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{15x^{10}} - \frac{\left(3+5x^2+x^4\right)^{3/2}}{36x^8} + \frac{173\left(3+5x^2\right.}{3240x^6}
\end{aligned}$$

Mathematica [A] time = 0.0370601, size = 84, normalized size = 0.64

$$\frac{10465\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) - \frac{6\sqrt{x^4+5x^2+3}\left(2641x^8-1370x^6+1176x^4+10800x^2+5184\right)}{x^{10}}}{155520}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

```
[Out] ((-6*Sqrt[3 + 5*x^2 + x^4]*(5184 + 10800*x^2 + 1176*x^4 - 1370*x^6 + 2641*x
^8))/x^10 + 10465*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x
^4])])/155520
```

Maple [A] time = 0.018, size = 152, normalized size = 1.2

$$-\frac{1}{15x^{10}}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{1}{36x^8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{173}{3240x^6}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{161}{2592x^4}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{805}{15552x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x)
```

```
[Out] -1/15*(x^4+5*x^2+3)^(3/2)/x^10-1/36*(x^4+5*x^2+3)^(3/2)/x^8+173/3240*(x^4+5
*x^2+3)^(3/2)/x^6-161/2592/x^4*(x^4+5*x^2+3)^(3/2)+805/15552/x^2*(x^4+5*x^2
+3)^(3/2)-2093/31104*(x^4+5*x^2+3)^(1/2)+2093/31104*arctanh(1/6*(5*x^2+6)*3
^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-805/31104*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 1.50393, size = 180, normalized size = 1.36

$$\frac{2093}{31104}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{161}{2592}\sqrt{x^4+5x^2+3}+\frac{805\sqrt{x^4+5x^2+3}}{5184x^2}-\frac{161\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{2592x^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] 2093/31104*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1
61/2592*sqrt(x^4 + 5*x^2 + 3) + 805/5184*sqrt(x^4 + 5*x^2 + 3)/x^2 - 161/25
92*(x^4 + 5*x^2 + 3)^(3/2)/x^4 + 173/3240*(x^4 + 5*x^2 + 3)^(3/2)/x^6 - 1/3
6*(x^4 + 5*x^2 + 3)^(3/2)/x^8 - 1/15*(x^4 + 5*x^2 + 3)^(3/2)/x^10
```

Fricas [A] time = 1.33106, size = 292, normalized size = 2.21

$$\frac{10465\sqrt{3}x^{10}\log\left(\frac{25x^2+2\sqrt{3}\left(5x^2+6\right)+2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}+6\right)+30}{x^2}\right)-15846x^{10}-6\left(2641x^8-1370x^6+1176x^4+10800x^2+5\right.}{155520x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/155520*(10465*sqrt(3)*x^10*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x
^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 15846*x^10 - 6*(2641*x^8 - 137
0*x^6 + 1176*x^4 + 10800*x^2 + 5184)*sqrt(x^4 + 5*x^2 + 3))/x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**11,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**11, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^11, x)
```

3.151 $\int x^4 \left(2+3x^2\right) \sqrt{3+5x^2+x^4}\, dx$

Optimal. Leaf size=322

$$-\frac{13\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}+\frac{1}{21}\left(7x^2+11\right)\sqrt{x^4+5x^2}$$

```
[Out] (-1924*x*(5 + Sqrt[13] + 2*x^2))/(105*Sqrt[3 + 5*x^2 + x^4]) + (13*x*Sqrt[3
 + 5*x^2 + x^4])/3 - (26*x^3*Sqrt[3 + 5*x^2 + x^4])/35 + (x^5*(11 + 7*x^2)*
Sqrt[3 + 5*x^2 + x^4])/21 + (962*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 -
Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE
[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(105*Sqrt[3 + 5*x
^2 + x^4]) - (13*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.273842, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1273, 1279, 1189, 1099, 1135}

$$\frac{1}{21}\left(7x^2+11\right)\sqrt{x^4+5x^2+3}x^5-\frac{26}{35}\sqrt{x^4+5x^2+3}x^3+\frac{13}{3}\sqrt{x^4+5x^2+3}x-\frac{1924\left(2x^2+\sqrt{13}+5\right)x}{105\sqrt{x^4+5x^2+3}}-\frac{13\sqrt{\frac{(5-\sqrt{13})x^2}{(5+\sqrt{13})x^2}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-1924*x*(5 + Sqrt[13] + 2*x^2))/(105*Sqrt[3 + 5*x^2 + x^4]) + (13*x*Sqrt[3
 + 5*x^2 + x^4])/3 - (26*x^3*Sqrt[3 + 5*x^2 + x^4])/35 + (x^5*(11 + 7*x^2)*
Sqrt[3 + 5*x^2 + x^4])/21 + (962*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 -
Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE
[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(105*Sqrt[3 + 5*x
^2 + x^4]) - (13*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1273

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*
p + 3)), x] + Dist[(2*p)/(c*(4*p + m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
```

```
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x^4\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx=\frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{63}\int\frac{x^4\left(-117-234x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx$$

$$=-\frac{26}{35}x^3\sqrt{3+5x^2+x^4}+\frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{315}\int\frac{x^2\left(-2106-4095x^2\right.}{\sqrt{3+5x^2+x^4}}$$

$$=\frac{13}{3}x\sqrt{3+5x^2+x^4}-\frac{26}{35}x^3\sqrt{3+5x^2+x^4}+\frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{945}\int$$

$$=\frac{13}{3}x\sqrt{3+5x^2+x^4}-\frac{26}{35}x^3\sqrt{3+5x^2+x^4}+\frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5x^2+x^4}-13\int\frac{}{\sqrt{}}$$

$$=-\frac{1924x\left(5+\sqrt{13}+2x^2\right)}{105\sqrt{3+5x^2+x^4}}+\frac{13}{3}x\sqrt{3+5x^2+x^4}-\frac{26}{35}x^3\sqrt{3+5x^2+x^4}+\frac{1}{21}x^5\left(11+7x\right.$$

Mathematica [C] time = 0.325432, size = 237, normalized size = 0.74

$$\frac{13i\sqrt{2}\left(148\sqrt{13}-635\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+70x^{11}+460x^9+604}{210\sqrt{x^4+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2730*x + 4082*x^3 + 460*x^5 + 604*x^7 + 460*x^9 + 70*x^11 - (1924*I)*Sqrt[
2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + S
qrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*S
qrt[13])/6] + (13*I)*Sqrt[2]*(-635 + 148*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*
x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2
/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(210*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.098, size = 260, normalized size = 0.8

$$\frac{x^7}{3}\sqrt{x^4+5x^2+3}+\frac{11x^5}{21}\sqrt{x^4+5x^2+3}-\frac{26x^3}{35}\sqrt{x^4+5x^2+3}+\frac{13x}{3}\sqrt{x^4+5x^2+3}-78\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/3*x^7*(x^4+5*x^2+3)^(1/2)+11/21*x^5*(x^4+5*x^2+3)^(1/2)-26/35*x^3*(x^4+5*
x^2+3)^(1/2)+13/3*x*(x^4+5*x^2+3)^(1/2)-78/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+
1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1
/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+46176/
35/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13
^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-30+6
*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))
^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5x^2+3}\left(3x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(3x^6+2x^4\right)\sqrt{x^4+5x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)*sqrt(x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \left(3x^2+2\right) \sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**4*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4, x)
```

3.152 $\int x^2\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx$

Optimal. Leaf size=305

$$\frac{2\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{\sqrt{x^4+5x^2+3}}+\frac{1}{35}\left(15x^2+29\right)\sqrt{\ldots}$$

```
[Out] (1247*x*(5 + Sqrt[13] + 2*x^2))/(210*Sqrt[3 + 5*x^2 + x^4]) - (4*x*Sqrt[3 +
 5*x^2 + x^4])/3 + (x^3*(29 + 15*x^2)*Sqrt[3 + 5*x^2 + x^4])/35 - (1247*Sqr
t[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(210*Sqrt[3 + 5*x^2 + x^4]) + (2*Sqrt[2/(3*(5 + Sqrt[13])
)]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[1
3])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])
/Sqrt[3 + 5*x^2 + x^4]
```

Rubi [A] time = 0.202803, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1273, 1279, 1189, 1099, 1135}

$$\frac{1}{35}\left(15x^2+29\right)\sqrt{x^4+5x^2+3}x^3-\frac{4}{3}\sqrt{x^4+5x^2+3}x+\frac{1247\left(2x^2+\sqrt{13}+5\right)x}{210\sqrt{x^4+5x^2+3}}+\frac{2\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (1247*x*(5 + Sqrt[13] + 2*x^2))/(210*Sqrt[3 + 5*x^2 + x^4]) - (4*x*Sqrt[3 +
 5*x^2 + x^4])/3 + (x^3*(29 + 15*x^2)*Sqrt[3 + 5*x^2 + x^4])/35 - (1247*Sqr
t[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(210*Sqrt[3 + 5*x^2 + x^4]) + (2*Sqrt[2/(3*(5 + Sqrt[13])
)]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[1
3])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])
/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1273

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*
p + 3)), x] + Dist[(2*p)/(c*(4*p + m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
```

```
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx &= \frac{1}{35}x^3\left(29+15x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{35}\int\frac{x^2\left(-51-140x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{4}{3}x\sqrt{3+5x^2+x^4}+\frac{1}{35}x^3\left(29+15x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{105}\int\frac{-420-1247x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{4}{3}x\sqrt{3+5x^2+x^4}+\frac{1}{35}x^3\left(29+15x^2\right)\sqrt{3+5x^2+x^4}+4\int\frac{1}{\sqrt{3+5x^2+x^4}}\,dx+\frac{12}{10}\\ &= \frac{1247x\left(5+\sqrt{13}+2x^2\right)}{210\sqrt{3+5x^2+x^4}}-\frac{4}{3}x\sqrt{3+5x^2+x^4}+\frac{1}{35}x^3\left(29+15x^2\right)\sqrt{3+5x^2+x^4}-\frac{124}{}\end{aligned}$$

Mathematica [C] time = 0.286797, size = 234, normalized size = 0.77

$$\frac{-i\sqrt{2}\left(1247\sqrt{13}-5395\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+4x\left(45x^8+312x^6+\right.}{420\sqrt{x^4+5x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (4*x*(-420 - 439*x^2 + 430*x^4 + 312*x^6 + 45*x^8) + (1247*I)*Sqrt[2]*(-5 +
 Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13]
```

```
+ 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])
/6] - I*Sqrt[2]*(-5395 + 1247*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 +
Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[
13])]*x], 19/6 + (5*Sqrt[13])/6])/(420*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.013, size = 243, normalized size = 0.8

$$\frac{3x^5}{7}\sqrt{x^4+5x^2+3}+\frac{29x^3}{35}\sqrt{x^4+5x^2+3}-\frac{4x}{3}\sqrt{x^4+5x^2+3}+24\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right.}}{\sqrt{-30+6\sqrt{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/7*x^5*(x^4+5*x^2+3)^(1/2)+29/35*x^3*(x^4+5*x^2+3)^(1/2)-4/3*x*(x^4+5*x^2+
3)^(1/2)+24/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5
/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(
1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-14964/35/(-30+6*13^(1/2))^(1/2)*(1-(-
5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3
)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/
6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5x^2+3}\left(3x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(3x^4+2x^2\right)\sqrt{x^4+5x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)*sqrt(x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2, x)
```

3.153 $\int \left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx$

Optimal. Leaf size=279

$$\frac{\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}-\frac{23x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}+\frac{1}{15}x\left(\right.$$

```
[Out] (-23*x*(5 + Sqrt[13] + 2*x^2))/(15*Sqrt[3 + 5*x^2 + x^4]) + (x*(25 + 9*x^2)
*Sqrt[3 + 5*x^2 + x^4])/15 + (23*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt
[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[Arc
Tan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(15*Sqrt[3 + 5*x^2 +
x^4]) + (Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13
])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.122301, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1176, 1189, 1099, 1135}

$$-\frac{23x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}+\frac{1}{15}x\left(9x^2+25\right)\sqrt{x^4+5x^2+3}+\frac{\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right.\right.}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```


```
[Out] (-23*x*(5 + Sqrt[13] + 2*x^2))/(15*Sqrt[3 + 5*x^2 + x^4]) + (x*(25 + 9*x^2)
*Sqrt[3 + 5*x^2 + x^4])/15 + (23*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt
[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[Arc
Tan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(15*Sqrt[3 + 5*x^2 +
x^4]) + (Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13
])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```


Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx &= \frac{1}{15}x\left(25+9x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{15}\int \frac{15-46x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= \frac{1}{15}x\left(25+9x^2\right)\sqrt{3+5x^2+x^4}-\frac{46}{15}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx+\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{23x\left(5+\sqrt{13}+2x^2\right)}{15\sqrt{3+5x^2+x^4}}+\frac{1}{15}x\left(25+9x^2\right)\sqrt{3+5x^2+x^4}+\frac{23\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x}{6+\left(5+\sqrt{13}\right)x}}}{}\end{aligned}$$

Mathematica [C] time = 0.290848, size = 229, normalized size = 0.82

$$\frac{i\sqrt{2}\left(23\sqrt{13}-130\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+2x\left(9x^6+70x^4+152x^2\right.}{30\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*x*(75 + 152*x^2 + 70*x^4 + 9*x^6) - (23*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[
(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellipti
cE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*
(-130 + 23*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 +
 Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5
*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.012, size = 226, normalized size = 0.8

$$\frac{3x^3}{5}\sqrt{x^4+5x^2+3}+\frac{5x}{3}\sqrt{x^4+5x^2+3}+6\,\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\,\text{EllipticF}\left(1/6\,x\sqrt{-30+\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/5*x^3*(x^4+5*x^2+3)^(1/2)+5/3*x*(x^4+5*x^2+3)^(1/2)+6/(-30+6*13^(1/2))^(1
/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^
4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^
(1/2))+552/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-
5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/
6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6
*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 5\,x^2 + 3}\left(3\,x^2 + 2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{x^4 + 5\,x^2 + 3}\left(3\,x^2 + 2\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(3x^2 + 2\right) \sqrt{x^4 + 5x^2 + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4 + 5\,x^2 + 3}\left(3\,x^2 + 2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

3.154 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx$

Optimal. Leaf size=284

$$\frac{8\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{x^4+5x^2+3}\left(2-x^2\right)}{x}$$

```
[Out] (9*x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - ((2 - x^2)*Sqrt[3
+ 5*x^2 + x^4])/x - (3*Sqrt[(3*(5 + Sqrt[13]))/2]*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sq
rt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4]) +
 (8*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt
[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6
]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi [A] time = 0.128488, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1271, 1189, 1099, 1135}

$$-\frac{\sqrt{x^4+5x^2+3}\left(2-x^2\right)}{x}+\frac{9x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}+\frac{8\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\right.\right.}{\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2,x]
```


```
[Out] (9*x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - ((2 - x^2)*Sqrt[3
+ 5*x^2 + x^4])/x - (3*Sqrt[(3*(5 + Sqrt[13]))/2]*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sq
rt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4]) +
 (8*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt
[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6
]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```


Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx &= -\frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x}-\frac{1}{3}\int \frac{-48-27x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x}+9\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx+16\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx\\ &= \frac{9x\left(5+\sqrt{13}+2x^2\right)}{2\sqrt{3+5x^2+x^4}}-\frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x}-\frac{3\sqrt{\frac{3}{2}}\left(5+\sqrt{13}\right)\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+(\right.}{}\end{aligned}$$

Mathematica [C] time = 0.304177, size = 231, normalized size = 0.81

$$\frac{-i\sqrt{2}\left(9\sqrt{13}-13\right)x\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+4\left(x^6+3x^4-7x^2-6\right.}{4x\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2,x]
```

```
[Out] (4*(-6 - 7*x^2 + 3*x^4 + x^6) + (9*I)*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 +
Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*A
rcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-13 +
 9*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[
13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[
13])/6])/(4*x*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.018, size = 225, normalized size = 0.8

$$x\sqrt{x^4+5x^2+3}+96\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\,\text{EllipticF}\left(1/6\,x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x)
```

```
[Out] x*(x^4+5*x^2+3)^(1/2)+96/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)
^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*
x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-324/(-30+6*13^(1/2))^(1/
2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4
+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^
(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*
39^(1/2)))-2*(x^4+5*x^2+3)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**2,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

3.155 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx$

Optimal. Leaf size=305

$$\frac{49\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{3\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}-\frac{\sqrt{x^4+5x^2+3}\left(2-9x^2\right)}{3x^3}-64$$

```
[Out] (32*x*(5 + Sqrt[13] + 2*x^2))/(9*Sqrt[3 + 5*x^2 + x^4]) - (64*Sqrt[3 + 5*x^
2 + x^4])/(9*x) - ((2 - 9*x^2)*Sqrt[3 + 5*x^2 + x^4])/(3*x^3) - (16*Sqrt[(2
*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(9*Sqrt[3 + 5*x^2 + x^4]) + (49*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sq
rt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3*Sqrt[6*(5 + Sqrt[13])]*S
qrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.158304, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1271, 1281, 1189, 1099, 1135}

$$-\frac{\sqrt{x^4+5x^2+3}\left(2-9x^2\right)}{3x^3}-\frac{64\sqrt{x^4+5x^2+3}}{9x}+\frac{32x\left(2x^2+\sqrt{13}+5\right)}{9\sqrt{x^4+5x^2+3}}+\frac{49\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{}\right.\right.}{3\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4,x]
```

```
[Out] (32*x*(5 + Sqrt[13] + 2*x^2))/(9*Sqrt[3 + 5*x^2 + x^4]) - (64*Sqrt[3 + 5*x^
2 + x^4])/(9*x) - ((2 - 9*x^2)*Sqrt[3 + 5*x^2 + x^4])/(3*x^3) - (16*Sqrt[(2
*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(9*Sqrt[3 + 5*x^2 + x^4]) + (49*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sq
rt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3*Sqrt[6*(5 + Sqrt[13])]*S
qrt[3 + 5*x^2 + x^4])
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
```

```
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2+3x^2\right)\sqrt{3+5x^2+x^4}}{x^4}\,dx &= -\frac{\left(2-9x^2\right)\sqrt{3+5x^2+x^4}}{3x^3}-\frac{1}{3}\int \frac{-64-49x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx\\
&= -\frac{64\sqrt{3+5x^2+x^4}}{9x}-\frac{\left(2-9x^2\right)\sqrt{3+5x^2+x^4}}{3x^3}+\frac{1}{9}\int \frac{147+64x^2}{\sqrt{3+5x^2+x^4}}\,dx\\
&= -\frac{64\sqrt{3+5x^2+x^4}}{9x}-\frac{\left(2-9x^2\right)\sqrt{3+5x^2+x^4}}{3x^3}+\frac{64}{9}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx+\frac{49}{3}\int \frac{}{}\\
&= \frac{32x\left(5+\sqrt{13}+2x^2\right)}{9\sqrt{3+5x^2+x^4}}-\frac{64\sqrt{3+5x^2+x^4}}{9x}-\frac{\left(2-9x^2\right)\sqrt{3+5x^2+x^4}}{3x^3}-\frac{16\sqrt{\frac{2}{3}}\left(5+\right.}{}
\end{aligned}$$

Mathematica [C] time = 0.313315, size = 237, normalized size = 0.78

$$\frac{-i\sqrt{2}\left(32\sqrt{13}-13\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}x^3\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-2\left(37x^6+191x^4+1\right.}{18x^3\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4,x]
```

```
[Out] (-2*(18 + 141*x^2 + 191*x^4 + 37*x^6) + (32*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3*
Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*El
lipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqr
t[2]*(-13 + 32*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*
Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 1
9/6 + (5*Sqrt[13])/6])/(18*x^3*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.017, size = 228, normalized size = 0.8

$$-\frac{37}{9x}\sqrt{x^4+5x^2+3}+98\,\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\,\mathrm{EllipticF}\left(1/6\,x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x)
```

```
[Out] -37/9*(x^4+5*x^2+3)^(1/2)/x+98/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2)
)*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*Elliptic
F(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-256/(-30+6*13^(1/2
))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2
)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),
5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2
)+1/6*39^(1/2)))-2/3*(x^4+5*x^2+3)^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5x^2+3}\left(3x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5x^2+3}\left(3x^2+2\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3x^2+2\right)\sqrt{x^4+5x^2+3}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**4,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

3.156 $\int x^5 \left(2+3x^2\right) \left(3+5x^2+x^4\right)^{3/2} dx$

Optimal. Leaf size=127

$$\frac{3}{14}\left(x^4+5x^2+3\right)^{5/2}x^4+\frac{\left(3313-1070x^2\right)\left(x^4+5x^2+3\right)^{5/2}}{1680}-\frac{2183}{768}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{28379\left(2x^2+5\right)\sqrt{x^4+}}{2048}$$

```
[Out] (28379*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/2048 - (2183*(5 + 2*x^2)*(3 + 5*x
^2 + x^4)^(3/2))/768 + (3*x^4*(3 + 5*x^2 + x^4)^(5/2))/14 + ((3313 - 1070*x
^2)*(3 + 5*x^2 + x^4)^(5/2))/1680 - (368927*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 +
 5*x^2 + x^4])])/4096
```

Rubi [A] time = 0.0958111, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 832, 779, 612, 621, 206}

$$\frac{3}{14}\left(x^4+5x^2+3\right)^{5/2}x^4+\frac{\left(3313-1070x^2\right)\left(x^4+5x^2+3\right)^{5/2}}{1680}-\frac{2183}{768}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{28379\left(2x^2+5\right)\sqrt{x^4+}}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```


```
[Out] (28379*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/2048 - (2183*(5 + 2*x^2)*(3 + 5*x
^2 + x^4)^(3/2))/768 + (3*x^4*(3 + 5*x^2 + x^4)^(5/2))/14 + ((3313 - 1070*x
^2)*(3 + 5*x^2 + x^4)^(5/2))/1680 - (368927*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 +
 5*x^2 + x^4])])/4096
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^5 \left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2} dx = \frac{1}{2}\operatorname{Subst}\left(\int x^2(2+3x)\left(3+5x+x^2\right)^{3/2} dx, x, x^2\right)$$

$$= \frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2} + \frac{1}{14}\operatorname{Subst}\left(\int \left(-18-\frac{107x}{2}\right)x\left(3+5x+x^2\right)^{3/2} dx, x, x$$

$$= \frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2} + \frac{\left(3313-1070x^2\right)\left(3+5x^2+x^4\right)^{5/2}}{1680} - \frac{2183}{96}\operatorname{Subst}\left(\int$$

$$= -\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2} + \frac{\left(3313-1070x^2\right)}{16}$$

$$= \frac{28379\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{2048} - \frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{3}{14}x^4\left(3-\right.$$

$$= \frac{28379\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{2048} - \frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{3}{14}x^4\left(3-\right.$$

$$= \frac{28379\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{2048} - \frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{3}{14}x^4\left(3-\right.$$

Mathematica [A] time = 0.0384189, size = 81, normalized size = 0.64

$$\frac{2\sqrt{x^4+5x^2+3}\left(46080x^{12}+323840x^{10}+482944x^8+154800x^6+283304x^4-1499570x^2+9546951\right)-38737335\,\text{t}}{430080}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(9546951 - 1499570*x^2 + 283304*x^4 + 154800*x^6 +
 482944*x^8 + 323840*x^10 + 46080*x^12) - 38737335*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])])/430080
```

Maple [A] time = 0.034, size = 138, normalized size = 1.1

$$\frac{3\,x^{12}}{14}\sqrt{x^4+5\,x^2+3}+\frac{253\,x^{10}}{168}\sqrt{x^4+5\,x^2+3}+\frac{539\,x^8}{240}\sqrt{x^4+5\,x^2+3}+\frac{3182317}{71680}\sqrt{x^4+5\,x^2+3}+\frac{5059\,x^4}{3840}\sqrt{x^4+5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 3/14*x^12*(x^4+5*x^2+3)^(1/2)+253/168*x^10*(x^4+5*x^2+3)^(1/2)+539/240*x^8*
(x^4+5*x^2+3)^(1/2)+3182317/71680*(x^4+5*x^2+3)^(1/2)+5059/3840*x^4*(x^4+5*
x^2+3)^(1/2)+645/896*x^6*(x^4+5*x^2+3)^(1/2)-149957/21504*x^2*(x^4+5*x^2+3)
^(1/2)-368927/4096*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A] time = 1.05815, size = 182, normalized size = 1.43

$$\frac{3}{14}\left(x^4+5x^2+3\right)^{\frac{5}{2}}x^4-\frac{107}{168}\left(x^4+5x^2+3\right)^{\frac{5}{2}}x^2-\frac{2183}{384}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2+\frac{3313}{1680}\left(x^4+5x^2+3\right)^{\frac{5}{2}}+\frac{28379}{1024}\sqrt{x^4+5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/14*(x^4 + 5*x^2 + 3)^(5/2)*x^4 - 107/168*(x^4 + 5*x^2 + 3)^(5/2)*x^2 - 21
83/384*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3313/1680*(x^4 + 5*x^2 + 3)^(5/2) + 28
379/1024*sqrt(x^4 + 5*x^2 + 3)*x^2 - 10915/768*(x^4 + 5*x^2 + 3)^(3/2) + 14
1895/2048*sqrt(x^4 + 5*x^2 + 3) - 368927/4096*log(2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) + 5)
```

Fricas [A] time = 1.2053, size = 240, normalized size = 1.89

$$\frac{1}{215040}\left(46080x^{12}+323840x^{10}+482944x^8+154800x^6+283304x^4-1499570x^2+9546951\right)\sqrt{x^4+5x^2+3}+\frac{3689}{409}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/215040*(46080*x^12 + 323840*x^10 + 482944*x^8 + 154800*x^6 + 283304*x^4 -
 1499570*x^2 + 9546951)*sqrt(x^4 + 5*x^2 + 3) + 368927/4096*log(-2*x^2 + 2*
sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [A] time = 1.12456, size = 109, normalized size = 0.86

$$\frac{1}{215040}\sqrt{x^4+5x^2+3}\left(2\left(4\left(2\left(8\left(10\left(36x^2+253\right)x^2+3773\right)x^2+9675\right)x^2+35413\right)x^2-749785\right)x^2+9546951\right)+\frac{3689}{409}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/215040*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(10*(36*x^2 + 253)*x^2 + 3773)*x
^2 + 9675)*x^2 + 35413)*x^2 - 749785)*x^2 + 9546951) + 368927/4096*log(2*x^
2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.157 $\int x^3 \left(2+3x^2\right) \left(3+5x^2+x^4\right)^{3/2} dx$

Optimal. Leaf size=106

$$-\frac{1}{40}\left(27-10x^2\right)\left(x^4+5x^2+3\right)^{5/2}+\frac{123}{128}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{4797\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{1024}+\frac{62361\tanh^{-1}\left(\frac{}{2}\right.}{2048}$$

```
[Out] (-4797*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/1024 + (123*(5 + 2*x^2)*(3 + 5*x^
2 + x^4)^(3/2))/128 - ((27 - 10*x^2)*(3 + 5*x^2 + x^4)^(5/2))/40 + (62361*A
rcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2048
```

Rubi [A] time = 0.0718341, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1251, 779, 612, 621, 206}

$$-\frac{1}{40}\left(27-10x^2\right)\left(x^4+5x^2+3\right)^{5/2}+\frac{123}{128}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{4797\left(2x^2+5\right)\sqrt{x^4+5x^2+3}}{1024}+\frac{62361\tanh^{-1}\left(\frac{}{2}\right.}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-4797*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/1024 + (123*(5 + 2*x^2)*(3 + 5*x^
2 + x^4)^(3/2))/128 - ((27 - 10*x^2)*(3 + 5*x^2 + x^4)^(5/2))/40 + (62361*A
rcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2048
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^3\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}dx=\frac{1}{2}\operatorname{Subst}\left(\int x(2+3x)\left(3+5x+x^2\right)^{3/2}dx,x,x^2\right)$$
$$=-\frac{1}{40}\left(27-10x^2\right)\left(3+5x^2+x^4\right)^{5/2}+\frac{123}{16}\operatorname{Subst}\left(\int\left(3+5x+x^2\right)^{3/2}dx,x,x^2\right)$$
$$=\frac{123}{128}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{1}{40}\left(27-10x^2\right)\left(3+5x^2+x^4\right)^{5/2}-\frac{4797}{256}\operatorname{Sub}$$
$$=-\frac{4797\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{1024}+\frac{123}{128}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{1}{40}\left(27-1\right.$$
$$=-\frac{4797\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{1024}+\frac{123}{128}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{1}{40}\left(27-1\right.$$
$$=-\frac{4797\left(5+2x^2\right)\sqrt{3+5x^2+x^4}}{1024}+\frac{123}{128}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{1}{40}\left(27-1\right.$$

Mathematica [A] time = 0.0309979, size = 76, normalized size = 0.72

$$\frac{2\sqrt{x^4+5x^2+3}\left(1280x^{10}+9344x^8+14960x^6+5064x^4+12390x^2-77229\right)+311805\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-77229 + 12390*x^2 + 5064*x^4 + 14960*x^6 + 9344*
x^8 + 1280*x^10) + 311805*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/1
0240
```

Maple [A] time = 0.016, size = 121, normalized size = 1.1

$$\frac{x^{10}}{4}\sqrt{x^4+5x^2+3}+\frac{73x^8}{40}\sqrt{x^4+5x^2+3}-\frac{77229}{5120}\sqrt{x^4+5x^2+3}+\frac{633x^4}{640}\sqrt{x^4+5x^2+3}+\frac{187x^6}{64}\sqrt{x^4+5x^2+3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 1/4*x^10*(x^4+5*x^2+3)^(1/2)+73/40*x^8*(x^4+5*x^2+3)^(1/2)-77229/5120*(x^4+
5*x^2+3)^(1/2)+633/640*x^4*(x^4+5*x^2+3)^(1/2)+187/64*x^6*(x^4+5*x^2+3)^(1/
2)+1239/512*x^2*(x^4+5*x^2+3)^(1/2)+62361/2048*ln(5/2+x^2+(x^4+5*x^2+3)^(1/
2))
```

Maxima [A] time = 0.970596, size = 159, normalized size = 1.5

$$\frac{1}{4}\left(x^4+5x^2+3\right)^{\frac{5}{2}}x^2+\frac{123}{64}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2-\frac{27}{40}\left(x^4+5x^2+3\right)^{\frac{5}{2}}-\frac{4797}{512}\sqrt{x^4+5x^2+3}x^2+\frac{615}{128}\left(x^4+5x^2+3\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^4 + 5*x^2 + 3)^(5/2)*x^2 + 123/64*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 27/4
0*(x^4 + 5*x^2 + 3)^(5/2) - 4797/512*sqrt(x^4 + 5*x^2 + 3)*x^2 + 615/128*(x
^4 + 5*x^2 + 3)^(3/2) - 23985/1024*sqrt(x^4 + 5*x^2 + 3) + 62361/2048*log(2
*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.31966, size = 204, normalized size = 1.92

$$\frac{1}{5120}\left(1280\,x^{10}+9344\,x^{8}+14960\,x^{6}+5064\,x^{4}+12390\,x^{2}-77229\right)\sqrt{x^{4}+5\,x^{2}+3}-\frac{62361}{2048}\log\left(-2\,x^{2}+2\,\sqrt{x^{4}+5\,x^{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5120*(1280*x^10 + 9344*x^8 + 14960*x^6 + 5064*x^4 + 12390*x^2 - 77229)*sq
rt(x^4 + 5*x^2 + 3) - 62361/2048*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3}\left(3x^{2}+2\right)\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [A] time = 1.09392, size = 100, normalized size = 0.94

$$\frac{1}{5120}\sqrt{x^{4}+5\,x^{2}+3}\left(2\left(4\left(2\left(8\left(10\,x^{2}+73\right)x^{2}+935\right)x^{2}+633\right)x^{2}+6195\right)x^{2}-77229\right)-\frac{62361}{2048}\log\left(2\,x^{2}-2\,\sqrt{x^{4}+5\,x^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/5120*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(10*x^2 + 73)*x^2 + 935)*x^2 + 633
)*x^2 + 6195)*x^2 - 77229) - 62361/2048*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3)
 + 5)
```

3.158 $\int x\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=99

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{32}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{429}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{5577}{512}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] (429*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 - (11*(5 + 2*x^2)*(3 + 5*x^2 +
x^4)^(3/2))/32 + (3*(3 + 5*x^2 + x^4)^(5/2))/10 - (5577*ArcTanh[(5 + 2*x^2)
/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rubi [A] time = 0.0583216, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1247, 640, 612, 621, 206}

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{32}\left(2x^2+5\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{429}{256}\left(2x^2+5\right)\sqrt{x^4+5x^2+3}-\frac{5577}{512}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (429*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 - (11*(5 + 2*x^2)*(3 + 5*x^2 +
x^4)^(3/2))/32 + (3*(3 + 5*x^2 + x^4)^(5/2))/10 - (5577*ArcTanh[(5 + 2*x^2)
/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x(2+3x^2)(3+5x^2+x^4)^{3/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int (2+3x)(3+5x+x^2)^{3/2}\,dx,x,x^2\right)\\
&= \frac{3}{10}(3+5x^2+x^4)^{5/2}-\frac{11}{4}\text{Subst}\left(\int (3+5x+x^2)^{3/2}\,dx,x,x^2\right)\\
&= -\frac{11}{32}(5+2x^2)(3+5x^2+x^4)^{3/2}+\frac{3}{10}(3+5x^2+x^4)^{5/2}+\frac{429}{64}\text{Subst}\left(\int \sqrt{3+5x+\right.\\
&= \frac{429}{256}(5+2x^2)\sqrt{3+5x^2+x^4}-\frac{11}{32}(5+2x^2)(3+5x^2+x^4)^{3/2}+\frac{3}{10}(3+5x^2+x^4)\\
&= \frac{429}{256}(5+2x^2)\sqrt{3+5x^2+x^4}-\frac{11}{32}(5+2x^2)(3+5x^2+x^4)^{3/2}+\frac{3}{10}(3+5x^2+x^4)\\
&= \frac{429}{256}(5+2x^2)\sqrt{3+5x^2+x^4}-\frac{11}{32}(5+2x^2)(3+5x^2+x^4)^{3/2}+\frac{3}{10}(3+5x^2+x^4)
\end{aligned}
$$

Mathematica [A] time = 0.0259414, size = 71, normalized size = 0.72

$$\frac{2\sqrt{x^4+5x^2+3}\left(384x^8+2960x^6+5304x^4+2170x^2+7581\right)-27885\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{2560}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(7581 + 2170*x^2 + 5304*x^4 + 2960*x^6 + 384*x^8)
- 27885*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2560
```

Maple [A] time = 0.014, size = 104, normalized size = 1.1

$$\frac{3x^8}{10}\sqrt{x^4+5x^2+3}+\frac{37x^6}{16}\sqrt{x^4+5x^2+3}+\frac{663x^4}{160}\sqrt{x^4+5x^2+3}+\frac{217x^2}{128}\sqrt{x^4+5x^2+3}+\frac{7581}{1280}\sqrt{x^4+5x^2+3}-\frac{557}{51}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 3/10*x^8*(x^4+5*x^2+3)^(1/2)+37/16*x^6*(x^4+5*x^2+3)^(1/2)+663/160*x^4*(x^4
+5*x^2+3)^(1/2)+217/128*x^2*(x^4+5*x^2+3)^(1/2)+7581/1280*(x^4+5*x^2+3)^(1/
2)-5577/512*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A] time = 0.945292, size = 136, normalized size = 1.37

$$-\frac{11}{16}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2+\frac{3}{10}\left(x^4+5x^2+3\right)^{\frac{5}{2}}+\frac{429}{128}\sqrt{x^4+5x^2+3}x^2-\frac{55}{32}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{2145}{256}\sqrt{x^4+5x^2+3}-\frac{55}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -11/16*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3/10*(x^4 + 5*x^2 + 3)^(5/2) + 429/128
*sqrt(x^4 + 5*x^2 + 3)*x^2 - 55/32*(x^4 + 5*x^2 + 3)^(3/2) + 2145/256*sqrt(
x^4 + 5*x^2 + 3) - 5577/512*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.24524, size = 180, normalized size = 1.82

$$\frac{1}{1280}\left(384\,x^{8}+2960\,x^{6}+5304\,x^{4}+2170\,x^{2}+7581\right)\sqrt{x^{4}+5\,x^{2}+3}+\frac{5577}{512}\,\log\left(-2\,x^{2}+2\,\sqrt{x^{4}+5\,x^{2}+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1280*(384*x^8 + 2960*x^6 + 5304*x^4 + 2170*x^2 + 7581)*sqrt(x^4 + 5*x^2 +
 3) + 5577/512*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(3x^{2}+2\right)\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [A] time = 1.10413, size = 90, normalized size = 0.91

$$\frac{1}{1280}\,\sqrt{x^{4}+5\,x^{2}+3}\left(2\left(4\left(2\left(24\,x^{2}+185\right)x^{2}+663\right)x^{2}+1085\right)x^{2}+7581\right)+\frac{5577}{512}\,\log\left(2\,x^{2}-2\,\sqrt{x^{4}+5\,x^{2}+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1280*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(24*x^2 + 185)*x^2 + 663)*x^2 + 1085)
*x^2 + 7581) + 5577/512*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.159 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=119

$$\frac{1}{48}\left(18x^2+61\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{1}{128}\left(199-74x^2\right)\sqrt{x^4+5x^2+3}+\frac{2401}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-3\sqrt{3}\tanh^{-1}\left(\frac{}{2\cdots}\right.$$

```
[Out] ((199 - 74*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 + ((61 + 18*x^2)*(3 + 5*x^2 + x^
4)^(3/2))/48 + (2401*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256 -
3*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

Rubi [A] time = 0.105921, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 814, 843, 621, 206, 724}

$$\frac{1}{48}\left(18x^2+61\right)\left(x^4+5x^2+3\right)^{3/2}+\frac{1}{128}\left(199-74x^2\right)\sqrt{x^4+5x^2+3}+\frac{2401}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-3\sqrt{3}\tanh^{-1}\left(\frac{}{2\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]
```

```
[Out] ((199 - 74*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 + ((61 + 18*x^2)*(3 + 5*x^2 + x^
4)^(3/2))/48 + (2401*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256 -
3*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(2+3x)\left(3+5x+x^2\right)^{3/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2} - \frac{1}{16}\,\text{Subst}\left(\int \frac{\left(-48+\frac{37x}{2}\right)\sqrt{3+5x+x^2}}{x}\,dx, x, \right.$$

$$= \frac{1}{128}\left(199-74x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{1}{64}\,\text{Subst}$$

$$= \frac{1}{128}\left(199-74x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2} + 9\,\text{Subst}\left(\int\right.$$

$$= \frac{1}{128}\left(199-74x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2} - 18\,\text{Subst}\left(\right.$$

$$= \frac{1}{128}\left(199-74x^2\right)\sqrt{3+5x^2+x^4} + \frac{1}{48}\left(61+18x^2\right)\left(3+5x^2+x^4\right)^{3/2} + \frac{2401}{256}\tanh$$

Mathematica [A] time = 0.0637445, size = 104, normalized size = 0.87

$$\frac{1}{384}\sqrt{x^4+5x^2+3}\left(144x^6+1208x^4+2650x^2+2061\right) + \frac{2401}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - 3\sqrt{3}\tanh^{-1}\left(\frac{5x^2+}{2\sqrt{3}\sqrt{x^4+5}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(2061 + 2650*x^2 + 1208*x^4 + 144*x^6))/384 + (2401*
ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256 - 3*Sqrt[3]*ArcTanh[(6
+ 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

Maple [A] time = 0.016, size = 117, normalized size = 1.

$$\frac{3x^6}{8}\sqrt{x^4+5x^2+3} + \frac{151x^4}{48}\sqrt{x^4+5x^2+3} + \frac{1325x^2}{192}\sqrt{x^4+5x^2+3} + \frac{687}{128}\sqrt{x^4+5x^2+3} + \frac{2401}{256}\ln\left(\frac{5}{2}+x^2+\sqrt{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x)
```

```
[Out] 3/8*x^6*(x^4+5*x^2+3)^(1/2)+151/48*x^4*(x^4+5*x^2+3)^(1/2)+1325/192*x^2*(x^
4+5*x^2+3)^(1/2)+687/128*(x^4+5*x^2+3)^(1/2)+2401/256*ln(5/2+x^2+(x^4+5*x^2
+3)^(1/2))-3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Maxima [A] time = 1.4835, size = 162, normalized size = 1.36

$$\frac{3}{8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2-\frac{37}{64}\sqrt{x^4+5x^2+3}x^2+\frac{61}{48}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-3\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{199}{128}\sqrt{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 37/64*sqrt(x^4 + 5*x^2 + 3)*x^2 + 61/48*(
x^4 + 5*x^2 + 3)^(3/2) - 3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2
+ 6/x^2 + 5) + 199/128*sqrt(x^4 + 5*x^2 + 3) + 2401/256*log(2*x^2 + 2*sqrt(
x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.29806, size = 300, normalized size = 2.52

$$\frac{1}{384}\left(144x^6+1208x^4+2650x^2+2061\right)\sqrt{x^4+5x^2+3}+3\sqrt{3}\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/384*(144*x^6 + 1208*x^4 + 2650*x^2 + 2061)*sqrt(x^4 + 5*x^2 + 3) + 3*sqrt
(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3
) - 6) + 30)/x^2) - 2401/256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}\left(3x^2+2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x, x)
```

3.160 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=122

$$-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{2x^2}+\frac{3}{16}(18x^2+109)\sqrt{x^4+5x^2+3}+\frac{609}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-12\sqrt{3}\tanh^{-1}\left(\frac{5x}{2\sqrt{3}\sqrt{x^4}}\right.$$

```
[Out] (3*(109 + 18*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 - ((2 - x^2)*(3 + 5*x^2 + x^4)^
(3/2))/(2*x^2) + (609*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32 -
12*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

Rubi [A] time = 0.106506, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1251, 812, 814, 843, 621, 206, 724}

$$-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{2x^2}+\frac{3}{16}(18x^2+109)\sqrt{x^4+5x^2+3}+\frac{609}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-12\sqrt{3}\tanh^{-1}\left(\frac{5x}{2\sqrt{3}\sqrt{x^4}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

```
[Out] (3*(109 + 18*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 - ((2 - x^2)*(3 + 5*x^2 + x^4)^
(3/2))/(2*x^2) + (609*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32 -
12*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
```

```
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{x^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(2+3x)\left(3+5x+x^2\right)^{3/2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{\left(2-x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{2x^2} - \frac{1}{4}\,\text{Subst}\left(\int \frac{(-48-27x)\sqrt{3+5x+x^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{3}{16}\left(109+18x^2\right)\sqrt{3+5x^2+x^4} - \frac{\left(2-x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{2x^2} + \frac{1}{16}\,\text{Subst}\left(\int \frac{57\ldots}{x\sqrt{3\ldots}}\right.$$

$$= \frac{3}{16}\left(109+18x^2\right)\sqrt{3+5x^2+x^4} - \frac{\left(2-x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{2x^2} + \frac{609}{32}\,\text{Subst}\left(\int \frac{\ldots}{\sqrt{3\ldots}}\right.$$

$$= \frac{3}{16}\left(109+18x^2\right)\sqrt{3+5x^2+x^4} - \frac{\left(2-x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{2x^2} + \frac{609}{16}\,\text{Subst}\left(\int \frac{1\ldots}{4-\ldots}\right.$$

$$= \frac{3}{16}\left(109+18x^2\right)\sqrt{3+5x^2+x^4} - \frac{\left(2-x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{2x^2} + \frac{609}{32}\tanh^{-1}\left(\frac{5\ldots}{2\sqrt{3\ldots}}\right.$$

Mathematica [A] time = 0.0542173, size = 107, normalized size = 0.88

$$\frac{\sqrt{x^4+5x^2+3}\left(8x^6+78x^4+271x^2-48\right)}{16x^2}+\frac{609}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-12\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-48 + 271*x^2 + 78*x^4 + 8*x^6))/(16*x^2) + (609*Ar
cTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32 - 12*Sqrt[3]*ArcTanh[(6 +
5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

Maple [A] time = 0.015, size = 117, normalized size = 1.

$$\frac{x^4}{2}\sqrt{x^4+5\,x^2+3}+\frac{39\,x^2}{8}\sqrt{x^4+5\,x^2+3}+\frac{271}{16}\sqrt{x^4+5\,x^2+3}+\frac{609}{32}\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5\,x^2+3}\right)-12\,\mathrm{Artanh}\left(1/6\,\frac{(5}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x)
```

```
[Out] 1/2*x^4*(x^4+5*x^2+3)^(1/2)+39/8*x^2*(x^4+5*x^2+3)^(1/2)+271/16*(x^4+5*x^2+
3)^(1/2)+609/32*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))-12*arctanh(1/6*(5*x^2+6)*3^
(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-3*(x^4+5*x^2+3)^(1/2)/x^2
```

Maxima [A] time = 1.46772, size = 162, normalized size = 1.33

$$\frac{27}{8}\sqrt{x^4+5\,x^2+3}x^2+\frac{1}{2}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-12\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{327}{16}\sqrt{x^4+5\,x^2+3}-\frac{(x^4+5\,x}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 27/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 12*sqrt(3)*l
og(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 327/16*sqrt(x^4 + 5*x
^2 + 3) - (x^4 + 5*x^2 + 3)^(3/2)/x^2 + 609/32*log(2*x^2 + 2*sqrt(x^4 + 5*x
^2 + 3) + 5)
```

Fricas [A] time = 1.34821, size = 325, normalized size = 2.66

$$\frac{1536\sqrt{3}x^2\log\left(\frac{25\,x^2-2\sqrt{3}\left(5\,x^2+6\right)-2\sqrt{x^4+5\,x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-2436\,x^2\log\left(-2\,x^2+2\sqrt{x^4+5\,x^2+3}-5\right)+1541\,x^2+8\left(8\,x^6+\right.}{128\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/128*(1536*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 +
5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 2436*x^2*log(-2*x^2 + 2*sqrt(x^4 +
5*x^2 + 3) - 5) + 1541*x^2 + 8*(8*x^6 + 78*x^4 + 271*x^2 - 48)*sqrt(x^4 + 5
*x^2 + 3))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**3,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+5x^2+3)^{\frac{3}{2}}(3x^2+2)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^3, x)
```

3.161 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=127

$$-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{4x^4}-\frac{3(28-19x^2)\sqrt{x^4+5x^2+3}}{8x^2}+\frac{453}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{127}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x}{2\sqrt{3}\sqrt{x}}\right.$$

```
[Out] (-3*(28 - 19*x^2)*Sqrt[3 + 5*x^2 + x^4])/(8*x^2) - ((2 - 3*x^2)*(3 + 5*x^2
+ x^4)^(3/2))/(4*x^4) + (453*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])]
)/16 - (127*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])
/8
```

Rubi [A] time = 0.108955, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1251, 812, 843, 621, 206, 724}

$$-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{4x^4}-\frac{3(28-19x^2)\sqrt{x^4+5x^2+3}}{8x^2}+\frac{453}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{127}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x}{2\sqrt{3}\sqrt{x}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

```
[Out] (-3*(28 - 19*x^2)*Sqrt[3 + 5*x^2 + x^4])/(8*x^2) - ((2 - 3*x^2)*(3 + 5*x^2
+ x^4)^(3/2))/(4*x^4) + (453*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])]
)/16 - (127*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])
/8
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{x^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(2+3x)\left(3+5x+x^2\right)^{3/2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4} - \frac{3}{16}\,\text{Subst}\left(\int \frac{(-56-38x)\sqrt{3+5x+x^2}}{x^2}\,dx, x, x^2\right)$$

$$= -\frac{3\left(28-19x^2\right)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4} + \frac{3}{32}\,\text{Subst}\left(\int \frac{50}{x\sqrt{3}}\right.$$

$$= -\frac{3\left(28-19x^2\right)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4} + \frac{453}{16}\,\text{Subst}\left(\int \frac{}{\sqrt{3}}\right.$$

$$= -\frac{3\left(28-19x^2\right)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4} + \frac{453}{8}\,\text{Subst}\left(\int \frac{}{4-}\right.$$

$$= -\frac{3\left(28-19x^2\right)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{\left(2-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{4x^4} + \frac{453}{16}\tanh^{-1}\left(\frac{5}{2\sqrt{3}}\right.$$

Mathematica [A] time = 0.0650273, size = 107, normalized size = 0.84

$$\frac{1}{16}\left(\frac{2\sqrt{x^4+5x^2+3}\left(6x^6+83x^4-86x^2-12\right)}{x^4} + 453\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right) - 254\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

```
[Out] ((2*Sqrt[3 + 5*x^2 + x^4]*(-12 - 86*x^2 + 83*x^4 + 6*x^6))/x^4 + 453*ArcTan
h[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])] - 254*Sqrt[3]*ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/16
```

Maple [A] time = 0.017, size = 117, normalized size = 0.9

$$\frac{83}{8}\sqrt{x^4+5x^2+3}+\frac{453}{16}\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)-\frac{43}{4x^2}\sqrt{x^4+5x^2+3}-\frac{127\sqrt{3}}{8}\mathrm{Artanh}\left(\frac{(5x^2+6)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x)
```

```
[Out] 83/8*(x^4+5*x^2+3)^(1/2)+453/16*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))-43/4*(x^4+5
*x^2+3)^(1/2)/x^2-127/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*
3^(1/2)-3/2*(x^4+5*x^2+3)^(1/2)/x^4+3/4*x^2*(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 1.48336, size = 185, normalized size = 1.46

$$\frac{7}{2}\sqrt{x^4+5x^2+3}x^2+\frac{1}{6}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{127}{8}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{197}{8}\sqrt{x^4+5x^2+3}-\frac{23\left(x^4+\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 7/2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/6*(x^4 + 5*x^2 + 3)^(3/2) - 127/8*sqrt(3)
*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 197/8*sqrt(x^4 + 5*
x^2 + 3) - 23/12*(x^4 + 5*x^2 + 3)^(3/2)/x^2 - 1/6*(x^4 + 5*x^2 + 3)^(5/2)/
x^4 + 453/16*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.37994, size = 320, normalized size = 2.52

$$\frac{1016\sqrt{3}x^4\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right)-1812x^4\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)+67x^4+8\left(6x^6+8\right.}{64x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/64*(1016*sqrt(3)*x^4*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5
*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 1812*x^4*log(-2*x^2 + 2*sqrt(x^4 + 5
*x^2 + 3) - 5) + 67*x^4 + 8*(6*x^6 + 83*x^4 - 86*x^2 - 12)*sqrt(x^4 + 5*x^2
 + 3))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(3x^2+2)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**5,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{4}+5 x^{2}+3\right)^{\frac{3}{2}} \left(3 x^{2}+2\right)}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^5, x)
```

3.162 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=127

$$-\frac{(7x^2+2)(x^4+5x^2+3)^{3/2}}{6x^6}-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{12x^2}+\frac{49}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{527\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

```
[Out] -((67 - 32*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^2) - ((2 + 7*x^2)*(3 + 5*x^2 +
 x^4)^(3/2))/(6*x^6) + (49*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/
4 - (527*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(24*Sqrt[3
])
```

Rubi [A] time = 0.106862, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1251, 810, 812, 843, 621, 206, 724}

$$-\frac{(7x^2+2)(x^4+5x^2+3)^{3/2}}{6x^6}-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{12x^2}+\frac{49}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{527\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```


```
[Out] -((67 - 32*x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^2) - ((2 + 7*x^2)*(3 + 5*x^2 +
 x^4)^(3/2))/(6*x^6) + (49*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/
4 - (527*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(24*Sqrt[3
])
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 810

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^
p*((d*g - e*f*(m + 2))*(c*d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d
*g) - e*(g*(m + 1)*(c*d^2 - b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x
))/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[p/(e^2*(m + 1)*
(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(
p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p +
 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*(2
*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m -
 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &&
LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 812

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2)
```

```
- d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + b*x + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p
 + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{x^7}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(2+3x)\left(3+5x+x^2\right)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{\left(2+7x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{6x^6} - \frac{1}{24}\operatorname{Subst}\left(\int \frac{(-134-64x)\sqrt{3+5x+x^2}}{x^2}\,dx, x, x^2\right.$$

$$= -\frac{\left(67-32x^2\right)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{\left(2+7x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{6x^6} + \frac{1}{48}\operatorname{Subst}\left(\int \frac{105}{x\sqrt{3}}\right.$$

$$= -\frac{\left(67-32x^2\right)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{\left(2+7x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{6x^6} + \frac{49}{4}\operatorname{Subst}\left(\int \frac{}{\sqrt{3+}}\right.$$

$$= -\frac{\left(67-32x^2\right)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{\left(2+7x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{6x^6} + \frac{49}{2}\operatorname{Subst}\left(\int \frac{1}{4-x}\right.$$

$$= -\frac{\left(67-32x^2\right)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{\left(2+7x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{6x^6} + \frac{49}{4}\tanh^{-1}\left(\frac{5+}{2\sqrt{3+}}\right.$$

Mathematica [A] time = 0.0515496, size = 107, normalized size = 0.84

$$\frac{1}{72}\left(\frac{6\sqrt{x^4+5x^2+3}\left(18x^6-141x^4-62x^2-12\right)}{x^6}+882\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-527\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```

```
[Out] ((6*Sqrt[3 + 5*x^2 + x^4]*(-12 - 62*x^2 - 141*x^4 + 18*x^6))/x^6 + 882*ArcT
anh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])] - 527*Sqrt[3]*ArcTanh[(6 + 5*x^2
)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/72
```

Maple [A] time = 0.017, size = 117, normalized size = 0.9

$$\frac{3}{2}\sqrt{x^4+5\,x^2+3}+\frac{49}{4}\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5\,x^2+3}\right)-\frac{47}{4\,x^2}\sqrt{x^4+5\,x^2+3}-\frac{527\,\sqrt{3}}{72}\mathrm{Artanh}\left(\frac{\left(5\,x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5\,x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x)
```

```
[Out] 3/2*(x^4+5*x^2+3)^(1/2)+49/4*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))-47/4*(x^4+5*x^
2+3)^(1/2)/x^2-527/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^
(1/2)-31/6*(x^4+5*x^2+3)^(1/2)/x^4-(x^4+5*x^2+3)^(1/2)/x^6
```

Maxima [A] time = 1.49001, size = 208, normalized size = 1.64

$$\frac{67}{36}\sqrt{x^4+5\,x^2+3}x^2+\frac{11}{54}\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}-\frac{527}{72}\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{431}{36}\sqrt{x^4+5\,x^2+3}-\frac{79\left(x^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 67/36*sqrt(x^4 + 5*x^2 + 3)*x^2 + 11/54*(x^4 + 5*x^2 + 3)^(3/2) - 527/72*sq
rt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 431/36*sqrt(x^
4 + 5*x^2 + 3) - 79/108*(x^4 + 5*x^2 + 3)^(3/2)/x^2 - 11/54*(x^4 + 5*x^2 +
3)^(5/2)/x^4 - 1/9*(x^4 + 5*x^2 + 3)^(5/2)/x^6 + 49/4*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.31379, size = 321, normalized size = 2.53

$$\frac{527\,\sqrt{3}x^6\log\left(\frac{25\,x^2-2\,\sqrt{3}\left(5\,x^2+6\right)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)-882\,x^6\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)-711\,x^6+6\left(18\,x^6-14\right.}{72\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/72*(527*sqrt(3)*x^6*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 882*x^6*log(-2*x^2 + 2*sqrt(x^4 + 5*x
^2 + 3) - 5) - 711*x^6 + 6*(18*x^6 - 141*x^4 - 62*x^2 - 12)*sqrt(x^4 + 5*x^
2 + 3))/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**7,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**7, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+5x^2+3)^{\frac{3}{2}}(3x^2+2)}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^7, x)
```

3.163 $\int x^4 \left(2+3x^2\right) \left(3+5x^2+x^4\right)^{3/2} dx$

Optimal. Leaf size=356

$$\frac{2105\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{143\sqrt{x^4+5x^2+3}}+\frac{1}{143}\left(33x^2+71\right)$$

```
[Out] (176723*x*(5 + Sqrt[13] + 2*x^2))/(4290*Sqrt[3 + 5*x^2 + x^4]) - (4210*x*Sq
rt[3 + 5*x^2 + x^4])/429 + (1251*x^3*Sqrt[3 + 5*x^2 + x^4])/715 - (x^5*(283
 + 272*x^2)*Sqrt[3 + 5*x^2 + x^4])/429 + (x^5*(71 + 33*x^2)*(3 + 5*x^2 + x^
4)^(3/2))/143 - (176723*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2
)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[
(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(4290*Sqrt[3 + 5*x^2 + x^4]) +
 (2105*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + S
qrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13]
)/6]*x], (-13 + 5*Sqrt[13])/6])/(143*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.256476, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1273, 1279, 1189, 1099, 1135}

$$\frac{1}{143}\left(33x^2+71\right)\left(x^4+5x^2+3\right)^{3/2}x^5-\frac{1}{429}\left(272x^2+283\right)\sqrt{x^4+5x^2+3}x^5+\frac{1251}{715}\sqrt{x^4+5x^2+3}x^3-\frac{4210}{429}\sqrt{x^4+5x^2+}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```


```
[Out] (176723*x*(5 + Sqrt[13] + 2*x^2))/(4290*Sqrt[3 + 5*x^2 + x^4]) - (4210*x*Sq
rt[3 + 5*x^2 + x^4])/429 + (1251*x^3*Sqrt[3 + 5*x^2 + x^4])/715 - (x^5*(283
 + 272*x^2)*Sqrt[3 + 5*x^2 + x^4])/429 + (x^5*(71 + 33*x^2)*(3 + 5*x^2 + x^
4)^(3/2))/143 - (176723*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2
)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[
(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(4290*Sqrt[3 + 5*x^2 + x^4]) +
 (2105*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + S
qrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13]
)/6]*x], (-13 + 5*Sqrt[13])/6])/(143*Sqrt[3 + 5*x^2 + x^4])
```


Rule 1273

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*
p + 3)), x] + Dist[(2*p)/(c*(4*p + m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
```

```
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x^4 \left(2+3x^2\right) \left(3+5x^2+x^4\right)^{3/2} dx = \frac{1}{143} x^5 \left(71+33x^2\right) \left(3+5x^2+x^4\right)^{3/2} + \frac{3}{143} \int x^4 \left(-69-272x^2\right) \sqrt{3+5x^2+x^4}$$

$$= -\frac{1}{429} x^5 \left(283+272x^2\right) \sqrt{3+5x^2+x^4} + \frac{1}{143} x^5 \left(71+33x^2\right) \left(3+5x^2+x^4\right)^{3/2} +$$

$$= \frac{1251}{715} x^3 \sqrt{3+5x^2+x^4} - \frac{1}{429} x^5 \left(283+272x^2\right) \sqrt{3+5x^2+x^4} + \frac{1}{143} x^5 \left(71+33\right.$$

$$= -\frac{4210}{429} x \sqrt{3+5x^2+x^4} + \frac{1251}{715} x^3 \sqrt{3+5x^2+x^4} - \frac{1}{429} x^5 \left(283+272x^2\right) \sqrt{3+5}$$

$$= -\frac{4210}{429} x \sqrt{3+5x^2+x^4} + \frac{1251}{715} x^3 \sqrt{3+5x^2+x^4} - \frac{1}{429} x^5 \left(283+272x^2\right) \sqrt{3+5}$$

$$= \frac{176723 x \left(5+\sqrt{13}+2x^2\right)}{4290 \sqrt{3+5x^2+x^4}} - \frac{4210}{429} x \sqrt{3+5x^2+x^4} + \frac{1251}{715} x^3 \sqrt{3+5x^2+x^4} - \frac{}{4}$$

Mathematica [C] time = 0.317198, size = 249, normalized size = 0.7

$$-i\sqrt{2} \left(176723\sqrt{13}-757315\right) \sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}} \sqrt{2x^2+\sqrt{13}+5} \text{EllipticF}\left(i \sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}} x\right), \frac{19}{6}+\frac{5\sqrt{13}}{6}\right) + 4x \left(495x^{14}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(-63150 - 93991*x^2 + 3055*x^4 + 29003*x^6 + 39650*x^8 + 24635*x^10 +
6015*x^12 + 495*x^14) + (176723*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[
13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSin
h[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-757315 +
176723*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqr
t[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqr
t[13])/6])/(8580*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.02, size = 294, normalized size = 0.8

$$\frac{3\,x^{11}}{13}\sqrt{x^4+5\,x^2+3}+\frac{236\,x^9}{143}\sqrt{x^4+5\,x^2+3}+\frac{1090\,x^7}{429}\sqrt{x^4+5\,x^2+3}+\frac{356\,x^5}{429}\sqrt{x^4+5\,x^2+3}+\frac{1251\,x^3}{715}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 3/13*x^11*(x^4+5*x^2+3)^(1/2)+236/143*x^9*(x^4+5*x^2+3)^(1/2)+1090/429*x^7*
(x^4+5*x^2+3)^(1/2)+356/429*x^5*(x^4+5*x^2+3)^(1/2)+1251/715*x^3*(x^4+5*x^2
+3)^(1/2)-4210/429*x*(x^4+5*x^2+3)^(1/2)+25260/143/(-30+6*13^(1/2))^(1/2)*(
1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x
^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)
)-2120676/715/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(
-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1
/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+
6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+5\,x^2+3\right)^{\frac{3}{2}}\left(3\,x^2+2\right)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(3\,x^{10}+17\,x^8+19\,x^6+6\,x^4\right)\sqrt{x^4+5\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^10 + 17*x^8 + 19*x^6 + 6*x^4)*sqrt(x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \left(3x^2 + 2\right) \left(x^4 + 5x^2 + 3\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**4*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4 + 5\,x^2 + 3\right)^{\frac{3}{2}} \left(3\,x^2 + 2\right) x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4, x)
```

3.164 $\int x^2\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=331

$$-\frac{353\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{33\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}+\frac{1}{99}\left(27x^2+67\right)\left(x^4+5x^2+\right.$$

```
[Out] (-49949*x*(5 + Sqrt[13] + 2*x^2))/(3465*Sqrt[3 + 5*x^2 + x^4]) + (353*x*Sqr
t[3 + 5*x^2 + x^4])/99 - (x^3*(911 + 890*x^2)*Sqrt[3 + 5*x^2 + x^4])/1155 +
 (x^3*(67 + 27*x^2)*(3 + 5*x^2 + x^4)^(3/2))/99 + (49949*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(3465*Sqrt[3 + 5*x^2 + x^4]) - (353*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (
5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqr
t[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(33*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*
x^2 + x^4])
```

Rubi [A] time = 0.213838, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1273, 1279, 1189, 1099, 1135}

$$\frac{1}{99}\left(27x^2+67\right)\left(x^4+5x^2+3\right)^{3/2}x^3-\frac{\left(890x^2+911\right)\sqrt{x^4+5x^2+3}x^3}{1155}+\frac{353}{99}\sqrt{x^4+5x^2+3}x-\frac{49949\left(2x^2+\sqrt{13}+5\right)x}{3465\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```


```
[Out] (-49949*x*(5 + Sqrt[13] + 2*x^2))/(3465*Sqrt[3 + 5*x^2 + x^4]) + (353*x*Sqr
t[3 + 5*x^2 + x^4])/99 - (x^3*(911 + 890*x^2)*Sqrt[3 + 5*x^2 + x^4])/1155 +
 (x^3*(67 + 27*x^2)*(3 + 5*x^2 + x^4)^(3/2))/99 + (49949*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(3465*Sqrt[3 + 5*x^2 + x^4]) - (353*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (
5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqr
t[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(33*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*
x^2 + x^4])
```


Rule 1273

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2))/(c*f*(4*p + m + 1)*(m + 4*
p + 3)), x] + Dist[(2*p)/(c*(4*p + m + 1)*(m + 4*p + 3)), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int x^2\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}dx=\frac{1}{99}x^3\left(67+27x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{33}\int x^2\left(-3-178x^2\right)\sqrt{3+5x^2+x^4}\,dx$$

$$=-\frac{x^3\left(911+890x^2\right)\sqrt{3+5x^2+x^4}}{1155}+\frac{1}{99}x^3\left(67+27x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{\int \frac{x^2}{}}{}$$

$$=\frac{353}{99}x\sqrt{3+5x^2+x^4}-\frac{x^3\left(911+890x^2\right)\sqrt{3+5x^2+x^4}}{1155}+\frac{1}{99}x^3\left(67+27x^2\right)(3$$

$$=\frac{353}{99}x\sqrt{3+5x^2+x^4}-\frac{x^3\left(911+890x^2\right)\sqrt{3+5x^2+x^4}}{1155}+\frac{1}{99}x^3\left(67+27x^2\right)(3$$

$$=-\frac{49949x\left(5+\sqrt{13}+2x^2\right)}{3465\sqrt{3+5x^2+x^4}}+\frac{353}{99}x\sqrt{3+5x^2+x^4}-\frac{x^3\left(911+890x^2\right)\sqrt{3+5x^2}}{1155}$$

Mathematica [C] time = 0.308416, size = 244, normalized size = 0.74

$$i\sqrt{2}\left(49949\sqrt{13}-212680\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+2x\left(945x^{12}+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*x*(37065 + 74681*x^2 + 69535*x^4 + 84962*x^6 + 50075*x^8 + 11795*x^10 +
945*x^12) - (49949*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/
(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 +
 Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-212680 + 49949*Sqrt[13
])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]
*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(69
30*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.015, size = 277, normalized size = 0.8

$$\frac{3x^9}{11}\sqrt{x^4+5x^2+3}+\frac{202x^7}{99}\sqrt{x^4+5x^2+3}+\frac{2378x^5}{693}\sqrt{x^4+5x^2+3}+\frac{478x^3}{385}\sqrt{x^4+5x^2+3}+\frac{353x}{99}\sqrt{x^4+5x^2+3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 3/11*x^9*(x^4+5*x^2+3)^(1/2)+202/99*x^7*(x^4+5*x^2+3)^(1/2)+2378/693*x^5*(x
^4+5*x^2+3)^(1/2)+478/385*x^3*(x^4+5*x^2+3)^(1/2)+353/99*x*(x^4+5*x^2+3)^(1
/2)-706/11/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/
6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1
/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+399592/385/(-30+6*13^(1/2))^(1/2)*(1-(
-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+
3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1
/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+5x^2+3\right)^{\frac{3}{2}}\left(3x^2+2\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(3x^8+17x^6+19x^4+6x^2\right)\sqrt{x^4+5x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^8 + 17*x^6 + 19*x^4 + 6*x^2)*sqrt(x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(3x^2+2\right) \left(x^4+5x^2+3\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+5\,x^2+3\right)^{\frac{3}{2}} \left(3\,x^2+2\right) x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2, x)
```

3.165 $\int \left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=308

$$\frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{\sqrt{x^4+5x^2+3}}+\frac{1}{3}x\left(x^2+3\right)\left(x^4+5x\right.$$

```
[Out] (203*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (x*(5 + 12*x^2)
*Sqrt[3 + 5*x^2 + x^4])/15 + (x*(3 + x^2)*(3 + 5*x^2 + x^4)^(3/2))/3 - (203
*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x
^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (
-13 + 5*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4]) + (5*Sqrt[2/(3*(5 + Sqrt[1
3]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqr
t[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/
6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi [A] time = 0.152772, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1176, 1189, 1099, 1135}

$$\frac{1}{3}x\left(x^2+3\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1}{15}x\left(12x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{203x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}}+\frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```


```
[Out] (203*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (x*(5 + 12*x^2)
*Sqrt[3 + 5*x^2 + x^4])/15 + (x*(3 + x^2)*(3 + 5*x^2 + x^4)^(3/2))/3 - (203
*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x
^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (
-13 + 5*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4]) + (5*Sqrt[2/(3*(5 + Sqrt[1
3]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqr
t[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/
6])/Sqrt[3 + 5*x^2 + x^4]
```


Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2} dx = \frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{21}\int \left(63-84x^2\right)\sqrt{3+5x^2+x^4}\,dx$$
$$= -\frac{1}{15}x\left(5+12x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{315}\int \frac{3150+}{\sqrt{3+5}}$$
$$= -\frac{1}{15}x\left(5+12x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+x^4\right)^{3/2}+10\int \frac{1}{\sqrt{3+5x^2}}$$
$$= \frac{203x\left(5+\sqrt{13}+2x^2\right)}{30\sqrt{3+5x^2+x^4}}-\frac{1}{15}x\left(5+12x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+\right.$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.012, size = 260, normalized size = 0.8

$$\frac{x^7}{3}\sqrt{x^4+5x^2+3}+\frac{8x^5}{3}\sqrt{x^4+5x^2+3}+\frac{26x^3}{5}\sqrt{x^4+5x^2+3}+\frac{8x}{3}\sqrt{x^4+5x^2+3}+60\,\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 1/3*x^7*(x^4+5*x^2+3)^(1/2)+8/3*x^5*(x^4+5*x^2+3)^(1/2)+26/5*x^3*(x^4+5*x^2
+3)^(1/2)+8/3*x*(x^4+5*x^2+3)^(1/2)+60/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*
```

```
13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*
EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-2436/5/(-3
0+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2)
)*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-30+6*13^(1
/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2)
,5/6*3^(1/2)+1/6*39^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+5x^2+3\right)^{\frac{3}{2}}\left(3x^2+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(3x^6+17x^4+19x^2+6\right)\sqrt{x^4+5x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^4+5x^2+3\right)^{\frac{3}{2}}\left(3x^2+2\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2), x)
```

3.166 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=312

$$\frac{19\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(5\sqrt{13}-13)\right)}{\sqrt{x^4+5x^2+3}}-\frac{(14-3x^2)(x^4+}{7x}$$

```
[Out] (412*x*(5 + Sqrt[13] + 2*x^2))/(35*Sqrt[3 + 5*x^2 + x^4]) + (x*(655 + 129*x
^2)*Sqrt[3 + 5*x^2 + x^4])/35 - ((14 - 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(7*x
) - (206*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
 Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[1
3])/6]*x], (-13 + 5*Sqrt[13])/6])/(35*Sqrt[3 + 5*x^2 + x^4]) + (19*Sqrt[3/(
2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*
(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 +
 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi [A] time = 0.150656, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1271, 1176, 1189, 1099, 1135}

$$-\frac{(14-3x^2)(x^4+5x^2+3)^{3/2}}{7x}+\frac{1}{35}x(129x^2+655)\sqrt{x^4+5x^2+3}+\frac{412x(2x^2+\sqrt{13}+5)}{35\sqrt{x^4+5x^2+3}}+\frac{19\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2}{(5+\sqrt{13})x^2}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^2,x]
```


```
[Out] (412*x*(5 + Sqrt[13] + 2*x^2))/(35*Sqrt[3 + 5*x^2 + x^4]) + (x*(655 + 129*x
^2)*Sqrt[3 + 5*x^2 + x^4])/35 - ((14 - 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(7*x
) - (206*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
 Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[1
3])/6]*x], (-13 + 5*Sqrt[13])/6])/(35*Sqrt[3 + 5*x^2 + x^4]) + (19*Sqrt[3/(
2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*
(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 +
 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```


Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
```

```
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{x^2}\,dx = -\frac{\left(14-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{7x} - \frac{3}{7}\int \left(-88-43x^2\right)\sqrt{3+5x^2+x^4}\,dx$$

$$= \frac{1}{35}x\left(655+129x^2\right)\sqrt{3+5x^2+x^4} - \frac{\left(14-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{7x} - \frac{1}{35}\int \frac{-1995-8}{\sqrt{3+5x^2}}$$

$$= \frac{1}{35}x\left(655+129x^2\right)\sqrt{3+5x^2+x^4} - \frac{\left(14-3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{7x} + \frac{824}{35}\int \frac{x^2}{\sqrt{3+5x^2}}$$

$$= \frac{412x\left(5+\sqrt{13}+2x^2\right)}{35\sqrt{3+5x^2+x^4}} + \frac{1}{35}x\left(655+129x^2\right)\sqrt{3+5x^2+x^4} - \frac{\left(14-3x^2\right)\left(3+5x^2+\right.}{7x}$$

Mathematica [C] time = 0.307957, size = 235, normalized size = 0.75

$$\frac{-i\sqrt{2}\left(412\sqrt{13}-65\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,x\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+30x^{10}+418x^8+2130}{70x\sqrt{x^4+5x^2+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^2,x]
```

```
[Out] (-1260 + 3884*x^4 + 2130*x^6 + 418*x^8 + 30*x^10 + (412*I)*Sqrt[2]*(-5 + Sq
rt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] +
 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/
6] - I*Sqrt[2]*(-65 + 412*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sq
rt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13
])]*x], 19/6 + (5*Sqrt[13])/6])/(70*x*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.016, size = 260, normalized size = 0.8

$$\frac{3x^5}{7}\sqrt{x^4+5x^2+3}+\frac{134x^3}{35}\sqrt{x^4+5x^2+3}+10x\sqrt{x^4+5x^2+3}+342\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{\right.}}}{\sqrt{-30+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x)
```

```
[Out] 3/7*x^5*(x^4+5*x^2+3)^(1/2)+134/35*x^3*(x^4+5*x^2+3)^(1/2)+10*x*(x^4+5*x^2+
3)^(1/2)+342/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-
5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^
(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-29664/35/(-30+6*13^(1/2))^(1/2)*(1-(
-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+
3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1
/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2
)))-6*(x^4+5*x^2+3)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}\left(3x^2+2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(3x^6+17x^4+19x^2+6\right)\sqrt{x^4+5x^2+3}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**2,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+5x^2+3)^{\frac{3}{2}}(3x^2+2)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2, x)
```

3.167 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4} \, dx$

Optimal. Leaf size=314

$$\frac{65\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{\left(10-9x^2\right)\left(x^4+\right.}{15x^3}$$

```
[Out] (949*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (13*(24 - 5*x^2
)*Sqrt[3 + 5*x^2 + x^4])/(15*x) - ((10 - 9*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(1
5*x^3) - (949*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5
+ Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[
13])/6]*x], (-13 + 5*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4]) + (65*Sqrt[2/
(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi [A] time = 0.164134, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1271, 1189, 1099, 1135}

$$-\frac{\left(10-9x^2\right)\left(x^4+5x^2+3\right)^{3/2}}{15x^3}-\frac{13\left(24-5x^2\right)\sqrt{x^4+5x^2+3}}{15x}+\frac{949x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}}+\frac{65\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^4,x]
```


```
[Out] (949*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (13*(24 - 5*x^2
)*Sqrt[3 + 5*x^2 + x^4])/(15*x) - ((10 - 9*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(1
5*x^3) - (949*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5
+ Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[
13])/6]*x], (-13 + 5*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4]) + (65*Sqrt[2/
(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```


Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
```

```
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = -\frac{(10-9x^2)(3+5x^2+x^4)^{3/2}}{15x^3} - \frac{1}{5}\int \frac{(-104-65x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx$$

$$= -\frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x} - \frac{(10-9x^2)(3+5x^2+x^4)^{3/2}}{15x^3} + \frac{1}{15}\int \frac{1950+949x^2}{\sqrt{3+5x^2+x^4}}$$

$$= -\frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x} - \frac{(10-9x^2)(3+5x^2+x^4)^{3/2}}{15x^3} + \frac{949}{15}\int \frac{x^2}{\sqrt{3+5x^2+}}$$

$$= \frac{949x(5+\sqrt{13}+2x^2)}{30\sqrt{3+5x^2+x^4}} - \frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x} - \frac{(10-9x^2)(3+5x^2+x^4)^{3/2}}{15x^3}$$

Mathematica [C] time = 0.3355, size = 247, normalized size = 0.79

$$\frac{-13i\sqrt{2}(73\sqrt{13}-65)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,x^3\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+4(9x^{10}+145x^8+1}{60x^3\sqrt{x^4+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^4,x]
```

```
[Out] (4*(-90 - 1155*x^2 - 1405*x^4 + 192*x^6 + 145*x^8 + 9*x^10) + (949*I)*Sqrt[
2]*(-5 + Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5
 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 +
(5*Sqrt[13])/6] - (13*I)*Sqrt[2]*(-65 + 73*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13
] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[
Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(60*x^3*Sqrt[3 + 5*x^2 +
 x^4])
```

Maple [A] time = 0.016, size = 260, normalized size = 0.8

$$-\frac{67}{3x}\sqrt{x^4+5x^2+3}+\frac{3x^3}{5}\sqrt{x^4+5x^2+3}+\frac{20x}{3}\sqrt{x^4+5x^2+3}+780\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{1}\right.}}{\sqrt{-30+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x)
```

```
[Out] -67/3*(x^4+5*x^2+3)^(1/2)/x+3/5*x^3*(x^4+5*x^2+3)^(1/2)+20/3*x*(x^4+5*x^2+3
)^(1/2)+780/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5
/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(
1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-11388/5/(-30+6*13^(1/2))^(1/2)*(1-(-5
/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)
^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6
*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
)-2*(x^4+5*x^2+3)^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}\left(3x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3x^6+17x^4+19x^2+6\right)\sqrt{x^4+5x^2+3}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**4,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}\left(3\,x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4, x)
```

3.168 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=331

$$\frac{103\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}-\frac{\left(2-5x^2\right)\left(x^4+5x^2+3\right)^{3/}}{5x^5}$$

```
[Out] (361*x*(5 + Sqrt[13] + 2*x^2))/(15*Sqrt[3 + 5*x^2 + x^4]) - (722*Sqrt[3 + 5
*x^2 + x^4])/(15*x) - ((40 - 87*x^2)*Sqrt[3 + 5*x^2 + x^4])/(5*x^3) - ((2 -
 5*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(5*x^5) - (361*Sqrt[(5 + Sqrt[13])/6]*Sqrt
[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2
)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(15*Sq
rt[3 + 5*x^2 + x^4]) + (103*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13
])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x
], (-13 + 5*Sqrt[13])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.202751, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1271, 1281, 1189, 1099, 1135}

$$-\frac{\left(2-5x^2\right)\left(x^4+5x^2+3\right)^{3/2}}{5x^5}-\frac{\left(40-87x^2\right)\sqrt{x^4+5x^2+3}}{5x^3}-\frac{722\sqrt{x^4+5x^2+3}}{15x}+\frac{361x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}+\frac{103\sqrt{\frac{(5-}{(5+}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^6,x]
```


```
[Out] (361*x*(5 + Sqrt[13] + 2*x^2))/(15*Sqrt[3 + 5*x^2 + x^4]) - (722*Sqrt[3 + 5
*x^2 + x^4])/(15*x) - ((40 - 87*x^2)*Sqrt[3 + 5*x^2 + x^4])/(5*x^3) - ((2 -
 5*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(5*x^5) - (361*Sqrt[(5 + Sqrt[13])/6]*Sqrt
[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2
)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(15*Sq
rt[3 + 5*x^2 + x^4]) + (103*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13
])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x
], (-13 + 5*Sqrt[13])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```


Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
```

```
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{x^6}\,dx = -\frac{\left(2-5x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{5x^5} - \frac{1}{5}\int \frac{\left(-120-87x^2\right)\sqrt{3+5x^2+x^4}}{x^4}\,dx$$

$$= -\frac{\left(40-87x^2\right)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{\left(2-5x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{5x^5} + \frac{1}{15}\int \frac{2166+1545x^2}{x^2\sqrt{3+5x^2+x^4}}$$

$$= -\frac{722\sqrt{3+5x^2+x^4}}{15x} - \frac{\left(40-87x^2\right)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{\left(2-5x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{5x^5} - \frac{1}{45}$$

$$= -\frac{722\sqrt{3+5x^2+x^4}}{15x} - \frac{\left(40-87x^2\right)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{\left(2-5x^2\right)\left(3+5x^2+x^4\right)^{3/2}}{5x^5} + \frac{7}{1}$$

$$= \frac{361x\left(5+\sqrt{13}+2x^2\right)}{15\sqrt{3+5x^2+x^4}} - \frac{722\sqrt{3+5x^2+x^4}}{15x} - \frac{\left(40-87x^2\right)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{(2-5x^2}{}$$

Mathematica [C] time = 0.316542, size = 244, normalized size = 0.74

$$\frac{-i\sqrt{2}\left(361\sqrt{13}-260\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,x^5\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+30x^{10}-634x^8-40}{30x^5\sqrt{x^4+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^6,x]
```

```
[Out] (-108 - 810*x^2 - 3438*x^4 - 4040*x^6 - 634*x^8 + 30*x^10 + (361*I)*Sqrt[2]
*(-5 + Sqrt[13])*x^5*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 +
 Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5
*Sqrt[13])/6] - I*Sqrt[2]*(-260 + 361*Sqrt[13])*x^5*Sqrt[(-5 + Sqrt[13] - 2
*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[
2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(30*x^5*Sqrt[3 + 5*x^2 + x^4]
)
```

Maple [A] time = 0.02, size = 259, normalized size = 0.8

$$-\frac{6}{5x^5}\sqrt{x^4+5x^2+3}-7\frac{\sqrt{x^4+5x^2+3}}{x^3}-\frac{392}{15x}\sqrt{x^4+5x^2+3}+618\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right.}}{\sqrt{-30+6\sqrt{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x)
```

```
[Out] -6/5/x^5*(x^4+5*x^2+3)^(1/2)-7*(x^4+5*x^2+3)^(1/2)/x^3-392/15*(x^4+5*x^2+3)
^(1/2)/x+618/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-
5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^
(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-8664/5/(-30+6*13^(1/2))^(1/2)*(1-(-5
/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)
^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6
*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
)+x*(x^4+5*x^2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}\left(3x^2+2\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(3x^6+17x^4+19x^2+6\right)\sqrt{x^4+5x^2+3}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)(x^4+5x^2+3)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**6,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^4+5x^2+3)^{\frac{3}{2}}(3x^2+2)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6, x)
```

3.169 $\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{a+bx^2+cx^4}\left(-16aBc-2cx^2(5bB-6Ac)-18Abc+15b^2B\right)}{48c^3}-\frac{\left(8aAc^2-12abBc-6Ab^2c+5b^3B\right)\tanh^{-1}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+b}}\right.}{32c^{7/2}}$$

```
[Out] (B*x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2*B - 18*A*b*c - 16*a*B*c -
2*c*(5*b*B - 6*A*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(48*c^3) - ((5*b^3*B - 6*
A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4])])/(32*c^(7/2))
```

Rubi [A] time = 0.202879, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1251, 832, 779, 621, 206}

$$\frac{\sqrt{a+bx^2+cx^4}\left(-16aBc-2cx^2(5bB-6Ac)-18Abc+15b^2B\right)}{48c^3}-\frac{\left(8aAc^2-12abBc-6Ab^2c+5b^3B\right)\tanh^{-1}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+b}}\right.}{32c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2*B - 18*A*b*c - 16*a*B*c -
2*c*(5*b*B - 6*A*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(48*c^3) - ((5*b^3*B - 6*
A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4])])/(32*c^(7/2))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
```

```
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\text{Subst}\left(\int \frac{x\left(-2aB-\frac{1}{2}(5bB-6Ac)x\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{6c}$$

$$= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{\left(5b^3B-6\right.}{}$$

$$= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{\left(5b^3B-6\right.}{}$$

$$= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\left(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2\right)\sqrt{a+bx^2+cx^4}}{48c^3} - \frac{\left(5b^3B-6\right.}{}$$

Mathematica [A] time = 0.10728, size = 139, normalized size = 0.91

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(4c\left(-4aB+3Acx^2+2Bcx^4\right)-2bc\left(9A+5Bx^2\right)+15b^2B\right)-3\left(8aAc^2-12abBc-6Ab^2c+5b^3B\right)\tan}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^2*B - 2*b*c*(9*A + 5*B*x^2) + 4*c*
(-4*a*B + 3*A*c*x^2 + 2*B*c*x^4)) - 3*(5*b^3*B - 6*A*b^2*c - 12*a*b*B*c + 8
*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(96*c
^(7/2))
```

Maple [B] time = 0.027, size = 286, normalized size = 1.9

$$\frac{Bx^4}{6\,c}\sqrt{cx^4+bx^2+a}-\frac{5\,Bbx^2}{24\,c^2}\sqrt{cx^4+bx^2+a}+\frac{5\,b^2B}{16\,c^3}\sqrt{cx^4+bx^2+a}-\frac{5\,b^3B}{32}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{7}{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*B*x^4*(c*x^4+b*x^2+a)^(1/2)/c-5/24*B*b/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)+5/
16*B*b^2/c^3*(c*x^4+b*x^2+a)^(1/2)-5/32*B*b^3/c^(7/2)*ln((1/2*b+c*x^2)/c^(1
/2)+(c*x^4+b*x^2+a)^(1/2))+3/8*B*b/c^(5/2)*a*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2))-1/3*B*a/c^2*(c*x^4+b*x^2+a)^(1/2)+1/4*A*x^2/c*(c*x^4+b*x^
2+a)^(1/2)-3/8*A*b/c^2*(c*x^4+b*x^2+a)^(1/2)+3/16*A*b^2/c^(5/2)*ln((1/2*b+c
*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*A*a/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/
2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55295, size = 725, normalized size = 4.74

$$\left[\frac{3\left(5\,Bb^3+8\,Aac^2-6\left(2\,Bab+Ab^2\right)c\right)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)+4\left(8\,B\right.}{192\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(5*B*b^3 + 8*A*a*c^2 - 6*(2*B*a*b + A*b^2)*c)*sqrt(c)*log(-8*c^2*
x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4
*a*c) + 4*(8*B*c^3*x^4 + 15*B*b^2*c - 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c^2
- 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/96*(3*(5*B*b^3 + 8*A*a*c^2
- 6*(2*B*a*b + A*b^2)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x
^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*B*c^3*x^4 + 15*B*b^2*c -
 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2 +
a))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.19961, size = 203, normalized size = 1.33

$$\frac{1}{48}\sqrt{cx^4+bx^2+a}\left(2\left(\frac{4\,Bx^2}{c}-\frac{5\,Bbc^2-6\,Ac^3}{c^4}\right)x^2+\frac{15\,Bb^2c-16\,Bac^2-18\,Abc^2}{c^4}\right)+\frac{\left(5\,Bb^3c-12\,Babc^2-6\,Ab^2c^2+8\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*(4*B*x^2/c - (5*B*b*c^2 - 6*A*c^3)/c^4)*x^2
 + (15*B*b^2*c - 16*B*a*c^2 - 18*A*b*c^2)/c^4) + 1/32*(5*B*b^3*c - 12*B*a*b
*c^2 - 6*A*b^2*c^2 + 8*A*a*c^3)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^
2 + a))*sqrt(c) - b))/c^(9/2)
```

3.170 $\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=100

$$\frac{(-4aBc-4Abc+3b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{\sqrt{a+bx^2+cx^4}\,(-4Ac+3bB-2Bcx^2)}{8c^2}$$

```
[Out] -((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + ((3*b^2*B
- 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(5/2))
```

Rubi [A] time = 0.0927491, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1251, 779, 621, 206}

$$\frac{(-4aBc-4Abc+3b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{\sqrt{a+bx^2+cx^4}\,(-4Ac+3bB-2Bcx^2)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + ((3*b^2*B
- 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(5/2))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{\left(3bB-4Ac-2Bcx^2\right)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{\left(3b^2B-4Abc-4aBc\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{16c^2}$$

$$= -\frac{\left(3bB-4Ac-2Bcx^2\right)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{\left(3b^2B-4Abc-4aBc\right)\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{8c^2}$$

$$= -\frac{\left(3bB-4Ac-2Bcx^2\right)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{\left(3b^2B-4Abc-4aBc\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}$$

Mathematica [A] time = 0.0476248, size = 101, normalized size = 1.01

$$\frac{\left(-4aBc-4Abc+3b^2B\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)+2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(4Ac-3bB+2Bcx^2\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*(-3*b*B + 4*A*c + 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (3*b^2*B
- 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(5/2))
```

Maple [B] time = 0.015, size = 176, normalized size = 1.8

$$\frac{Bx^2}{4c}\sqrt{cx^4+bx^2+a}-\frac{3\,bB}{8\,c^2}\sqrt{cx^4+bx^2+a}+\frac{3\,b^2B}{16}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}-\frac{aB}{4}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*x^2/c*(c*x^4+b*x^2+a)^(1/2)-3/8*B*b/c^2*(c*x^4+b*x^2+a)^(1/2)+3/16*B*
b^2/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*B*a/c^(3/2)
*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/2*A/c*(c*x^4+b*x^2+a)^(1
/2)-1/4*A*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48283, size = 545, normalized size = 5.45

$$\left[-\frac{\left(3\,Bb^2-4\,(Ba+Ab)c\right)\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)-4\left(2\,Bc^2x^2-3\,Bbc\right.}{32\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*B*b^2 - 4*(B*a + A*b)*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^
2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(2*B*c^2*x
^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^3, -1/16*((3*B*b^2 - 4*(
B*a + A*b)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt
(c*x^4 + b*x^2 + a))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.15289, size = 132, normalized size = 1.32

$$\frac{1}{8}\sqrt{cx^4+bx^2+a}\left(\frac{2\,Bx^2}{c}-\frac{3\,Bb-4\,Ac}{c^2}\right)-\frac{\left(3\,Bb^2-4\,Bac-4\,Abc\right)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*B*x^2/c - (3*B*b - 4*A*c)/c^2) - 1/16*(3*B*b
^2 - 4*B*a*c - 4*A*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*
sqrt(c) - b))/c^(5/2)
```

3.171 $\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=76

$$\frac{B\sqrt{a+bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

```
[Out] (B*Sqrt[a + b*x^2 + c*x^4])/(2*c) - ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rubi [A] time = 0.0639266, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1247, 640, 621, 206}

$$\frac{B\sqrt{a+bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[a + b*x^2 + c*x^4])/(2*c) - ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$
$$= \frac{B\sqrt{a+bx^2+cx^4}}{2c} + \frac{(-bB+2Ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4c}$$
$$= \frac{B\sqrt{a+bx^2+cx^4}}{2c} + \frac{(-bB+2Ac)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2c}$$
$$= \frac{B\sqrt{a+bx^2+cx^4}}{2c} - \frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Mathematica [A] time = 0.0252939, size = 78, normalized size = 1.03

$$\frac{1}{2}\left(\frac{(2Ac-bB)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}} + \frac{B\sqrt{a+bx^2+cx^4}}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((B*Sqrt[a + b*x^2 + c*x^4])/c + ((-(b*B) + 2*A*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*c^(3/2)))/2
```

Maple [A] time = 0.01, size = 93, normalized size = 1.2

$$\frac{B}{2c}\sqrt{cx^4+bx^2+a} - \frac{bB}{4}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}} + \frac{A}{2}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*B*(c*x^4+b*x^2+a)^(1/2)/c-1/4*B*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))+1/2*A*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^
(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.50833, size = 421, normalized size = 5.54

$$\left[\frac{4\sqrt{cx^4+bx^2+a}Bc-(Bb-2Ac)\sqrt{c}\log\left(-8c^2x^4-8bcx^2-b^2-4\sqrt{cx^4+bx^2+a}\left(2cx^2+b\right)\sqrt{c}-4ac\right)}{8c^2}, \frac{2\sqrt{cx^4+bx^2+}\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(c*x^4 + b*x^2 + a)*B*c - (B*b - 2*A*c)*sqrt(c)*log(-8*c^2*x^4
- 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c
))/c^2, 1/4*(2*sqrt(c*x^4 + b*x^2 + a)*B*c + (B*b - 2*A*c)*sqrt(-c)*arctan(
1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c
)))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.17761, size = 93, normalized size = 1.22

$$\frac{\sqrt{cx^4+bx^2+a}B}{2c}+\frac{(Bb-2Ac)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{4c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4 + b*x^2 + a)*B/c + 1/4*(B*b - 2*A*c)*log(abs(-2*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2)
```

3.172 $\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=90

$$\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -(A*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a])
 + (B*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c
])
```

Rubi [A] time = 0.0939564, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1251, 843, 621, 206, 724}

$$\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a])
 + (B*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c
])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}A\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)+\frac{1}{2}B\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\ &= -\left(A\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)\right)+B\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)\\ &= -\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}+\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.0290851, size = 89, normalized size = 0.99

$$\frac{1}{2}\left(\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}-\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-((A*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a])
+ (B*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c])/2
```

Maple [A] time = 0.013, size = 76, normalized size = 0.8

$$\frac{B}{2}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}-\frac{A}{2}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*B*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*A/a^(1/2)
*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04862, size = 1211, normalized size = 13.46

$$\left[\frac{Ba\sqrt{c}\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)+A\sqrt{ac}\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(b\right.}{x^4}\right.}{4\,ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 +
 a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + A*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4 +
8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/
(a*c), -1/4*(2*B*a*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - A*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4
 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4
))/(a*c), 1/4*(2*A*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2
*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + B*a*sqrt(c)*log(-8*c^2*x^4 - 8*b*
c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(a*
c), 1/2*(A*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt
(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - B*a*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^
2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x), x)
```

3.173 $\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=80

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}$$

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(2*a*x^2) + ((A*b - 2*a*B)*ArcTanh[(2*a + b*x^
2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rubi [A] time = 0.0828435, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1251, 806, 724, 206}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```


```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(2*a*x^2) + ((A*b - 2*a*B)*ArcTanh[(2*a + b*x^
2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} - \frac{(Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4a}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} + \frac{(Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2a}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} + \frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

Mathematica [A] time = 0.0364715, size = 82, normalized size = 1.02

$$\frac{1}{2}\left(\frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}} - \frac{A\sqrt{a+bx^2+cx^4}}{ax^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-((A*Sqrt[a + b*x^2 + c*x^4])/(a*x^2)) + ((A*b - 2*a*B)*ArcTanh[(2*a + b*x
^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)))/2
```

Maple [A] time = 0.016, size = 104, normalized size = 1.3

$$-\frac{B}{2}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}} - \frac{A}{2x^2a}\sqrt{cx^4+bx^2+a} + \frac{Ab}{4}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*B/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-1/2*A*(c
*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*A*b/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*
x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77008, size = 460, normalized size = 5.75

$$\left[-\frac{(2\,Ba-Ab)\sqrt{a}x^2\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)+4\,\sqrt{cx^4+bx^2+a}Aa}{8\,a^2x^2},\ \frac{(2\,Ba-Ab)\sqrt{-a}x^2\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*B*a - A*b)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sq
rt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 +
b*x^2 + a)*A*a)/(a^2*x^2), 1/4*((2*B*a - A*b)*sqrt(-a)*x^2*arctan(1/2*sqrt(
c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*sq
rt(c*x^4 + b*x^2 + a)*A*a)/(a^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**3*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^3), x)
```

3.174 $\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=124

$$-\frac{\left(-4aAc-4abB+3Ab^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}$$

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(4*a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^2 +
c*x^4])/(8*a^2*x^2) - ((3*A*b^2 - 4*a*b*B - 4*a*A*c)*ArcTanh[(2*a + b*x^2)/
(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

Rubi [A] time = 0.145204, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1251, 834, 806, 724, 206}

$$-\frac{\left(-4aAc-4abB+3Ab^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(4*a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^2 +
c*x^4])/(8*a^2*x^2) - ((3*A*b^2 - 4*a*b*B - 4*a*A*c)*ArcTanh[(2*a + b*x^2)/
(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3Ab-4aB)+Acx}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4a}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} + \frac{\left(3Ab^2-4abB-4aAc\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+}}\right.}{16a^2}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{\left(3Ab^2-4abB-4aAc\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,d\right.}{8a^2}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{\left(3Ab^2-4abB-4aAc\right)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx}}\right.}{16a^{5/2}}$$

Mathematica [A] time = 0.076057, size = 107, normalized size = 0.86

$$\frac{\left(4aAc+4abB-3Ab^2\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{\sqrt{a+bx^2+cx^4}\left(3Abx^2-2a\left(A+2Bx^2\right)\right)}{8a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(3*A*b*x^2 - 2*a*(A + 2*B*x^2)))/(8*a^2*x^4) + ((-
3*A*b^2 + 4*a*b*B + 4*a*A*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^
2 + c*x^4])])/(16*a^(5/2))
```

Maple [A] time = 0.018, size = 194, normalized size = 1.6

$$-\frac{A}{4\,ax^4}\sqrt{cx^4+bx^2+a}+\frac{3\,Ab}{8\,a^2x^2}\sqrt{cx^4+bx^2+a}-\frac{3\,Ab^2}{16}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{Ac}{4}\ln\left(\frac{1}{x^2}\left(2\,a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*A*(c*x^4+b*x^2+a)^(1/2)/a/x^4+3/8*A*b/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)-3/
16*A*b^2/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/4*A*
```

```
c/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-1/2*B/a/x^2*(
c*x^4+b*x^2+a)^(1/2)+1/4*B*b/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a
)^(1/2))/x^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.37909, size = 593, normalized size = 4.78

$$\left[\frac{\left(4\,Bab-3\,Ab^2+4\,Aac\right)\sqrt{a}x^4\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2+a}\left(2\,Aa^2+\left(4\,Ba^2-\right.\right.}{32\,a^3x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 +
8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) -
 4*sqrt(c*x^4 + b*x^2 + a)*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2))/(a^3*x^4),
-1/16*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b
*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*sqrt(c*x^4
+ b*x^2 + a)*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2))/(a^3*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**5*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2+A}{\sqrt{cx^4+bx^2+a}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^5), x)
```

3.175 $\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=177

$$-\frac{\sqrt{a+bx^2+cx^4}\left(-16aAc-18abB+15Ab^2\right)}{48a^3x^2}+\frac{\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{(5Ab-$$

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(6*a*x^6) + ((5*A*b - 6*a*B)*Sqrt[a + b*x^2 +
c*x^4])/(24*a^2*x^4) - ((15*A*b^2 - 18*a*b*B - 16*a*A*c)*Sqrt[a + b*x^2 + c
*x^4])/(48*a^3*x^2) + ((5*A*b^3 - 6*a*b^2*B - 12*a*A*b*c + 8*a^2*B*c)*ArcTa
nh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(7/2))
```

Rubi [A] time = 0.238614, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1251, 834, 806, 724, 206}

$$-\frac{\sqrt{a+bx^2+cx^4}\left(-16aAc-18abB+15Ab^2\right)}{48a^3x^2}+\frac{\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{(5Ab-$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```


```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(6*a*x^6) + ((5*A*b - 6*a*B)*Sqrt[a + b*x^2 +
c*x^4])/(24*a^2*x^4) - ((15*A*b^2 - 18*a*b*B - 16*a*A*c)*Sqrt[a + b*x^2 + c
*x^4])/(48*a^3*x^2) + ((5*A*b^3 - 6*a*b^2*B - 12*a*A*b*c + 8*a^2*B*c)*ArcTa
nh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(7/2))
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{A+Bx}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5Ab-6aB)+2Acx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{6a}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}\left(15Ab^2-18abB-16aAc\right)+\frac{1}{2}(5Ab-6aB)}{x^2\sqrt{a+bx+cx^2}}\right.}{12a^2}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15Ab^2-18abB-16aAc\right)\sqrt{a+bx^2+cx}}{48a^3x^2}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15Ab^2-18abB-16aAc\right)\sqrt{a+bx^2+cx}}{48a^3x^2}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{\left(15Ab^2-18abB-16aAc\right)\sqrt{a+bx^2+cx}}{48a^3x^2}$$

Mathematica [A] time = 0.115826, size = 148, normalized size = 0.84

$$\frac{\sqrt{a+bx^2+cx^4}\left(-4a^2\left(2A+3Bx^2\right)+2a\left(5Abx^2+8Acx^4+9bBx^4\right)-15Ab^2x^4\right)}{48a^3x^6} + \frac{\left(8a^2Bc-12aAbc-6ab^2B+5Ab^3\right)\text{ta}}{32a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-15*A*b^2*x^4 - 4*a^2*(2*A + 3*B*x^2) + 2*a*(5*A*
b*x^2 + 9*b*B*x^4 + 8*A*c*x^4)))/(48*a^3*x^6) + ((5*A*b^3 - 6*a*b^2*B - 12*
a*A*b*c + 8*a^2*B*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^
4])])/(32*a^(7/2))
```

Maple [A] time = 0.019, size = 311, normalized size = 1.8

$$-\frac{B}{4\,a{x}^{4}}\sqrt{c{x}^{4}+b{x}^{2}+a}+\frac{3\,bB}{8\,{a}^{2}{x}^{2}}\sqrt{c{x}^{4}+b{x}^{2}+a}-\frac{3\,{b}^{2}B}{16}\ln\left(\frac{1}{{x}^{2}}\left(2\,a+b{x}^{2}+2\,\sqrt{a}\sqrt{c{x}^{4}+b{x}^{2}+a}\right)\right){a}^{-\frac{5}{2}}+\frac{Bc}{4}\ln\left(\frac{1}{{x}^{2}}\left(2\,a+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*B/a/x^4*(c*x^4+b*x^2+a)^(1/2)+3/8*B*b/a^2/x^2*(c*x^4+b*x^2+a)^(1/2)-3/
16*B*b^2/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/4*B*
c/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-1/6*A*(c*x^4+
b*x^2+a)^(1/2)/a/x^6+5/24*A*b/a^2/x^4*(c*x^4+b*x^2+a)^(1/2)-5/16*A*b^2/a^3/
x^2*(c*x^4+b*x^2+a)^(1/2)+5/32*A*b^3/a^(7/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4
+b*x^2+a)^(1/2))/x^2)-3/8*A*b/a^(5/2)*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^
2+a)^(1/2))/x^2)+1/3*A/a^2*c/x^2*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.14775, size = 779, normalized size = 4.4

$$\left[\frac{3\left(6\,Bab^2-5\,Ab^3-4\left(2\,Ba^2-3\,Aab\right)c\right)\sqrt{a}x^6\log\left(-\frac{\left(b^2+4\,ac\right)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\left(bx^2+2\,a\right)\sqrt{a}+8\,a^2}{x^4}\right)+4\left(\left(18\,Ba^2b-15\,Aa\right.\right.}{192\,a^4x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(a)*x^6*log(-
((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sq
rt(a) + 8*a^2)/x^4) + 4*((18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^4 - 8*A*a
^3 - 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6), 1/96*
(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(-a)*x^6*arctan(1/2*
sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) +
 2*((18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^4 - 8*A*a^3 - 2*(6*B*a^3 - 5*A
*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{x^7\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**7*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^7} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^7), x)
```

3.176 $\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=403

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}\sqrt{c}(4bB-5Ac)-9aBc-10Abc+8b^2B\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{11/4}\sqrt{a+bx^2+cx^4}}+$$

```
[Out] -((4*b*B - 5*A*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (B*x^3*Sqrt[a + b*x
^2 + c*x^4])/(5*c) + ((8*b^2*B - 10*A*b*c - 9*a*B*c)*x*Sqrt[a + b*x^2 + c*x
^4])/(15*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(8*b^2*B - 10*A*b*c -
9*a*B*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(15*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^2*B - 10*A*b*c
- 9*a*B*c + Sqrt[a]*Sqrt[c]*(4*b*B - 5*A*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(11/4)*Sqrt[a + b*x^2 + c*x
^4])
```

Rubi [A] time = 0.283668, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1279, 1197, 1103, 1195}

$$\frac{x\sqrt{a+bx^2+cx^4}\left(-9aBc-10Abc+8b^2B\right)}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}\sqrt{c}(4bB-5Ac)-9aBc-10Abc+8b^2\right.}{30c^{11/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] -((4*b*B - 5*A*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^2) + (B*x^3*Sqrt[a + b*x
^2 + c*x^4])/(5*c) + ((8*b^2*B - 10*A*b*c - 9*a*B*c)*x*Sqrt[a + b*x^2 + c*x
^4])/(15*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(8*b^2*B - 10*A*b*c -
9*a*B*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(15*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^2*B - 10*A*b*c
- 9*a*B*c + Sqrt[a]*Sqrt[c]*(4*b*B - 5*A*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(11/4)*Sqrt[a + b*x^2 + c*x
^4])
```


Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} - \frac{\int \frac{x^2\left(3aB+(4bB-5Ac)x^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx}{5c}$$

$$= -\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{\int \frac{a(4bB-5Ac)+\left(8b^2B-10Abc-9aBc\right)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{15c^2}$$

$$= -\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} - \frac{\left(\sqrt{a}\left(8b^2B-10Abc-9aBc\right)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+c}}$$

$$= -\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{\left(8b^2B-10Abc-9aBc\right)x\sqrt{a+bx^2+cx^{}}}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Mathematica [C] time = 2.16367, size = 532, normalized size = 1.32

$$-i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(2b^2\left(4B\sqrt{b^2-4ac}+5Ac\right)+bc\left(17aB-10A\sqrt{b^2-4ac}\right)-ac\left(9B\sqrt{b^2-4ac}+10A\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b*B + 5*A*c + 3*B*c*x^2)*(a + b*
x^2 + c*x^4) + I*(8*b^2*B - 10*A*b*c - 9*a*B*c)*(-b + Sqrt[b^2 - 4*a*c])*Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2
*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])] - I*(-8*b^3*B + b*c*(17*a*B - 10*A*Sqrt[b^2 - 4*a*c]) + 2
*b^2*(5*A*c + 4*B*Sqrt[b^2 - 4*a*c]) - a*c*(10*A*c + 9*B*Sqrt[b^2 - 4*a*c])
```

```
)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b
 - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcS
inh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
- Sqrt[b^2 - 4*a*c])])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x
^2 + c*x^4])
```

Maple [B] time = 0.048, size = 815, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] B*(1/5/c*x^3*(c*x^4+b*x^2+a)^(1/2)-4/15*b/c^2*x*(c*x^4+b*x^2+a)^(1/2)+1/15*
b/c^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-3/5/c*a+8/15*b^2/c^2)*a*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(
4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^
2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+A*
(1/3/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/c*
a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(
b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^4}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^6 + Ax^4}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^6 + A*x^4)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \left(A + Bx^2\right)}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^4}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

3.177 $\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=336

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B\sqrt{c}-3Ac+2bB\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}(2b\ldots}{3c^{3/2}\left(\sqrt{a}+\sqrt{\ldots}\right.}$$

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - ((2*b*B - 3*A*c)*x*Sqrt[a + b*x^2 + c
*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(2*b*B - 3*A*c)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/
4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(2*b*B + Sqrt[a]*B*Sqrt[c] - 3*A*c)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*
c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.149233, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1279, 1197, 1103, 1195}

$$-\frac{x\sqrt{a+bx^2+cx^4}(2bB-3Ac)}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}B\sqrt{c}-3Ac+2bB\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\ldots\right.}{6c^{7/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - ((2*b*B - 3*A*c)*x*Sqrt[a + b*x^2 + c
*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(2*b*B - 3*A*c)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/
4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(2*b*B + Sqrt[a]*B*Sqrt[c] - 3*A*c)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*
c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```


Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
```

```
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx^2\right)}{\sqrt{a+bx^2+cx^4}}dx = \frac{Bx\sqrt{a+bx^2+cx^4}}{3c} - \frac{\int \frac{aB+(2bB-3Ac)x^2}{\sqrt{a+bx^2+cx^4}}dx}{3c}$$

$$= \frac{Bx\sqrt{a+bx^2+cx^4}}{3c} + \frac{\left(\sqrt{a}(2bB-3Ac)\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}dx}{3c^{3/2}} - \frac{\left(\sqrt{a}\left(2bB+\sqrt{a}B\sqrt{c}-3Ac\right)\right)\int \frac{1}{\sqrt{a+bx^2+}}$$

$$= \frac{Bx\sqrt{a+bx^2+cx^4}}{3c} - \frac{(2bB-3Ac)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{\sqrt[4]{a}(2bB-3Ac)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\,I}{3c^{7/4}\sqrt{a+bx^2+}}$$

Mathematica [C] time = 1.38167, size = 479, normalized size = 1.43

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(-3Ac\sqrt{b^2-4ac}+2bB\sqrt{b^2-4ac}+2aBc+3Abc-2b^2B\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*B*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) - I*(2*b*B - 3
*A*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^
2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x]
, (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-2*b^2*B + 3*A*b*c
+ 2*a*B*c + 2*b*B*Sqrt[b^2 - 4*a*c] - 3*A*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2
- 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sq
rt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4
*a*c])])/(12*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.01, size = 607, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] B*(1/3/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/
c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/
c)^(1/2))))-1/2*A*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elli
pticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*
c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(Bx^2 + A)x^2}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^4 + Ax^2}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x^2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \left(A + Bx^2\right)}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(Bx^2 + A\right)x^2}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

3.178 $\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=283

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(\right.}{c^{3/4}\sqrt{a+bx}}$$

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
B*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(B + (A*Sqrt[c])/Sqrt[a])*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.0828396, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1197, 1103, 1195}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\right.}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
B*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(B + (A*Sqrt[c])/Sqrt[a])*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
```

```
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \left(A+\frac{\sqrt{a}B}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx - \frac{\left(\sqrt{a}B\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}}$$

$$= \frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}B\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{\left(\sqrt{a}B+\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.255628, size = 302, normalized size = 1.07

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)+B\left(\sqrt{b^2\ldots}\right.\right.}{2\sqrt{2}c\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(B*(-b + Sqrt[b^2 - 4*a*c])*Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])] + (b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*Ellipt
icF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.009, size = 362, normalized size = 1.3

$$-\frac{aB\sqrt{2}}{2}\sqrt{4-2\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*B*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2)))+1/4*A*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*
x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
```

```
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.179 $\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=312

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B+A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan\right.}{a^{3/4}\sqrt{a+bx^2+}}$$

```
[Out] -((A*Sqrt[a + b*x^2 + c*x^4])/(a*x)) + (A*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4]
)/(a*(Sqrt[a] + Sqrt[c]*x^2)) - (A*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) +
 ((Sqrt[a]*B + A*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/
(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.1314, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1281, 1197, 1103, 1195}

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}B+A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{a^{3/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2 + c*x^4])/(a*x)) + (A*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4]
)/(a*(Sqrt[a] + Sqrt[c]*x^2)) - (A*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) +
 ((Sqrt[a]*B + A*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/
(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{ax} - \frac{\int \frac{-aB-Acx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{ax} + \left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx - \frac{\left(A\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{ax} + \frac{A\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{a^{3/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] time = 1.07488, size = 448, normalized size = 1.44

$$\frac{-ix\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}\left(A\left(\sqrt{b^2-4ac}-b\right)+2aB\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\right),\frac{\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4ac}}\right)}{4ax\sqrt{\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-4*A*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4) + I*A*(-b + Sqrt[
b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*El
lipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(2*a*B + A*(-b + Sqrt[b^2 - 4*a*c])
)*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2
*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a + b*
x^2 + c*x^4])
```

Maple [A] time = 0.014, size = 386, normalized size = 1.2

$$\frac{B\sqrt{2}}{4}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\text{EllipticF}\left(\frac{x\sqrt{2}}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+A*(-1/a*(c*x^4+b*x^2+a)^(1/2)/x-1/2*c*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*
a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),
1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}\left(Bx^2 + A\right)}{cx^6 + bx^4 + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(c*x^6 + b*x^4 + a*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{x^2\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**2*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

3.180 $\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=376

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}A\sqrt{c}-3aB+2Ab\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{(2Ab-3aB)\sqrt{a+bx^2}}{3a^2x}$$

```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(3*a*x^3) + ((2*A*b - 3*a*B)*Sqrt[a + b*x^2 +
c*x^4])/(3*a^2*x) - ((2*A*b - 3*a*B)*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*
a^2*(Sqrt[a] + Sqrt[c]*x^2)) + ((2*A*b - 3*a*B)*c^(1/4)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(7/4)*Sqrt[a + b*
x^2 + c*x^4]) - ((2*A*b - 3*a*B + Sqrt[a]*A*Sqrt[c])*c^(1/4)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*a^(7/4)*Sqrt[a
 + b*x^2 + c*x^4])
```

Rubi [A] time = 0.227337, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1281, 1197, 1103, 1195}

$$\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{\sqrt{c}x(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}A\sqrt{c}-3aB+2Ab\right)\text{F}}{6a^{7/4}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```


```
[Out] -(A*Sqrt[a + b*x^2 + c*x^4])/(3*a*x^3) + ((2*A*b - 3*a*B)*Sqrt[a + b*x^2 +
c*x^4])/(3*a^2*x) - ((2*A*b - 3*a*B)*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*
a^2*(Sqrt[a] + Sqrt[c]*x^2)) + ((2*A*b - 3*a*B)*c^(1/4)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(7/4)*Sqrt[a + b*
x^2 + c*x^4]) - ((2*A*b - 3*a*B + Sqrt[a]*A*Sqrt[c])*c^(1/4)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*A
rcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*a^(7/4)*Sqrt[a
 + b*x^2 + c*x^4])
```


Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
```

```
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} - \frac{\int \frac{2Ab-3aB+Acx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx}{3a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x} + \frac{\int \frac{-aAc-(2Ab-3aB)cx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3a^2}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x} + \frac{\left((2Ab-3aB)\sqrt{c}\right)\int \frac{1-\frac{\sqrt{c}x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3a^{3/2}} - \ldots$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x} - \frac{(2Ab-3aB)\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)} + \ldots$$

Mathematica [C] time = 0.695821, size = 373, normalized size = 0.99

$$\frac{-\frac{4\left(a+bx^2+cx^4\right)\left(a\left(A+3Bx^2\right)-2Abx^2\right)}{x^3} + \frac{i\sqrt{2}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(\left(2A\left(b\sqrt{b^2-4ac}+ac-b^2\right)+3aB\left(b-\sqrt{b^2-4ac}\right)\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{2}x\sqrt{\ldots}\right.\right.\ldots}{\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}}}{12a^2\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-4*(a + b*x^2 + c*x^4)*(-2*A*b*x^2 + a*(A + 3*B*x^2)))/x^3 + (I*Sqrt[2]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(-((2*A*b - 3*a*B)*(-b + Sqrt[b^2 - 4*a*c])
*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (3*a*B*(b - Sqrt[b^2 - 4*a*c]) +
2*A*(-b^2 + a*c + b*Sqrt[b^2 - 4*a*c]))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(12*a^2*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.018, size = 656, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] B*(-1/a*(c*x^4+b*x^2+a)^(1/2)/x-1/2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2)
)/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2
*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+A*(-1/3/a*(c*x^4+b*x^2+a)^(1/
2)/x^3+2/3/a^2*b*(c*x^4+b*x^2+a)^(1/2)/x-1/12/a*c*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+
b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)+1/3*b*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+
a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/
2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(
1/2))/a/c)^(1/2))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}\left(Bx^2 + A\right)}{cx^8 + bx^6 + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(B*x^2 + A)/(c*x^8 + b*x^6 + a*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + Bx^2}{x^4\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Bx^2 + A}{\sqrt{cx^4 + bx^2 + a}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

3.181 $\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=98

$$\frac{3}{8}\sqrt{x^4+5x^2+3}x^6-\frac{89}{48}\sqrt{x^4+5x^2+3}x^4-\frac{1}{384}\left(24243-3802x^2\right)\sqrt{x^4+5x^2+3}+\frac{32801}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] (-89*x^4*Sqrt[3 + 5*x^2 + x^4])/48 + (3*x^6*Sqrt[3 + 5*x^2 + x^4])/8 - ((24
243 - 3802*x^2)*Sqrt[3 + 5*x^2 + x^4])/384 + (32801*ArcTanh[(5 + 2*x^2)/(2*
Sqrt[3 + 5*x^2 + x^4])])/256
```

Rubi [A] time = 0.086537, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1251, 832, 779, 621, 206}

$$\frac{3}{8}\sqrt{x^4+5x^2+3}x^6-\frac{89}{48}\sqrt{x^4+5x^2+3}x^4-\frac{1}{384}\left(24243-3802x^2\right)\sqrt{x^4+5x^2+3}+\frac{32801}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-89*x^4*Sqrt[3 + 5*x^2 + x^4])/48 + (3*x^6*Sqrt[3 + 5*x^2 + x^4])/8 - ((24
243 - 3802*x^2)*Sqrt[3 + 5*x^2 + x^4])/384 + (32801*ArcTanh[(5 + 2*x^2)/(2*
Sqrt[3 + 5*x^2 + x^4])])/256
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7\left(2+3x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3(2+3x)}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\ &= \frac{3}{8}x^6\sqrt{3+5x^2+x^4}+\frac{1}{8}\,\text{Subst}\left(\int \frac{\left(-27-\frac{89x}{2}\right)x^2}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\ &= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4}+\frac{3}{8}x^6\sqrt{3+5x^2+x^4}+\frac{1}{24}\,\text{Subst}\left(\int \frac{x\left(267+\frac{1901x}{4}\right)}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\ &= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4}+\frac{3}{8}x^6\sqrt{3+5x^2+x^4}-\frac{1}{384}\left(24243-3802x^2\right)\sqrt{3+5x^2+x^4}+\frac{32801}{256}\,\text{S}\\ &= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4}+\frac{3}{8}x^6\sqrt{3+5x^2+x^4}-\frac{1}{384}\left(24243-3802x^2\right)\sqrt{3+5x^2+x^4}+\frac{32801}{128}\,\text{S}\\ &= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4}+\frac{3}{8}x^6\sqrt{3+5x^2+x^4}-\frac{1}{384}\left(24243-3802x^2\right)\sqrt{3+5x^2+x^4}+\frac{32801}{256}\,\text{t}\end{aligned}$$

Mathematica [A] time = 0.0312108, size = 66, normalized size = 0.67

$$\frac{1}{768}\left(2\sqrt{x^4+5x^2+3}\left(144x^6-712x^4+3802x^2-24243\right)+98403\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(-24243 + 3802*x^2 - 712*x^4 + 144*x^6) + 98403*Ar
cTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/768
```

Maple [A] time = 0.016, size = 87, normalized size = 0.9

$$\frac{3x^6}{8}\sqrt{x^4+5x^2+3}-\frac{89x^4}{48}\sqrt{x^4+5x^2+3}+\frac{1901x^2}{192}\sqrt{x^4+5x^2+3}-\frac{8081}{128}\sqrt{x^4+5x^2+3}+\frac{32801}{256}\ln\left(\frac{5}{2}+x^2+\sqrt{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/8*x^6*(x^4+5*x^2+3)^(1/2)-89/48*x^4*(x^4+5*x^2+3)^(1/2)+1901/192*x^2*(x^4
+5*x^2+3)^(1/2)-8081/128*(x^4+5*x^2+3)^(1/2)+32801/256*ln(5/2+x^2+(x^4+5*x^
```

```
2+3)^(1/2))
```

Maxima [A] time = 0.949552, size = 122, normalized size = 1.24

$$\frac{3}{8}\sqrt{x^4+5x^2+3}x^6-\frac{89}{48}\sqrt{x^4+5x^2+3}x^4+\frac{1901}{192}\sqrt{x^4+5x^2+3}x^2-\frac{8081}{128}\sqrt{x^4+5x^2+3}+\frac{32801}{256}\log\left(2x^2+2\sqrt{x^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(x^4 + 5*x^2 + 3)*x^6 - 89/48*sqrt(x^4 + 5*x^2 + 3)*x^4 + 1901/192*
sqrt(x^4 + 5*x^2 + 3)*x^2 - 8081/128*sqrt(x^4 + 5*x^2 + 3) + 32801/256*log(
2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.37426, size = 165, normalized size = 1.68

$$\frac{1}{384}\left(144x^6-712x^4+3802x^2-24243\right)\sqrt{x^4+5x^2+3}-\frac{32801}{256}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(144*x^6 - 712*x^4 + 3802*x^2 - 24243)*sqrt(x^4 + 5*x^2 + 3) - 32801/
256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**7*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [A] time = 1.1422, size = 81, normalized size = 0.83

$$\frac{1}{384}\sqrt{x^4+5x^2+3}\left(2\left(4\left(18x^2-89\right)x^2+1901\right)x^2-24243\right)-\frac{32801}{256}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 - 89)*x^2 + 1901)*x^2 - 24243) -
32801/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.182 $\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=77

$$\frac{1}{2}\sqrt{x^4+5x^2+3}x^4+\frac{3}{16}\left(89-14x^2\right)\sqrt{x^4+5x^2+3}-\frac{1083}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] (x^4*Sqrt[3 + 5*x^2 + x^4])/2 + (3*(89 - 14*x^2)*Sqrt[3 + 5*x^2 + x^4])/16
- (1083*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Rubi [A] time = 0.0662238, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1251, 832, 779, 621, 206}

$$\frac{1}{2}\sqrt{x^4+5x^2+3}x^4+\frac{3}{16}\left(89-14x^2\right)\sqrt{x^4+5x^2+3}-\frac{1083}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (x^4*Sqrt[3 + 5*x^2 + x^4])/2 + (3*(89 - 14*x^2)*Sqrt[3 + 5*x^2 + x^4])/16
- (1083*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(2+3x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(2+3x)}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}x^4\sqrt{3+5x^2+x^4}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{\left(-18-\frac{63x}{2}\right)x}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}x^4\sqrt{3+5x^2+x^4}+\frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4}-\frac{1083}{32}\operatorname{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}x^4\sqrt{3+5x^2+x^4}+\frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4}-\frac{1083}{16}\operatorname{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= \frac{1}{2}x^4\sqrt{3+5x^2+x^4}+\frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4}-\frac{1083}{32}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0215752, size = 61, normalized size = 0.79

$$\frac{1}{32}\left(2\sqrt{x^4+5x^2+3}\left(8x^4-42x^2+267\right)-1083\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*Sqrt[3 + 5*x^2 + x^4]*(267 - 42*x^2 + 8*x^4) - 1083*ArcTanh[(5 + 2*x^2)/
(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Maple [A] time = 0.012, size = 70, normalized size = 0.9

$$\frac{x^4}{2}\sqrt{x^4+5x^2+3}-\frac{21x^2}{8}\sqrt{x^4+5x^2+3}+\frac{267}{16}\sqrt{x^4+5x^2+3}-\frac{1083}{32}\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/2*x^4*(x^4+5*x^2+3)^(1/2)-21/8*x^2*(x^4+5*x^2+3)^(1/2)+267/16*(x^4+5*x^2+
3)^(1/2)-1083/32*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A] time = 0.960459, size = 99, normalized size = 1.29

$$\frac{1}{2}\sqrt{x^4+5x^2+3}\,x^4-\frac{21}{8}\sqrt{x^4+5x^2+3}\,x^2+\frac{267}{16}\sqrt{x^4+5x^2+3}-\frac{1083}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^4 + 5*x^2 + 3)*x^4 - 21/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 267/16*sqr
t(x^4 + 5*x^2 + 3) - 1083/32*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.31517, size = 139, normalized size = 1.81

$$\frac{1}{16}\left(8\,x^4-42\,x^2+267\right)\sqrt{x^4+5\,x^2+3}+\frac{1083}{32}\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*x^4 - 42*x^2 + 267)*sqrt(x^4 + 5*x^2 + 3) + 1083/32*log(-2*x^2 + 2*
sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [A] time = 1.13247, size = 72, normalized size = 0.94

$$\frac{1}{16}\,\sqrt{x^4+5\,x^2+3}\left(2\left(4\,x^2-21\right)x^2+267\right)+\frac{1083}{32}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 - 21)*x^2 + 267) + 1083/32*log(2*x^2 -
 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.183 $\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=56

$$\frac{149}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{1}{8}\left(37-6x^2\right)\sqrt{x^4+5x^2+3}$$

```
[Out] -((37 - 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + (149*ArcTanh[(5 + 2*x^2)/(2*Sqrt[
3 + 5*x^2 + x^4])])/16
```

Rubi [A] time = 0.0449077, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1251, 779, 621, 206}

$$\frac{149}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{1}{8}\left(37-6x^2\right)\sqrt{x^4+5x^2+3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] -((37 - 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + (149*ArcTanh[(5 + 2*x^2)/(2*Sqrt[
3 + 5*x^2 + x^4])])/16
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3\left(2+3x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x(2+3x)}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$
$$= -\frac{1}{8}\left(37-6x^2\right)\sqrt{3+5x^2+x^4}+\frac{149}{16}\,\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$
$$= -\frac{1}{8}\left(37-6x^2\right)\sqrt{3+5x^2+x^4}+\frac{149}{8}\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)$$
$$= -\frac{1}{8}\left(37-6x^2\right)\sqrt{3+5x^2+x^4}+\frac{149}{16}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

Mathematica [A] time = 0.0157702, size = 56, normalized size = 1.

$$\frac{1}{16}\left(2\sqrt{x^4+5x^2+3}\left(6x^2-37\right)+149\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*(-37 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4] + 149*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3
+ 5*x^2 + x^4])])/16
```

Maple [A] time = 0.011, size = 53, normalized size = 1.

$$\frac{3\,x^2}{4}\sqrt{x^4+5\,x^2+3}-\frac{37}{8}\sqrt{x^4+5\,x^2+3}+\frac{149}{16}\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5\,x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/4*x^2*(x^4+5*x^2+3)^(1/2)-37/8*(x^4+5*x^2+3)^(1/2)+149/16*ln(5/2+x^2+(x^4
+5*x^2+3)^(1/2))
```

Maxima [A] time = 0.968043, size = 76, normalized size = 1.36

$$\frac{3}{4}\sqrt{x^4+5\,x^2+3}x^2-\frac{37}{8}\sqrt{x^4+5\,x^2+3}+\frac{149}{16}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - 37/8*sqrt(x^4 + 5*x^2 + 3) + 149/16*log(2*x
^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.28019, size = 123, normalized size = 2.2

$$\frac{1}{8}\sqrt{x^4+5\,x^2+3}\left(6\,x^2-37\right)-\frac{149}{16}\log\left(-2\,x^2+2\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*log(-2*x^2 + 2*sqrt(x^4 + 5
*x^2 + 3) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [A] time = 1.11393, size = 62, normalized size = 1.11

$$\frac{1}{8}\sqrt{x^4+5\,x^2+3}\left(6\,x^2-37\right)-\frac{149}{16}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*log(2*x^2 - 2*sqrt(x^4 + 5*
x^2 + 3) + 5)
```

3.184 $\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=49

$$\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{11}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 - (11*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x
^4])])/4
```

Rubi [A] time = 0.0323246, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1247, 640, 621, 206}

$$\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{11}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 - (11*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x
^4])])/4
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= \frac{3}{2}\sqrt{3+5x^2+x^4} - \frac{11}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= \frac{3}{2}\sqrt{3+5x^2+x^4} - \frac{11}{2}\operatorname{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)$$
$$= \frac{3}{2}\sqrt{3+5x^2+x^4} - \frac{11}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

Mathematica [A] time = 0.0095622, size = 49, normalized size = 1.

$$\frac{3}{2}\sqrt{x^4+5x^2+3} - \frac{11}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 - (11*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x
^4])])/4
```

Maple [A] time = 0.01, size = 36, normalized size = 0.7

$$\frac{3}{2}\sqrt{x^4+5\,x^2+3} - \frac{11}{4}\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5\,x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/2*(x^4+5*x^2+3)^(1/2)-11/4*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A] time = 0.952726, size = 53, normalized size = 1.08

$$\frac{3}{2}\sqrt{x^4+5\,x^2+3} - \frac{11}{4}\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) - 11/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.39899, size = 103, normalized size = 2.1

$$\frac{3}{2}\sqrt{x^4+5\,x^2+3} + \frac{11}{4}\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(3x^2+2\right)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [A] time = 1.11169, size = 53, normalized size = 1.08

$$\frac{3}{2}\sqrt{x^4+5x^2+3}+\frac{11}{4}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.185 $\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=69

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

```
[Out] (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2 - ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/Sqrt[3]
```

Rubi [A] time = 0.0624769, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1251, 843, 621, 206, 724}

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2 - ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/Sqrt[3]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\ &= \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)+\operatorname{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\ &= -\left(2\operatorname{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\right)+3\operatorname{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\ &= \frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-\frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0156079, size = 69, normalized size = 1.

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2 - ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/Sqrt[3]
```

Maple [A] time = 0.01, size = 52, normalized size = 0.8

$$\frac{3}{2}\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5\,x^2+3}\right)-\frac{\sqrt{3}}{3}\operatorname{Artanh}\left(\frac{\left(5\,x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5\,x^2+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/2*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))-1/3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+
5*x^2+3)^(1/2))*3^(1/2)
```

Maxima [A] time = 1.43019, size = 78, normalized size = 1.13

$$-\frac{1}{3}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{3}{2}\log\left(2\,x^2+2\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 3/2*log
(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.46738, size = 203, normalized size = 2.94

$$\frac{1}{3}\sqrt{3}\log\left(\frac{25x^2 - 2\sqrt{3}\left(5x^2+6\right) - 2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right) + 30}{x^2}\right) - \frac{3}{2}\log\left(-2x^2 + 2\sqrt{x^4+5x^2+3} - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(
5*sqrt(3) - 6) + 30)/x^2) - 3/2*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x*sqrt(x**4 + 5*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\sqrt{x^4+5x^2+3}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x), x)
```

3.186 $\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(3*x^2) - (2*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 +
 5*x^2 + x^4])])/(3*Sqrt[3])
```

Rubi [A] time = 0.0500117, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1251, 806, 724, 206}

$$-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(3*x^2) - (2*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 +
 5*x^2 + x^4])])/(3*Sqrt[3])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{2+3x}{x^2\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{3+5x^2+x^4}}{3x^2}+\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\ &= -\frac{\sqrt{3+5x^2+x^4}}{3x^2}-\frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\ &= -\frac{\sqrt{3+5x^2+x^4}}{3x^2}-\frac{2\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0152075, size = 62, normalized size = 1.

$$-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(3*x^2) - (2*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 +
 5*x^2 + x^4])])/(3*Sqrt[3])
```

Maple [A] time = 0.013, size = 49, normalized size = 0.8

$$-\frac{2\sqrt{3}}{9}\operatorname{Artanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right)-\frac{1}{3x^2}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -2/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3*(x^4+5*
x^2+3)^(1/2)/x^2
```

Maxima [A] time = 1.42467, size = 69, normalized size = 1.11

$$-\frac{2}{9}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 1/3*sqr
t(x^4 + 5*x^2 + 3)/x^2
```

Fricas [A] time = 1.40014, size = 200, normalized size = 3.23

$$\frac{2\sqrt{3}x^2\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right)-3x^2-3\sqrt{x^4+5x^2+3}}{9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2
 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 3*x^2 - 3*sqrt(x^4 + 5*x^2 + 3))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{3x^2+2}{x^3\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**3*sqrt(x**4 + 5*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{3x^2+2}{\sqrt{x^4+5x^2+3}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^3), x)
```

3.187 $\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}+\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(6*x^4) - Sqrt[3 + 5*x^2 + x^4]/(12*x^2) + (Sqrt[3]*
ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/8
```

Rubi [A] time = 0.0699484, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1251, 834, 806, 724, 206}

$$-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}+\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(6*x^4) - Sqrt[3 + 5*x^2 + x^4]/(12*x^2) + (Sqrt[3]*
ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/8
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^3\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{6x^4} - \frac{1}{12}\text{Subst}\left(\int \frac{-3+2x}{x^2\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{6x^4} - \frac{\sqrt{3+5x^2+x^4}}{12x^2} - \frac{3}{8}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{6x^4} - \frac{\sqrt{3+5x^2+x^4}}{12x^2} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{6x^4} - \frac{\sqrt{3+5x^2+x^4}}{12x^2} + \frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0227131, size = 67, normalized size = 0.81

$$\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right) - \frac{\left(x^2+2\right)\sqrt{x^4+5x^2+3}}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -((2 + x^2)*Sqrt[3 + 5*x^2 + x^4])/(12*x^4) + (Sqrt[3]*ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/8
```

Maple [A] time = 0.013, size = 66, normalized size = 0.8

$$\frac{\sqrt{3}}{8}\text{Artanh}\left(\frac{\left(5\,x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5\,x^2+3}}\right) - \frac{1}{6\,x^4}\sqrt{x^4+5\,x^2+3} - \frac{1}{12\,x^2}\sqrt{x^4+5\,x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/6*(x^4+5*x
^2+3)^(1/2)/x^4-1/12*(x^4+5*x^2+3)^(1/2)/x^2
```

Maxima [A] time = 1.44558, size = 92, normalized size = 1.11

$$\frac{1}{8}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2} + \frac{6}{x^2} + 5\right) - \frac{\sqrt{x^4+5\,x^2+3}}{12\,x^2} - \frac{\sqrt{x^4+5\,x^2+3}}{6\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 1/12*sqr
t(x^4 + 5*x^2 + 3)/x^2 - 1/6*sqrt(x^4 + 5*x^2 + 3)/x^4
```

Fricas [A] time = 1.39538, size = 215, normalized size = 2.59

$$\frac{3\sqrt{3}x^4\log\left(\frac{25\,x^2+2\,\sqrt{3}\left(5\,x^2+6\right)+2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}+6\right)+30}{x^2}\right)-2\,x^4-2\,\sqrt{x^4+5\,x^2+3}\left(x^2+2\right)}{24\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*sqrt(3)*x^4*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x^
2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*(x^2 +
2))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{3x^2+2}{x^5\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**5/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**5*sqrt(x**4 + 5*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^5), x)
```

3.188 $\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=104

$$\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}-\frac{61\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{216\sqrt{3}}$$

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(9*x^6) - Sqrt[3 + 5*x^2 + x^4]/(54*x^4) + (13*Sqrt[
3 + 5*x^2 + x^4])/(108*x^2) - (61*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5
*x^2 + x^4])])/(216*Sqrt[3])
```

Rubi [A] time = 0.0881354, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1251, 834, 806, 724, 206}

$$\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}-\frac{61\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{216\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -Sqrt[3 + 5*x^2 + x^4]/(9*x^6) - Sqrt[3 + 5*x^2 + x^4]/(54*x^4) + (13*Sqrt[
3 + 5*x^2 + x^4])/(108*x^2) - (61*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5
*x^2 + x^4])])/(216*Sqrt[3])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{2+3x}{x^4\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{1}{18}\,\text{Subst}\left(\int \frac{-2+4x}{x^3\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{\sqrt{3+5x^2+x^4}}{54x^4} + \frac{1}{108}\,\text{Subst}\left(\int \frac{-39-2x}{x^2\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{\sqrt{3+5x^2+x^4}}{54x^4} + \frac{13\sqrt{3+5x^2+x^4}}{108x^2} + \frac{61}{216}\,\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x\ldots\right.$$
$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{\sqrt{3+5x^2+x^4}}{54x^4} + \frac{13\sqrt{3+5x^2+x^4}}{108x^2} - \frac{61}{108}\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5\ldots}{\sqrt{3+5x\ldots}}\right.$$
$$= -\frac{\sqrt{3+5x^2+x^4}}{9x^6} - \frac{\sqrt{3+5x^2+x^4}}{54x^4} + \frac{13\sqrt{3+5x^2+x^4}}{108x^2} - \frac{61\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{216\sqrt{3}}$$

Mathematica [A] time = 0.024487, size = 77, normalized size = 0.74

$$\frac{6\sqrt{x^4+5x^2+3}\left(13x^4-2x^2-12\right)-61\sqrt{3}x^6\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{648x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (6*Sqrt[3 + 5*x^2 + x^4]*(-12 - 2*x^2 + 13*x^4) - 61*Sqrt[3]*x^6*ArcTanh[(6
 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(648*x^6)
```

Maple [A] time = 0.013, size = 83, normalized size = 0.8

$$-\frac{61\sqrt{3}}{648}\text{Artanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right)-\frac{1}{9x^6}\sqrt{x^4+5x^2+3}-\frac{1}{54x^4}\sqrt{x^4+5x^2+3}+\frac{13}{108x^2}\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -61/648*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/9*(x^4
+5*x^2+3)^(1/2)/x^6-1/54*(x^4+5*x^2+3)^(1/2)/x^4+13/108*(x^4+5*x^2+3)^(1/2)
/x^2
```

Maxima [A] time = 1.4628, size = 115, normalized size = 1.11

$$-\frac{61}{648}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -61/648*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 13/1
08*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/54*sqrt(x^4 + 5*x^2 + 3)/x^4 - 1/9*sqrt(x^
4 + 5*x^2 + 3)/x^6
```

Fricas [A] time = 1.46903, size = 235, normalized size = 2.26

$$\frac{61\sqrt{3}x^6\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}(5\sqrt{3}-6)+30}{x^2}\right)+78x^6+6(13x^4-2x^2-12)\sqrt{x^4+5x^2+3}}{648x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/648*(61*sqrt(3)*x^6*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 78*x^6 + 6*(13*x^4 - 2*x^2 - 12)*sqrt
(x^4 + 5*x^2 + 3))/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x^7\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**7/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**7*sqrt(x**4 + 5*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\sqrt{x^4+5x^2+3}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")

[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^7), x)
```

3.189 $\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=298

$$\frac{5\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{\sqrt{x^4+5x^2+3}}+\frac{3}{5}\sqrt{x^4+5x^2+3}x$$

```
[Out] (419*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (10*x*Sqrt[3 +
5*x^2 + x^4])/3 + (3*x^3*Sqrt[3 + 5*x^2 + x^4])/5 - (419*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(30*Sqrt[3 + 5*x^2 + x^4]) + (5*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5
- Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellipti
cF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2
+ x^4]
```

Rubi [A] time = 0.175286, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1279, 1189, 1099, 1135}

$$\frac{3}{5}\sqrt{x^4+5x^2+3}x^3-\frac{10}{3}\sqrt{x^4+5x^2+3}x+\frac{419\left(2x^2+\sqrt{13}+5\right)x}{30\sqrt{x^4+5x^2+3}}+\frac{5\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(t\right.}{\sqrt{x^4+5x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```


```
[Out] (419*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (10*x*Sqrt[3 +
5*x^2 + x^4])/3 + (3*x^3*Sqrt[3 + 5*x^2 + x^4])/5 - (419*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(30*Sqrt[3 + 5*x^2 + x^4]) + (5*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5
- Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellipti
cF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2
+ x^4]
```


Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(2+3x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx &= \frac{3}{5}x^3\sqrt{3+5x^2+x^4}-\frac{1}{5}\int \frac{x^2\left(27+50x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{10}{3}x\sqrt{3+5x^2+x^4}+\frac{3}{5}x^3\sqrt{3+5x^2+x^4}+\frac{1}{15}\int \frac{150+419x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{10}{3}x\sqrt{3+5x^2+x^4}+\frac{3}{5}x^3\sqrt{3+5x^2+x^4}+10\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx+\frac{419}{15}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= \frac{419x\left(5+\sqrt{13}+2x^2\right)}{30\sqrt{3+5x^2+x^4}}-\frac{10}{3}x\sqrt{3+5x^2+x^4}+\frac{3}{5}x^3\sqrt{3+5x^2+x^4}-\frac{419\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)}{6+\left(5+\sqrt{13}\right)}}}{}\end{aligned}$$

Mathematica [C] time = 0.304325, size = 229, normalized size = 0.77

$$\frac{-i\sqrt{2}\left(419\sqrt{13}-1795\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+4x\left(9x^6-5x^4-223x\right.}{60\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (4*x*(-150 - 223*x^2 - 5*x^4 + 9*x^6) + (419*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqr
t[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellip
ticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2
]*(-1795 + 419*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt
[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6
+ (5*Sqrt[13])/6])/(60*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.02, size = 226, normalized size = 0.8

$$\frac{3x^3}{5}\sqrt{x^4+5x^2+3}-\frac{10x}{3}\sqrt{x^4+5x^2+3}+60\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\,\text{EllipticF}\left(1/6\,x\sqrt{-30\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 3/5*x^3*(x^4+5*x^2+3)^(1/2)-10/3*x*(x^4+5*x^2+3)^(1/2)+60/(-30+6*13^(1/2))^
(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(
x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*3
9^(1/2))-5028/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1
-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF
(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-3
0+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)x^4}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3x^6+2x^4}{\sqrt{x^4+5x^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)/sqrt(x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4(3x^2+2)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**4*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)x^4}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3), x)
```

3.190 $\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=270

$$-\frac{\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(5\sqrt{13}-13)\right)}{\sqrt{x^4+5x^2+3}}+\sqrt{x^4+5x^2+3}x$$

```
[Out] (-4*x*(5 + Sqrt[13] + 2*x^2))/Sqrt[3 + 5*x^2 + x^4] + x*Sqrt[3 + 5*x^2 + x^
4] + (2*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13
])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4] - (Sqrt[3/(2*(5 + Sq
rt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
 Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[1
3])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi [A] time = 0.119849, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1279, 1189, 1099, 1135}

$$\sqrt{x^4+5x^2+3}x-\frac{4\left(2x^2+\sqrt{13}+5\right)x}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}(-\right.}{\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-4*x*(5 + Sqrt[13] + 2*x^2))/Sqrt[3 + 5*x^2 + x^4] + x*Sqrt[3 + 5*x^2 + x^
4] + (2*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13
])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4] - (Sqrt[3/(2*(5 + Sq
rt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
 Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[1
3])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(2+3x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx &= x\sqrt{3+5x^2+x^4}-\frac{1}{3}\int \frac{9+24x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= x\sqrt{3+5x^2+x^4}-3\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx-8\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{4x\left(5+\sqrt{13}+2x^2\right)}{\sqrt{3+5x^2+x^4}}+x\sqrt{3+5x^2+x^4}+\frac{2\sqrt{\frac{2}{3}}\left(5+\sqrt{13}\right)\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\text{ta}\right.}{\sqrt{3+5x^2+x^4}}\end{aligned}$$

Mathematica [C] time = 0.292998, size = 222, normalized size = 0.82

$$\frac{i\sqrt{2}\left(4\sqrt{13}-17\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+2x\left(x^4+5x^2+3\right)-4i\sqrt{2}\left(\right.}{2\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*x*(3 + 5*x^2 + x^4) - (4*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13]
- 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sq
rt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-17 + 4*Sqrt[1
3])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2
]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(2
*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.014, size = 208, normalized size = 0.8

$$x\sqrt{x^4+5x^2+3}-18\,\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\,\text{EllipticF}\left(1/6\,x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\sqrt{3}\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] x*(x^4+5*x^2+3)^(1/2)-18/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)
^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*
x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+288/(-30+6*13^(1/2))^(1/
2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4
+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^
(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*
39^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3\,x^2+2)x^2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^4+2\,x^2}{\sqrt{x^4+5\,x^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)/sqrt(x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\,(3x^2+2)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3\,x^2+2)x^2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")

[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3), x)
```

3.191 $\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}} \, dx$

Optimal. Leaf size=257

$$\frac{\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{\sqrt{x^4+5x^2+3}}+\frac{3x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}$$

```
[Out] (3*x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(3*(5 + Sqrt
[13]))/2]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
 Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[1
3])/6])/(2*Sqrt[3 + 5*x^2 + x^4]) + (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (
5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellip
ticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^
2 + x^4]
```

Rubi [A] time = 0.0767277, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1189, 1099, 1135}

$$\frac{3x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}+\frac{\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(3*(5 + Sqrt
[13]))/2]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
 Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[1
3])/6])/(2*Sqrt[3 + 5*x^2 + x^4]) + (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (
5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellip
ticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^
2 + x^4]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx = 2\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx + 3\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= \frac{3x\left(5+\sqrt{13}+2x^2\right)}{2\sqrt{3+5x^2+x^4}} - \frac{\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)\right.}{2\sqrt{3+5x^2+x^4}}$$

Mathematica [C] time = 0.143619, size = 159, normalized size = 0.62

$$\frac{i\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\left(\left(11-3\sqrt{13}\right)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+3\left(\sqrt{13}-5\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{}{5+}}\right.\right.\right.}{2\sqrt{2}\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] ((I/2)*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*
x^2]*(3*(-5 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6
 + (5*Sqrt[13])/6] + (11 - 3*Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt
[13])]*x], 19/6 + (5*Sqrt[13])/6]))/(Sqrt[2]*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.011, size = 194, normalized size = 0.8

$$-108\,\frac{\sqrt{1-\left(-5/6+1/6\,\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\,\sqrt{13}\right)x^2}\left(\text{EllipticF}\left(1/6\,x\sqrt{-30+6\,\sqrt{13}},5/6\,\sqrt{3}+1/6\,\sqrt{39}\right)-\text{Elliptic}\right.}{\sqrt{-30+6\,\sqrt{13}}\sqrt{x^4+5\,x^2+3}\left(\sqrt{13}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -108/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*
13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-30
+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2
))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))+12/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*
13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*
EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

3.192 $\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=278

$$\frac{\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{\sqrt{x^4+5x^2+3}}+\frac{x\left(2x^2+\sqrt{13}+5\right)}{3\sqrt{x^4+5x^2+3}}-$$

```
[Out] (x*(5 + Sqrt[13] + 2*x^2))/(3*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2 +
x^4])/(3*x) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5
 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt
[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3*Sqrt[3 + 5*x^2 + x^4]) + (Sqrt[3/(2*
(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rubi [A] time = 0.122658, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1281, 1189, 1099, 1135}

$$\frac{x\left(2x^2+\sqrt{13}+5\right)}{3\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}+\frac{\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+\right.\right.}{\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^2*Sqrt[3 + 5*x^2 + x^4]),x]
```


```
[Out] (x*(5 + Sqrt[13] + 2*x^2))/(3*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2 +
x^4])/(3*x) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5
 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt
[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3*Sqrt[3 + 5*x^2 + x^4]) + (Sqrt[3/(2*
(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```


Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx &= -\frac{2\sqrt{3+5x^2+x^4}}{3x}-\frac{1}{3}\int \frac{-9-2x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{2\sqrt{3+5x^2+x^4}}{3x}+\frac{2}{3}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx+3\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx\\ &= \frac{x\left(5+\sqrt{13}+2x^2\right)}{3\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{3x}-\frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\right.}{3\sqrt{3+5x^2+}}\end{aligned}$$

Mathematica [C] time = 0.274852, size = 224, normalized size = 0.81

$$\frac{-i\sqrt{2}\left(4+\sqrt{13}\right)x\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-4\left(x^4+5x^2+3\right)+i\sqrt{2}}{6x\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/(x^2*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (-4*(3 + 5*x^2 + x^4) + I*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2
*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[
2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(4 + Sqrt[13])*x*S
qrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ell
ipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(6*x*Sq
rt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.017, size = 211, normalized size = 0.8

$$18\,\frac{\sqrt{1-\left(-5/6+1/6\sqrt{13}\right)x^2}\sqrt{1-\left(-5/6-1/6\sqrt{13}\right)x^2}\text{EllipticF}\left(1/6\,x\sqrt{-30+6\sqrt{13}},5/6\sqrt{3}+1/6\sqrt{39}\right)}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5\,x^2+3}}-\frac{2}{3\,x}\sqrt{x^4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 18/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13
^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/
2),5/6*3^(1/2)+1/6*39^(1/2))-2/3*(x^4+5*x^2+3)^(1/2)/x-24/(-30+6*13^(1/2))^
(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(
x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6
*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1
/6*39^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)}{x^6+5\,x^4+3\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/(x^6 + 5*x^4 + 3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x^2\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**2/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**2*sqrt(x**4 + 5*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2), x)
```

3.193 $\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=302

$$-\frac{\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{9\sqrt{x^4+5x^2+3}}+\frac{7x\left(2x^2+\sqrt{13}+5\right)}{54\sqrt{x^4+5x^2+3}}$$

```
[Out] (7*x*(5 + Sqrt[13] + 2*x^2))/(54*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2
 + x^4])/(9*x^3) - (7*Sqrt[3 + 5*x^2 + x^4])/(27*x) - (7*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(54*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 -
Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF
[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(9*Sqrt[3 + 5*x^2
 + x^4])
```

Rubi [A] time = 0.16332, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1281, 1189, 1099, 1135}

$$\frac{7x\left(2x^2+\sqrt{13}+5\right)}{54\sqrt{x^4+5x^2+3}}-\frac{7\sqrt{x^4+5x^2+3}}{27x}-\frac{2\sqrt{x^4+5x^2+3}}{9x^3}-\frac{\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}(5\right.\right.}{9\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^4*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (7*x*(5 + Sqrt[13] + 2*x^2))/(54*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2
 + x^4])/(9*x^3) - (7*Sqrt[3 + 5*x^2 + x^4])/(27*x) - (7*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(54*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 -
Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF
[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(9*Sqrt[3 + 5*x^2
 + x^4])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx &= -\frac{2\sqrt{3+5x^2+x^4}}{9x^3}-\frac{1}{9}\int \frac{-7+2x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{2\sqrt{3+5x^2+x^4}}{9x^3}-\frac{7\sqrt{3+5x^2+x^4}}{27x}+\frac{1}{27}\int \frac{-6+7x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{2\sqrt{3+5x^2+x^4}}{9x^3}-\frac{7\sqrt{3+5x^2+x^4}}{27x}-\frac{2}{9}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx+\frac{7}{27}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= \frac{7x\left(5+\sqrt{13}+2x^2\right)}{54\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{9x^3}-\frac{7\sqrt{3+5x^2+x^4}}{27x}-\frac{7\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\,(6\cdots}{\cdots}\end{aligned}$$

Mathematica [C] time = 0.306831, size = 237, normalized size = 0.78

$$\frac{-i\sqrt{2}\left(7\sqrt{13}-47\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,x^3\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-4\left(7x^6+41x^4+51x^2\cdots\right.}{108x^3\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/(x^4*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (-4*(18 + 51*x^2 + 41*x^4 + 7*x^6) + (7*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3*Sqrt
[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellipt
icE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]
*(-47 + 7*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[
5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 +
 (5*Sqrt[13])/6])/(108*x^3*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.017, size = 228, normalized size = 0.8

$$-\frac{7}{27x}\sqrt{x^4+5x^2+3}-\frac{28}{3\sqrt{-30+6\sqrt{13}}\left(\sqrt{13}+5\right)}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-30+\cdots}}{6}\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -7/27*(x^4+5*x^2+3)^(1/2)/x-28/3/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/
2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1
/2)+5)*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-El
lipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-2/9*(x^4+5*
x^2+3)^(1/2)/x^3-4/3/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/
2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-
30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)}{x^8+5\,x^6+3\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/(x^8 + 5*x^6 + 3*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x^4\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**4/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**4*sqrt(x**4 + 5*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5\,x^2+3}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4), x)
```

3.194 $\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{(47x^2+33)\,x^2}{13\sqrt{x^4+5x^2+3}}+\frac{133}{26}\sqrt{x^4+5x^2+3}-\frac{41}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

```
[Out] -(x^2*(33 + 47*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (133*Sqrt[3 + 5*x^2 + x^4
])/26 - (41*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/4
```

Rubi [A] time = 0.0576965, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1251, 818, 640, 621, 206}

$$-\frac{(47x^2+33)\,x^2}{13\sqrt{x^4+5x^2+3}}+\frac{133}{26}\sqrt{x^4+5x^2+3}-\frac{41}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -(x^2*(33 + 47*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (133*Sqrt[3 + 5*x^2 + x^4
])/26 - (41*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/4
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(2+3x)}{(3+5x+x^2)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{1}{13}\operatorname{Subst}\left(\int \frac{33+\frac{133x}{2}}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{2}\operatorname{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0243875, size = 72, normalized size = 0.94

$$\frac{78x^4+1198x^2-533\sqrt{x^4+5x^2+3}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)+798}{52\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (798 + 1198*x^2 + 78*x^4 - 533*Sqrt[3 + 5*x^2 + x^4]*ArcTanh[(5 + 2*x^2)/(2
*Sqrt[3 + 5*x^2 + x^4])])/(52*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.019, size = 91, normalized size = 1.2

$$\frac{3x^4}{2}\frac{1}{\sqrt{x^4+5x^2+3}} + \frac{41x^2}{4}\frac{1}{\sqrt{x^4+5x^2+3}} - \frac{133}{8}\frac{1}{\sqrt{x^4+5x^2+3}} + \frac{1330x^2+3325}{104}\frac{1}{\sqrt{x^4+5x^2+3}} - \frac{41}{4}\ln\left(\frac{5}{2}+x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 3/2*x^4/(x^4+5*x^2+3)^(1/2)+41/4*x^2/(x^4+5*x^2+3)^(1/2)-133/8/(x^4+5*x^2+3
)^(1/2)+665/104*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)-41/4*ln(5/2+x^2+(x^4+5*x^2+3)
^(1/2))
```

Maxima [A] time = 0.965985, size = 99, normalized size = 1.29

$$\frac{3\,x^4}{2\,\sqrt{x^4+5\,x^2+3}}+\frac{599\,x^2}{26\,\sqrt{x^4+5\,x^2+3}}+\frac{399}{26\,\sqrt{x^4+5\,x^2+3}}-\frac{41}{4}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*x^4/sqrt(x^4 + 5*x^2 + 3) + 599/26*x^2/sqrt(x^4 + 5*x^2 + 3) + 399/26/s
qrt(x^4 + 5*x^2 + 3) - 41/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.45239, size = 232, normalized size = 3.01

$$\frac{1811\,x^4+9055\,x^2+1066\left(x^4+5\,x^2+3\right)\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)+4\left(39\,x^4+599\,x^2+399\right)\sqrt{x^4+5\,x^2+3}+}{104\left(x^4+5\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/104*(1811*x^4 + 9055*x^2 + 1066*(x^4 + 5*x^2 + 3)*log(-2*x^2 + 2*sqrt(x^4
 + 5*x^2 + 3) - 5) + 4*(39*x^4 + 599*x^2 + 399)*sqrt(x^4 + 5*x^2 + 3) + 543
3)/(x^4 + 5*x^2 + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5\left(3x^2+2\right)}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [A] time = 1.12356, size = 70, normalized size = 0.91

$$\frac{\left(39\,x^2+599\right)x^2+399}{26\,\sqrt{x^4+5\,x^2+3}}+\frac{41}{4}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/26*((39*x^2 + 599)*x^2 + 399)/sqrt(x^4 + 5*x^2 + 3) + 41/4*log(2*x^2 - 2*
sqrt(x^4 + 5*x^2 + 3) + 5)
```

3.195 $\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=56

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}$$

```
[Out] -(33 + 47*x^2)/(13*Sqrt[3 + 5*x^2 + x^4]) + (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[
3 + 5*x^2 + x^4])])/2
```

Rubi [A] time = 0.0434781, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1251, 777, 621, 206}

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -(33 + 47*x^2)/(13*Sqrt[3 + 5*x^2 + x^4]) + (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[
3 + 5*x^2 + x^4])])/2
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(2+3x)}{(3+5x+x^2)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}} + \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}} + 3\operatorname{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)$$
$$= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}} + \frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

Mathematica [A] time = 0.124872, size = 54, normalized size = 0.96

$$\frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right) - \frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -(33 + 47*x^2)/(13*Sqrt[3 + 5*x^2 + x^4]) + (3*Log[5 + 2*x^2 + 2*Sqrt[3 + 5
*x^2 + x^4]])/2
```

Maple [B] time = 0.011, size = 95, normalized size = 1.7

$$-\frac{3x^2}{2}\frac{1}{\sqrt{x^4+5x^2+3}} + \frac{15}{4}\frac{1}{\sqrt{x^4+5x^2+3}} - \frac{150x^2+375}{52}\frac{1}{\sqrt{x^4+5x^2+3}} + \frac{3}{2}\ln\left(\frac{5}{2}+x^2+\sqrt{x^4+5x^2+3}\right) + \frac{10x^2+12}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -3/2*x^2/(x^4+5*x^2+3)^(1/2)+15/4/(x^4+5*x^2+3)^(1/2)-75/52*(2*x^2+5)/(x^4+
5*x^2+3)^(1/2)+3/2*ln(5/2+x^2+(x^4+5*x^2+3)^(1/2))+2/13/(x^4+5*x^2+3)^(1/2)
*(5*x^2+6)
```

Maxima [A] time = 0.959788, size = 76, normalized size = 1.36

$$-\frac{47x^2}{13\sqrt{x^4+5x^2+3}} - \frac{33}{13\sqrt{x^4+5x^2+3}} + \frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -47/13*x^2/sqrt(x^4 + 5*x^2 + 3) - 33/13/sqrt(x^4 + 5*x^2 + 3) + 3/2*log(2*
x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A] time = 1.23437, size = 209, normalized size = 3.73

$$-\frac{94\,x^4+470\,x^2+39\left(x^4+5\,x^2+3\right)\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)+2\,\sqrt{x^4+5\,x^2+3}\left(47\,x^2+33\right)+282}{26\left(x^4+5\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/26*(94*x^4 + 470*x^2 + 39*(x^4 + 5*x^2 + 3)*log(-2*x^2 + 2*sqrt(x^4 + 5*
x^2 + 3) - 5) + 2*sqrt(x^4 + 5*x^2 + 3)*(47*x^2 + 33) + 282)/(x^4 + 5*x^2 +
 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3\left(3x^2+2\right)}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [A] time = 1.14948, size = 62, normalized size = 1.11

$$-\frac{47\,x^2+33}{13\,\sqrt{x^4+5\,x^2+3}}-\frac{3}{2}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/13*(47*x^2 + 33)/sqrt(x^4 + 5*x^2 + 3) - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*
x^2 + 3) + 5)
```

3.196 $\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=25

$$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])

Rubi [A] time = 0.0191952, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1247, 636}

$$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

[In] Int[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]

[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{(3+5x+x^2)^{3/2}} \, dx, x, x^2\right)\\ &= \frac{8+11x^2}{13\sqrt{3+5x^2+x^4}}\end{aligned}$$

Mathematica [A] time = 0.0954649, size = 25, normalized size = 1.

$$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]

```
[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.005, size = 22, normalized size = 0.9

$$\frac{11\,x^{2}+8}{13}\frac{1}{\sqrt{x^{4}+5\,x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 1/13*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 0.952234, size = 43, normalized size = 1.72

$$\frac{11\,x^{2}}{13\,\sqrt{x^{4}+5\,x^{2}+3}}+\frac{8}{13\,\sqrt{x^{4}+5\,x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 11/13*x^2/sqrt(x^4 + 5*x^2 + 3) + 8/13/sqrt(x^4 + 5*x^2 + 3)
```

Fricas [B] time = 1.33069, size = 113, normalized size = 4.52

$$\frac{11\,x^{4}+55\,x^{2}+\sqrt{x^{4}+5\,x^{2}+3}\left(11\,x^{2}+8\right)+33}{13\,\left(x^{4}+5\,x^{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/13*(11*x^4 + 55*x^2 + sqrt(x^4 + 5*x^2 + 3)*(11*x^2 + 8) + 33)/(x^4 + 5*x
^2 + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(3x^{2}+2\right)}{\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [A] time = 1.16028, size = 28, normalized size = 1.12

$$\frac{11\,x^2+8}{13\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13*(11*x^2 + 8)/sqrt(x^4 + 5*x^2 + 3)
```

3.197 $\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

```
[Out] -(7 + 8*x^2)/(39*Sqrt[3 + 5*x^2 + x^4]) - ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sq
rt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```

Rubi [A] time = 0.0570446, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1251, 822, 12, 724, 206}

$$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -(7 + 8*x^2)/(39*Sqrt[3 + 5*x^2 + x^4]) - ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sq
rt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x\left(3+5x+x^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}} - \frac{1}{39}\text{Subst}\left(\int -\frac{13}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}} + \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}} - \frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0293901, size = 66, normalized size = 1.

$$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}} - \frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -(7 + 8*x^2)/(39*Sqrt[3 + 5*x^2 + x^4]) - ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sq
rt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```

Maple [A] time = 0.018, size = 67, normalized size = 1.

$$-\frac{8\,x^2+20}{39}\frac{1}{\sqrt{x^4+5\,x^2+3}} + \frac{1}{3}\frac{1}{\sqrt{x^4+5\,x^2+3}} - \frac{\sqrt{3}}{9}\text{Artanh}\left(\frac{\left(5\,x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5\,x^2+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -4/39*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)+1/3/(x^4+5*x^2+3)^(1/2)-1/9*arctanh(1/6
*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Maxima [A] time = 1.43624, size = 88, normalized size = 1.33

$$-\frac{8\,x^2}{39\,\sqrt{x^4+5\,x^2+3}}-\frac{1}{9}\,\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{7}{39\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/39*x^2/sqrt(x^4 + 5*x^2 + 3) - 1/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^
2 + 3)/x^2 + 6/x^2 + 5) - 7/39/sqrt(x^4 + 5*x^2 + 3)
```

Fricas [B] time = 1.37649, size = 281, normalized size = 4.26

$$-\frac{24\,x^4-13\,\sqrt{3}\left(x^4+5\,x^2+3\right)\log\left(\frac{25\,x^2-2\,\sqrt{3}\left(5\,x^2+6\right)-2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)+120\,x^2+3\,\sqrt{x^4+5\,x^2+3}\left(8\,x^2+7\right)+72}{117\,\left(x^4+5\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/117*(24*x^4 - 13*sqrt(3)*(x^4 + 5*x^2 + 3)*log((25*x^2 - 2*sqrt(3)*(5*x^
2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 120*x^2 + 3*s
qrt(x^4 + 5*x^2 + 3)*(8*x^2 + 7) + 72)/(x^4 + 5*x^2 + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x), x)
```

3.198 $\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=90

$$-\frac{8x^2+7}{39x^2\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{39x^2}+\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

```
[Out] -(7 + 8*x^2)/(39*x^2*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2 + x^4])/(39
*x^2) + ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```

Rubi [A] time = 0.0707172, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1251, 822, 806, 724, 206}

$$-\frac{8x^2+7}{39x^2\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{39x^2}+\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
```


```
[Out] -(7 + 8*x^2)/(39*x^2*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2 + x^4])/(39
*x^2) + ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^2\left(3+5x+x^2\right)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}} - \frac{1}{39}\text{Subst}\left(\int \frac{-6+8x}{x^2\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{39x^2} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{39x^2} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{39x^2} + \frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0183794, size = 88, normalized size = 0.98

$$\frac{-6x^4-54x^2+13\sqrt{3}\sqrt{x^4+5x^2+3}x^2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)-39}{117x^2\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-39 - 54*x^2 - 6*x^4 + 13*Sqrt[3]*x^2*Sqrt[3 + 5*x^2 + x^4]*ArcTanh[(6 + 5
*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(117*x^2*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.013, size = 84, normalized size = 0.9

$$-\frac{1}{3}\frac{1}{\sqrt{x^4+5x^2+3}}-\frac{2x^2+5}{39}\frac{1}{\sqrt{x^4+5x^2+3}}+\frac{\sqrt{3}}{9}\text{Artanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6}\frac{1}{\sqrt{x^4+5x^2+3}}\right)-\frac{1}{3x^2}\frac{1}{\sqrt{x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -1/3/(x^4+5*x^2+3)^(1/2)-1/39*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)+1/9*arctanh(1/6
*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3/x^2/(x^4+5*x^2+3)^(1/2)
```

Maxima [A] time = 1.66629, size = 111, normalized size = 1.23

$$-\frac{2\,x^2}{39\,\sqrt{x^4+5\,x^2+3}}+\frac{1}{9}\,\sqrt{3}\log\left(\frac{2\,\sqrt{3}\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{6}{13\,\sqrt{x^4+5\,x^2+3}}-\frac{1}{3\,\sqrt{x^4+5\,x^2+3}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/39*x^2/sqrt(x^4 + 5*x^2 + 3) + 1/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^
2 + 3)/x^2 + 6/x^2 + 5) - 6/13/sqrt(x^4 + 5*x^2 + 3) - 1/3/(sqrt(x^4 + 5*x^
2 + 3)*x^2)
```

Fricas [A] time = 1.349, size = 308, normalized size = 3.42

$$-\frac{6\,x^6+30\,x^4-13\,\sqrt{3}\left(x^6+5\,x^4+3\,x^2\right)\log\left(\frac{25\,x^2+2\,\sqrt{3}\left(5\,x^2+6\right)+2\,\sqrt{x^4+5\,x^2+3}\left(5\,\sqrt{3}+6\right)+30}{x^2}\right)+18\,x^2+3\left(2\,x^4+18\,x^2+13\right)\sqrt{x^4+}}{117\left(x^6+5\,x^4+3\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/117*(6*x^6 + 30*x^4 - 13*sqrt(3)*(x^6 + 5*x^4 + 3*x^2)*log((25*x^2 + 2*s
qrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) + 1
8*x^2 + 3*(2*x^4 + 18*x^2 + 13)*sqrt(x^4 + 5*x^2 + 3))/(x^6 + 5*x^4 + 3*x^2
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x^3\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**3*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^3), x)
```

3.199 $\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=307

$$\frac{11\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{13\sqrt{x^4+5x^2+3}}+\frac{\left(11x^2+8\right)x^3}{13\sqrt{x^4+5x^2+3}}-$$

```
[Out] (43*x*(5 + Sqrt[13] + 2*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (x^3*(8 + 11*x^2
))/(13*Sqrt[3 + 5*x^2 + x^4]) - (11*x*Sqrt[3 + 5*x^2 + x^4])/13 - (43*Sqrt[
(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(
6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 +
5*Sqrt[13])/6])/(13*Sqrt[3 + 5*x^2 + x^4]) + (11*Sqrt[3/(2*(5 + Sqrt[13]))]
*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13]
)*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(
13*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.163282, antiderivative size = 307, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1275, 1279, 1189, 1099, 1135}

$$\frac{\left(11x^2+8\right)x^3}{13\sqrt{x^4+5x^2+3}}-\frac{11}{13}\sqrt{x^4+5x^2+3}x+\frac{43\left(2x^2+\sqrt{13}+5\right)x}{13\sqrt{x^4+5x^2+3}}+\frac{11\sqrt{\frac{3}{2(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\right.}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```


```
[Out] (43*x*(5 + Sqrt[13] + 2*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (x^3*(8 + 11*x^2
))/(13*Sqrt[3 + 5*x^2 + x^4]) - (11*x*Sqrt[3 + 5*x^2 + x^4])/13 - (43*Sqrt[
(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(
6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 +
5*Sqrt[13])/6])/(13*Sqrt[3 + 5*x^2 + x^4]) + (11*Sqrt[3/(2*(5 + Sqrt[13]))]
*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13]
)*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(
13*Sqrt[3 + 5*x^2 + x^4])
```


Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
```

```
3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(2+3x^2\right)}{\left(3+5x^2+x^4\right)^{3/2}}\,dx &= \frac{x^3\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{1}{13}\int \frac{x^2\left(-24-33x^2\right)}{\sqrt{3+5x^2+x^4}}\,dx\\
&= \frac{x^3\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}-\frac{11}{13}x\sqrt{3+5x^2+x^4}-\frac{1}{39}\int \frac{-99-258x^2}{\sqrt{3+5x^2+x^4}}\,dx\\
&= \frac{x^3\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}-\frac{11}{13}x\sqrt{3+5x^2+x^4}+\frac{33}{13}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx+\frac{86}{13}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,d\\
&= \frac{43x\left(5+\sqrt{13}+2x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{x^3\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}-\frac{11}{13}x\sqrt{3+5x^2+x^4}-\frac{43\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{\right.}}{6+\left(5+\sqrt{\right.}}}}{}
\end{aligned}$$

Mathematica [C] time = 0.300614, size = 219, normalized size = 0.71

$$\frac{-i\sqrt{2}\left(43\sqrt{13}-182\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-2x\left(47x^2+33\right)+43}{26\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-2*x*(33 + 47*x^2) + (43*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] -
2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt
```

```
[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-182 + 43*Sqrt[1
3])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2
]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(2
6*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.021, size = 240, normalized size = 0.8

$$-6\frac{1}{\sqrt{x^4+5x^2+3}}\left(\frac{19x^3}{26}+\frac{15x}{26}\right)+\frac{198}{13\sqrt{-30+6\sqrt{13}}}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-30+6}}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -6*(19/26*x^3+15/26*x)/(x^4+5*x^2+3)^(1/2)+198/13/(-30+6*13^(1/2))^(1/2)*(1
-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^
2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
-3096/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-
1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*
(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^
(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-4*(-5/26*x^3-3/13*x)/(x^4+5*x^2+3)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)x^4}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(3x^6+2x^4)\sqrt{x^4+5x^2+3}}{x^8+10x^6+31x^4+30x^2+9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4)*sqrt(x^4 + 5*x^2 + 3)/(x^8 + 10*x^6 + 31*x^4 + 30*
x^2 + 9), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (3x^2 + 2)}{(x^4 + 5x^2 + 3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**4*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3\,x^2 + 2)x^4}{(x^4 + 5\,x^2 + 3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2), x)
```

3.200 $\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=286

$$-\frac{4\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right),\frac{1}{6}(5\sqrt{13}-13)\right)}{13\sqrt{x^4+5x^2+3}}-\frac{11x(2x^2+\sqrt{13}+5}{26\sqrt{x^4+5x^2+3}}$$

```
[Out] (-11*x*(5 + Sqrt[13] + 2*x^2))/(26*Sqrt[3 + 5*x^2 + x^4]) + (x*(8 + 11*x^2)
)/(13*Sqrt[3 + 5*x^2 + x^4]) + (11*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sq
rt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[A
rcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(26*Sqrt[3 + 5*x^2
+ x^4]) - (4*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 +
(5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sq
rt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(13*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.122732, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1275, 1189, 1099, 1135}

$$-\frac{11x(2x^2+\sqrt{13}+5)}{26\sqrt{x^4+5x^2+3}}+\frac{x(11x^2+8)}{13\sqrt{x^4+5x^2+3}}-\frac{4\sqrt{\frac{2}{3(5+\sqrt{13})}}\sqrt{\frac{(5-\sqrt{13})x^2+6}{(5+\sqrt{13})x^2+6}}\left((5+\sqrt{13})x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}(5+\sqrt{13})}x\right)|\frac{1}{6}\right.}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```


```
[Out] (-11*x*(5 + Sqrt[13] + 2*x^2))/(26*Sqrt[3 + 5*x^2 + x^4]) + (x*(8 + 11*x^2)
)/(13*Sqrt[3 + 5*x^2 + x^4]) + (11*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sq
rt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[A
rcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(26*Sqrt[3 + 5*x^2
+ x^4]) - (4*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 +
(5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sq
rt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(13*Sqrt[3 + 5*x^2 + x^4])
```


Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2\left(2+3x^2\right)}{\left(3+5x^2+x^4\right)^{3/2}}\,dx = \frac{x\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{1}{13}\int \frac{-8-11x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= \frac{x\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}-\frac{8}{13}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx-\frac{11}{13}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{11x\left(5+\sqrt{13}+2x^2\right)}{26\sqrt{3+5x^2+x^4}}+\frac{x\left(8+11x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{11\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)\right.}{26\sqrt{3\,-}}$$

Mathematica [C] time = 0.287375, size = 219, normalized size = 0.77

$$\frac{i\sqrt{2}\left(11\sqrt{13}-39\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+4x\left(11x^2+8\right)-11i\sqrt{2}\left(\right.}{52\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(8 + 11*x^2) - (11*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*
x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2
/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-39 + 11*Sqrt[13])
*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*E
llipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(52*S
qrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.014, size = 240, normalized size = 0.8

$$-6\,\frac{1}{\sqrt{x^4+5\,x^2+3}}\left(-\frac{5\,x^3}{26}-3/13\,x\right)-\frac{48}{13\,\sqrt{-30+6\,\sqrt{13}}}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-3}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -6*(-5/26*x^3-3/13*x)/(x^4+5*x^2+3)^(1/2)-48/13/(-30+6*13^(1/2))^(1/2)*(1-(
-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+
3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+3
96/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6
*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-3
0+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/
2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-4*(1/13*x^3+5/26*x)/(x^4+5*x^2+3)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3\,x^2+2)x^2}{(x^4+5\,x^2+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(3\,x^4+2\,x^2)\sqrt{x^4+5\,x^2+3}}{x^8+10\,x^6+31\,x^4+30\,x^2+9}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((3*x^4 + 2*x^2)*sqrt(x^4 + 5*x^2 + 3)/(x^8 + 10*x^6 + 31*x^4 + 30*
x^2 + 9), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\,(3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x^2+2)x^2}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2), x)
```

3.201 $\int \frac{2+3x^2}{\left(3+5x^2+x^4\right)^{3/2}} \, dx$

Optimal. Leaf size=282

$$\frac{11\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{13\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}+\frac{4x\left(2x^2+\sqrt{13}+5\right)}{39\sqrt{x^4+5x^2+3}}-\frac{x\left(8\right.}{39\sqrt{x^4}}$$

```
[Out] (4*x*(5 + Sqrt[13] + 2*x^2))/(39*Sqrt[3 + 5*x^2 + x^4]) - (x*(7 + 8*x^2))/(
39*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sq
rt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[A
rcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(39*Sqrt[3 + 5*x^2
+ x^4]) + (11*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 +
(5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sq
rt[13])/6])/(13*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.109434, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1178, 1189, 1099, 1135}

$$\frac{4x\left(2x^2+\sqrt{13}+5\right)}{39\sqrt{x^4+5x^2+3}}-\frac{x\left(8x^2+7\right)}{39\sqrt{x^4+5x^2+3}}+\frac{11\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right.\right.}{13\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(5 + Sqrt[13] + 2*x^2))/(39*Sqrt[3 + 5*x^2 + x^4]) - (x*(7 + 8*x^2))/(
39*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sq
rt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[A
rcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(39*Sqrt[3 + 5*x^2
+ x^4]) + (11*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 +
(5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sq
rt[13])/6])/(13*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{2+3x^2}{\left(3+5x^2+x^4\right)^{3/2}}\,dx = -\frac{x\left(7+8x^2\right)}{39\sqrt{3+5x^2+x^4}}-\frac{1}{39}\int \frac{-33-8x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{x\left(7+8x^2\right)}{39\sqrt{3+5x^2+x^4}}+\frac{8}{39}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx+\frac{11}{13}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= \frac{4x\left(5+\sqrt{13}+2x^2\right)}{39\sqrt{3+5x^2+x^4}}-\frac{x\left(7+8x^2\right)}{39\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{\frac{2}{3}}\left(5+\sqrt{13}\right)\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)}{39\sqrt{3+5}}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(2 + 3*x^2)/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [A] time = 0.013, size = 240, normalized size = 0.9

$$-6\frac{1}{\sqrt{x^4+5x^2+3}}\left(1/13x^3+\frac{5x}{26}\right)+\frac{66}{13\sqrt{-30+6\sqrt{13}}}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-30}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -6*(1/13*x^3+5/26*x)/(x^4+5*x^2+3)^(1/2)+66/13/(-30+6*13^(1/2))^(1/2)*(1-(-
5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3
```

```
)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-96
/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*1
3^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-30+
6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2)
)^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-4*(-19/78*x-5/78*x^3)/(x^4+5*x^2+3)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)}{x^8+10\,x^6+31\,x^4+30\,x^2+9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/(x^8 + 10*x^6 + 31*x^4 + 30*x^2
+ 9), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

3.202 $\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=309

$$-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{39\sqrt{x^4+5x^2+3}}+\frac{19x\left(2x^2+\sqrt{13}+\ldots\right.}{234\sqrt{x^4+5x^2+\ldots}}$$

```
[Out] (19*x*(5 + Sqrt[13] + 2*x^2))/(234*Sqrt[3 + 5*x^2 + x^4]) - (7 + 8*x^2)/(39
*x*Sqrt[3 + 5*x^2 + x^4]) - (19*Sqrt[3 + 5*x^2 + x^4])/(117*x) - (19*Sqrt[(
5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/(234*Sqrt[3 + 5*x^2 + x^4]) - (4*Sqrt[2/(3*(5 + Sqrt[13]))]*
Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])
*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3
9*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.163558, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1277, 1281, 1189, 1099, 1135}

$$\frac{19x\left(2x^2+\sqrt{13}+5\right)}{234\sqrt{x^4+5x^2+3}}-\frac{19\sqrt{x^4+5x^2+3}}{117x}-\frac{8x^2+7}{39x\sqrt{x^4+5x^2+3}}-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\tan^{-1}\left(\ldots\right.\right.}{39\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^2*(3 + 5*x^2 + x^4)^(3/2)),x]
```


```
[Out] (19*x*(5 + Sqrt[13] + 2*x^2))/(234*Sqrt[3 + 5*x^2 + x^4]) - (7 + 8*x^2)/(39
*x*Sqrt[3 + 5*x^2 + x^4]) - (19*Sqrt[3 + 5*x^2 + x^4])/(117*x) - (19*Sqrt[(
5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/(234*Sqrt[3 + 5*x^2 + x^4]) - (4*Sqrt[2/(3*(5 + Sqrt[13]))]*
Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])
*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3
9*Sqrt[3 + 5*x^2 + x^4])
```


Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
```

```
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx = -\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}}-\frac{1}{39}\int \frac{-19+8x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}}-\frac{19\sqrt{3+5x^2+x^4}}{117x}+\frac{1}{117}\int \frac{-24+19x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}}-\frac{19\sqrt{3+5x^2+x^4}}{117x}+\frac{19}{117}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx-\frac{8}{39}\int \frac{1}{\sqrt{3+5x^2+}}$$

$$= \frac{19x\left(5+\sqrt{13}+2x^2\right)}{234\sqrt{3+5x^2+x^4}}-\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}}-\frac{19\sqrt{3+5x^2+x^4}}{117x}-\frac{19\sqrt{\frac{1}{6}}\left(5+\sqrt{13}\right)\sqrt{\frac{6+(5}{6+(5}}}{}$$

Mathematica [C] time = 0.315317, size = 228, normalized size = 0.74

$$\frac{-i\sqrt{2}\left(19\sqrt{13}-143\right)x\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-4\left(19x^4+119x^2+\right.}{468x\sqrt{x^4+5x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/(x^2*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-4*(78 + 119*x^2 + 19*x^4) + (19*I)*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 + S
qrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*Ar
```

```
cSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-143 +
 19*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt
[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt
[13])/6])/(468*x*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.02, size = 257, normalized size = 0.8

$$-6\frac{1}{\sqrt{x^4+5x^2+3}}\left(-\frac{19x}{78}-\frac{5x^3}{78}\right)-\frac{16}{13\sqrt{-30+6\sqrt{13}}}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-30+6}}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] -6*(-19/78*x-5/78*x^3)/(x^4+5*x^2+3)^(1/2)-16/13/(-30+6*13^(1/2))^(1/2)*(1-
(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2
+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-
76/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6
*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(13^(1/2)+5)*(EllipticF(1/6*x*(-3
0+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/
2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-2/9*(x^4+5*x^2+3)^(1/2)/x-4*(19/234*x^
3+40/117*x)/(x^4+5*x^2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5x^2+3}\left(3x^2+2\right)}{x^{10}+10x^8+31x^6+30x^4+9x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/(x^10 + 10*x^8 + 31*x^6 + 30*x^4
 + 9*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x^2\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**2/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**2*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2), x)
```

3.203 $\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{5\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)\text{EllipticF}\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right),\frac{1}{6}\left(5\sqrt{13}-13\right)\right)}{351\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}-\frac{133x\left(2x^2+\sqrt{13}+5\right)}{1053\sqrt{x^4+5x^2+3}}+\frac{266\cdots}{}$$

```
[Out] (-133*x*(5 + Sqrt[13] + 2*x^2))/(1053*Sqrt[3 + 5*x^2 + x^4]) - (7 + 8*x^2)/
(39*x^3*Sqrt[3 + 5*x^2 + x^4]) - (5*Sqrt[3 + 5*x^2 + x^4])/(351*x^3) + (266
*Sqrt[3 + 5*x^2 + x^4])/(1053*x) + (133*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5
 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellipt
icE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(1053*Sqrt[3 +
 5*x^2 + x^4]) - (5*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(351*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rubi [A] time = 0.205836, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1277, 1281, 1189, 1099, 1135}

$$-\frac{133x\left(2x^2+\sqrt{13}+5\right)}{1053\sqrt{x^4+5x^2+3}}+\frac{266\sqrt{x^4+5x^2+3}}{1053x}-\frac{5\sqrt{x^4+5x^2+3}}{351x^3}-\frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}-\frac{5\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+\cdots\right.}{351\sqrt{6\left(\cdots\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^4*(3 + 5*x^2 + x^4)^(3/2)),x]
```


```
[Out] (-133*x*(5 + Sqrt[13] + 2*x^2))/(1053*Sqrt[3 + 5*x^2 + x^4]) - (7 + 8*x^2)/
(39*x^3*Sqrt[3 + 5*x^2 + x^4]) - (5*Sqrt[3 + 5*x^2 + x^4])/(351*x^3) + (266
*Sqrt[3 + 5*x^2 + x^4])/(1053*x) + (133*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5
 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellipt
icE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(1053*Sqrt[3 +
 5*x^2 + x^4]) - (5*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(351*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```


Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
```

```
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1189

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1099

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[((2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]*EllipticF[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)/(b + q)])/(2*
a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1135

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[(x*(b + q + 2*c*x^2))/(2*c*Sqrt[a + b*x^2 + c*x^
4]), x] - Simp[(Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]*EllipticE[ArcTan[Rt[(b + q)/(2*a), 2]*x], (2*q)
/(b + q)])/(2*c*Sqrt[a + b*x^2 + c*x^4]), x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx = -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{1}{39}\int \frac{-5+24x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{3+5x^2+x^4}}{351x^3}+\frac{1}{351}\int \frac{-266-5x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{3+5x^2+x^4}}{351x^3}+\frac{266\sqrt{3+5x^2+x^4}}{1053x}-\frac{\int \frac{15+266x^2}{\sqrt{3+5x^2+x^4}}\,dx}{1053}$$

$$= -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{3+5x^2+x^4}}{351x^3}+\frac{266\sqrt{3+5x^2+x^4}}{1053x}-\frac{5}{351}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,d$$

$$= -\frac{133x\left(5+\sqrt{13}+2x^2\right)}{1053\sqrt{3+5x^2+x^4}}-\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{3+5x^2+x^4}}{351x^3}+\frac{266\sqrt{3+5x^2+x^4}}{1053x}+$$

Mathematica [C] time = 0.321328, size = 234, normalized size = 0.72

$$\frac{i\sqrt{2}\left(133\sqrt{13}-650\right)\sqrt{\frac{-2x^2+\sqrt{13}-5}{\sqrt{13}-5}}\sqrt{2x^2+\sqrt{13}+5}x^3\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right),\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+532x^6+2630x^4+}{2106x^3\sqrt{x^4+5x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/(x^4*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-468 + 1014*x^2 + 2630*x^4 + 532*x^6 - (133*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3
*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*E
llipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sq
rt[2]*(-650 + 133*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13]
)]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x]
, 19/6 + (5*Sqrt[13])/6])/(2106*x^3*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A] time = 0.02, size = 274, normalized size = 0.8

$$\frac{23}{81\,x}\sqrt{x^4+5\,x^2+3}-6\,\frac{1}{\sqrt{x^4+5\,x^2+3}}\left(\frac{19\,x^3}{234}+\frac{40\,x}{117}\right)-\frac{10}{117\,\sqrt{-30+6\,\sqrt{13}}}\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\mathrm{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x)
```

```
[Out] 23/81*(x^4+5*x^2+3)^(1/2)/x-6*(19/234*x^3+40/117*x)/(x^4+5*x^2+3)^(1/2)-10/
117/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*1
3^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1
/2),5/6*3^(1/2)+1/6*39^(1/2))+1064/117/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*
13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/
(13^(1/2)+5)*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/
2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-2/27*
(x^4+5*x^2+3)^(1/2)/x^3-4*(-40/351*x^3-343/702*x)/(x^4+5*x^2+3)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^4+5\,x^2+3}\left(3\,x^2+2\right)}{x^{12}+10\,x^{10}+31\,x^8+30\,x^6+9\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/(x^12 + 10*x^10 + 31*x^8 + 30*x^
6 + 9*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x^2+2}{x^4\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**4/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**4*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3\,x^2+2}{\left(x^4+5\,x^2+3\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4), x)
```

3.204 $\int (fx)^{3/2}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{9/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{9}{4};-\frac{1}{2},-\frac{1}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right.}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2, 9/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*e*(f*x)^(9/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[9/4, -1/2
, -1/2, 13/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(9*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.385335, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{9/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{9}{4};-\frac{1}{2},-\frac{1}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right.}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(3/2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (2*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2, 9/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*e*(f*x)^(9/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[9/4, -1/2
, -1/2, 13/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(9*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```


Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (fx)^{3/2}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx=\int\left(d(fx)^{3/2}\sqrt{a+bx^2+cx^4}+\frac{e(fx)^{7/2}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$

$$=d\int (fx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx+\frac{e\int (fx)^{7/2}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$

$$=\frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int (fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{\left(e\sqrt{a+bx^2\right.}}{}$$

$$=\frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2e(fx)^{9/2}}{}$$

Mathematica [A] time = 0.974693, size = 430, normalized size = 1.45

$$\frac{2f\sqrt{fx}\left(2x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\left(-79abce+130ac^2d-39b^2cd+21b\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^(3/2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*f*Sqrt[f*x]*(5*(a + b*x^2 + c*x^4)*(-14*b^2*e + 2*b*c*(13*d + 5*e*x^2) +
 c*(36*a*e + 65*c*d*x^2 + 45*c*e*x^4)) + 10*a*(-13*b*c*d + 7*b^2*e - 18*a*c
*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2,
 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4
*a*c])] + 2*(-39*b^2*c*d + 130*a*c^2*d + 21*b^3*e - 79*a*b*c*e)*x^2*Sqrt[(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4,
(-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(
2925*c^2*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int (fx)^{\frac{3}{2}}\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)\left(fx\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(efx^3 + dfx\right)\sqrt{cx^4 + bx^2 + a}\sqrt{fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*f*x^3 + d*f*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(fx\right)^{\frac{3}{2}}\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f*x)**(3/2)*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)\left(fx\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^(3/2), x)
```

3.205 $\int \sqrt{fx}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{7/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{7}{4};-\frac{1}{2},-\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2, 7/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*e*(f*x)^(7/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[7/4, -1/2
, -1/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(7*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.322517, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{7/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{7}{4};-\frac{1}{2},-\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f*x]*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2, 7/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*e*(f*x)^(7/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[7/4, -1/2
, -1/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(7*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sqrt{fx}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx = \int \left(d\sqrt{fx}\sqrt{a+bx^2+cx^4}+\frac{e(fx)^{5/2}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$

$$= d\int \sqrt{fx}\sqrt{a+bx^2+cx^4}\,dx+\frac{e\int (fx)^{5/2}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int \sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{\left(e\sqrt{a+bx^2+cx^4}\right)}{f^2\sqrt{}}$$

$$= \frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2e(fx)^{7/2}\sqrt{a+}}{}$$

Mathematica [A] time = 5.73658, size = 386, normalized size = 1.3

$$\frac{2x\sqrt{fx}\left(6x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\left(14ace-5b^2e+11bcd\right)+14a\sqrt{\frac{-\sqrt{b^2-4}}{b-\sqrt{}}}\right.}{1617c\sqrt{a+b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[f*x]*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*x*Sqrt[f*x]*(21*(11*c*d + 2*b*e + 7*c*e*x^2)*(a + b*x^2 + c*x^4) + 14*a*
(22*c*d - 3*b*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])] + 6*(11*b*c*d - 5*b^2*e + 14*a*c*e)*x^2*Sqrt[(b - Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(1617*c
*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.04, size = 0, normalized size = 0.

$$\int \sqrt{fx}\left(ex^2+d\right)\sqrt{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)\sqrt{fx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)\sqrt{fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{fx}\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f*x)*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.206 $\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx$

Optimal. Leaf size=295

$$\frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\right.}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2, 5/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])]) + (2*e*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1
/2, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(5*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.321355, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\right.}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[f*x],x]
```


```
[Out] (2*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2, 5/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])]) + (2*e*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1
/2, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(5*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])])
```


Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \int \left(\frac{d\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}+\frac{e(fx)^{3/2}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$

$$= d\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx+\frac{e\int (fx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{\sqrt{fx}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{\left(e\sqrt{a+bx^2+cx^4}\right)\int (fx)^{3/2}\sqrt{1}}{f^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}$$

$$= \frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2e(fx)^{5/2}\sqrt{a+bx}}{5f}$$

Mathematica [A] time = 0.746919, size = 386, normalized size = 1.31

$$\frac{2x\left(2x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\left(10ace-3b^2e+9bcd\right)+10a\sqrt{\frac{-\sqrt{b^2-4ac}+}{b-\sqrt{b^2-}}}\right.}{225c\sqrt{fx}\sqrt{a+b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[f*x],x]
```

```
[Out] (2*x*(5*(9*c*d + 2*b*e + 5*c*e*x^2)*(a + b*x^2 + c*x^4) + 10*a*(18*c*d - b*
e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2,
1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*
a*c])] + 2*(9*b*c*d - 3*b^2*e + 10*a*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(225*c*Sqrt[f*x]*Sqrt[a
 + b*x^2 + c*x^4])
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (ex^2+d)\sqrt{cx^4+bx^2+a}\frac{1}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x)
```

```
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/sqrt(f*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)\sqrt{fx}}{fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex^2)\sqrt{a + bx^2 + cx^4}}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(1/2)/(f*x)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/sqrt(f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/sqrt(f*x), x)
```

3.207 $\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx$

Optimal. Leaf size=295

$$\frac{2e(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2d\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\right.}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (-2*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]) + (2*e*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2,
-1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])])/(3*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.317338, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2e(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2d\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\right.}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(f*x)^(3/2),x]
```

```
[Out] (-2*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]) + (2*e*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2,
-1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])])/(3*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx = \int \left(\frac{d\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}+\frac{e\sqrt{fx}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$

$$= d\int \frac{\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx+\frac{e\int \sqrt{fx}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{(fx)^{3/2}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{\left(e\sqrt{a+bx^2+cx^4}\right)\int \sqrt{fx}\sqrt{1+\frac{}{b-}}}{f^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1}}$$

$$= -\frac{2d\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2e(fx)^{3/2}\sqrt{a+bx^2+c}}{3f^3\sqrt{1}}$$

Mathematica [A] time = 0.912697, size = 370, normalized size = 1.25

$$\frac{x\left(12x^4\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}(be+14cd)F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+28x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-}}{\sqrt{}}}\right.}{147(fx)^{3/2}\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(f*x)^(3/2),x]
```

```
[Out] (x*(-42*(7*d - e*x^2)*(a + b*x^2 + c*x^4) + 28*(7*b*d + 2*a*e)*x^2*Sqrt[(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (
-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 12
*(14*c*d + b*e)*x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])]))/(147*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)\sqrt{cx^4+bx^2+a}\,(fx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x)
```

```
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)}{\left(fx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)/(f*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)\sqrt{fx}}{f^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(f^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)\sqrt{a + bx^2 + cx^4}}{\left(fx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(1/2)/(f*x)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/(f*x)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.208 $\int (fx)^{3/2}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=299

$$\frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{9/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{9}{4};-\frac{3}{2},-\frac{3}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*a*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2, 9/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*
f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]) + (2*a*e*(f*x)^(9/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[9/4,
-3/2, -3/2, 13/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])])/(9*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.347235, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{9/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{9}{4};-\frac{3}{2},-\frac{3}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2, 9/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*
f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]) + (2*a*e*(f*x)^(9/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[9/4,
-3/2, -3/2, 13/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])])/(9*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (fx)^{3/2}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}dx=\int\left(d(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}+\frac{e(fx)^{7/2}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right)dx$$

$$=d\int (fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}dx+\frac{e\int (fx)^{7/2}\left(a+bx^2+cx^4\right)^{3/2}dx}{f^2}$$

$$=\frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int (fx)^{3/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{(a}{}$$

$$=\frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2ae(}{}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int (fx)^{\frac{3}{2}}\left(ex^2+d\right)\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)(fx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cefx^7 + (cd + be)fx^5 + (bd + ae)fx^3 + adfx\right)\sqrt{cx^4 + bx^2 + a}\sqrt{fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral((c*e*f*x^7 + (c*d + b*e)*f*x^5 + (b*d + a*e)*f*x^3 + a*d*f*x)*sqrt
(c*x^4 + b*x^2 + a)*sqrt(f*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)\left(fx\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^(3/2), x)
```

3.209 $\int \sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=299

$$\frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{7/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{7}{4};-\frac{3}{2},-\frac{3}{2};\frac{11}{4};-\frac{2}{b-\sqrt{}}\right.}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}}}$$

```
[Out] (2*a*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2, 7/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*
f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]) + (2*a*e*(f*x)^(7/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[7/4,
-3/2, -3/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])])/(7*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.354368, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{7/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{7}{4};-\frac{3}{2},-\frac{3}{2};\frac{11}{4};-\frac{2}{b-\sqrt{}}\right.}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2, 7/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*
f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]) + (2*a*e*(f*x)^(7/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[7/4,
-3/2, -3/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])])/(7*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}dx=\int\left(d\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}+\frac{e(fx)^{5/2}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right)dx$$

$$=d\int\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}dx+\frac{e\int(fx)^{5/2}\left(a+bx^2+cx^4\right)^{3/2}dx}{f^2}$$

$$=\frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int\sqrt{fx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{\left(ae\sqrt{a+\right.}{}$$

$$=\frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2ae(fx)^{7/}}{}$$

Mathematica [A] time = 6.19388, size = 490, normalized size = 1.64

$$\frac{2x\sqrt{fx}\left(12x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\left(420a^2c^2e-309ab^2ce+684abc^2d-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*Sqrt[f*x]*(7*(a + b*x^2 + c*x^4)*(-108*b^3*e + 12*b^2*c*(19*d + 7*e*x^
2) + b*c*(624*a*e + 7*c*x^2*(323*d + 231*e*x^2)) + c^2*(77*c*x^4*(19*d + 15
*e*x^2) + a*(3971*d + 2415*e*x^2))) + 28*a*(-57*b^2*c*d + 836*a*c^2*d + 27*
b^3*e - 156*a*b*c*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*A
ppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + 12*(-95*b^3*c*d + 684*a*b*c^2*d + 45*b^4*e - 309
*a*b^2*c*e + 420*a^2*c^2*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b -
 Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(153615*c^2*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int\sqrt{fx}\left(ex^2+d\right)\left(cx^4+bx^2+a\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} \left(ex^2 + d\right) \sqrt{fx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*sqrt(f*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cex^6 + (cd + be)x^4 + (bd + ae)x^2 + ad\right)\sqrt{cx^4 + bx^2 + a}\sqrt{fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 + b
*x^2 + a)*sqrt(f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{fx} \left(d + ex^2\right) \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(f*x)*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.210 $\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx$

Optimal. Leaf size=297

$$\frac{2ad\sqrt{fx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},\right.}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*a*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2, 5/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]) + (2*a*e*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2
, -3/2, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])])/(5*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.35108, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2ad\sqrt{fx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},\right.}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[f*x],x]
```

```
[Out] (2*a*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2, 5/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]) + (2*a*e*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2
, -3/2, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])])/(5*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx = \int \left(\frac{d(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}} + \frac{e(fx)^{3/2}(a+bx^2+cx^4)^{3/2}}{f^2}\right)dx$$

$$= d\int \frac{(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx + \frac{e\int (fx)^{3/2}(a+bx^2+cx^4)^{3/2}\,dx}{f^2}$$

$$= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{\sqrt{fx}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(ae\sqrt{a+bx^2+cx^4}\right)\int (f\ldots}{f^2\sqrt{1\ldots}}$$

$$= \frac{2ad\sqrt{fx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2ae(fx)^{5/2}\sqrt{a\ldots}}{\ldots}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[f*x],x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int (ex^2+d)\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\frac{1}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x)
```

```
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}(ex^2+d)}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/sqrt(f*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cex^6 + (cd + be)x^4 + (bd + ae)x^2 + ad\right)\sqrt{cx^4 + bx^2 + a}\sqrt{fx}}{fx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 + b
*x^2 + a)*sqrt(f*x)/(f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{\sqrt{fx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(3/2)/(f*x)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)/sqrt(f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)}{\sqrt{fx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/sqrt(f*x), x)
```

3.211 $\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx$

Optimal. Leaf size=297

$$\frac{2ae(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2ad\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},\dots\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (-2*a*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*a*e*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3
/2, -3/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])/(3*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.347938, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2ae(fx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2ad\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},\dots\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x]
```


```
[Out] (-2*a*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*a*e*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3
/2, -3/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])/(3*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```


Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \int \left(\frac{d(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}} + \frac{e\sqrt{fx}(a+bx^2+cx^4)^{3/2}}{f^2}\right)dx$$

$$= d\int \frac{(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx + \frac{e\int \sqrt{fx}(a+bx^2+cx^4)^{3/2}\,dx}{f^2}$$

$$= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{(fx)^{3/2}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(ae\sqrt{a+bx^2+cx^4}\right)\int \sqrt{fx}}{f^2\sqrt{1+\frac{}{b-}}}$$

$$= -\frac{2ad\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{2ae(fx)^{3/2}\sqrt{a+bx}}{3f}$$

Mathematica [A] time = 1.05684, size = 447, normalized size = 1.51

$$\frac{x\left(24x^4\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\left(36abce+420ac^2d+15b^2cd-5b^3e\right)-56a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x]
```

```
[Out] (x*(14*(a + b*x^2 + c*x^4)*(a*c*(-1155*d + 209*e*x^2) + x^2*(12*b^2*e + 7*c
^2*x^2*(15*d + 11*e*x^2) + b*c*(195*d + 119*e*x^2))) - 56*a*(-240*b*c*d + 3
*b^2*e - 44*a*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2
 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])] + 24*(15*b^2*c*d + 420*a*c^2*d - 5*b^3*e + 36*a*
b*c*e)*x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/
4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])]))/(8085*c*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int (ex^2+d)\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(fx\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x)
```

```
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)}{\left(fx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)/(f*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(cex^6 + (cd + be)x^4 + (bd + ae)x^2 + ad\right)\sqrt{cx^4 + bx^2 + a}\sqrt{fx}}{f^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 + b
*x^2 + a)*sqrt(f*x)/(f^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{\left(fx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(3/2)/(f*x)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)/(f*x)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.212 $\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}{9f^3\sqrt{a+}}$$

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[9/4, 1/2, 1/2, 13/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(9*
f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.328178, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}{9f^3\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^(3/2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[9/4, 1/2, 1/2, 13/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(9*
f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx=\int \left(\frac{d(fx)^{3/2}}{\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{7/2}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$

$$=d\int \frac{(fx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx+\frac{e\int \frac{(fx)^{7/2}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2}$$

$$=\frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{3/2}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\right.}{f}$$

$$=\frac{2d(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{}}{}$$

Mathematica [A] time = 0.62466, size = 354, normalized size = 1.19

$$\frac{2f\sqrt{fx}\left(x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}(5cd-3be)F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-5ae\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{}\right.}{25c\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^(3/2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*f*Sqrt[f*x]*(5*e*(a + b*x^2 + c*x^4) - 5*a*e*Sqrt[(b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (5*c*d - 3*b*e)*x^2*S
qrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2,
 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)]))/(25*c*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)\left(fx\right)^{\frac{3}{2}}\frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(fx)^{\frac{3}{2}}}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(efx^3 + dfx)\sqrt{fx}}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*f*x^3 + d*f*x)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^{\frac{3}{2}}(d + ex^2)}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f*x)**(3/2)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(fx)^{\frac{3}{2}}}{\sqrt{cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.213 $\int \frac{\sqrt{fx}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}}}{7f^3\sqrt{a}}$$

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*
f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.322115, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}}}{7f^3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f*x]*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*
f^3*Sqrt[a + b*x^2 + c*x^4])
```


Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{fx}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \left(\frac{d\sqrt{fx}}{\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{5/2}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$

$$= d\int \frac{\sqrt{fx}}{\sqrt{a+bx^2+cx^4}}\,dx+\frac{e\int \frac{(fx)^{5/2}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{fx}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c}{b+\sqrt{b}}}\right.}{f^2\sqrt{a}}$$

$$= \frac{2d(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{1+}}{}$$

Mathematica [A] time = 5.1534, size = 242, normalized size = 0.81

$$\frac{2\sqrt{fx}\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\left(7dxF_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+3ex^3F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4a}}\right.\right.}{21\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f*x]*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[f*x]*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(7*d*x*Ap
pellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])] + 3*e*x^3*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*Sqrt[a +
 b*x^2 + c*x^4])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (ex^2+d)\sqrt{fx}\frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)\sqrt{fx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)\sqrt{fx}}{\sqrt{cx^4+bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*x^2 + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{fx}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f*x)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.214 $\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=295

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F}{5f^3\sqrt{a+bx^2}}$$

```
[Out] (2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[a + b*x^2 +
c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f^3*S
qrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.325269, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F}{5f^3\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[a + b*x^2 +
c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f^3*S
qrt[a + b*x^2 + c*x^4])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}dx = \int \left(\frac{d}{\sqrt{fx}\sqrt{a+bx^2+cx^4}} + \frac{e(fx)^{3/2}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$

$$= d\int \frac{1}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}dx + \frac{e\int \frac{(fx)^{3/2}}{\sqrt{a+bx^2+cx^4}}dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}dx}{\sqrt{a+bx^2+cx^4}} + \frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\right.}{}$$

$$= \frac{2d\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{5/2}}{}$$

Mathematica [A] time = 0.222462, size = 241, normalized size = 0.82

$$\frac{2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\left(5dxF_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+ex^3F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\right)}{5\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/(Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(5*d*x*AppellF1[1/4
, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])] + e*x^3*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*Sqrt[f*x]*Sqrt[a + b*x
^2 + c*x^4])
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int (ex^2+d)\frac{1}{\sqrt{fx}}\frac{1}{\sqrt{cx^4+bx^2+a}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\sqrt{cx^4+bx^2+a}\sqrt{fx}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)\sqrt{fx}}{cfx^5 + bfx^3 + afx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c*f*x^5 + b*f*x^3 +
 a*f*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{\sqrt{fx}\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(f*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/(sqrt(f*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + bx^2 + a}\sqrt{fx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)), x)
```

3.215 $\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}} dx$

Optimal. Leaf size=295

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1(}{f\sqrt{fx}\sqrt{a+}}$$

```
[Out] (-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*Sqrt[a + b*x^2
+ c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f^3
*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.324836, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1(}{f\sqrt{fx}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/((f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```


```
[Out] (-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*Sqrt[a + b*x^2
+ c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f^3
*Sqrt[a + b*x^2 + c*x^4])
```


Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx = \int \left(\frac{d}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}} + \frac{e\sqrt{fx}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$

$$= d\int \frac{1}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx + \frac{e\int \frac{\sqrt{fx}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}} + \frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4a}}}\right.}{}$$

$$= -\frac{2d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{3/2}\sqrt{\cdots}}{}$$

Mathematica [A] time = 0.685725, size = 356, normalized size = 1.21

$$\frac{2x\left(7x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}(ae+bd)F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+9cdx^4\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4a\cdots}}{\sqrt{b^2\cdots}}}\cdots\right.}{21a(fx)^{3/2}\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/((f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*x*(-21*d*(a + b*x^2 + c*x^4) + 7*(b*d + a*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 9*c*d*x^4*Sqrt[(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4,
 (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/
(21*a*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.037, size = 0, normalized size = 0.

$$\int (ex^2+d)(fx)^{-\frac{3}{2}}\frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + bx^2 + a}\,(fx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)\sqrt{fx}}{cf^2x^6 + bf^2x^4 + af^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c*f^2*x^6 + b*f^2*x
^4 + a*f^2*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex^2}{(fx)^{\frac{3}{2}}\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(f*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/((f*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\sqrt{cx^4 + bx^2 + a}\,(fx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)), x)
```

3.216 $\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=303

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}{9af^3\sqrt{a+}}$$

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*f*Sqrt[a + b*
x^2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[9/4, 3/2, 3/2, 13/4,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
9*a*f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.34412, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}{9af^3\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*f*Sqrt[a + b*
x^2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[9/4, 3/2, 3/2, 13/4,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
9*a*f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \left(\frac{d(fx)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}+\frac{e(fx)^{7/2}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$= d\int \frac{(fx)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx+\frac{e\int \frac{(fx)^{7/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{3/2}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\right.}{}$$

$$= \frac{2d(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}}{}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)(fx)^{\frac{3}{2}}\left(cx^4+bx^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)(fx)^{\frac{3}{2}}}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(efx^3 + dfx\right)\sqrt{cx^4 + bx^2 + a}\sqrt{fx}}{c^2x^8 + 2\,bcx^6 + \left(b^2 + 2\,ac\right)x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral((e*f*x^3 + d*f*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)/(c^2*x^8 + 2*b
*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)\left(fx\right)^{\frac{3}{2}}}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.217 $\int \frac{\sqrt{fx}\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=303

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}\cdots}}{7af^3\sqrt{\cdots}}$$

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*f*Sqrt[a + b*
x^2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 3/2, 3/2, 11/4,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
7*a*f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.348842, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}\cdots}}{7af^3\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f*x]*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*f*Sqrt[a + b*
x^2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 3/2, 3/2, 11/4,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
7*a*f^3*Sqrt[a + b*x^2 + c*x^4])
```


Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{fx}\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \left(\frac{d\sqrt{fx}}{\left(a+bx^2+cx^4\right)^{3/2}} + \frac{e(fx)^{5/2}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$= d\int \frac{\sqrt{fx}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx + \frac{e\int \frac{(fx)^{5/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{fx}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}} + \frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1-\right.}{}$$

$$= \frac{2d(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)^{7/2}\sqrt{1}}{}$$

Mathematica [A] time = 5.75118, size = 397, normalized size = 1.31

$$\frac{x\sqrt{fx}\left(7\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\left(-3abe+2acd+b^2d\right)+9cx^2\sqrt{\frac{-\sqrt{b^2-4ac}+b}{b-\sqrt{b^2-4a}}}\right.}{21a\left(4ac-b^2\right)\sqrt{a+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[f*x]*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[f*x]*(-21*b^2*d + 21*b*(a*e - c*d*x^2) + 42*a*c*(d + e*x^2) + 7*(b^
2*d + 2*a*c*d - 3*a*b*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])] + 9*c*(b*d - 2*a*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*a*(-b^2 + 4
*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int (ex^2+d)\sqrt{fx}\left(cx^4+bx^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)\sqrt{fx}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x^2 + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}(ex^2+d)\sqrt{fx}}{c^2x^8+2\,bcx^6+(b^2+2\,ac)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c^2*x^8 + 2*b*c*x^6
 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)\sqrt{fx}}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.218 $\int \frac{d+ex^2}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=301

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F\cdots}{5af^3\sqrt{a+bx\cdots}}$$

```
[Out] (2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*Sqrt[a + b*x^2
+ c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*f
^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.349815, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F\cdots}{5af^3\sqrt{a+bx\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*Sqrt[a + b*x^2
+ c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*f
^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \left(\frac{d}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}} + \frac{e(fx)^{3/2}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$= d\int \frac{1}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx + \frac{e\int \frac{(fx)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{fx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}} + \frac{\left(e\sqrt{1+\frac{}{b-}}\right.}{}$$

$$= \frac{2d\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{a+bx^2+cx^4}} + \frac{2e(fx)}{}$$

Mathematica [F] time = 0, size = 0, normalized size = 0.

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^2)/(Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)),x]

[Out] $Aborted
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int (ex^2+d)\frac{1}{\sqrt{fx}}\left(cx^4+bx^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)

[Out] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\sqrt{fx}}{c^2fx^9+2\,bcfx^7+\left(b^2+2\,ac\right)fx^5+2\,abfx^3+a^2fx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c^2*f*x^9 + 2*b*c*f
*x^7 + (b^2 + 2*a*c)*f*x^5 + 2*a*b*f*x^3 + a^2*f*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(f*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2+d}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)), x)
```

3.219 $\int \frac{d+ex^2}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=301

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\right.}{af\sqrt{fx}\sqrt{a+}}$$

```
[Out] (-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*Sqrt[f*x]*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a
*f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.351231, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1335, 1141, 510}

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\right.}{af\sqrt{fx}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] (-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*Sqrt[f*x]*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a
*f^3*Sqrt[a + b*x^2 + c*x^4])
```


Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx=\int\left(\frac{d}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}+\frac{e\sqrt{fx}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$=d\int\frac{1}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx+\frac{e\int\frac{\sqrt{fx}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx}{f^2}$$

$$=\frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int\frac{1}{(fx)^{3/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}+\frac{\left(e\sqrt{1+\frac{}{b-}}\right.}{}$$

$$=-\frac{2d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{fx}\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{3/2}}{}$$

Mathematica [A] time = 1.05257, size = 460, normalized size = 1.53

$$-\frac{x\left(7x^2\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\left(2a^2ce+ab^2e+9abcd-3b^3d\right)-21\left(a^2c\left(8\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -(x*(-21*(-3*b^2*d*x^2*(b + c*x^2) + a^2*c*(8*d - 2*e*x^2) + a*(10*c^2*d*x^
4 + b^2*(-2*d + e*x^2) + b*c*x^2*(11*d + e*x^2))) + 7*(-3*b^3*d + 9*a*b*c*d
 + a*b^2*e + 2*a^2*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 9*c*(3*b^2*d - 10*a*c*d - a*b*e)*x^4*Sqrt
[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11
/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
))/(21*a^2*(b^2 - 4*a*c)*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \left(ex^2+d\right)\left(fx\right)^{-\frac{3}{2}}\left(cx^4+bx^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} \left(fx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}\left(ex^2 + d\right)\sqrt{fx}}{c^2f^2x^{10} + 2\,bcf^2x^8 + \left(b^2 + 2\,ac\right)f^2x^6 + 2\,abf^2x^4 + a^2f^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*sqrt(f*x)/(c^2*f^2*x^10 + 2*b*
c*f^2*x^8 + (b^2 + 2*a*c)*f^2*x^6 + 2*a*b*f^2*x^4 + a^2*f^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(f*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^2 + d}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} \left(fx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)), x)
```

3.220 $\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=243

$$\frac{(fx)^{m+7} \left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{f^7(m+7)}+\frac{a^2(fx)^{m+3}(ae+3bd)}{f^3(m+3)}+\frac{a^3d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+9} \left(6abce+3ac^2d+3b^2cd+b^3e\right)}{f^9(m+9)}$$

```
[Out] (a^3*d*(f*x)^(1 + m))/(f*(1 + m)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m))/(f^3*
(3 + m)) + (3*a*(b^2*d + a*c*d + a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((b^
3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + ((3
*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (3
*c*(b*c*d + b^2*e + a*c*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (c^2*(c*d + 3*
b*e)*(f*x)^(13 + m))/(f^13*(13 + m)) + (c^3*e*(f*x)^(15 + m))/(f^15*(15 + m
))
```

Rubi [A] time = 0.176117, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1261}

$$\frac{(fx)^{m+7} \left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{f^7(m+7)}+\frac{a^2(fx)^{m+3}(ae+3bd)}{f^3(m+3)}+\frac{a^3d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+9} \left(6abce+3ac^2d+3b^2cd+b^3e\right)}{f^9(m+9)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```


```
[Out] (a^3*d*(f*x)^(1 + m))/(f*(1 + m)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m))/(f^3*
(3 + m)) + (3*a*(b^2*d + a*c*d + a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((b^
3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + ((3
*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (3
*c*(b*c*d + b^2*e + a*c*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (c^2*(c*d + 3*
b*e)*(f*x)^(13 + m))/(f^13*(13 + m)) + (c^3*e*(f*x)^(15 + m))/(f^15*(15 + m
))
```


Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^3 \, dx = \int \left(a^3d(fx)^m+\frac{a^2(3bd+ae)(fx)^{2+m}}{f^2}+\frac{3a\left(b^2d+acd+abe\right)(fx)^{4+m}}{f^4}+\frac{\left(b^3d+\right.}{}\right.$$

$$=\frac{a^3d(fx)^{1+m}}{f(1+m)}+\frac{a^2(3bd+ae)(fx)^{3+m}}{f^3(3+m)}+\frac{3a\left(b^2d+acd+abe\right)(fx)^{5+m}}{f^5(5+m)}+\frac{\left(b^3d+6\right.}{}$$

Mathematica [A] time = 0.341344, size = 191, normalized size = 0.79

$$x(fx)^m\left(\frac{x^6\left(3a^2ce+3ab^2e+6abcd+b^3d\right)}{m+7}+\frac{a^2x^2(ae+3bd)}{m+3}+\frac{a^3d}{m+1}+\frac{x^8\left(6abce+3ac^2d+3b^2cd+b^3e\right)}{m+9}+\frac{3cx^{10}\left(ace+\right.}{m+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] x*(f*x)^m*((a^3*d)/(1 + m) + (a^2*(3*b*d + a*e)*x^2)/(3 + m) + (3*a*(b^2*d
+ a*c*d + a*b*e)*x^4)/(5 + m) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e
)*x^6)/(7 + m) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^8)/(9 + m)
+ (3*c*(b*c*d + b^2*e + a*c*e)*x^10)/(11 + m) + (c^2*(c*d + 3*b*e)*x^12)/(1
3 + m) + (c^3*e*x^14)/(15 + m))
```

Maple [B] time = 0.008, size = 1935, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] x*(c^3*e*m^7*x^14+49*c^3*e*m^6*x^14+3*b*c^2*e*m^7*x^12+c^3*d*m^7*x^12+973*c
^3*e*m^5*x^14+153*b*c^2*e*m^6*x^12+51*c^3*d*m^6*x^12+10045*c^3*e*m^4*x^14+3
*a*c^2*e*m^7*x^10+3*b^2*c*e*m^7*x^10+3*b*c^2*d*m^7*x^10+3135*b*c^2*e*m^5*x^
12+1045*c^3*d*m^5*x^12+57379*c^3*e*m^3*x^14+159*a*c^2*e*m^6*x^10+159*b^2*c*
e*m^6*x^10+159*b*c^2*d*m^6*x^10+33165*b*c^2*e*m^4*x^12+11055*c^3*d*m^4*x^12
+177331*c^3*e*m^2*x^14+6*a*b*c*e*m^7*x^8+3*a*c^2*d*m^7*x^8+3375*a*c^2*e*m^5
*x^10+b^3*e*m^7*x^8+3*b^2*c*d*m^7*x^8+3375*b^2*c*e*m^5*x^10+3375*b*c^2*d*m^
5*x^10+193017*b*c^2*e*m^3*x^12+64339*c^3*d*m^3*x^12+264207*c^3*e*m*x^14+330
*a*b*c*e*m^6*x^8+165*a*c^2*d*m^6*x^8+36795*a*c^2*e*m^4*x^10+55*b^3*e*m^6*x^
8+165*b^2*c*d*m^6*x^8+36795*b^2*c*e*m^4*x^10+36795*b*c^2*d*m^4*x^10+604827*
b*c^2*e*m^2*x^12+201609*c^3*d*m^2*x^12+135135*c^3*e*x^14+3*a^2*c*e*m^7*x^6+
3*a*b^2*e*m^7*x^6+6*a*b*c*d*m^7*x^6+7278*a*b*c*e*m^5*x^8+3639*a*c^2*d*m^5*x
^8+219417*a*c^2*e*m^3*x^10+b^3*d*m^7*x^6+1213*b^3*e*m^5*x^8+3639*b^2*c*d*m^
5*x^8+219417*b^2*c*e*m^3*x^10+219417*b*c^2*d*m^3*x^10+909765*b*c^2*e*m*x^12
+303255*c^3*d*m*x^12+171*a^2*c*e*m^6*x^6+171*a*b^2*e*m^6*x^6+342*a*b*c*d*m^
6*x^6+82338*a*b*c*e*m^4*x^8+41169*a*c^2*d*m^4*x^8+700461*a*c^2*e*m^2*x^10+5
7*b^3*d*m^6*x^6+13723*b^3*e*m^4*x^8+41169*b^2*c*d*m^4*x^8+700461*b^2*c*e*m^
2*x^10+700461*b*c^2*d*m^2*x^10+467775*b*c^2*e*x^12+155925*c^3*d*x^12+3*a^2*
b*e*m^7*x^4+3*a^2*c*d*m^7*x^4+3927*a^2*c*e*m^5*x^6+3*a*b^2*d*m^7*x^4+3927*a
*b^2*e*m^5*x^6+7854*a*b*c*d*m^5*x^6+507282*a*b*c*e*m^3*x^8+253641*a*c^2*d*m
^3*x^8+1067445*a*c^2*e*m*x^10+1309*b^3*d*m^5*x^6+84547*b^3*e*m^3*x^8+253641
*b^2*c*d*m^3*x^8+1067445*b^2*c*e*m*x^10+1067445*b*c^2*d*m*x^10+177*a^2*b*e*
m^6*x^4+177*a^2*c*d*m^6*x^4+46431*a^2*c*e*m^4*x^6+177*a*b^2*d*m^6*x^4+46431
*a*b^2*e*m^4*x^6+92862*a*b*c*d*m^4*x^6+1662558*a*b*c*e*m^2*x^8+831279*a*c^2
*d*m^2*x^8+552825*a*c^2*e*x^10+15477*b^3*d*m^4*x^6+277093*b^3*e*m^2*x^8+831
279*b^2*c*d*m^2*x^8+552825*b^2*c*e*x^10+552825*b*c^2*d*x^10+a^3*e*m^7*x^2+3
*a^2*b*d*m^7*x^2+4239*a^2*b*e*m^5*x^4+4239*a^2*c*d*m^5*x^4+299145*a^2*c*e*m
^3*x^6+4239*a*b^2*d*m^5*x^4+299145*a*b^2*e*m^3*x^6+598290*a*b*c*d*m^3*x^6+2
582010*a*b*c*e*m*x^8+1291005*a*c^2*d*m*x^8+99715*b^3*d*m^3*x^6+430335*b^3*e
*m*x^8+1291005*b^2*c*d*m*x^8+61*a^3*e*m^6*x^2+183*a^2*b*d*m^6*x^2+52725*a^2
*b*e*m^4*x^4+52725*a^2*c*d*m^4*x^4+1020033*a^2*c*e*m^2*x^6+52725*a*b^2*d*m^
4*x^4+1020033*a*b^2*e*m^2*x^6+2040066*a*b*c*d*m^2*x^6+1351350*a*b*c*e*x^8+6
75675*a*c^2*d*x^8+340011*b^3*d*m^2*x^6+225225*b^3*e*x^8+675675*b^2*c*d*x^8+
a^3*d*m^7+1525*a^3*e*m^5*x^2+4575*a^2*b*d*m^5*x^2+360537*a^2*b*e*m^3*x^4+36
0537*a^2*c*d*m^3*x^4+1632285*a^2*c*e*m*x^6+360537*a*b^2*d*m^3*x^4+1632285*a
*b^2*e*m*x^6+3264570*a*b*c*d*m*x^6+544095*b^3*d*m*x^6+63*a^3*d*m^6+20065*a^
3*e*m^4*x^2+60195*a^2*b*d*m^4*x^2+1311363*a^2*b*e*m^2*x^4+1311363*a^2*c*d*m
^2*x^4+868725*a^2*c*e*x^6+1311363*a*b^2*d*m^2*x^4+868725*a*b^2*e*x^6+173745
0*a*b*c*d*x^6+289575*b^3*d*x^6+1645*a^3*d*m^5+147859*a^3*e*m^3*x^2+443577*a
```

```
^2*b*d*m^3*x^2+2215701*a^2*b*e*m*x^4+2215701*a^2*c*d*m*x^4+2215701*a*b^2*d*
m*x^4+22995*a^3*d*m^4+594439*a^3*e*m^2*x^2+1783317*a^2*b*d*m^2*x^2+1216215*
a^2*b*e*x^4+1216215*a^2*c*d*x^4+1216215*a*b^2*d*x^4+185059*a^3*d*m^3+114085
5*a^3*e*m*x^2+3422565*a^2*b*d*m*x^2+852957*a^3*d*m^2+675675*a^3*e*x^2+20270
25*a^2*b*d*x^2+2071215*a^3*d*m+2027025*a^3*d)*(f*x)^m/(1+m)/(3+m)/(5+m)/(7+
m)/(9+m)/(11+m)/(13+m)/(15+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.47846, size = 3271, normalized size = 13.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((c^3*e*m^7 + 49*c^3*e*m^6 + 973*c^3*e*m^5 + 10045*c^3*e*m^4 + 57379*c^3*e*
m^3 + 177331*c^3*e*m^2 + 264207*c^3*e*m + 135135*c^3*e)*x^15 + ((c^3*d + 3*
b*c^2*e)*m^7 + 51*(c^3*d + 3*b*c^2*e)*m^6 + 1045*(c^3*d + 3*b*c^2*e)*m^5 +
11055*(c^3*d + 3*b*c^2*e)*m^4 + 155925*c^3*d + 467775*b*c^2*e + 64339*(c^3*
d + 3*b*c^2*e)*m^3 + 201609*(c^3*d + 3*b*c^2*e)*m^2 + 303255*(c^3*d + 3*b*c
^2*e)*m)*x^13 + 3*((b*c^2*d + (b^2*c + a*c^2)*e)*m^7 + 53*(b*c^2*d + (b^2*c
 + a*c^2)*e)*m^6 + 1125*(b*c^2*d + (b^2*c + a*c^2)*e)*m^5 + 12265*(b*c^2*d
+ (b^2*c + a*c^2)*e)*m^4 + 184275*b*c^2*d + 73139*(b*c^2*d + (b^2*c + a*c^2
)*e)*m^3 + 233487*(b*c^2*d + (b^2*c + a*c^2)*e)*m^2 + 184275*(b^2*c + a*c^2
)*e + 355815*(b*c^2*d + (b^2*c + a*c^2)*e)*m)*x^11 + ((3*(b^2*c + a*c^2)*d
+ (b^3 + 6*a*b*c)*e)*m^7 + 55*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^6
 + 1213*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^5 + 13723*(3*(b^2*c + a
*c^2)*d + (b^3 + 6*a*b*c)*e)*m^4 + 84547*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*
b*c)*e)*m^3 + 277093*(3*(b^2*c + a*c^2)*d + (b^3 + 6*a*b*c)*e)*m^2 + 675675
*(b^2*c + a*c^2)*d + 225225*(b^3 + 6*a*b*c)*e + 430335*(3*(b^2*c + a*c^2)*d
 + (b^3 + 6*a*b*c)*e)*m)*x^9 + (((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m
^7 + 57*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^6 + 1309*((b^3 + 6*a*b*
c)*d + 3*(a*b^2 + a^2*c)*e)*m^5 + 15477*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2
*c)*e)*m^4 + 99715*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^3 + 340011*(
(b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*m^2 + 289575*(b^3 + 6*a*b*c)*d + 8
68725*(a*b^2 + a^2*c)*e + 544095*((b^3 + 6*a*b*c)*d + 3*(a*b^2 + a^2*c)*e)*
m)*x^7 + 3*((a^2*b*e + (a*b^2 + a^2*c)*d)*m^7 + 59*(a^2*b*e + (a*b^2 + a^2*
c)*d)*m^6 + 1413*(a^2*b*e + (a*b^2 + a^2*c)*d)*m^5 + 17575*(a^2*b*e + (a*b^
2 + a^2*c)*d)*m^4 + 405405*a^2*b*e + 120179*(a^2*b*e + (a*b^2 + a^2*c)*d)*m
^3 + 437121*(a^2*b*e + (a*b^2 + a^2*c)*d)*m^2 + 405405*(a*b^2 + a^2*c)*d +
738567*(a^2*b*e + (a*b^2 + a^2*c)*d)*m)*x^5 + ((3*a^2*b*d + a^3*e)*m^7 + 61
*(3*a^2*b*d + a^3*e)*m^6 + 1525*(3*a^2*b*d + a^3*e)*m^5 + 20065*(3*a^2*b*d
+ a^3*e)*m^4 + 2027025*a^2*b*d + 675675*a^3*e + 147859*(3*a^2*b*d + a^3*e)*
m^3 + 594439*(3*a^2*b*d + a^3*e)*m^2 + 1140855*(3*a^2*b*d + a^3*e)*m)*x^3 +
 (a^3*d*m^7 + 63*a^3*d*m^6 + 1645*a^3*d*m^5 + 22995*a^3*d*m^4 + 185059*a^3*
```

```
d*m^3 + 852957*a^3*d*m^2 + 2071215*a^3*d*m + 2027025*a^3*d)*x)*(f*x)^m/(m^8
 + 64*m^7 + 1708*m^6 + 24640*m^5 + 208054*m^4 + 1038016*m^3 + 2924172*m^2 +
 4098240*m + 2027025)
```

Sympy [A] time = 14.7837, size = 11538, normalized size = 47.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Piecewise(((-a**3*d/(14*x**14) - a**3*e/(12*x**12) - a**2*b*d/(4*x**12) - 3
*a**2*b*e/(10*x**10) - 3*a**2*c*d/(10*x**10) - 3*a**2*c*e/(8*x**8) - 3*a*b*
*2*d/(10*x**10) - 3*a*b**2*e/(8*x**8) - 3*a*b*c*d/(4*x**8) - a*b*c*e/x**6 -
 a*c**2*d/(2*x**6) - 3*a*c**2*e/(4*x**4) - b**3*d/(8*x**8) - b**3*e/(6*x**6
) - b**2*c*d/(2*x**6) - 3*b**2*c*e/(4*x**4) - 3*b*c**2*d/(4*x**4) - 3*b*c**
2*e/(2*x**2) - c**3*d/(2*x**2) + c**3*e*log(x))/f**15, Eq(m, -15)), ((-a**3
*d/(12*x**12) - a**3*e/(10*x**10) - 3*a**2*b*d/(10*x**10) - 3*a**2*b*e/(8*x
**8) - 3*a**2*c*d/(8*x**8) - a**2*c*e/(2*x**6) - 3*a*b**2*d/(8*x**8) - a*b*
*2*e/(2*x**6) - a*b*c*d/x**6 - 3*a*b*c*e/(2*x**4) - 3*a*c**2*d/(4*x**4) - 3
*a*c**2*e/(2*x**2) - b**3*d/(6*x**6) - b**3*e/(4*x**4) - 3*b**2*c*d/(4*x**4
) - 3*b**2*c*e/(2*x**2) - 3*b*c**2*d/(2*x**2) + 3*b*c**2*e*log(x) + c**3*d*
log(x) + c**3*e*x**2/2)/f**13, Eq(m, -13)), ((-a**3*d/(10*x**10) - a**3*e/(
8*x**8) - 3*a**2*b*d/(8*x**8) - a**2*b*e/(2*x**6) - a**2*c*d/(2*x**6) - 3*a
**2*c*e/(4*x**4) - a*b**2*d/(2*x**6) - 3*a*b**2*e/(4*x**4) - 3*a*b*c*d/(2*x
**4) - 3*a*b*c*e/x**2 - 3*a*c**2*d/(2*x**2) + 3*a*c**2*e*log(x) - b**3*d/(4
*x**4) - b**3*e/(2*x**2) - 3*b**2*c*d/(2*x**2) + 3*b**2*c*e*log(x) + 3*b*c*
*2*d*log(x) + 3*b*c**2*e*x**2/2 + c**3*d*x**2/2 + c**3*e*x**4/4)/f**11, Eq(
m, -11)), ((-a**3*d/(8*x**8) - a**3*e/(6*x**6) - a**2*b*d/(2*x**6) - 3*a**2
*b*e/(4*x**4) - 3*a**2*c*d/(4*x**4) - 3*a**2*c*e/(2*x**2) - 3*a*b**2*d/(4*x
**4) - 3*a*b**2*e/(2*x**2) - 3*a*b*c*d/x**2 + 6*a*b*c*e*log(x) + 3*a*c**2*d
*log(x) + 3*a*c**2*e*x**2/2 - b**3*d/(2*x**2) + b**3*e*log(x) + 3*b**2*c*d*
log(x) + 3*b**2*c*e*x**2/2 + 3*b*c**2*d*x**2/2 + 3*b*c**2*e*x**4/4 + c**3*d
*x**4/4 + c**3*e*x**6/6)/f**9, Eq(m, -9)), ((-a**3*d/(6*x**6) - a**3*e/(4*x
**4) - 3*a**2*b*d/(4*x**4) - 3*a**2*b*e/(2*x**2) - 3*a**2*c*d/(2*x**2) + 3*
a**2*c*e*log(x) - 3*a*b**2*d/(2*x**2) + 3*a*b**2*e*log(x) + 6*a*b*c*d*log(x
) + 3*a*b*c*e*x**2 + 3*a*c**2*d*x**2/2 + 3*a*c**2*e*x**4/4 + b**3*d*log(x)
+ b**3*e*x**2/2 + 3*b**2*c*d*x**2/2 + 3*b**2*c*e*x**4/4 + 3*b*c**2*d*x**4/4
 + b*c**2*e*x**6/2 + c**3*d*x**6/6 + c**3*e*x**8/8)/f**7, Eq(m, -7)), ((-a*
*3*d/(4*x**4) - a**3*e/(2*x**2) - 3*a**2*b*d/(2*x**2) + 3*a**2*b*e*log(x) +
 3*a**2*c*d*log(x) + 3*a**2*c*e*x**2/2 + 3*a*b**2*d*log(x) + 3*a*b**2*e*x**
2/2 + 3*a*b*c*d*x**2 + 3*a*b*c*e*x**4/2 + 3*a*c**2*d*x**4/4 + a*c**2*e*x**6
/2 + b**3*d*x**2/2 + b**3*e*x**4/4 + 3*b**2*c*d*x**4/4 + b**2*c*e*x**6/2 +
b*c**2*d*x**6/2 + 3*b*c**2*e*x**8/8 + c**3*d*x**8/8 + c**3*e*x**10/10)/f**5
, Eq(m, -5)), ((-a**3*d/(2*x**2) + a**3*e*log(x) + 3*a**2*b*d*log(x) + 3*a*
*2*b*e*x**2/2 + 3*a**2*c*d*x**2/2 + 3*a**2*c*e*x**4/4 + 3*a*b**2*d*x**2/2 +
 3*a*b**2*e*x**4/4 + 3*a*b*c*d*x**4/2 + a*b*c*e*x**6 + a*c**2*d*x**6/2 + 3*
a*c**2*e*x**8/8 + b**3*d*x**4/4 + b**3*e*x**6/6 + b**2*c*d*x**6/2 + 3*b**2*
c*e*x**8/8 + 3*b*c**2*d*x**8/8 + 3*b*c**2*e*x**10/10 + c**3*d*x**10/10 + c*
*3*e*x**12/12)/f**3, Eq(m, -3)), ((a**3*d*log(x) + a**3*e*x**2/2 + 3*a**2*b
*d*x**2/2 + 3*a**2*b*e*x**4/4 + 3*a**2*c*d*x**4/4 + a**2*c*e*x**6/2 + 3*a*b
**2*d*x**4/4 + a*b**2*e*x**6/2 + a*b*c*d*x**6 + 3*a*b*c*e*x**8/4 + 3*a*c**2
*d*x**8/8 + 3*a*c**2*e*x**10/10 + b**3*d*x**6/6 + b**3*e*x**8/8 + 3*b**2*c*
d*x**8/8 + 3*b**2*c*e*x**10/10 + 3*b*c**2*d*x**10/10 + b*c**2*e*x**12/4 + c
**3*d*x**12/12 + c**3*e*x**14/14)/f, Eq(m, -1)), (a**3*d*f**m*m**7*x*x**m/(
m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924
```

172*m**2 + 4098240*m + 2027025) + 63*a**3*d*f**m*m**6*x*x**m/(m**8 + 64*m**
7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40
98240*m + 2027025) + 1645*a**3*d*f**m*m**5*x*x**m/(m**8 + 64*m**7 + 1708*m*
*6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2
027025) + 22995*a**3*d*f**m*m**4*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640
*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
185059*a**3*d*f**m*m**3*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2
08054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 852957*a*
*3*d*f**m*m**2*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2071215*a**3*d*f**
m*m*x*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016
*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2027025*a**3*d*f**m*x*x**m/(m
8 + 64*m7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29241
72*m**2 + 4098240*m + 2027025) + a**3*e*f**m*m**7*x**3*x**m/(m**8 + 64*m**7
 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409
8240*m + 2027025) + 61*a**3*e*f**m*m**6*x**3*x**m/(m**8 + 64*m**7 + 1708*m*
*6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2
027025) + 1525*a**3*e*f**m*m**5*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 246
40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 20065*a**3*e*f**m*m**4*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 14785
9*a**3*e*f**m*m**3*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208
054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 594439*a**3
*e*f**m*m**2*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1140855*a**3*e*f*
*m*m*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103
8016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 675675*a**3*e*f**m*x**3*x
m/(m8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
 2924172*m**2 + 4098240*m + 2027025) + 3*a**2*b*d*f**m*m**7*x**3*x**m/(m**8
 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*
m**2 + 4098240*m + 2027025) + 183*a**2*b*d*f**m*m**6*x**3*x**m/(m**8 + 64*m
7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 4575*a**2*b*d*f**m*m**5*x**3*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409824
0*m + 2027025) + 60195*a**2*b*d*f**m*m**4*x**3*x**m/(m**8 + 64*m**7 + 1708*
m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
 2027025) + 443577*a**2*b*d*f**m*m**3*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6
 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202
7025) + 1783317*a**2*b*d*f**m*m**2*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 3422565*a**2*b*d*f**m*m*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2
027025*a**2*b*d*f**m*x**3*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2
08054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a**2*b*
e*f**m*m**7*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 177*a**2*b*e*f**m*
m**6*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103
8016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 4239*a**2*b*e*f**m*m**5*x
5*xm/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m
3 + 2924172*m2 + 4098240*m + 2027025) + 52725*a**2*b*e*f**m*m**4*x**5*x
m/(m8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
 2924172*m**2 + 4098240*m + 2027025) + 360537*a**2*b*e*f**m*m**3*x**5*x**m/
(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292
4172*m**2 + 4098240*m + 2027025) + 1311363*a**2*b*e*f**m*m**2*x**5*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 2215701*a**2*b*e*f**m*m*x**5*x**m/(m**8 + 6
4*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2
 + 4098240*m + 2027025) + 1216215*a**2*b*e*f**m*x**5*x**m/(m**8 + 64*m**7 +
 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982

40*m + 2027025) + 3*a**2*c*d*f**m*m**7*x**5*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 177*a**2*c*d*f**m*m**6*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 246
40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 4239*a**2*c*d*f**m*m**5*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 5272
5*a**2*c*d*f**m*m**4*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2
08054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 360537*a*
*2*c*d*f**m*m**3*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20805
4*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1311363*a**2*
c*d*f**m*m**2*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m
4 + 1038016*m3 + 2924172*m**2 + 4098240*m + 2027025) + 2215701*a**2*c*d
*f**m*m*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 +
1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1216215*a**2*c*d*f**m*
x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*
m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a**2*c*e*f**m*m**7*x**7*x**m
/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29
24172*m**2 + 4098240*m + 2027025) + 171*a**2*c*e*f**m*m**6*x**7*x**m/(m**8
+ 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m
2 + 4098240*m + 2027025) + 3927*a2*c*e*f**m*m**5*x**7*x**m/(m**8 + 64*m
7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 46431*a**2*c*e*f**m*m**4*x**7*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 299145*a**2*c*e*f**m*m**3*x**7*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
+ 2027025) + 1020033*a**2*c*e*f**m*m**2*x**7*x**m/(m**8 + 64*m**7 + 1708*m
6 + 24640*m5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 1632285*a**2*c*e*f**m*m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 868725*a**2*c*e*f**m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*a*
b**2*d*f**m*m**7*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20805
4*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 177*a*b**2*d*
f**m*m**6*x**5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4
+ 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 4239*a*b**2*d*f**m*m
5*x5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038
016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 52725*a*b**2*d*f**m*m**4*x
5*xm/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m
3 + 2924172*m2 + 4098240*m + 2027025) + 360537*a*b**2*d*f**m*m**3*x**5*
x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3
+ 2924172*m**2 + 4098240*m + 2027025) + 1311363*a*b**2*d*f**m*m**2*x**5*x**
m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2
924172*m**2 + 4098240*m + 2027025) + 2215701*a*b**2*d*f**m*m*x**5*x**m/(m**
8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172
*m**2 + 4098240*m + 2027025) + 1216215*a*b**2*d*f**m*x**5*x**m/(m**8 + 64*m
7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 3*a*b**2*e*f**m*m**7*x**7*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
+ 2027025) + 171*a*b**2*e*f**m*m**6*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6
+ 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027
025) + 3927*a*b**2*e*f**m*m**5*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 2464
0*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
46431*a*b**2*e*f**m*m**4*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2991
45*a*b**2*e*f**m*m**3*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1020033*
a*b**2*e*f**m*m**2*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208
054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1632285*a*b
2*e*fm*m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*

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*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 868725*a*b**2*e*f
**m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038
016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 6*a*b*c*d*f**m*m**7*x**7*x
**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
 2924172*m**2 + 4098240*m + 2027025) + 342*a*b*c*d*f**m*m**6*x**7*x**m/(m**
8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172
*m**2 + 4098240*m + 2027025) + 7854*a*b*c*d*f**m*m**5*x**7*x**m/(m**8 + 64*
m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
 4098240*m + 2027025) + 92862*a*b*c*d*f**m*m**4*x**7*x**m/(m**8 + 64*m**7 +
 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 598290*a*b*c*d*f**m*m**3*x**7*x**m/(m**8 + 64*m**7 + 1708
*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
 + 2027025) + 2040066*a*b*c*d*f**m*m**2*x**7*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 3264570*a*b*c*d*f**m*m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 246
40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
 + 1737450*a*b*c*d*f**m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 6*a*b*c
*e*f**m*m**7*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 330*a*b*c*e*f**m*
m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103
8016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 7278*a*b*c*e*f**m*m**5*x*
*9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m*
*3 + 2924172*m**2 + 4098240*m + 2027025) + 82338*a*b*c*e*f**m*m**4*x**9*x**
m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2
924172*m**2 + 4098240*m + 2027025) + 507282*a*b*c*e*f**m*m**3*x**9*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 1662558*a*b*c*e*f**m*m**2*x**9*x**m/(m**8 +
 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m*
*2 + 4098240*m + 2027025) + 2582010*a*b*c*e*f**m*m*x**9*x**m/(m**8 + 64*m**
7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40
98240*m + 2027025) + 1351350*a*b*c*e*f**m*x**9*x**m/(m**8 + 64*m**7 + 1708*
m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
 2027025) + 3*a*c**2*d*f**m*m**7*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24
640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
 + 165*a*c**2*d*f**m*m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3639
*a*c**2*d*f**m*m**5*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20
8054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 41169*a*c*
*2*d*f**m*m**4*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*
m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 253641*a*c**2*d
*f**m*m**3*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4
 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 831279*a*c**2*d*f**
m*m**2*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1
038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1291005*a*c**2*d*f**m*m
*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016
*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 675675*a*c**2*d*f**m*x**9*x**
m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2
924172*m**2 + 4098240*m + 2027025) + 3*a*c**2*e*f**m*m**7*x**11*x**m/(m**8
 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m
**2 + 4098240*m + 2027025) + 159*a*c**2*e*f**m*m**6*x**11*x**m/(m**8 + 64*m
**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 3375*a*c**2*e*f**m*m**5*x**11*x**m/(m**8 + 64*m**7 +
 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 40982
40*m + 2027025) + 36795*a*c**2*e*f**m*m**4*x**11*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
 + 2027025) + 219417*a*c**2*e*f**m*m**3*x**11*x**m/(m**8 + 64*m**7 + 1708*m
**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 700461*a*c**2*e*f**m*m**2*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6
```

+ 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202
7025) + 1067445*a*c**2*e*f**m*m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24
640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 552825*a*c**2*e*f**m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
+ 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + b**3*
d*f**m*m**7*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**
4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 57*b**3*d*f**m*m**
6*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801
6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1309*b**3*d*f**m*m**5*x**7*x
m/(m8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
2924172*m**2 + 4098240*m + 2027025) + 15477*b**3*d*f**m*m**4*x**7*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 99715*b**3*d*f**m*m**3*x**7*x**m/(m**8 + 64
*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2
+ 4098240*m + 2027025) + 340011*b**3*d*f**m*m**2*x**7*x**m/(m**8 + 64*m**7
+ 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098
240*m + 2027025) + 544095*b**3*d*f**m*m*x**7*x**m/(m**8 + 64*m**7 + 1708*m*
*6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2
027025) + 289575*b**3*d*f**m*x**7*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + b
3*e*fm*m**7*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054
*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 55*b**3*e*f**m
*m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1213*b**3*e*f**m*m**5*x*
*9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m*
*3 + 2924172*m**2 + 4098240*m + 2027025) + 13723*b**3*e*f**m*m**4*x**9*x**m
/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29
24172*m**2 + 4098240*m + 2027025) + 84547*b**3*e*f**m*m**3*x**9*x**m/(m**8
+ 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m
2 + 4098240*m + 2027025) + 277093*b3*e*f**m*m**2*x**9*x**m/(m**8 + 64*m
7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 430335*b**3*e*f**m*m*x**9*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
+ 2027025) + 225225*b**3*e*f**m*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24
640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 3*b**2*c*d*f**m*m**7*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
+ 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 165*b*
*2*c*d*f**m*m**6*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20805
4*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3639*b**2*c*d
*f**m*m**5*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4
+ 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 41169*b**2*c*d*f**m
*m**4*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 253641*b**2*c*d*f**m*m**
3*x**9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801
6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 831279*b**2*c*d*f**m*m**2*x*
*9*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m*
*3 + 2924172*m**2 + 4098240*m + 2027025) + 1291005*b**2*c*d*f**m*m*x**9*x**
m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2
924172*m**2 + 4098240*m + 2027025) + 675675*b**2*c*d*f**m*x**9*x**m/(m**8 +
64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m*
*2 + 4098240*m + 2027025) + 3*b**2*c*e*f**m*m**7*x**11*x**m/(m**8 + 64*m**7
+ 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409
8240*m + 2027025) + 159*b**2*c*e*f**m*m**6*x**11*x**m/(m**8 + 64*m**7 + 170
8*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m
+ 2027025) + 3375*b**2*c*e*f**m*m**5*x**11*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 36795*b**2*c*e*f**m*m**4*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 219417*b**2*c*e*f**m*m**3*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 246

40*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025)
+ 700461*b**2*c*e*f**m*m**2*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1
067445*b**2*c*e*f**m*m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5
+ 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 552825
*b**2*c*e*f**m*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054
*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3*b*c**2*d*f**
m*m**7*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 +
1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 159*b*c**2*d*f**m*m**6
*x**11*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 103801
6*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 3375*b*c**2*d*f**m*m**5*x**1
1*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**
3 + 2924172*m**2 + 4098240*m + 2027025) + 36795*b*c**2*d*f**m*m**4*x**11*x*
*m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
2924172*m**2 + 4098240*m + 2027025) + 219417*b*c**2*d*f**m*m**3*x**11*x**m/
(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292
4172*m**2 + 4098240*m + 2027025) + 700461*b*c**2*d*f**m*m**2*x**11*x**m/(m*
*8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 292417
2*m**2 + 4098240*m + 2027025) + 1067445*b*c**2*d*f**m*m*x**11*x**m/(m**8 +
64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**
2 + 4098240*m + 2027025) + 552825*b*c**2*d*f**m*x**11*x**m/(m**8 + 64*m**7
+ 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098
240*m + 2027025) + 3*b*c**2*e*f**m*m**7*x**13*x**m/(m**8 + 64*m**7 + 1708*m
6 + 24640*m5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 153*b*c**2*e*f**m*m**6*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 202702
5) + 3135*b*c**2*e*f**m*m**5*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640
*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
33165*b*c**2*e*f**m*m**4*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1930
17*b*c**2*e*f**m*m**3*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 +
208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 604827*
b*c**2*e*f**m*m**2*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 20
8054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 909765*b*c
2*e*fm*m*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m
4 + 1038016*m3 + 2924172*m**2 + 4098240*m + 2027025) + 467775*b*c**2*e*
f**m*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + c**3*d*f**m*m**7*x**13*x
m/(m8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
2924172*m**2 + 4098240*m + 2027025) + 51*c**3*d*f**m*m**6*x**13*x**m/(m**8
+ 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*
m**2 + 4098240*m + 2027025) + 1045*c**3*d*f**m*m**5*x**13*x**m/(m**8 + 64*m
7 + 1708*m6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 +
4098240*m + 2027025) + 11055*c**3*d*f**m*m**4*x**13*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409824
0*m + 2027025) + 64339*c**3*d*f**m*m**3*x**13*x**m/(m**8 + 64*m**7 + 1708*m
6 + 24640*m5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 201609*c**3*d*f**m*m**2*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 +
24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20270
25) + 303255*c**3*d*f**m*m*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m
5 + 208054*m4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 15
5925*c**3*d*f**m*x**13*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2080
54*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + c**3*e*f**m*
m**7*x**15*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 49*c**3*e*f**m*m**6*x**1
5*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**
3 + 2924172*m**2 + 4098240*m + 2027025) + 973*c**3*e*f**m*m**5*x**15*x**m/(
m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924
172*m**2 + 4098240*m + 2027025) + 10045*c**3*e*f**m*m**4*x**15*x**m/(m**8 +

```
 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m*
*2 + 4098240*m + 2027025) + 57379*c**3*e*f**m*m**3*x**15*x**m/(m**8 + 64*m*
*7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4
098240*m + 2027025) + 177331*c**3*e*f**m*m**2*x**15*x**m/(m**8 + 64*m**7 +
1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 409824
0*m + 2027025) + 264207*c**3*e*f**m*m*x**15*x**m/(m**8 + 64*m**7 + 1708*m**
6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 20
27025) + 135135*c**3*e*f**m*x**15*x**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*
m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025), Tr
ue))
```

Giac [B] time = 1.18964, size = 3802, normalized size = 15.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] ((f*x)^m*c^3*m^7*x^15*e + 49*(f*x)^m*c^3*m^6*x^15*e + (f*x)^m*c^3*d*m^7*x^1
3 + 3*(f*x)^m*b*c^2*m^7*x^13*e + 973*(f*x)^m*c^3*m^5*x^15*e + 51*(f*x)^m*c^
3*d*m^6*x^13 + 153*(f*x)^m*b*c^2*m^6*x^13*e + 10045*(f*x)^m*c^3*m^4*x^15*e
+ 3*(f*x)^m*b*c^2*d*m^7*x^11 + 1045*(f*x)^m*c^3*d*m^5*x^13 + 3*(f*x)^m*b^2*
c*m^7*x^11*e + 3*(f*x)^m*a*c^2*m^7*x^11*e + 3135*(f*x)^m*b*c^2*m^5*x^13*e +
 57379*(f*x)^m*c^3*m^3*x^15*e + 159*(f*x)^m*b*c^2*d*m^6*x^11 + 11055*(f*x)^
m*c^3*d*m^4*x^13 + 159*(f*x)^m*b^2*c*m^6*x^11*e + 159*(f*x)^m*a*c^2*m^6*x^1
1*e + 33165*(f*x)^m*b*c^2*m^4*x^13*e + 177331*(f*x)^m*c^3*m^2*x^15*e + 3*(f
*x)^m*b^2*c*d*m^7*x^9 + 3*(f*x)^m*a*c^2*d*m^7*x^9 + 3375*(f*x)^m*b*c^2*d*m^
5*x^11 + 64339*(f*x)^m*c^3*d*m^3*x^13 + (f*x)^m*b^3*m^7*x^9*e + 6*(f*x)^m*a
*b*c*m^7*x^9*e + 3375*(f*x)^m*b^2*c*m^5*x^11*e + 3375*(f*x)^m*a*c^2*m^5*x^1
1*e + 193017*(f*x)^m*b*c^2*m^3*x^13*e + 264207*(f*x)^m*c^3*m*x^15*e + 165*(
f*x)^m*b^2*c*d*m^6*x^9 + 165*(f*x)^m*a*c^2*d*m^6*x^9 + 36795*(f*x)^m*b*c^2*
d*m^4*x^11 + 201609*(f*x)^m*c^3*d*m^2*x^13 + 55*(f*x)^m*b^3*m^6*x^9*e + 330
*(f*x)^m*a*b*c*m^6*x^9*e + 36795*(f*x)^m*b^2*c*m^4*x^11*e + 36795*(f*x)^m*a
*c^2*m^4*x^11*e + 604827*(f*x)^m*b*c^2*m^2*x^13*e + 135135*(f*x)^m*c^3*x^15
*e + (f*x)^m*b^3*d*m^7*x^7 + 6*(f*x)^m*a*b*c*d*m^7*x^7 + 3639*(f*x)^m*b^2*c
*d*m^5*x^9 + 3639*(f*x)^m*a*c^2*d*m^5*x^9 + 219417*(f*x)^m*b*c^2*d*m^3*x^11
 + 303255*(f*x)^m*c^3*d*m*x^13 + 3*(f*x)^m*a*b^2*m^7*x^7*e + 3*(f*x)^m*a^2*
c*m^7*x^7*e + 1213*(f*x)^m*b^3*m^5*x^9*e + 7278*(f*x)^m*a*b*c*m^5*x^9*e + 2
19417*(f*x)^m*b^2*c*m^3*x^11*e + 219417*(f*x)^m*a*c^2*m^3*x^11*e + 909765*(
f*x)^m*b*c^2*m*x^13*e + 57*(f*x)^m*b^3*d*m^6*x^7 + 342*(f*x)^m*a*b*c*d*m^6*
x^7 + 41169*(f*x)^m*b^2*c*d*m^4*x^9 + 41169*(f*x)^m*a*c^2*d*m^4*x^9 + 70046
1*(f*x)^m*b*c^2*d*m^2*x^11 + 155925*(f*x)^m*c^3*d*x^13 + 171*(f*x)^m*a*b^2*
m^6*x^7*e + 171*(f*x)^m*a^2*c*m^6*x^7*e + 13723*(f*x)^m*b^3*m^4*x^9*e + 823
38*(f*x)^m*a*b*c*m^4*x^9*e + 700461*(f*x)^m*b^2*c*m^2*x^11*e + 700461*(f*x)
^m*a*c^2*m^2*x^11*e + 467775*(f*x)^m*b*c^2*x^13*e + 3*(f*x)^m*a*b^2*d*m^7*x
^5 + 3*(f*x)^m*a^2*c*d*m^7*x^5 + 1309*(f*x)^m*b^3*d*m^5*x^7 + 7854*(f*x)^m*
a*b*c*d*m^5*x^7 + 253641*(f*x)^m*b^2*c*d*m^3*x^9 + 253641*(f*x)^m*a*c^2*d*m
^3*x^9 + 1067445*(f*x)^m*b*c^2*d*m*x^11 + 3*(f*x)^m*a^2*b*m^7*x^5*e + 3927*
(f*x)^m*a*b^2*m^5*x^7*e + 3927*(f*x)^m*a^2*c*m^5*x^7*e + 84547*(f*x)^m*b^3*
m^3*x^9*e + 507282*(f*x)^m*a*b*c*m^3*x^9*e + 1067445*(f*x)^m*b^2*c*m*x^11*e
 + 1067445*(f*x)^m*a*c^2*m*x^11*e + 177*(f*x)^m*a*b^2*d*m^6*x^5 + 177*(f*x)
^m*a^2*c*d*m^6*x^5 + 15477*(f*x)^m*b^3*d*m^4*x^7 + 92862*(f*x)^m*a*b*c*d*m^
4*x^7 + 831279*(f*x)^m*b^2*c*d*m^2*x^9 + 831279*(f*x)^m*a*c^2*d*m^2*x^9 + 5
52825*(f*x)^m*b*c^2*d*x^11 + 177*(f*x)^m*a^2*b*m^6*x^5*e + 46431*(f*x)^m*a*
b^2*m^4*x^7*e + 46431*(f*x)^m*a^2*c*m^4*x^7*e + 277093*(f*x)^m*b^3*m^2*x^9*
e + 1662558*(f*x)^m*a*b*c*m^2*x^9*e + 552825*(f*x)^m*b^2*c*x^11*e + 552825*
```

```
(f*x)^m*a*c^2*x^11*e + 3*(f*x)^m*a^2*b*d*m^7*x^3 + 4239*(f*x)^m*a*b^2*d*m^5
*x^5 + 4239*(f*x)^m*a^2*c*d*m^5*x^5 + 99715*(f*x)^m*b^3*d*m^3*x^7 + 598290*
(f*x)^m*a*b*c*d*m^3*x^7 + 1291005*(f*x)^m*b^2*c*d*m*x^9 + 1291005*(f*x)^m*a
*c^2*d*m*x^9 + (f*x)^m*a^3*m^7*x^3*e + 4239*(f*x)^m*a^2*b*m^5*x^5*e + 29914
5*(f*x)^m*a*b^2*m^3*x^7*e + 299145*(f*x)^m*a^2*c*m^3*x^7*e + 430335*(f*x)^m
*b^3*m*x^9*e + 2582010*(f*x)^m*a*b*c*m*x^9*e + 183*(f*x)^m*a^2*b*d*m^6*x^3
+ 52725*(f*x)^m*a*b^2*d*m^4*x^5 + 52725*(f*x)^m*a^2*c*d*m^4*x^5 + 340011*(f
*x)^m*b^3*d*m^2*x^7 + 2040066*(f*x)^m*a*b*c*d*m^2*x^7 + 675675*(f*x)^m*b^2*
c*d*x^9 + 675675*(f*x)^m*a*c^2*d*x^9 + 61*(f*x)^m*a^3*m^6*x^3*e + 52725*(f*
x)^m*a^2*b*m^4*x^5*e + 1020033*(f*x)^m*a*b^2*m^2*x^7*e + 1020033*(f*x)^m*a^
2*c*m^2*x^7*e + 225225*(f*x)^m*b^3*x^9*e + 1351350*(f*x)^m*a*b*c*x^9*e + (f
*x)^m*a^3*d*m^7*x + 4575*(f*x)^m*a^2*b*d*m^5*x^3 + 360537*(f*x)^m*a*b^2*d*m
^3*x^5 + 360537*(f*x)^m*a^2*c*d*m^3*x^5 + 544095*(f*x)^m*b^3*d*m*x^7 + 3264
570*(f*x)^m*a*b*c*d*m*x^7 + 1525*(f*x)^m*a^3*m^5*x^3*e + 360537*(f*x)^m*a^2
*b*m^3*x^5*e + 1632285*(f*x)^m*a*b^2*m*x^7*e + 1632285*(f*x)^m*a^2*c*m*x^7*
e + 63*(f*x)^m*a^3*d*m^6*x + 60195*(f*x)^m*a^2*b*d*m^4*x^3 + 1311363*(f*x)^
m*a*b^2*d*m^2*x^5 + 1311363*(f*x)^m*a^2*c*d*m^2*x^5 + 289575*(f*x)^m*b^3*d*
x^7 + 1737450*(f*x)^m*a*b*c*d*x^7 + 20065*(f*x)^m*a^3*m^4*x^3*e + 1311363*(
f*x)^m*a^2*b*m^2*x^5*e + 868725*(f*x)^m*a*b^2*x^7*e + 868725*(f*x)^m*a^2*c*
x^7*e + 1645*(f*x)^m*a^3*d*m^5*x + 443577*(f*x)^m*a^2*b*d*m^3*x^3 + 2215701
*(f*x)^m*a*b^2*d*m*x^5 + 2215701*(f*x)^m*a^2*c*d*m*x^5 + 147859*(f*x)^m*a^3
*m^3*x^3*e + 2215701*(f*x)^m*a^2*b*m*x^5*e + 22995*(f*x)^m*a^3*d*m^4*x + 17
83317*(f*x)^m*a^2*b*d*m^2*x^3 + 1216215*(f*x)^m*a*b^2*d*x^5 + 1216215*(f*x)
^m*a^2*c*d*x^5 + 594439*(f*x)^m*a^3*m^2*x^3*e + 1216215*(f*x)^m*a^2*b*x^5*e
 + 185059*(f*x)^m*a^3*d*m^3*x + 3422565*(f*x)^m*a^2*b*d*m*x^3 + 1140855*(f*
x)^m*a^3*m*x^3*e + 852957*(f*x)^m*a^3*d*m^2*x + 2027025*(f*x)^m*a^2*b*d*x^3
 + 675675*(f*x)^m*a^3*x^3*e + 2071215*(f*x)^m*a^3*d*m*x + 2027025*(f*x)^m*a
^3*d*x)/(m^8 + 64*m^7 + 1708*m^6 + 24640*m^5 + 208054*m^4 + 1038016*m^3 + 2
924172*m^2 + 4098240*m + 2027025)
```

3.221 $\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=155

$$\frac{a^2 d (fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+5} \left(2abe+2acd+b^2 d\right)}{f^5 (m+5)}+\frac{(fx)^{m+7} \left(2ace+b^2 e+2bcd\right)}{f^7 (m+7)}+\frac{a (fx)^{m+3} (ae+2bd)}{f^3 (m+3)}+\frac{c (fx)^{m+9} (2be+c}{f^9 (m+9)}$$

```
[Out] (a^2*d*(f*x)^(1 + m))/(f*(1 + m)) + (a*(2*b*d + a*e)*(f*x)^(3 + m))/(f^3*(3
 + m)) + ((b^2*d + 2*a*c*d + 2*a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((2*b*
c*d + b^2*e + 2*a*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + (c*(c*d + 2*b*e)*(f*x
)^(9 + m))/(f^9*(9 + m)) + (c^2*e*(f*x)^(11 + m))/(f^11*(11 + m))
```

Rubi [A] time = 0.0994351, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {1261}

$$\frac{a^2 d (fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+5} \left(2abe+2acd+b^2 d\right)}{f^5 (m+5)}+\frac{(fx)^{m+7} \left(2ace+b^2 e+2bcd\right)}{f^7 (m+7)}+\frac{a (fx)^{m+3} (ae+2bd)}{f^3 (m+3)}+\frac{c (fx)^{m+9} (2be+c}{f^9 (m+9)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*d*(f*x)^(1 + m))/(f*(1 + m)) + (a*(2*b*d + a*e)*(f*x)^(3 + m))/(f^3*(3
 + m)) + ((b^2*d + 2*a*c*d + 2*a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((2*b*
c*d + b^2*e + 2*a*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + (c*(c*d + 2*b*e)*(f*x
)^(9 + m))/(f^9*(9 + m)) + (c^2*e*(f*x)^(11 + m))/(f^11*(11 + m))
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2 d (fx)^m+\frac{a(2bd+ae)(fx)^{2+m}}{f^2}+\frac{\left(b^2 d+2acd+2abe\right) (fx)^{4+m}}{f^4}+\frac{(2bcd\right. \\ &= \frac{a^2 d (fx)^{1+m}}{f(1+m)}+\frac{a(2bd+ae)(fx)^{3+m}}{f^3 (3+m)}+\frac{\left(b^2 d+2acd+2abe\right) (fx)^{5+m}}{f^5 (5+m)}+\frac{(2bcd\end{aligned}$$

Mathematica [A] time = 0.148003, size = 117, normalized size = 0.75

$$x (fx)^m \left(\frac{a^2 d}{m+1}+\frac{x^6 \left(2ace+b^2 e+2bcd\right)}{m+7}+\frac{x^4 \left(2abe+2acd+b^2 d\right)}{m+5}+\frac{a x^2 (ae+2bd)}{m+3}+\frac{c x^8 (2be+cd)}{m+9}+\frac{c^2 e x^{10}}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] x*(f*x)^m*((a^2*d)/(1 + m) + (a*(2*b*d + a*e)*x^2)/(3 + m) + ((b^2*d + 2*a*
c*d + 2*a*b*e)*x^4)/(5 + m) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^6)/(7 + m) + (
c*(c*d + 2*b*e)*x^8)/(9 + m) + (c^2*e*x^10)/(11 + m))
```

Maple [B] time = 0.008, size = 783, normalized size = 5.1

$$\frac{\left(c^2em^5x^{10} + 25\,c^2em^4x^{10} + 2\,bcem^5x^8 + c^2dm^5x^8 + 230\,c^2em^3x^{10} + 54\,bcem^4x^8 + 27\,c^2dm^4x^8 + 950\,c^2em^2x^{10} + 2\,acem^5x \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] x*(c^2*e*m^5*x^10+25*c^2*e*m^4*x^10+2*b*c*e*m^5*x^8+c^2*d*m^5*x^8+230*c^2*e
*m^3*x^10+54*b*c*e*m^4*x^8+27*c^2*d*m^4*x^8+950*c^2*e*m^2*x^10+2*a*c*e*m^5*
x^6+b^2*e*m^5*x^6+2*b*c*d*m^5*x^6+524*b*c*e*m^3*x^8+262*c^2*d*m^3*x^8+1689*
c^2*e*m*x^10+58*a*c*e*m^4*x^6+29*b^2*e*m^4*x^6+58*b*c*d*m^4*x^6+2244*b*c*e*
m^2*x^8+1122*c^2*d*m^2*x^8+945*c^2*e*x^10+2*a*b*e*m^5*x^4+2*a*c*d*m^5*x^4+6
04*a*c*e*m^3*x^6+b^2*d*m^5*x^4+302*b^2*e*m^3*x^6+604*b*c*d*m^3*x^6+4082*b*c
*e*m*x^8+2041*c^2*d*m*x^8+62*a*b*e*m^4*x^4+62*a*c*d*m^4*x^4+2732*a*c*e*m^2*
x^6+31*b^2*d*m^4*x^4+1366*b^2*e*m^2*x^6+2732*b*c*d*m^2*x^6+2310*b*c*e*x^8+1
155*c^2*d*x^8+a^2*e*m^5*x^2+2*a*b*d*m^5*x^2+700*a*b*e*m^3*x^4+700*a*c*d*m^3
*x^4+5154*a*c*e*m*x^6+350*b^2*d*m^3*x^4+2577*b^2*e*m*x^6+5154*b*c*d*m*x^6+3
3*a^2*e*m^4*x^2+66*a*b*d*m^4*x^2+3460*a*b*e*m^2*x^4+3460*a*c*d*m^2*x^4+2970
*a*c*e*x^6+1730*b^2*d*m^2*x^4+1485*b^2*e*x^6+2970*b*c*d*x^6+a^2*d*m^5+406*a
^2*e*m^3*x^2+812*a*b*d*m^3*x^2+6978*a*b*e*m*x^4+6978*a*c*d*m*x^4+3489*b^2*d
*m*x^4+35*a^2*d*m^4+2262*a^2*e*m^2*x^2+4524*a*b*d*m^2*x^2+4158*a*b*e*x^4+41
58*a*c*d*x^4+2079*b^2*d*x^4+470*a^2*d*m^3+5353*a^2*e*m*x^2+10706*a*b*d*m*x^
2+3010*a^2*d*m^2+3465*a^2*e*x^2+6930*a*b*d*x^2+9129*a^2*d*m+10395*a^2*d)*(f
*x)^m/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.38458, size = 1416, normalized size = 9.14

$$\frac{\left(\left(c^2em^5 + 25\,c^2em^4 + 230\,c^2em^3 + 950\,c^2em^2 + 1689\,c^2em + 945\,c^2e\right)x^{11} + \left(\left(c^2d + 2\,bce\right)m^5 + 27\left(c^2d + 2\,bce\right)m^4 + 262 \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*e*m^5 + 25*c^2*e*m^4 + 230*c^2*e*m^3 + 950*c^2*e*m^2 + 1689*c^2*e*m +
 945*c^2*e)*x^11 + ((c^2*d + 2*b*c*e)*m^5 + 27*(c^2*d + 2*b*c*e)*m^4 + 262*
```

```
(c^2*d + 2*b*c*e)*m^3 + 1155*c^2*d + 2310*b*c*e + 1122*(c^2*d + 2*b*c*e)*m^
2 + 2041*(c^2*d + 2*b*c*e)*m)*x^9 + ((2*b*c*d + (b^2 + 2*a*c)*e)*m^5 + 29*(
2*b*c*d + (b^2 + 2*a*c)*e)*m^4 + 302*(2*b*c*d + (b^2 + 2*a*c)*e)*m^3 + 2970
*b*c*d + 1366*(2*b*c*d + (b^2 + 2*a*c)*e)*m^2 + 1485*(b^2 + 2*a*c)*e + 2577
*(2*b*c*d + (b^2 + 2*a*c)*e)*m)*x^7 + ((2*a*b*e + (b^2 + 2*a*c)*d)*m^5 + 31
*(2*a*b*e + (b^2 + 2*a*c)*d)*m^4 + 350*(2*a*b*e + (b^2 + 2*a*c)*d)*m^3 + 41
58*a*b*e + 1730*(2*a*b*e + (b^2 + 2*a*c)*d)*m^2 + 2079*(b^2 + 2*a*c)*d + 34
89*(2*a*b*e + (b^2 + 2*a*c)*d)*m)*x^5 + ((2*a*b*d + a^2*e)*m^5 + 33*(2*a*b*
d + a^2*e)*m^4 + 406*(2*a*b*d + a^2*e)*m^3 + 6930*a*b*d + 3465*a^2*e + 2262
*(2*a*b*d + a^2*e)*m^2 + 5353*(2*a*b*d + a^2*e)*m)*x^3 + (a^2*d*m^5 + 35*a^
2*d*m^4 + 470*a^2*d*m^3 + 3010*a^2*d*m^2 + 9129*a^2*d*m + 10395*a^2*d)*x)*(
f*x)^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

Sympy [A] time = 6.13589, size = 4190, normalized size = 27.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Piecewise(((-a**2*d/(10*x**10) - a**2*e/(8*x**8) - a*b*d/(4*x**8) - a*b*e/(
3*x**6) - a*c*d/(3*x**6) - a*c*e/(2*x**4) - b**2*d/(6*x**6) - b**2*e/(4*x**
4) - b*c*d/(2*x**4) - b*c*e/x**2 - c**2*d/(2*x**2) + c**2*e*log(x))/f**11,
Eq(m, -11)), ((-a**2*d/(8*x**8) - a**2*e/(6*x**6) - a*b*d/(3*x**6) - a*b*e/
(2*x**4) - a*c*d/(2*x**4) - a*c*e/x**2 - b**2*d/(4*x**4) - b**2*e/(2*x**2)
- b*c*d/x**2 + 2*b*c*e*log(x) + c**2*d*log(x) + c**2*e*x**2/2)/f**9, Eq(m,
-9)), ((-a**2*d/(6*x**6) - a**2*e/(4*x**4) - a*b*d/(2*x**4) - a*b*e/x**2 -
a*c*d/x**2 + 2*a*c*e*log(x) - b**2*d/(2*x**2) + b**2*e*log(x) + 2*b*c*d*log
(x) + b*c*e*x**2 + c**2*d*x**2/2 + c**2*e*x**4/4)/f**7, Eq(m, -7)), ((-a**2
*d/(4*x**4) - a**2*e/(2*x**2) - a*b*d/x**2 + 2*a*b*e*log(x) + 2*a*c*d*log(x
) + a*c*e*x**2 + b**2*d*log(x) + b**2*e*x**2/2 + b*c*d*x**2 + b*c*e*x**4/2
+ c**2*d*x**4/4 + c**2*e*x**6/6)/f**5, Eq(m, -5)), ((-a**2*d/(2*x**2) + a**
2*e*log(x) + 2*a*b*d*log(x) + a*b*e*x**2 + a*c*d*x**2 + a*c*e*x**4/2 + b**2
*d*x**2/2 + b**2*e*x**4/4 + b*c*d*x**4/2 + b*c*e*x**6/3 + c**2*d*x**6/6 + c
**2*e*x**8/8)/f**3, Eq(m, -3)), ((a**2*d*log(x) + a**2*e*x**2/2 + a*b*d*x**
2 + a*b*e*x**4/2 + a*c*d*x**4/2 + a*c*e*x**6/3 + b**2*d*x**4/4 + b**2*e*x**
6/6 + b*c*d*x**6/3 + b*c*e*x**8/4 + c**2*d*x**8/8 + c**2*e*x**10/10)/f, Eq(
m, -1)), (a**2*d*f**m*m**5*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 35*a**2*d*f**m*m**4*x*x**m/(m**6 + 36*m**5
+ 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 470*a**2*d*f**m*m*
*3*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 3010*a**2*d*f**m*m**2*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
 + 12139*m**2 + 19524*m + 10395) + 9129*a**2*d*f**m*m*x*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10395*a**2*d*f**
m*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10
395) + a**2*e*f**m*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 33*a**2*e*f**m*m**4*x**3*x**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 406*a**2*e*f**m
*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 2262*a**2*e*f**m*m**2*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 +
3480*m**3 + 12139*m**2 + 19524*m + 10395) + 5353*a**2*e*f**m*m*x**3*x**m/(m
**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3465
*a**2*e*f**m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
+ 19524*m + 10395) + 2*a*b*d*f**m*m**5*x**3*x**m/(m**6 + 36*m**5 + 505*m**4
 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 66*a*b*d*f**m*m**4*x**3*x**m
/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 8
```

```
12*a*b*d*f**m*m**3*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139
*m**2 + 19524*m + 10395) + 4524*a*b*d*f**m*m**2*x**3*x**m/(m**6 + 36*m**5 +
 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10706*a*b*d*f**m*m*
x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 6930*a*b*d*f**m*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 +
12139*m**2 + 19524*m + 10395) + 2*a*b*e*f**m*m**5*x**5*x**m/(m**6 + 36*m**5
 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 62*a*b*e*f**m*m**
4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
 10395) + 700*a*b*e*f**m*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m
**3 + 12139*m**2 + 19524*m + 10395) + 3460*a*b*e*f**m*m**2*x**5*x**m/(m**6
+ 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6978*a*b
*e*f**m*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 4158*a*b*e*f**m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a*c*d*f**m*m**5*x**5*x**m/(m**6
 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 62*a*c*
d*f**m*m**4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 +
 19524*m + 10395) + 700*a*c*d*f**m*m**3*x**5*x**m/(m**6 + 36*m**5 + 505*m**
4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3460*a*c*d*f**m*m**2*x**5*x
**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
+ 6978*a*c*d*f**m*m*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1213
9*m**2 + 19524*m + 10395) + 4158*a*c*d*f**m*x**5*x**m/(m**6 + 36*m**5 + 505
*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2*a*c*e*f**m*m**5*x**7*
x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
 + 58*a*c*e*f**m*m**4*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12
139*m**2 + 19524*m + 10395) + 604*a*c*e*f**m*m**3*x**7*x**m/(m**6 + 36*m**5
 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2732*a*c*e*f**m*m
**2*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 5154*a*c*e*f**m*m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m
**3 + 12139*m**2 + 19524*m + 10395) + 2970*a*c*e*f**m*x**7*x**m/(m**6 + 36*
m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + b**2*d*f**m*m
**5*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 31*b**2*d*f**m*m**4*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 350*b**2*d*f**m*m**3*x**5*x**m/(m**
6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1730*b
**2*d*f**m*m**2*x**5*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m*
*2 + 19524*m + 10395) + 3489*b**2*d*f**m*m*x**5*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 2079*b**2*d*f**m*x**5*x*
*m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
 b**2*e*f**m*m**5*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + 29*b**2*e*f**m*m**4*x**7*x**m/(m**6 + 36*m**5 + 5
05*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 302*b**2*e*f**m*m**3*
x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1
0395) + 1366*b**2*e*f**m*m**2*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m
**3 + 12139*m**2 + 19524*m + 10395) + 2577*b**2*e*f**m*m*x**7*x**m/(m**6 +
36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 1485*b**2*
e*f**m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1952
4*m + 10395) + 2*b*c*d*f**m*m**5*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 348
0*m**3 + 12139*m**2 + 19524*m + 10395) + 58*b*c*d*f**m*m**4*x**7*x**m/(m**6
 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 604*b*c
*d*f**m*m**3*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
+ 19524*m + 10395) + 2732*b*c*d*f**m*m**2*x**7*x**m/(m**6 + 36*m**5 + 505*m
**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 5154*b*c*d*f**m*m*x**7*x*
*m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
 2970*b*c*d*f**m*x**7*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
**2 + 19524*m + 10395) + 2*b*c*e*f**m*m**5*x**9*x**m/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 54*b*c*e*f**m*m**4*x**9*
x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
 + 524*b*c*e*f**m*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 2244*b*c*e*f**m*m**2*x**9*x**m/(m**6 + 36*m*
```

```
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4082*b*c*e*f**m
*m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
+ 10395) + 2310*b*c*e*f**m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
 + 12139*m**2 + 19524*m + 10395) + c**2*d*f**m*m**5*x**9*x**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 27*c**2*d*f**m*
m**4*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*
m + 10395) + 262*c**2*d*f**m*m**3*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395) + 1122*c**2*d*f**m*m**2*x**9*x**m/(
m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 204
1*c**2*d*f**m*m*x**9*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m*
*2 + 19524*m + 10395) + 1155*c**2*d*f**m*x**9*x**m/(m**6 + 36*m**5 + 505*m*
*4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + c**2*e*f**m*m**5*x**11*x**
m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
25*c**2*e*f**m*m**4*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 230*c**2*e*f**m*m**3*x**11*x**m/(m**6 + 36*m**
5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 950*c**2*e*f**m*
m**2*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 1689*c**2*e*f**m*m*x**11*x**m/(m**6 + 36*m**5 + 505*m**4 + 34
80*m**3 + 12139*m**2 + 19524*m + 10395) + 945*c**2*e*f**m*x**11*x**m/(m**6
+ 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395), True))
```

Giac [B] time = 1.13304, size = 1590, normalized size = 10.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((f*x)^m*c^2*m^5*x^11*e + 25*(f*x)^m*c^2*m^4*x^11*e + (f*x)^m*c^2*d*m^5*x^9
 + 2*(f*x)^m*b*c*m^5*x^9*e + 230*(f*x)^m*c^2*m^3*x^11*e + 27*(f*x)^m*c^2*d*
m^4*x^9 + 54*(f*x)^m*b*c*m^4*x^9*e + 950*(f*x)^m*c^2*m^2*x^11*e + 2*(f*x)^m
*b*c*d*m^5*x^7 + 262*(f*x)^m*c^2*d*m^3*x^9 + (f*x)^m*b^2*m^5*x^7*e + 2*(f*x
)^m*a*c*m^5*x^7*e + 524*(f*x)^m*b*c*m^3*x^9*e + 1689*(f*x)^m*c^2*m*x^11*e +
 58*(f*x)^m*b*c*d*m^4*x^7 + 1122*(f*x)^m*c^2*d*m^2*x^9 + 29*(f*x)^m*b^2*m^4
*x^7*e + 58*(f*x)^m*a*c*m^4*x^7*e + 2244*(f*x)^m*b*c*m^2*x^9*e + 945*(f*x)^
m*c^2*x^11*e + (f*x)^m*b^2*d*m^5*x^5 + 2*(f*x)^m*a*c*d*m^5*x^5 + 604*(f*x)^
m*b*c*d*m^3*x^7 + 2041*(f*x)^m*c^2*d*m*x^9 + 2*(f*x)^m*a*b*m^5*x^5*e + 302*
(f*x)^m*b^2*m^3*x^7*e + 604*(f*x)^m*a*c*m^3*x^7*e + 4082*(f*x)^m*b*c*m*x^9*
e + 31*(f*x)^m*b^2*d*m^4*x^5 + 62*(f*x)^m*a*c*d*m^4*x^5 + 2732*(f*x)^m*b*c*
d*m^2*x^7 + 1155*(f*x)^m*c^2*d*x^9 + 62*(f*x)^m*a*b*m^4*x^5*e + 1366*(f*x)^
m*b^2*m^2*x^7*e + 2732*(f*x)^m*a*c*m^2*x^7*e + 2310*(f*x)^m*b*c*x^9*e + 2*(
f*x)^m*a*b*d*m^5*x^3 + 350*(f*x)^m*b^2*d*m^3*x^5 + 700*(f*x)^m*a*c*d*m^3*x^
5 + 5154*(f*x)^m*b*c*d*m*x^7 + (f*x)^m*a^2*m^5*x^3*e + 700*(f*x)^m*a*b*m^3*
x^5*e + 2577*(f*x)^m*b^2*m*x^7*e + 5154*(f*x)^m*a*c*m*x^7*e + 66*(f*x)^m*a*
b*d*m^4*x^3 + 1730*(f*x)^m*b^2*d*m^2*x^5 + 3460*(f*x)^m*a*c*d*m^2*x^5 + 297
0*(f*x)^m*b*c*d*x^7 + 33*(f*x)^m*a^2*m^4*x^3*e + 3460*(f*x)^m*a*b*m^2*x^5*e
 + 1485*(f*x)^m*b^2*x^7*e + 2970*(f*x)^m*a*c*x^7*e + (f*x)^m*a^2*d*m^5*x +
812*(f*x)^m*a*b*d*m^3*x^3 + 3489*(f*x)^m*b^2*d*m*x^5 + 6978*(f*x)^m*a*c*d*m
*x^5 + 406*(f*x)^m*a^2*m^3*x^3*e + 6978*(f*x)^m*a*b*m*x^5*e + 35*(f*x)^m*a^
2*d*m^4*x + 4524*(f*x)^m*a*b*d*m^2*x^3 + 2079*(f*x)^m*b^2*d*x^5 + 4158*(f*x
)^m*a*c*d*x^5 + 2262*(f*x)^m*a^2*m^2*x^3*e + 4158*(f*x)^m*a*b*x^5*e + 470*(
f*x)^m*a^2*d*m^3*x + 10706*(f*x)^m*a*b*d*m*x^3 + 5353*(f*x)^m*a^2*m*x^3*e +
 3010*(f*x)^m*a^2*d*m^2*x + 6930*(f*x)^m*a*b*d*x^3 + 3465*(f*x)^m*a^2*x^3*e
 + 9129*(f*x)^m*a^2*d*m*x + 10395*(f*x)^m*a^2*d*x)/(m^6 + 36*m^5 + 505*m^4
+ 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

3.222 $\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=83

$$\frac{(fx)^{m+3}(ae+bd)}{f^3(m+3)}+\frac{ad(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+5}(be+cd)}{f^5(m+5)}+\frac{ce(fx)^{m+7}}{f^7(m+7)}$$

```
[Out] (a*d*(f*x)^(1 + m))/(f*(1 + m)) + ((b*d + a*e)*(f*x)^(3 + m))/(f^3*(3 + m))
 + ((c*d + b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (c*e*(f*x)^(7 + m))/(f^7*(7
+ m))
```

Rubi [A] time = 0.0474501, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1261}

$$\frac{(fx)^{m+3}(ae+bd)}{f^3(m+3)}+\frac{ad(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+5}(be+cd)}{f^5(m+5)}+\frac{ce(fx)^{m+7}}{f^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d*(f*x)^(1 + m))/(f*(1 + m)) + ((b*d + a*e)*(f*x)^(3 + m))/(f^3*(3 + m))
 + ((c*d + b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (c*e*(f*x)^(7 + m))/(f^7*(7
+ m))
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (fx)^m \left(d+ex^2\right)\left(a+bx^2+cx^4\right) dx &= \int \left(ad(fx)^m+\frac{(bd+ae)(fx)^{2+m}}{f^2}+\frac{(cd+be)(fx)^{4+m}}{f^4}+\frac{ce(fx)^{6+m}}{f^6}\right) dx\\ &= \frac{ad(fx)^{1+m}}{f(1+m)}+\frac{(bd+ae)(fx)^{3+m}}{f^3(3+m)}+\frac{(cd+be)(fx)^{5+m}}{f^5(5+m)}+\frac{ce(fx)^{7+m}}{f^7(7+m)}\end{aligned}$$

Mathematica [A] time = 0.0494136, size = 59, normalized size = 0.71

$$x(fx)^m\left(\frac{x^2(ae+bd)}{m+3}+\frac{ad}{m+1}+\frac{x^4(be+cd)}{m+5}+\frac{cex^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] x*(f*x)^m*((a*d)/(1 + m) + ((b*d + a*e)*x^2)/(3 + m) + ((c*d + b*e)*x^4)/(5
 + m) + (c*e*x^6)/(7 + m))
```

Maple [B] time = 0.003, size = 221, normalized size = 2.7

$$\frac{\left(cem^3x^6 + 9\,cem^2x^6 + bem^3x^4 + cdm^3x^4 + 23\,cemx^6 + 11\,bem^2x^4 + 11\,cdm^2x^4 + 15\,cex^6 + aem^3x^2 + bdm^3x^2 + 31\,bem\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x)
```

```
[Out] x*(c*e*m^3*x^6+9*c*e*m^2*x^6+b*e*m^3*x^4+c*d*m^3*x^4+23*c*e*m*x^6+11*b*e*m^
2*x^4+11*c*d*m^2*x^4+15*c*e*x^6+a*e*m^3*x^2+b*d*m^3*x^2+31*b*e*m*x^4+31*c*d
*m*x^4+13*a*e*m^2*x^2+13*b*d*m^2*x^2+21*b*e*x^4+21*c*d*x^4+a*d*m^3+47*a*e*m
*x^2+47*b*d*m*x^2+15*a*d*m^2+35*a*e*x^2+35*b*d*x^2+71*a*d*m+105*a*d)*(f*x)^
m/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.66453, size = 414, normalized size = 4.99

$$\frac{\left(\left(cem^3 + 9\,cem^2 + 23\,cem + 15\,ce\right)x^7 + \left((cd + be)m^3 + 11\,(cd + be)m^2 + 21\,cd + 21\,be + 31\,(cd + be)m\right)x^5 + \left((bd + ae)\right.\right.}{m^4 + 16\,m^3 + 86\,m^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] ((c*e*m^3 + 9*c*e*m^2 + 23*c*e*m + 15*c*e)*x^7 + ((c*d + b*e)*m^3 + 11*(c*d
 + b*e)*m^2 + 21*c*d + 21*b*e + 31*(c*d + b*e)*m)*x^5 + ((b*d + a*e)*m^3 +
13*(b*d + a*e)*m^2 + 35*b*d + 35*a*e + 47*(b*d + a*e)*m)*x^3 + (a*d*m^3 + 1
5*a*d*m^2 + 71*a*d*m + 105*a*d)*x)*(f*x)^m/(m^4 + 16*m^3 + 86*m^2 + 176*m +
 105)
```

Sympy [A] time = 2.0871, size = 1056, normalized size = 12.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise(((-a*d/(6*x**6) - a*e/(4*x**4) - b*d/(4*x**4) - b*e/(2*x**2) - c*
d/(2*x**2) + c*e*log(x))/f**7, Eq(m, -7)), ((-a*d/(4*x**4) - a*e/(2*x**2) -
```

```
 b*d/(2*x**2) + b*e*log(x) + c*d*log(x) + c*e*x**2/2)/f**5, Eq(m, -5)), ((-
a*d/(2*x**2) + a*e*log(x) + b*d*log(x) + b*e*x**2/2 + c*d*x**2/2 + c*e*x**4
/4)/f**3, Eq(m, -3)), ((a*d*log(x) + a*e*x**2/2 + b*d*x**2/2 + b*e*x**4/4 +
 c*d*x**4/4 + c*e*x**6/6)/f, Eq(m, -1)), (a*d*f**m*m**3*x*x**m/(m**4 + 16*m
**3 + 86*m**2 + 176*m + 105) + 15*a*d*f**m*m**2*x*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 71*a*d*f**m*m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176
*m + 105) + 105*a*d*f**m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
a*e*f**m*m**3*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 13*a*e*f
**m*m**2*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 47*a*e*f**m*m
*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*a*e*f**m*x**3*x**m
/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b*d*f**m*m**3*x**3*x**m/(m**4 +
 16*m**3 + 86*m**2 + 176*m + 105) + 13*b*d*f**m*m**2*x**3*x**m/(m**4 + 16*m
**3 + 86*m**2 + 176*m + 105) + 47*b*d*f**m*m*x**3*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 35*b*d*f**m*x**3*x**m/(m**4 + 16*m**3 + 86*m**2 + 17
6*m + 105) + b*e*f**m*m**3*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 10
5) + 11*b*e*f**m*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
31*b*e*f**m*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 21*b*e*f
**m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + c*d*f**m*m**3*x**5
*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*c*d*f**m*m**2*x**5*x**m
/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*c*d*f**m*m*x**5*x**m/(m**4 +
 16*m**3 + 86*m**2 + 176*m + 105) + 21*c*d*f**m*x**5*x**m/(m**4 + 16*m**3 +
 86*m**2 + 176*m + 105) + c*e*f**m*m**3*x**7*x**m/(m**4 + 16*m**3 + 86*m**2
 + 176*m + 105) + 9*c*e*f**m*m**2*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176
*m + 105) + 23*c*e*f**m*m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105
) + 15*c*e*f**m*x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

Giac [B] time = 1.10684, size = 473, normalized size = 5.7

$$\left(fx\right)^{m} cm^3x^7e + 9 \left(fx\right)^{m} cm^2x^7e + \left(fx\right)^{m} cdm^3x^5 + \left(fx\right)^{m} bm^3x^5e + 23 \left(fx\right)^{m} cmx^7e + 11 \left(fx\right)^{m} cdm^2x^5 + 11 \left(fx\right)^{m} bm$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] ((f*x)^m*c*m^3*x^7*e + 9*(f*x)^m*c*m^2*x^7*e + (f*x)^m*c*d*m^3*x^5 + (f*x)^
m*b*m^3*x^5*e + 23*(f*x)^m*c*m*x^7*e + 11*(f*x)^m*c*d*m^2*x^5 + 11*(f*x)^m*
b*m^2*x^5*e + 15*(f*x)^m*c*x^7*e + (f*x)^m*b*d*m^3*x^3 + 31*(f*x)^m*c*d*m*x
^5 + (f*x)^m*a*m^3*x^3*e + 31*(f*x)^m*b*m*x^5*e + 13*(f*x)^m*b*d*m^2*x^3 +
21*(f*x)^m*c*d*x^5 + 13*(f*x)^m*a*m^2*x^3*e + 21*(f*x)^m*b*x^5*e + (f*x)^m*
a*d*m^3*x + 47*(f*x)^m*b*d*m*x^3 + 47*(f*x)^m*a*m*x^3*e + 15*(f*x)^m*a*d*m^
2*x + 35*(f*x)^m*b*d*x^3 + 35*(f*x)^m*a*x^3*e + 71*(f*x)^m*a*d*m*x + 105*(f
*x)^m*a*d*x)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

3.223 $\int \frac{(fx)^m(d+ex^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=194

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 -
4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)])/((b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rubi [A] time = 0.298859, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1285, 364}

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 -
4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)])/((b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rule 1285

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4} dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(fx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2} dx + \frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(fx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2} d$$
$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)f(1+m)} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{}{b}\right.}{\left(b+\sqrt{b^2-4ac}\right)f(1+m)}$$

Mathematica [A] time = 0.268756, size = 156, normalized size = 0.8

$$\frac{x(fx)^m\left(\left(d\sqrt{b^2-4ac}-2ae+bd\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(d\sqrt{b^2-4ac}+2ae-bd\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)\right)}{2a(m+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*(f*x)^m*((b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, (1 + m
)/2, (3 + m)/2, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (-(b*d) + Sqrt[b^2 -
4*a*c]*d + 2*a*e)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]))/(2*a*Sqrt[b^2 - 4*a*c]*(1 + m))
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (ex^2+d)}{cx^4+bx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)(fx)^m}{cx^4+bx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ex^2+d)(fx)^m}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m (d + ex^2)}{a + bx^2 + cx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(fx)^m}{cx^4 + bx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.224 $\int \frac{(fx)^m(d+ex^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=392

$$\frac{c(fx)^{m+1}\left(b\left(d(1-m)\sqrt{b^2-4ac}+4ae\right)-2a\left(e(1-m)\sqrt{b^2-4ac}+2cd(3-m)\right)+b^2(d-dm)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4}}\right.}{2af(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}$$

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 -
 4*a*c)*f*(a + b*x^2 + c*x^4)) + (c*(b*(4*a*e + Sqrt[b^2 - 4*a*c]*d*(1 - m)
) - 2*a*(Sqrt[b^2 - 4*a*c]*e*(1 - m) + 2*c*d*(3 - m)) + b^2*(d - d*m))*(f*x
)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m))
- (c*(b*(4*a*e - Sqrt[b^2 - 4*a*c]*d*(1 - m)) + 2*a*(Sqrt[b^2 - 4*a*c]*e*(1
 - m) - 2*c*d*(3 - m)) + b^2*d*(1 - m))*(f*x)^(1 + m)*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*
c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rubi [A] time = 2.646, antiderivative size = 358, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1277, 1285, 364}

$$\frac{c(fx)^{m+1}\left((1-m)\sqrt{b^2-4ac}(bd-2ae)+4abe-4acd(3-m)+b^2(d-dm)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2af(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(fx)^{m+1}\left(-(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 -
 4*a*c)*f*(a + b*x^2 + c*x^4)) + (c*(4*a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - 2*a
*e)*(1 - m) - 4*a*c*d*(3 - m) + b^2*(d - d*m))*(f*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2
 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)) - (c*(4*a*b*e - Sqrt[b^2
 - 4*a*c]*(b*d - 2*a*e)*(1 - m) - 4*a*c*d*(3 - m) + b^2*(d - d*m))*(f*x)^(1
 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1285

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
```

```
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(fx)^m\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^2}dx = \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)f\left(a+bx^2+cx^4\right)} - \frac{\int \frac{(fx)^m\left(-b^2d(1-m)+2acd(3-m)-abe(1+m)-c(bd-2ae)(1-m\right.}{a+bx^2+cx^4}}{2a\left(b^2-4ac\right)}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)f\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(4abe+b^2d(1-m)+\sqrt{b^2-4ac}(bd-2ae)(\right.\right.}{4a\left(b^2-\right.}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)f\left(a+bx^2+cx^4\right)} + \frac{c\left(4abe+b^2d(1-m)+\sqrt{b^2-4ac}(bd-2ae)(1\right.}{2a\left(b^2-4ac\right)^{\cdots}}$$

Mathematica [C] time = 0.217344, size = 160, normalized size = 0.41

$$\frac{x(fx)^m\left(d(m+3)F_1\left(\frac{m+1}{2};2,2;\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+e(m+1)x^2F_1\left(\frac{m+3}{2};2,2;\frac{m+5}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\right)}{a^2(m+1)(m+3)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(f*x)^m*(d*(3 + m)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^2*App
ellF1[(3 + m)/2, 2, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(a^2*(1 + m)*(3 + m))
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(ex^2+d\right)}{\left(cx^4+bx^2+a\right)^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)\left(fx\right)^m}{\left(cx^4+bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)\left(fx\right)^m}{c^2x^8+2\,bcx^6+\left(b^2+2\,ac\right)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a
*b*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)\left(fx\right)^m}{\left(cx^4+bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

3.225 $\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=319

$$\frac{ad(fx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{ae(fx)^{m+3}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+3}{2};-\frac{3}{2},-\frac{3}{2};\right.}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{}}$$

```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3/2, -3/2,
(3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (a*e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x
^4]*AppellF1[(3 + m)/2, -3/2, -3/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[1 + (2*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.399319, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1335, 1141, 510}

$$\frac{ad(fx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{ae(fx)^{m+3}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+3}{2};-\frac{3}{2},-\frac{3}{2};\right.}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3/2, -3/2,
(3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (a*e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x
^4]*AppellF1[(3 + m)/2, -3/2, -3/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[1 + (2*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \int \left(d(fx)^m\left(a+bx^2+cx^4\right)^{3/2}+\frac{e(fx)^{2+m}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right)dx$$

$$= d\int (fx)^m\left(a+bx^2+cx^4\right)^{3/2}\,dx+\frac{e\int (fx)^{2+m}\left(a+bx^2+cx^4\right)^{3/2}\,dx}{f^2}$$

$$= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int (fx)^m\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{\left(ae\sqrt{a\right.}\cdots}{\cdots}$$

$$= \frac{ad(fx)^{1+m}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1+m}{2};-\frac{3}{2},-\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{ae(\cdots}{\cdots}$$

Mathematica [A] time = 0.541473, size = 466, normalized size = 1.46

$$\frac{x(fx)^m\sqrt{a+bx^2+cx^4}\left((m+1)x^2\left(\left(m^2+12m+35\right)(ae+bd)F_1\left(\frac{m+3}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+5}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+(m+3)x^2\left(\cdots\right.\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*Sqrt[a + b*x^2 + c*x^4]*(a*d*(105 + 71*m + 15*m^2 + m^3)*AppellF
1[(1 + m)/2, -1/2, -1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*x^2*((b*d + a*e)*(35 + 12*m + m^
2)*AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (3 + m)*x^2*((c*d + b*e)*(7 +
m)*AppellF1[(5 + m)/2, -1/2, -1/2, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + c*e*(5 + m)*x^2*AppellF1[(7 +
m)/2, -1/2, -1/2, (9 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])]))))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int (fx)^m\left(ex^2+d\right)\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)(fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cex^6+(cd+be)x^4+(bd+ae)x^2+ad\right)\sqrt{cx^4+bx^2+a}\,(fx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^6 + (c*d + b*e)*x^4 + (b*d + a*e)*x^2 + a*d)*sqrt(c*x^4 + b
*x^2 + a)*(f*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)(fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*(f*x)^m, x)
```

3.226 $\int (fx)^m \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=317

$$\frac{d(fx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{e(fx)^{m+3}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+3}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+5}{2};-\ldots\right.}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c\ldots}{\sqrt{b^2-\ldots}}}}$$

```
[Out] (d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (3
 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x^4]*
AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[1 + (2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.360776, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1335, 1141, 510}

$$\frac{d(fx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{e(fx)^{m+3}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+3}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+5}{2};-\ldots\right.}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c\ldots}{\sqrt{b^2-\ldots}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (3
 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x^4]*
AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[1 + (2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx = \int \left(d(fx)^m\sqrt{a+bx^2+cx^4}+\frac{e(fx)^{2+m}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$

$$= d\int (fx)^m\sqrt{a+bx^2+cx^4}\,dx+\frac{e\int (fx)^{2+m}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int (fx)^m\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{\left(e\sqrt{a+bx^2+\right.}{}$$

$$= \frac{d(fx)^{1+m}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{e(fx}{}$$

Mathematica [A] time = 0.25582, size = 267, normalized size = 0.84

$$\frac{x(fx)^m\sqrt{a+bx^2+cx^4}\left(d(m+3)F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+e(m+1)x^2F_1\left(\frac{m+3}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+5}{2};-\frac{2}{b+\sqrt{}}\right.\right.}{(m+1)(m+3)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*(f*x)^m*Sqrt[a + b*x^2 + c*x^4]*(d*(3 + m)*AppellF1[(1 + m)/2, -1/2, -1/
2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])] + e*(1 + m)*x^2*AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/((1 +
m)*(3 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [F] time = 0.008, size = 0, normalized size = 0.

$$\int (fx)^m \left(ex^2+d\right)\sqrt{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2+a}\left(ex^2+d\right)(fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\left(fx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(fx\right)^m\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m, x)
```

3.227 $\int \frac{(fx)^m\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=317

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c}{\sqrt{b^2-}}}}{f^3}$$

```
[Out] (d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*(1
+ m)*Sqrt[a + b*x^2 + c*x^4]) + (e*(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 +
 m)/2, 1/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.352481, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1335, 1141, 510}

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c}{\sqrt{b^2-}}}}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*(1
+ m)*Sqrt[a + b*x^2 + c*x^4]) + (e*(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 +
 m)/2, 1/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```


Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(fx)^m\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx=\int\left(\frac{d(fx)^m}{\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{2+m}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$

$$=d\int\frac{(fx)^m}{\sqrt{a+bx^2+cx^4}}\,dx+\frac{e\int\frac{(fx)^{2+m}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2}$$

$$=\frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int\frac{(fx)^m}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{\sqrt{a+bx^2+cx^4}}+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c}{b+\sqrt{b}}}\right.}{f^2\sqrt{a}}$$

$$=\frac{d(fx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{2};\frac{1}{2},\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{3+m}}{}$$

Mathematica [A] time = 0.275698, size = 267, normalized size = 0.84

$$\frac{x(fx)^m\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\left(d(m+3)F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+e(m+1)x^2F_1\left(\frac{m+3}{2};\frac{1}{2},\frac{1}{2};\right.\right.}{(m+1)(m+3)\sqrt{a+bx^2+cx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*(f*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(d*(3 + m)*
AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^2*AppellF1[(3 + m)/2, 1
/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])]))/((1 + m)*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F] time = 0.012, size = 0, normalized size = 0.

$$\int\left(fx\right)^m\left(ex^2+d\right)\frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^2+d\right)\left(fx\right)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)\left(fx\right)^m}{\sqrt{cx^4+bx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^m\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)\left(fx\right)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

3.228 $\int \frac{(fx)^m(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=323

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}}\cdots}}{af^3(m\cdots}$$

```
[Out] (d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*(
1 + m)*Sqrt[a + b*x^2 + c*x^4]) + (e*(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3
 + m)/2, 3/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])])/(a*f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.388058, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1335, 1141, 510}

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}}\cdots}}{af^3(m\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*(
1 + m)*Sqrt[a + b*x^2 + c*x^4]) + (e*(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3
 + m)/2, 3/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])])/(a*f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1141

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[(a^IntPart[p]*(a + b*x^2 + c*x^4)^FracPart[p])/((1 + (2*c*x^2)/(b
+ Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^2)/(b - Rt[b^2 - 4*a*c, 2]))
^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p*(1 + (
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(fx)^m\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \left(\frac{d(fx)^m}{\left(a+bx^2+cx^4\right)^{3/2}}+\frac{e(fx)^{2+m}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$= d\int \frac{(fx)^m}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx+\frac{e\int \frac{(fx)^{2+m}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^m}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^2+cx^4}}+\frac{\left(e\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\right.\cdots}{\cdots}$$

$$= \frac{d(fx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{2};\frac{3}{2},\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af(1+m)\sqrt{a+bx^2+cx^4}}+\frac{e(f\cdots}{\cdots}$$

Mathematica [A] time = 0.42826, size = 307, normalized size = 0.95

$$\frac{x(fx)^m\left(\sqrt{b^2-4ac}-b-2cx^2\right)\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}\right)^{3/2}\left(d(m+3)F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\cdots\right.}{(m+1)(m+3)\left(\sqrt{b^2-4ac}-b\right)\left(a+bx^2+cx^4\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]))^(3/2)*(d*(3 + m)*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/
2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
+ e*(1 + m)*x^2*AppellF1[(3 + m)/2, 3/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/((-b + Sqrt[b^2 - 4
*a*c])*(1 + m)*(3 + m)*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int (fx)^m\left(ex^2+d\right)\left(cx^4+bx^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)(fx)^m}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{cx^4+bx^2+a}(ex^2+d)(fx)^m}{c^2x^8+2\,bcx^6+(b^2+2\,ac)x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*(f*x)^m/(c^2*x^8 + 2*b*c*x^6 +
 (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m(d+ex^2)}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)(fx)^m}{(cx^4+bx^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.229 $\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=134

$$-\frac{a^2 e \log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)}+\frac{a^{3/2} d \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}+\frac{d^4 \log\left(d+ex^2\right)}{2e^3\left(ae^2+cd^2\right)}-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}$$

```
[Out] -(d*x^2)/(2*c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 + a*e^2)
) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.178927, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 1629, 635, 205, 260}

$$-\frac{a^2 e \log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)}+\frac{a^{3/2} d \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}+\frac{d^4 \log\left(d+ex^2\right)}{2e^3\left(ae^2+cd^2\right)}-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(d*x^2)/(2*c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 + a*e^2)
) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^9}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4}{(d+ex)\left(a+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{d}{ce^2}+\frac{x}{ce}+\frac{d^4}{e^2\left(cd^2+ae^2\right)(d+ex)}+\frac{a^2(d-ex)}{c\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx,x,x^2\right)\\
&= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2+ae^2\right)}+\frac{a^2\operatorname{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx,x,x^2\right)}{2c\left(cd^2+ae^2\right)}\\
&= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2+ae^2\right)}+\frac{\left(a^2d\right)\operatorname{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{2c\left(cd^2+ae^2\right)}-\frac{\left(a^2e\right)\operatorname{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)}{2c\left(cd^2+ae^2\right)}\\
&= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(cd^2+ae^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2+ae^2\right)}-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(cd^2+ae^2\right)}
\end{aligned}
$$

Mathematica [A] time = 0.0611195, size = 134, normalized size = 1.

$$
-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)}+\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(ae^2+cd^2\right)}-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(d*x^2)/(2*c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 + a*e^2)
) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2))
```

Maple [A] time = 0.011, size = 122, normalized size = 0.9

$$
\frac{x^4}{4\,ce}-\frac{dx^2}{2\,ce^2}-\frac{a^2e\ln\left(cx^4+a\right)}{4\,c^2\left(ae^2+cd^2\right)}+\frac{da^2}{\left(2\,ae^2+2\,cd^2\right)c}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}+\frac{d^4\ln\left(ex^2+d\right)}{2\,e^3\left(ae^2+cd^2\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4*x^4/c/e-1/2*d*x^2/c/e^2-1/4*a^2*e*ln(c*x^4+a)/c^2/(a*e^2+c*d^2)+1/2*a^2
/(a*e^2+c*d^2)/c*d/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))+1/2*d^4*ln(e*x^2+d
)/e^3/(a*e^2+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 24.9758, size = 563, normalized size = 4.2

$$\left[\frac{acde^3\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)-a^2e^4\log\left(cx^4+a\right)+2\,c^2d^4\log\left(ex^2+d\right)+\left(c^2d^2e^2+ace^4\right)x^4-2\left(c^2d^3e+acde^3\right)x^2}{4\left(c^3d^2e^3+ac^2e^5\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(a*c*d*e^3*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)
) - a^2*e^4*log(c*x^4 + a) + 2*c^2*d^4*log(e*x^2 + d) + (c^2*d^2*e^2 + a*c*
e^4)*x^4 - 2*(c^2*d^3*e + a*c*d*e^3)*x^2)/(c^3*d^2*e^3 + a*c^2*e^5), 1/4*(2
*a*c*d*e^3*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) - a^2*e^4*log(c*x^4 + a) + 2
*c^2*d^4*log(e*x^2 + d) + (c^2*d^2*e^2 + a*c*e^4)*x^4 - 2*(c^2*d^3*e + a*c*
d*e^3)*x^2)/(c^3*d^2*e^3 + a*c^2*e^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09969, size = 163, normalized size = 1.22

$$\frac{d^4\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e^3+ae^5\right)}-\frac{a^2e\log\left(cx^4+a\right)}{4\left(c^3d^2+ac^2e^2\right)}+\frac{a^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(c^2d^2+ace^2\right)\sqrt{ac}}+\frac{\left(cx^4e-2\,cdx^2\right)e^{(-2)}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(x^2*e + d))/(c*d^2*e^3 + a*e^5) - 1/4*a^2*e*log(c*x^4 + a)/
(c^3*d^2 + a*c^2*e^2) + 1/2*a^2*d*arctan(c*x^2/sqrt(a*c))/((c^2*d^2 + a*c*e
^2)*sqrt(a*c)) + 1/4*(c*x^4*e - 2*c*d*x^2)*e^(-2)/c^2
```

3.230 $\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=118

$$-\frac{a^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2+cd^2\right)}-\frac{ad\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}+\frac{x^2}{2ce}$$

```
[Out] x^2/(2*c*e) - (a^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c^(3/2)*(c*d^2 +
 a*e^2)) - (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 + a*e^2)) - (a*d*Log[a + c*x^
4])/(4*c*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.151951, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 1629, 635, 205, 260}

$$-\frac{a^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2+cd^2\right)}-\frac{ad\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}+\frac{x^2}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x^2/(2*c*e) - (a^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c^(3/2)*(c*d^2 +
 a*e^2)) - (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 + a*e^2)) - (a*d*Log[a + c*x^
4])/(4*c*(c*d^2 + a*e^2))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{ce}-\frac{d^3}{e(cd^2+ae^2)(d+ex)}-\frac{a(ae+cdx)}{c(cd^2+ae^2)(a+cx^2)}\right)dx, x, x^2\right)\\
&= \frac{x^2}{2ce}-\frac{d^3\log(d+ex^2)}{2e^2(cd^2+ae^2)}-\frac{a\operatorname{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx, x, x^2\right)}{2c(cd^2+ae^2)}\\
&= \frac{x^2}{2ce}-\frac{d^3\log(d+ex^2)}{2e^2(cd^2+ae^2)}-\frac{(ad)\operatorname{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}-\frac{(a^2e)\operatorname{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2c(cd^2+ae^2)}\\
&= \frac{x^2}{2ce}-\frac{a^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2c^{3/2}(cd^2+ae^2)}-\frac{d^3\log(d+ex^2)}{2e^2(cd^2+ae^2)}-\frac{ad\log(a+cx^4)}{4c(cd^2+ae^2)}
\end{aligned}
$$

Mathematica [A] time = 0.0954535, size = 99, normalized size = 0.84

$$
\frac{-\frac{2a^{3/2}e^3\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{c^{3/2}}+\frac{e\left(2x^2(ae^2+cd^2)-ade\log(a+cx^4)\right)}{c}-2d^3\log(d+ex^2)}{4e^2(ae^2+cd^2)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((-2*a^(3/2)*e^3*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2) - 2*d^3*Log[d + e*x
^2] + (e*(2*(c*d^2 + a*e^2)*x^2 - a*d*e*Log[a + c*x^4]))/c)/(4*e^2*(c*d^2 +
 a*e^2))
```

Maple [A] time = 0.007, size = 108, normalized size = 0.9

$$
\frac{x^2}{2\,ce}-\frac{ad\ln(cx^4+a)}{4\,(ae^2+cd^2)\,c}-\frac{a^2e}{(2\,ae^2+2\,cd^2)\,c}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{d^3\ln(ex^2+d)}{2\,e^2\,(ae^2+cd^2)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/2*x^2/c/e-1/4*a*d*ln(c*x^4+a)/c/(a*e^2+c*d^2)-1/2*a^2/(a*e^2+c*d^2)/c*e/(
a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 9.13428, size = 446, normalized size = 3.78

$$\left[\frac{ae^3\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)-ade^2\log\left(cx^4+a\right)-2\,cd^3\log\left(ex^2+d\right)+2\left(cd^2e+ae^3\right)x^2}{4\left(c^2d^2e^2+ace^4\right)},-\frac{2\,ae^3\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(a*e^3*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) -
a*d*e^2*log(c*x^4 + a) - 2*c*d^3*log(e*x^2 + d) + 2*(c*d^2*e + a*e^3)*x^2)/
(c^2*d^2*e^2 + a*c*e^4), -1/4*(2*a*e^3*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a)
+ a*d*e^2*log(c*x^4 + a) + 2*c*d^3*log(e*x^2 + d) - 2*(c*d^2*e + a*e^3)*x^2
)/(c^2*d^2*e^2 + a*c*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08914, size = 142, normalized size = 1.2

$$-\frac{d^3\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e^2+ae^4\right)}-\frac{a^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(c^2d^2+ace^2\right)\sqrt{ac}}+\frac{x^2e^{(-1)}}{2\,c}-\frac{ad\log\left(cx^4+a\right)}{4\left(c^2d^2+ace^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*d^3*log(abs(x^2*e + d))/(c*d^2*e^2 + a*e^4) - 1/2*a^2*arctan(c*x^2/sqr
t(a*c))*e/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) + 1/2*x^2*e^(-1)/c - 1/4*a*d*log(
c*x^4 + a)/(c^2*d^2 + a*c*e^2)
```

3.231 $\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=105

$$\frac{d^2\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)}+\frac{ae\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}-\frac{\sqrt{a}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(ae^2+cd^2\right)}$$

```
[Out] -(Sqrt[a]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[c]*(c*d^2 + a*e^2)) + (d
^2*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)) + (a*e*Log[a + c*x^4])/(4*c*(c*d^2
 + a*e^2))
```

Rubi [A] time = 0.136745, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 1629, 635, 205, 260}

$$\frac{d^2\log\left(d+ex^2\right)}{2e\left(ae^2+cd^2\right)}+\frac{ae\log\left(a+cx^4\right)}{4c\left(ae^2+cd^2\right)}-\frac{\sqrt{a}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(Sqrt[a]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[c]*(c*d^2 + a*e^2)) + (d
^2*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)) + (a*e*Log[a + c*x^4])/(4*c*(c*d^2
 + a*e^2))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{d^2}{\left(cd^2+ae^2\right)(d+ex)} - \frac{a(d-ex)}{\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right) dx, x, x^2\right)\\
&= \frac{d^2\log\left(d+ex^2\right)}{2e\left(cd^2+ae^2\right)} - \frac{a\operatorname{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)}\\
&= \frac{d^2\log\left(d+ex^2\right)}{2e\left(cd^2+ae^2\right)} - \frac{(ad)\operatorname{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)} + \frac{(ae)\operatorname{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)}\\
&= -\frac{\sqrt{a}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(cd^2+ae^2\right)} + \frac{d^2\log\left(d+ex^2\right)}{2e\left(cd^2+ae^2\right)} + \frac{ae\log\left(a+cx^4\right)}{4c\left(cd^2+ae^2\right)}
\end{aligned}
$$

Mathematica [A] time = 0.0347916, size = 77, normalized size = 0.73

$$\frac{-2\sqrt{a}\sqrt{c}de\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right) + ae^2\log\left(a+cx^4\right) + 2cd^2\log\left(d+ex^2\right)}{4ace^3+4c^2d^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + 2*c*d^2*Log[d + e*x
^2] + a*e^2*Log[a + c*x^4])/(4*c^2*d^2*e + 4*a*c*e^3)
```

Maple [A] time = 0.008, size = 92, normalized size = 0.9

$$\frac{ae\ln\left(cx^4+a\right)}{4\,c\left(ae^2+cd^2\right)} - \frac{ad}{2\,ae^2+2\,cd^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}} + \frac{d^2\ln\left(ex^2+d\right)}{2\,e\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4*a*e*ln(c*x^4+a)/c/(a*e^2+c*d^2)-1/2*a/(a*e^2+c*d^2)*d/(a*c)^(1/2)*arcta
n(c*x^2/(a*c)^(1/2))+1/2*d^2*ln(e*x^2+d)/e/(a*e^2+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 5.37463, size = 365, normalized size = 3.48

$$\left[\frac{cde\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)+ae^2\log\left(cx^4+a\right)+2\,cd^2\log\left(ex^2+d\right)}{4\left(c^2d^2e+ace^3\right)},-\frac{2\,cde\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)-ae^2\log\left(cx^4+a\right)-\cdots}{4\left(c^2d^2e+ace^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(c*d*e*sqrt(-a/c)*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) +
a*e^2*log(c*x^4 + a) + 2*c*d^2*log(e*x^2 + d))/(c^2*d^2*e + a*c*e^3), -1/4*
(2*c*d*e*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) - a*e^2*log(c*x^4 + a) - 2*c*d
^2*log(e*x^2 + d))/(c^2*d^2*e + a*c*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1149, size = 122, normalized size = 1.16

$$\frac{ae\log\left(cx^4+a\right)}{4\left(c^2d^2+ace^2\right)}+\frac{d^2\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e+ae^3\right)}-\frac{ad\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(cd^2+ae^2\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*a*e*log(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*d^2*log(abs(x^2*e + d))/(c
*d^2*e + a*e^3) - 1/2*a*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c
))
```

3.232 $\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=96

$$-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}+\frac{d\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}+\frac{\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(ae^2+cd^2\right)}$$

```
[Out] (Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[c]*(c*d^2 + a*e^2)) - (d*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) + (d*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
)
```

Rubi [A] time = 0.0944008, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 801, 635, 205, 260}

$$-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}+\frac{d\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}+\frac{\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[c]*(c*d^2 + a*e^2)) - (d*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) + (d*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
)
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{de}{\left(cd^2+ae^2\right)(d+ex)}+\frac{ae+cdx}{\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx, x, x^2\right)\\
&= -\frac{d\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)}+\frac{\operatorname{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)}\\
&= -\frac{d\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)}+\frac{(cd)\operatorname{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)}+\frac{(ae)\operatorname{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)}\\
&= \frac{\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}\left(cd^2+ae^2\right)}-\frac{d\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)}+\frac{d\log\left(a+cx^4\right)}{4\left(cd^2+ae^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.0392544, size = 66, normalized size = 0.69

$$\frac{d\log\left(a+cx^4\right)+\frac{2\sqrt{a}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{c}}-2d\log\left(d+ex^2\right)}{4ae^2+4cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((2*Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[c] - 2*d*Log[d + e*x^2] +
 d*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

Maple [A] time = 0.007, size = 83, normalized size = 0.9

$$\frac{d\ln\left(cx^4+a\right)}{4\,ae^2+4\,cd^2}+\frac{ae}{2\,ae^2+2\,cd^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{d\ln\left(ex^2+d\right)}{2\,ae^2+2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4*d*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2/(a*e^2+c*d^2)*a*e/(a*c)^(1/2)*arctan(c*
x^2/(a*c)^(1/2))-1/2*d*ln(e*x^2+d)/(a*e^2+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.3, size = 315, normalized size = 3.28

$$\left[\frac{e\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)+d\log\left(cx^4+a\right)-2\,d\log\left(ex^2+d\right)}{4\left(cd^2+ae^2\right)}, \frac{2\,e\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)+d\log\left(cx^4+a\right)-2\,d\log\left(ex^2+\right.}{4\left(cd^2+ae^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(e*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + d*lo
g(c*x^4 + a) - 2*d*log(e*x^2 + d))/(c*d^2 + a*e^2), 1/4*(2*e*sqrt(a/c)*arct
an(c*x^2*sqrt(a/c)/a) + d*log(c*x^4 + a) - 2*d*log(e*x^2 + d))/(c*d^2 + a*e
^2)]
```

Sympy [B] time = 144.776, size = 932, normalized size = 9.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] -d*log(x**2 + (16*a**2*c*d**3*e**4/(a*e**2 + c*d**2)**2 - 2*a**2*d*e**4/(a*
e**2 + c*d**2) + 32*a*c**2*d**5*e**2/(a*e**2 + c*d**2)**2 - 4*a*c*d**3*e**2
/(a*e**2 + c*d**2) + 3*a*d*e**2 + 16*c**3*d**7/(a*e**2 + c*d**2)**2 - 2*c**
2*d**5/(a*e**2 + c*d**2) - 5*c*d**3)/(a*e**3 + 9*c*d**2*e))/(2*(a*e**2 + c*
d**2)) + (d/(4*(a*e**2 + c*d**2)) - e*sqrt(-a*c)/(4*c*(a*e**2 + c*d**2)))*l
og(x**2 + (64*a**2*c*d*e**4*(d/(4*(a*e**2 + c*d**2)) - e*sqrt(-a*c)/(4*c*(a
*e**2 + c*d**2)))**2 + 4*a**2*e**4*(d/(4*(a*e**2 + c*d**2)) - e*sqrt(-a*c)/
(4*c*(a*e**2 + c*d**2))) + 128*a*c**2*d**3*e**2*(d/(4*(a*e**2 + c*d**2)) -
e*sqrt(-a*c)/(4*c*(a*e**2 + c*d**2)))**2 + 8*a*c*d**2*e**2*(d/(4*(a*e**2 +
c*d**2)) - e*sqrt(-a*c)/(4*c*(a*e**2 + c*d**2))) + 3*a*d*e**2 + 64*c**3*d**
5*(d/(4*(a*e**2 + c*d**2)) - e*sqrt(-a*c)/(4*c*(a*e**2 + c*d**2)))**2 + 4*c
**2*d**4*(d/(4*(a*e**2 + c*d**2)) - e*sqrt(-a*c)/(4*c*(a*e**2 + c*d**2))) -
 5*c*d**3)/(a*e**3 + 9*c*d**2*e)) + (d/(4*(a*e**2 + c*d**2)) + e*sqrt(-a*c)
/(4*c*(a*e**2 + c*d**2)))*log(x**2 + (64*a**2*c*d*e**4*(d/(4*(a*e**2 + c*d*
*2)) + e*sqrt(-a*c)/(4*c*(a*e**2 + c*d**2)))**2 + 4*a**2*e**4*(d/(4*(a*e**2
 + c*d**2)) + e*sqrt(-a*c)/(4*c*(a*e**2 + c*d**2))) + 128*a*c**2*d**3*e**2*
(d/(4*(a*e**2 + c*d**2)) + e*sqrt(-a*c)/(4*c*(a*e**2 + c*d**2)))**2 + 8*a*c
*d**2*e**2*(d/(4*(a*e**2 + c*d**2)) + e*sqrt(-a*c)/(4*c*(a*e**2 + c*d**2)))
 + 3*a*d*e**2 + 64*c**3*d**5*(d/(4*(a*e**2 + c*d**2)) + e*sqrt(-a*c)/(4*c*(
a*e**2 + c*d**2)))**2 + 4*c**2*d**4*(d/(4*(a*e**2 + c*d**2)) + e*sqrt(-a*c)
/(4*c*(a*e**2 + c*d**2))) - 5*c*d**3)/(a*e**3 + 9*c*d**2*e))
```

Giac [A] time = 1.11379, size = 116, normalized size = 1.21

$$-\frac{de\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e+ae^3\right)}+\frac{a\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(cd^2+ae^2\right)\sqrt{ac}}+\frac{d\log\left(cx^4+a\right)}{4\left(cd^2+ae^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*d*e*log(abs(x^2*e + d))/(c*d^2*e + a*e^3) + 1/2*a*arctan(c*x^2/sqrt(a*
c))*e/((c*d^2 + a*e^2)*sqrt(a*c)) + 1/4*d*log(c*x^4 + a)/(c*d^2 + a*e^2)
```

3.233 $\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=96

$$\frac{e\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}-\frac{e\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}+\frac{\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}$$

```
[Out] (Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)) + (e*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) - (e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
)
```

Rubi [A] time = 0.0636733, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1248, 706, 31, 635, 205, 260}

$$\frac{e\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)}-\frac{e\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)}+\frac{\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)) + (e*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) - (e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
)
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{cd-cex}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)} + \frac{e^2\operatorname{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)}\\
&= \frac{e\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)} + \frac{(cd)\operatorname{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)} - \frac{(ce)\operatorname{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)}\\
&= \frac{\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^2+ae^2\right)} + \frac{e\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)} - \frac{e\log\left(a+cx^4\right)}{4\left(cd^2+ae^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.0367347, size = 67, normalized size = 0.7

$$\frac{\frac{2\sqrt{c}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}} - e\log\left(a+cx^4\right) + 2e\log\left(d+ex^2\right)}{4ae^2+4cd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((2*Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[a] + 2*e*Log[d + e*x^2] -
 e*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

Maple [A] time = 0.006, size = 83, normalized size = 0.9

$$-\frac{e\ln\left(cx^4+a\right)}{4\,ae^2+4\,cd^2} + \frac{cd}{2\,ae^2+2\,cd^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}} + \frac{e\ln\left(ex^2+d\right)}{2\,ae^2+2\,cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/4*e*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2*c/(a*e^2+c*d^2)*d/(a*c)^(1/2)*arctan(c
*x^2/(a*c)^(1/2))+1/2*e*ln(e*x^2+d)/(a*e^2+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.81944, size = 319, normalized size = 3.32

$$\left[\frac{d\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)-e\log\left(cx^4+a\right)+2\,e\log\left(ex^2+d\right)}{4\left(cd^2+ae^2\right)}, -\frac{2\,d\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right)+e\log\left(cx^4+a\right)-2\,e\log\left(ex^2+\right.}{4\left(cd^2+ae^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(d*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - e*lo
g(c*x^4 + a) + 2*e*log(e*x^2 + d))/(c*d^2 + a*e^2), -1/4*(2*d*sqrt(c/a)*arc
tan(a*sqrt(c/a)/(c*x^2)) + e*log(c*x^4 + a) - 2*e*log(e*x^2 + d))/(c*d^2 +
a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09925, size = 115, normalized size = 1.2

$$\frac{cd\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(cd^2+ae^2\right)\sqrt{ac}}-\frac{e\log\left(cx^4+a\right)}{4\left(cd^2+ae^2\right)}+\frac{e^2\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e+ae^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*c*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/4*e*log(c*x
^4 + a)/(c*d^2 + a*e^2) + 1/2*e^2*log(abs(x^2*e + d))/(c*d^2*e + a*e^3)
```

3.234 $\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=114

$$-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)}-\frac{cd\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}-\frac{\sqrt{c}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}+\frac{\log(x)}{ad}$$

```
[Out] -(Sqrt[c]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)) + Lo
g[x]/(a*d) - (e^2*Log[d + e*x^2])/(2*d*(c*d^2 + a*e^2)) - (c*d*Log[a + c*x^
4])/(4*a*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.124906, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 894, 635, 205, 260}

$$-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)}-\frac{cd\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}-\frac{\sqrt{c}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(ae^2+cd^2\right)}+\frac{\log(x)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(Sqrt[c]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)) + Lo
g[x]/(a*d) - (e^2*Log[d + e*x^2])/(2*d*(c*d^2 + a*e^2)) - (c*d*Log[a + c*x^
4])/(4*a*(c*d^2 + a*e^2))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(d+ex)(a+cx^2)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{adx} - \frac{e^3}{d(cd^2+ae^2)(d+ex)} - \frac{c(ae+cdx)}{a(cd^2+ae^2)(a+cx^2)}\right)dx, x, x^2\right)\\ &= \frac{\log(x)}{ad} - \frac{e^2\log(d+ex^2)}{2d(cd^2+ae^2)} - \frac{c\operatorname{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx, x, x^2\right)}{2a(cd^2+ae^2)}\\ &= \frac{\log(x)}{ad} - \frac{e^2\log(d+ex^2)}{2d(cd^2+ae^2)} - \frac{(c^2d)\operatorname{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2a(cd^2+ae^2)} - \frac{(ce)\operatorname{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}\\ &= -\frac{\sqrt{c}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}(cd^2+ae^2)} + \frac{\log(x)}{ad} - \frac{e^2\log(d+ex^2)}{2d(cd^2+ae^2)} - \frac{cd\log(a+cx^4)}{4a(cd^2+ae^2)}\end{aligned}$$

Mathematica [A] time = 0.0710177, size = 134, normalized size = 1.18

$$\frac{-cd^2\log(a+cx^4) + 2\sqrt{a}\sqrt{c}de\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\sqrt{a}\sqrt{c}de\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right) - 2ae^2\log(d+ex^2) + 4ae^2\log(x) + 4cd}{4a^2de^2 + 4acd^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[a]*
Sqrt[c]*d*e*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*c*d^2*Log[x] + 4*a*
e^2*Log[x] - 2*a*e^2*Log[d + e*x^2] - c*d^2*Log[a + c*x^4])/(4*a*c*d^3 + 4*
a^2*d*e^2)
```

Maple [A] time = 0.01, size = 101, normalized size = 0.9

$$-\frac{cd\ln(cx^4+a)}{4(ae^2+cd^2)a} - \frac{ec}{2ae^2+2cd^2}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}} + \frac{\ln(x)}{ad} - \frac{e^2\ln(ex^2+d)}{2d(ae^2+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/4*c*d*ln(c*x^4+a)/a/(a*e^2+c*d^2)-1/2*c/(a*e^2+c*d^2)*e/(a*c)^(1/2)*arct
an(c*x^2/(a*c)^(1/2))+ln(x)/a/d-1/2*e^2*ln(e*x^2+d)/d/(a*e^2+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 36.2875, size = 439, normalized size = 3.85

$$\left[\frac{ade\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)-cd^2\log\left(cx^4+a\right)-2\,ae^2\log\left(ex^2+d\right)+4\left(cd^2+ae^2\right)\log\left(x\right)}{4\left(acd^3+a^2de^2\right)},\frac{2\,ade\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d*e*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) -
c*d^2*log(c*x^4 + a) - 2*a*e^2*log(e*x^2 + d) + 4*(c*d^2 + a*e^2)*log(x))/(
a*c*d^3 + a^2*d*e^2), 1/4*(2*a*d*e*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) -
c*d^2*log(c*x^4 + a) - 2*a*e^2*log(e*x^2 + d) + 4*(c*d^2 + a*e^2)*log(x))/(
a*c*d^3 + a^2*d*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.0956, size = 138, normalized size = 1.21

$$-\frac{cd\log\left(cx^4+a\right)}{4\left(acd^2+a^2e^2\right)}-\frac{c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(cd^2+ae^2\right)\sqrt{ac}}-\frac{e^3\log\left(\left|x^2e+d\right|\right)}{2\left(cd^3e+ade^3\right)}+\frac{\log\left(x^2\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*c*d*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) - 1/2*c*arctan(c*x^2/sqrt(a*c))
*e/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/2*e^3*log(abs(x^2*e + d))/(c*d^3*e + a*d
*e^3) + 1/2*log(x^2)/(a*d)
```

3.235 $\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=129

$$-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}-\frac{e\log(x)}{ad^2}-\frac{1}{2adx^2}$$

```
[Out] -1/(2*a*d*x^2) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^
2 + a*e^2)) - (e*Log[x])/(a*d^2) + (e^3*Log[d + e*x^2])/(2*d^2*(c*d^2 + a*e
^2)) + (c*e*Log[a + c*x^4])/(4*a*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.149769, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 894, 635, 205, 260}

$$-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}-\frac{e\log(x)}{ad^2}-\frac{1}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/(2*a*d*x^2) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*d^
2 + a*e^2)) - (e*Log[x])/(a*d^2) + (e^3*Log[d + e*x^2])/(2*d^2*(c*d^2 + a*e
^2)) + (c*e*Log[a + c*x^4])/(4*a*(c*d^2 + a*e^2))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx^2}-\frac{e}{ad^2x}+\frac{e^4}{d^2\left(cd^2+ae^2\right)(d+ex)}-\frac{c^2(d-ex)}{a\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx,x,x\right.\\
&= -\frac{1}{2adx^2}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)}-\frac{c^2\,\text{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx,x,x^2\right)}{2a\left(cd^2+ae^2\right)}\\
&= -\frac{1}{2adx^2}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)}-\frac{\left(c^2d\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx,x,x^2\right)}{2a\left(cd^2+ae^2\right)}+\frac{\left(c^2e\right)\text{Subst}\left(\right.}{2a\left(c\right.}\\
&= -\frac{1}{2adx^2}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^2+ae^2\right)}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(cd^2+ae^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.103158, size = 169, normalized size = 1.31

$$\frac{2c^{3/2}d^3x^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)+2c^{3/2}d^3x^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)+\sqrt{a}\left(-4ex^2\log(x)\left(ae^2+cd^2\right)+cd^2ex^2\log\left(a+cx^4\right)+2a\right.}{4a^{3/2}d^2x^2\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (2*c^(3/2)*d^3*x^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(3/2)*d^3*
x^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*(-2*c*d^3 - 2*a*d*e^2
 - 4*e*(c*d^2 + a*e^2)*x^2*Log[x] + 2*a*e^3*x^2*Log[d + e*x^2] + c*d^2*e*x^
2*Log[a + c*x^4]))/(4*a^(3/2)*d^2*(c*d^2 + a*e^2)*x^2)
```

Maple [A] time = 0.01, size = 119, normalized size = 0.9

$$\frac{ec\ln\left(cx^4+a\right)}{4\left(ae^2+cd^2\right)a}-\frac{c^2d}{\left(2\,ae^2+2\,cd^2\right)a}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}-\frac{1}{2\,adx^2}-\frac{\ln(x)\,e}{d^2a}+\frac{e^3\ln\left(ex^2+d\right)}{2\,d^2\left(ae^2+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4*c*e*ln(c*x^4+a)/a/(a*e^2+c*d^2)-1/2*c^2/(a*e^2+c*d^2)/a*d/(a*c)^(1/2)*a
rctan(c*x^2/(a*c)^(1/2))-1/2/a/d/x^2-e*ln(x)/a/d^2+1/2*e^3*ln(e*x^2+d)/d^2/
(a*e^2+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 178.174, size = 574, normalized size = 4.45

$$\left[\frac{cd^3x^2\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)+cd^2ex^2\log\left(cx^4+a\right)+2\,ae^3x^2\log\left(ex^2+d\right)-2\,cd^3-2\,ade^2-4\left(cd^2e+ae^3\right)x^2\log\left(x\right)}{4\left(acd^4+a^2d^2e^2\right)x^2},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(c*d^3*x^2*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)
) + c*d^2*e*x^2*log(c*x^4 + a) + 2*a*e^3*x^2*log(e*x^2 + d) - 2*c*d^3 - 2*a
*d*e^2 - 4*(c*d^2*e + a*e^3)*x^2*log(x))/((a*c*d^4 + a^2*d^2*e^2)*x^2), 1/4
*(2*c*d^3*x^2*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) + c*d^2*e*x^2*log(c*x^4
 + a) + 2*a*e^3*x^2*log(e*x^2 + d) - 2*c*d^3 - 2*a*d*e^2 - 4*(c*d^2*e + a*e
^3)*x^2*log(x))/((a*c*d^4 + a^2*d^2*e^2)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10962, size = 178, normalized size = 1.38

$$-\frac{c^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(acd^2+a^2e^2\right)\sqrt{ac}}+\frac{ce\log\left(cx^4+a\right)}{4\left(acd^2+a^2e^2\right)}+\frac{e^4\log\left(\left|x^2e+d\right|\right)}{2\left(cd^4e+ad^2e^3\right)}-\frac{e\log\left(x^2\right)}{2\,ad^2}+\frac{x^2e-d}{2\,ad^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*c^2*d*arctan(c*x^2/sqrt(a*c))/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) + 1/4*c*
e*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) + 1/2*e^4*log(abs(x^2*e + d))/(c*d^4*e
 + a*d^2*e^3) - 1/2*e*log(x^2)/(a*d^2) + 1/2*(x^2*e - d)/(a*d^2*x^2)
```

3.236 $\int \frac{1}{x^5(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=156

$$\frac{c^2 d \log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)}+\frac{c^{3/2} e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}-\frac{\log(x)\left(cd^2-ae^2\right)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)}+\frac{e}{2ad^2x^2}-\frac{1}{4adx^4}$$

```
[Out] -1/(4*a*d*x^4) + e/(2*a*d^2*x^2) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*a^(3/2)*(c*d^2 + a*e^2)) - ((c*d^2 - a*e^2)*Log[x])/(a^2*d^3) - (e^4*L
og[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)) + (c^2*d*Log[a + c*x^4])/(4*a^2*(c*d
^2 + a*e^2))
```

Rubi [A] time = 0.183466, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1252, 894, 635, 205, 260}

$$\frac{c^2 d \log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)}+\frac{c^{3/2} e \tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}-\frac{\log(x)\left(cd^2-ae^2\right)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)}+\frac{e}{2ad^2x^2}-\frac{1}{4adx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/(4*a*d*x^4) + e/(2*a*d^2*x^2) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*a^(3/2)*(c*d^2 + a*e^2)) - ((c*d^2 - a*e^2)*Log[x])/(a^2*d^3) - (e^4*L
og[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)) + (c^2*d*Log[a + c*x^4])/(4*a^2*(c*d
^2 + a*e^2))
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x^5(d+ex^2)(a+cx^4)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(d+ex)(a+cx^2)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{adx^3} - \frac{e}{ad^2x^2} + \frac{-cd^2+ae^2}{a^2d^3x} - \frac{e^5}{d^3(cd^2+ae^2)(d+ex)} + \frac{c^2(ae+cdx)}{a^2(cd^2+ae^2)(a+\right.\right.$$
$$= -\frac{1}{4adx^4} + \frac{e}{2ad^2x^2} - \frac{(cd^2-ae^2)\log(x)}{a^2d^3} - \frac{e^4\log(d+ex^2)}{2d^3(cd^2+ae^2)} + \frac{c^2\operatorname{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx, x, x^2\right)}{2a^2(cd^2+ae^2)}$$
$$= -\frac{1}{4adx^4} + \frac{e}{2ad^2x^2} - \frac{(cd^2-ae^2)\log(x)}{a^2d^3} - \frac{e^4\log(d+ex^2)}{2d^3(cd^2+ae^2)} + \frac{(c^3d)\operatorname{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right.}{2a^2(cd^2+ae^2)}$$
$$= -\frac{1}{4adx^4} + \frac{e}{2ad^2x^2} + \frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{3/2}(cd^2+ae^2)} - \frac{(cd^2-ae^2)\log(x)}{a^2d^3} - \frac{e^4\log(d+ex^2)}{2d^3(cd^2+ae^2)} + \frac{c^2d\log(}{4a^2(cd^2}$$

Mathematica [A] time = 0.0946543, size = 209, normalized size = 1.34

$$-\frac{a^2d^2e^2 - 2a^2de^3x^2 + 2a^2e^4x^4\log(d+ex^2) - 4a^2e^4x^4\log(x) + 2\sqrt{a}c^{3/2}d^3ex^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right) + 2\sqrt{a}c^{3/2}d^3ex^4\tan^{-1}\left(\frac{\sqrt{}}{}\right.}{4a^2d^3x^4(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(a*c*d^4 + a^2*d^2*e^2 - 2*a*c*d^3*e*x^2 - 2*a^2*d*e^3*x^2 + 2*Sqrt[a]*c^(
3/2)*d^3*e*x^4*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[a]*c^(3/2)*
d^3*e*x^4*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*c^2*d^4*x^4*Log[x] -
4*a^2*e^4*x^4*Log[x] + 2*a^2*e^4*x^4*Log[d + e*x^2] - c^2*d^4*x^4*Log[a + c
*x^4])/(4*a^2*d^3*(c*d^2 + a*e^2)*x^4)
```

Maple [A] time = 0.013, size = 145, normalized size = 0.9

$$\frac{c^2d\ln(cx^4+a)}{4(ae^2+cd^2)a^2} + \frac{c^2e}{(2ae^2+2cd^2)a}\arctan\left(cx^2\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}} - \frac{1}{4adx^4} + \frac{\ln(x)e^2}{d^3a} - \frac{\ln(x)c}{da^2} + \frac{e}{2d^2ax^2} - \frac{e^4\ln(ex^2+d)}{2d^3(ae^2+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4*c^2*d*ln(c*x^4+a)/a^2/(a*e^2+c*d^2)+1/2*c^2/(a*e^2+c*d^2)/a*e/(a*c)^(1/
2)*arctan(c*x^2/(a*c)^(1/2))-1/4/a/d/x^4+1/d^3/a*ln(x)*e^2-1/d/a^2*ln(x)*c+
1/2*e/a/d^2/x^2-1/2*e^4*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09408, size = 227, normalized size = 1.46

$$\frac{c^2 d \log\left(cx^4 + a\right)}{4\left(a^2cd^2 + a^3e^2\right)} + \frac{c^2 \arctan\left(\frac{cx^2}{\sqrt{ac}}\right) e}{2\left(acd^2 + a^2e^2\right)\sqrt{ac}} - \frac{e^5 \log\left(\left|x^2e + d\right|\right)}{2\left(cd^5e + ad^3e^3\right)} - \frac{\left(cd^2 - ae^2\right)\log\left(x^2\right)}{2\,a^2d^3} + \frac{3\,cd^2x^4 - 3\,ax^4e^2 + 2\,adx^2e - ad^2}{4\,a^2d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*c^2*d*log(c*x^4 + a)/(a^2*c*d^2 + a^3*e^2) + 1/2*c^2*arctan(c*x^2/sqrt(
a*c))*e/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) - 1/2*e^5*log(abs(x^2*e + d))/(c*d^
5*e + a*d^3*e^3) - 1/2*(c*d^2 - a*e^2)*log(x^2)/(a^2*d^3) + 1/4*(3*c*d^2*x^
4 - 3*a*x^4*e^2 + 2*a*d*x^2*e - a*d^2)/(a^2*d^3*x^4)
```

3.237 $\int \frac{x^8}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=359

$$-\frac{a^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}+\frac{a^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}-\frac{a^{5/4}\left(\sqrt{c}d-\sqrt{ae}\right.}{2\sqrt{2}c^{7/4}}$$

```
[Out] -((d*x)/(c*e^2)) + x^3/(3*c*e) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(
5/2)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + (a^(5/4)*(Sqrt
[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7
/4)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) +
(a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.345867, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{a^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}+\frac{a^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}-\frac{a^{5/4}\left(\sqrt{c}d-\sqrt{ae}\right.}{2\sqrt{2}c^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((d + e*x^2)*(a + c*x^4)),x]
```


```
[Out] -((d*x)/(c*e^2)) + x^3/(3*c*e) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(
5/2)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + (a^(5/4)*(Sqrt
[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7
/4)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) +
(a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```


Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
```

```
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^8}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(-\frac{d}{ce^2}+\frac{x^2}{ce}+\frac{d^4}{e^2\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{a^2\left(d-ex^2\right)}{c\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{a^2\int \frac{d-ex^2}{a+cx^4}\,dx}{c\left(cd^2+ae^2\right)}+\frac{d^4\int \frac{1}{d+ex^2}\,dx}{e^2\left(cd^2+ae^2\right)}$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2+ae^2\right)}+\frac{\left(a^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c^2\left(cd^2+ae^2\right)}+\frac{\left(a^2\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\cdots}{2c^2\left(cd^2+ae^2\right)}$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2+ae^2\right)}+\frac{\left(a^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c^2\left(cd^2+ae^2\right)}+\frac{\left(a^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \cdots}{4c^2\left(cd^2+\cdots\right)}$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2+ae^2\right)}-\frac{a^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(cd^2+ae^2\right)}+\frac{a^{5/4}\cdots}{\cdots}$$

$$= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2+ae^2\right)}-\frac{a^{7/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{7/4}\left(cd^2+ae^2\right)}+\frac{a^{7/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\tan^{-1}\cdots}{2\sqrt{2}c^{7/4}\left(cd^2\cdots\right)}$$

Mathematica [A] time = 0.379146, size = 344, normalized size = 0.96

$$-3\sqrt{2}ae^{5/2}\left(a^{3/4}e+\sqrt[4]{a}\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+3\sqrt{2}ae^{5/2}\left(a^{3/4}e+\sqrt[4]{a}\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-24*c^(3/4)*d*Sqrt[e]*(c*d^2 + a*e^2)*x + 8*c^(3/4)*e^(3/2)*(c*d^2 + a*e^2
)*x^3 + 24*c^(7/4)*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 6*Sqrt[2]*a^(5/4)*
e^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
- 6*Sqrt[2]*a^(5/4)*e^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*
c^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*a*e^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*a*e^(5/2)
*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(24*c^(7/4)*e^(5/2)*(c*d^2 + a*e^2))
```

Maple [A] time = 0.013, size = 405, normalized size = 1.1

$$\frac{x^3}{3ce}-\frac{dx}{ce^2}+\frac{ad\sqrt{2}}{\left(8ae^2+8cd^2\right)c}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{ad\sqrt{2}}{\left(4ae^2+4cd^2\right)c}\sqrt[4]{\frac{a}{c}}\arctan\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/3*x^3/c/e-d*x/c/e^2+1/8*a/(a*e^2+c*d^2)/c*d*(1/c*a)^(1/4)*2^(1/2)*ln((x^2
+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a
)^(1/2)))+1/4*a/(a*e^2+c*d^2)/c*d*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c
*a)^(1/4)*x+1)+1/4*a/(a*e^2+c*d^2)/c*d*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(1/c*a)^(1/4)*x-1)-1/8*a^2/(a*e^2+c*d^2)/c^2*e/(1/c*a)^(1/4)*2^(1/2)*ln((x
^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c
*a)^(1/2)))-1/4*a^2/(a*e^2+c*d^2)/c^2*e/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(1/c*a)^(1/4)*x+1)-1/4*a^2/(a*e^2+c*d^2)/c^2*e/(1/c*a)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(1/c*a)^(1/4)*x-1)+1/e^2*d^4/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*
x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 52.3569, size = 8402, normalized size = 23.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*c*d^3*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) +
 4*(c*d^2*e + a*e^3)*x^3 - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e + (c^5
*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2
+ a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2
*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*
c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^
2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*
d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^
8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5
*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*
c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2
 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e + (c^5*d^4 +
2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e
^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 +
a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 -
 a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^2*e^3 +
 a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4
*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sq
rt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^
4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4
*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*
c^3*e^4))) - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4
*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^
11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7
*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^
2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^
5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10
*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a
^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a
^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 +
4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4
))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^
2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8
+ 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))
/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x -
(a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*
sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^
2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e
- (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^
2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c
^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) - 1
2*(c*d^3 + a*d*e^2)*x)/(c^2*d^2*e^2 + a*c*e^4), 1/12*(12*c*d^3*sqrt(d/e)*ar
ctan(e*x*sqrt(d/e)/d) + 4*(c*d^2*e + a*e^3)*x^3 - 3*(c^2*d^2*e^2 + a*c*e^4)
*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2
*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*
d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a
^2*c^3*e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 +
(c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^
2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c
^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 +
a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*
a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^
5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((
2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 -
2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4
```

```
 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*
e^4))*log(-(a^3*c*d^2 - a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^
4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 +
 a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*
e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3
*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*
d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 +
 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d
*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c
*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^
3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*l
og(-(a^3*c*d^2 - a^4*e^2)*x + (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2
*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^
4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a
^4*c^7*e^8)))*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*s
qrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2
 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^
4*d^2*e^2 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c
^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^
2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d
^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-(a^
3*c*d^2 - a^4*e^2)*x - (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c^7*d^4*e + 2*a*c^6*
d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^1
1*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*
e^8)))*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a
^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^
2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e
^2 + a^2*c^3*e^4))) - 12*(c*d^3 + a*d*e^2)*x)/(c^2*d^2*e^2 + a*c*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10859, size = 490, normalized size = 1.36

$$\frac{d^{\frac{7}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2e^2+ae^4} + \frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d-\left(ac^3\right)^{\frac{3}{4}}ae\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)} + \frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d-\left(ac^3\right)^{\frac{3}{4}}ae\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{\cdots}\right.}{2\left(\frac{a}{c}\cdots\right.}\right.}{2\left(\sqrt{2}c^5d^2+\sqrt{2}ac^4e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e^2 + a*e^4) + 1/2*((a*c^
3)^(1/4)*a*c^2*d - (a*c^3)^(3/4)*a*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/
c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/2*((a*c^3)
```

```
^(1/4)*a*c^2*d - (a*c^3)^(3/4)*a*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)
^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/4*((a*c^3)^(
1/4)*a*c^2*d + (a*c^3)^(3/4)*a*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) - 1/4*((a*c^3)^(1/4)*a*c^2*d + (a
*c^3)^(3/4)*a*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^5*
d^2 + sqrt(2)*a*c^4*e^2) + 1/3*(c^2*x^3*e^2 - 3*c^2*d*x*e)*e^(-3)/c^3
```

3.238 $\int \frac{x^6}{(d+ex^2)(a+cx^4)} dx$

Optimal. Leaf size=345

$$-\frac{a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{2}c^{5/4}}$$

```
[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 + a*e^2)) +
 (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/
(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) - (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) -
(a^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) + (a^(3/4)*(Sqrt[c]*d - S
qrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2
]*c^(5/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.302142, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\left(\sqrt{a}e+\sqrt{c}d\right)}{2\sqrt{2}c^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 + a*e^2)) +
 (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/
(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) - (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) -
(a^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) + (a^(3/4)*(Sqrt[c]*d - S
qrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2
]*c^(5/4)*(c*d^2 + a*e^2))
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
```

```
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx &= \int \left(\frac{1}{ce}-\frac{d^3}{e\left(cd^2+ae^2\right)\left(d+ex^2\right)}-\frac{a\left(ae+cdx^2\right)}{c\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx \\
&= \frac{x}{ce}-\frac{a\int \frac{ae+cdx^2}{a+cx^4}\,dx}{c\left(cd^2+ae^2\right)}-\frac{d^3\int \frac{1}{d+ex^2}\,dx}{e\left(cd^2+ae^2\right)} \\
&= \frac{x}{ce}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)}+\frac{\left(a\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)}-\frac{\left(a\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)} \\
&= \frac{x}{ce}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)}-\frac{\left(a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}-\frac{\left(a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-\cdots}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{\sqrt[4]{c}}}}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)} \\
&= \frac{x}{ce}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)}-\frac{a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)}+\frac{a^{3/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\cdots}{4\sqrt{2}c\cdots} \\
&= \frac{x}{ce}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)}+\frac{a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{5/4}\left(cd^2+ae^2\right)}-\frac{a^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1\cdots\right.}{2\sqrt{2}c^{5/4}\left(cd^2+ae\cdots\right.}
\end{aligned}
$$

Mathematica [A] time = 0.246014, size = 373, normalized size = 1.08

$$-\frac{\left(a^{3/4}cd-a^{5/4}\sqrt{c}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}+\frac{\left(a^{3/4}cd-a^{5/4}\sqrt{c}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{7/4}\left(ae^2+cd^2\right)}-\frac{\left(a^{3/4}cd+a^{5/4}\right.}{2\sqrt{2}c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 + a*e^2)) -
 ((a^(3/4)*c*d + a^(5/4)*Sqrt[c]*e)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*c^(1/4)*
x)/(Sqrt[2]*a^(1/4))])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - ((a^(3/4)*c*d
+ a^(5/4)*Sqrt[c]*e)*ArcTan[(Sqrt[2]*a^(1/4) + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4
))])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*
e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/
4)*(c*d^2 + a*e^2)) + ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*e)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

Maple [A] time = 0.01, size = 387, normalized size = 1.1

$$\frac{x}{ce}-\frac{ae\sqrt{2}}{\left(4\,ae^2+4\,cd^2\right)c}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)-\frac{ae\sqrt{2}}{\left(8\,ae^2+8\,cd^2\right)c}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] x/c/e-1/4*a/(a*e^2+c*d^2)/c*e*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^
(1/4)*x-1)-1/8*a/(a*e^2+c*d^2)/c*e*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1
/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))-1
/4*a/(a*e^2+c*d^2)/c*e*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x
+1)-1/8*a/(a*e^2+c*d^2)/c*d/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2
^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))-1/4*a/(a
*e^2+c*d^2)/c*d/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)-1/4
*a/(a*e^2+c*d^2)/c*d/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1
)-1/e*d^3/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 8.30664, size = 8208, normalized size = 23.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*c*d^2*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) +
(c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c
^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*
d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 +
 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^
2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^
2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d
^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2
*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^
4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4
*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) - (c^2*d^2*e + a*c*
e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^
3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c
^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2
+ a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 -
 (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d
^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^
6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 +
a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a
*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*
d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) + (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2
*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4
*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^
3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*l
og(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^5 + 2*a*c
^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)
/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c
^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt
(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*
a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2
*e^2 + a^2*c^2*e^4))) - (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e - (c^4*d^4 +
 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*
e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a
^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 -
a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*
c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c
^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(
2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 -
2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 +
 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^
4))) + 4*(c*d^2 + a*e^2)*x)/(c^2*d^2*e + a*c*e^3), -1/4*(4*c*d^2*sqrt(d/e)*
arctan(e*x*sqrt(d/e)/d) - (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4
 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^
5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 +
 a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2
- a^3*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^
2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a
*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(
-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4
- 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4
 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*
e^4))) + (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^
2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 +
 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(
c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a
```

```
^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt
(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*
a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^
4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2
 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*
e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) - (c^2*d^2
*e + a*c*e^3)*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*
sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2
+ 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3
*d^2*e^2 + a^2*c^2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x + (a^2*c^2*d^2*e - a^
3*c*e^3 + (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 -
2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 +
 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d
^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*
d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8
)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) + (c^2*d^2*e + a*c*e^3)*sqr
t(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^
4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e
^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^
2*e^4))*log(-(a^2*c*d^2 - a^3*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^
5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 +
 a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^
6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*
e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6
*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*
a*c^3*d^2*e^2 + a^2*c^2*e^4))) - 4*(c*d^2 + a*e^2)*x)/(c^2*d^2*e + a*c*e^3)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20113, size = 450, normalized size = 1.3

$$-\frac{d^{\frac{5}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2e+ae^3}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right.\right.}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right.}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e + a*e^3) - 1/2*((a*c^3
)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/2*((a*c^3)^(1/4
)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
```

```
a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) + x*e^(-1)/c - 1/4*((a*c^
3)^(1/4)*a*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) + 1/4*((a*c^3)^(1/4)*a*c*e - (a*c
^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2
+ sqrt(2)*a*c^3*e^2)
```

3.239 $\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=336

$$\frac{\sqrt[4]{a}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\text{ta}}{2\sqrt{2}c^{3/4}\left(a\right.}$$

```
[Out] (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)) + (a^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) + (a^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*
(c*d^2 + a*e^2))
```

Rubi [A] time = 0.274359, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{c}d-\sqrt{a}e\right)\text{ta}}{2\sqrt{2}c^{3/4}\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)*(a + c*x^4)),x]
```


```
[Out] (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)) + (a^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) + (a^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*
(c*d^2 + a*e^2))
```


Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
```

```
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(\frac{d^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{a\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= -\frac{a\int \frac{d-ex^2}{a+cx^4}\,dx}{cd^2+ae^2} + \frac{d^2\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)} - \frac{\left(a\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)} - \frac{\left(a\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c\left(cd^2+ae^2\right)}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)} - \frac{\left(a\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c\left(cd^2+ae^2\right)} - \frac{\left(a\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4c\left(cd^2+ae^2\right)} + \cdots$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)} + \frac{\sqrt[4]{a}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)} - \frac{\sqrt[4]{a}\left(\sqrt{c}d+\sqrt{a}e\right)\log\cdots}{4\sqrt{2}c^{3}\cdots}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)} + \frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)} - \frac{a^{3/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}c^{3/4}\left(cd^2+ae^2\right)} + \cdots$$

Mathematica [A] time = 0.155594, size = 233, normalized size = 0.69

$$\frac{\sqrt{2}\sqrt[4]{a}\sqrt{e}\left(\left(\sqrt{a}e+\sqrt{c}d\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right)+2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\right.\right.}{8c^{3/4}\sqrt{e}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*c^(3/4)*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*a^(1/4)*Sqrt[e]*(2
*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (-2*Sqrt
[c]*d + 2*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (Sqrt[c]*d +
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*c^(3/4)*Sqrt[e]*(c*
d^2 + a*e^2))
```

Maple [A] time = 0.007, size = 363, normalized size = 1.1

$$-\frac{d\sqrt{2}}{8\,ae^2+8\,cd^2}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)-\frac{d\sqrt{2}}{4\,ae^2+4\,cd^2}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)-\frac{}{4\,ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/8/(a*e^2+c*d^2)*d*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+
(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))-1/4/(a*e^2+c*d^
2)*d*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)-1/4/(a*e^2+c*d
^2)*d*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+1/8*a/(a*e^2+
c*d^2)*e/c/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1
/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4*a/(a*e^2+c*d^2)*e/c/(
1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4*a/(a*e^2+c*d^2)*
e/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+d^2/(a*e^2+c*d^
2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.69988, size = 7710, normalized size = 22.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*((c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2
 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 + (c^
4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2
 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*
e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4
)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2
+ 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2
*d^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d
^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)
))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2*d
^3 - a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d
^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*
e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2
*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^
8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))
)/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) + (c*d^2 + a*e^2)*sqrt((2*a*d*e
 - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e
^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^
2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2
 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2
*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*
e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e
- (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^
2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) - (c*d^2 +
a*e^2)*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2
*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^
4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2
*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c
^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^
7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e
^8)))*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*
d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4
*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*
c*e^4))) + 2*d*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)))/
(c*d^2 + a*e^2), 1/4*(4*d*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + (c*d^2 + a*e^
2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4
 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^
4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4))*log(-(c*d^2 - a*e^2)*x + (c^2*d^3 - a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d
^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^
8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))
)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4
- 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4
 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^
2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c
^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x - (c^2*d^3 - a*c*d*e^2 + (c
^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^
2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2
 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^
```

```
2*d^2*e^2 + a^2*c*e^4))) + (c*d^2 + a*e^2)*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c
^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*
d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8
)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^2 - a*e^2)*x + (c^2*
d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*
d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4
*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d
^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)
))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*
e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*
e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d
^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-(c*d^
2 - a*e^2)*x - (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^
2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6
*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e
 - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e
^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^
2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))))/(c*d^2 +
a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15636, size = 441, normalized size = 1.31

$$\frac{d^{\frac{3}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2+ae^2} - \frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)} - \frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2 + a*e^2) - 1/2*((a*c^3)^(
1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/2*((a*c^3)^(1/4)*c
^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c
)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d +
 (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4
*d^2 + sqrt(2)*a*c^3*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log
(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*
e^2)
```

3.240 $\int \frac{x^2}{(d+ex^2)(a+cx^4)} dx$

Optimal. Leaf size=337

$$\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2+cd^2}-\frac{(\cdots}{\cdots}-$$

```
[Out] -((Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)) - ((Sqrt[c
]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4
)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]
*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)
*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.267139, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2+cd^2}-\frac{(\cdots}{\cdots}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)*(a + c*x^4)),x]
```


```
[Out] -((Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)) - ((Sqrt[c
]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4
)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]
*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)
*(c*d^2 + a*e^2))
```


Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
```

```
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(-\frac{de}{\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{ae+cdx^2}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{ae+cdx^2}{a+cx^4}\,dx}{cd^2+ae^2} - \frac{(de)\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}$$

$$= -\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2} - \frac{\left(d-\frac{\sqrt{ae}}{\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)} + \frac{\left(d+\frac{\sqrt{ae}}{\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)}$$

$$= -\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2} + \frac{\left(\sqrt[4]{c}\left(d-\frac{\sqrt{ae}}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{a}\left(cd^2+ae^2\right)} + \frac{\left(\sqrt[4]{c}\left(d-\frac{\sqrt{ae}}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}-2x}{-\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,\ldots}{4\sqrt{2}\sqrt[4]{a}\left(cd^2+ae^2\right)}$$

$$= -\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2} + \frac{\sqrt[4]{c}\left(d-\frac{\sqrt{ae}}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\left(cd^2+ae^2\right)} - \frac{\sqrt[4]{c}\left(d-\frac{\sqrt{ae}}{\sqrt{c}}\right)\log\left(\sqrt{a}+\ldots\right.}{4\sqrt{2}\sqrt[4]{a}\left(c\ldots\right.}$$

$$= -\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2+ae^2} - \frac{\sqrt[4]{c}\left(d+\frac{\sqrt{ae}}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\left(cd^2+ae^2\right)} + \frac{\sqrt[4]{c}\left(d+\frac{\sqrt{ae}}{\sqrt{c}}\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\left(cd^2+ae^2\right)} + \ldots$$

Mathematica [A] time = 0.13345, size = 232, normalized size = 0.69

$$\frac{\sqrt{2}\left(\left(\sqrt{c}d-\sqrt{a}e\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right)-2\left(\sqrt{a}e+\sqrt{c}d\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{a}}\right.}{8\sqrt[4]{a}\sqrt[4]{c}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a^(1/4)*c^(1/4)*Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*(
-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqr
t[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (Sqrt[c]*d -
Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(1/4)*c^(1/4)*(c*d
^2 + a*e^2))
```

Maple [A] time = 0.008, size = 351, normalized size = 1.

$$\frac{e\sqrt{2}}{4\,ae^2+4\,cd^2}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)+\frac{e\sqrt{2}}{8\,ae^2+8\,cd^2}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/4/(a*e^2+c*d^2)*e*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)
+1/8/(a*e^2+c*d^2)*e*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+
(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4/(a*e^2+c*d^
2)*e*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/8/(a*e^2+c*d
^2)*d/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/
(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4/(a*e^2+c*d^2)*d/(1/c*a)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4/(a*e^2+c*d^2)*d/(1/c*a)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)-d*e/(a*e^2+c*d^2)/(d*e)^(1/2
)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.91362, size = 7464, normalized size = 22.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*s
qrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6
*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 +
2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^
4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4
*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
)) - (c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqr
t(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a
^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2
+ a^2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*
a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*
c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4
- 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e
^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)))
 + (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(
-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3
*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^
2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c
^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*
e^8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 -
2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) -
 (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(
c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c
^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^
2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2*
c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5
*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^
8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*
a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 +
 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) - 2
*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(c*d^2 + a*e^2),
 -1/4*((c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*s
qrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6
*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 +
2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^
4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4
*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
)) - (c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqr
t(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a
^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2
+ a^2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*
a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*
c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4
- 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e
^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)))
 + (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(
-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3
```

```
*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4))*log(-(c*d^2 - a*e^2)*x + (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^
2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c
^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*
e^8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 -
2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) -
 (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(
c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c
^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^
2*e^4))*log(-(c*d^2 - a*e^2)*x - (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2*
c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5
*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^
8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*
a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 +
 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) + 4
*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(c*d^2 + a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13798, size = 454, normalized size = 1.35

$$-\frac{\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{cd^2+ae^2}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}(\cdots\right.}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\cdots\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c*d^2 + a*e^2) + 1/2*((a*c^3)^(
1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/
4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)
*a*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqr
t(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*a*c*e - (a*c^3)^
(3/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^2 +
sqrt(2)*a^2*c^2*e^2)
```

3.241 $\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=336

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\text{t}}{2\sqrt{2}a^{3/4}\left(\right.}$$

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2))
```

Rubi [A] time = 0.272933, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {1171, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\text{t}}{2\sqrt{2}a^{3/4}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2))
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
```

```
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx=\int \left(\frac{e^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$=\frac{c\int \frac{d-ex^2}{a+cx^4}\,dx}{cd^2+ae^2}+\frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}$$

$$=\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)}$$

$$=\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)}+\frac{\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)}-\frac{\left(\sqrt[4]{c}\left(\sqrt{c}d\right.\right.}{4}$$

$$=\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{c}d+\sqrt{a}e\right)\log}{4\sqrt{2}a^{3}}$$

$$=\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)}-\frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)}+\frac{\sqrt[4]{c}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{a}}\right.}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)}$$

Mathematica [A] time = 0.143788, size = 234, normalized size = 0.7

$$\frac{8a^{3/4}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\sqrt{2}\sqrt[4]{c}\sqrt{d}\left(-\left(\sqrt{a}e+\sqrt{c}d\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)\right)+(2}{8a^{3/4}\sqrt{d}\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*((
-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d +
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(3/4)*Sqrt[d]*(c*
d^2 + a*e^2))
```

Maple [A] time = 0.007, size = 363, normalized size = 1.1

$$\frac{cd\sqrt{2}}{\left(8\,ae^2+8\,cd^2\right)a}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)+\frac{cd\sqrt{2}}{\left(4\,ae^2+4\,cd^2\right)a}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] 1/8*c/(a*e^2+c*d^2)*d*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/
2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1/4*c/(a*e^2
+c*d^2)*d*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)+1/4*c/(
a*e^2+c*d^2)*d*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)-1/
8/(a*e^2+c*d^2)*e/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/
c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))-1/4/(a*e^2+c*d^2)*
e/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)-1/4/(a*e^2+c*d^2)
*e/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)+e^2/(a*e^2+c*d^2
)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 4.45999, size = 7792, normalized size = 23.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e
^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*
d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a
^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*
e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*
d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 +
 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2
+ a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a
^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^
4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d
^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^
4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
 a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^
2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^
5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d
^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (
a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a
^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^
2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e
^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4
- 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2
*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*
e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*
e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4
)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c
^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*
a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2
 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*c^2*d^3 -
 a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 -
 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - 2*e*sqrt(-e/d)*log((e*x^2 + 2*d
*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(c*d^2 + a*e^2), 1/4*(4*e*sqrt(e/d)*arctan
(x*sqrt(e/d)) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 -
 a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 -
 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*
d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e +
 (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*
d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2
 - a*c*e^2)*x - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3
 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*
a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*
c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^
2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4
```

```
*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*
d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-
(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 +
6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e
^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3
*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d
^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)
*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6
*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2
*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^
4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))
)/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-(c^2*d^2 - a*c*e^2)*x - (a*
c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(
c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6
*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4
+ 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/
(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^
7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))))/(c*d^2 + a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15282, size = 458, normalized size = 1.36

$$\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)\right.}{4\left(\sqrt{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/
2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*
((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^
2*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2
)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*
d^2 + a*e^2)*sqrt(d))
```

3.242 $\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=348

$$-\frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{a}e+\sqrt{\ldots}\right.}{2\sqrt{2}a\ldots}$$

```
[Out] -(1/(a*d*x)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^
2)) + (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2
)) - (c^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(Sqrt[c]*
d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(5/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.299538, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{c}d-\sqrt{a}e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{a}e+\sqrt{\ldots}\right.}{2\sqrt{2}a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^
2)) + (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2
)) - (c^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(Sqrt[c]*
d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(5/4)*(c*d^2 + a*e^2))
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
```

```
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(\frac{1}{adx^2} - \frac{e^3}{d\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{c\left(ae+cdx^2\right)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= -\frac{1}{adx} - \frac{c\int \frac{ae+cdx^2}{a+cx^4}\,dx}{a\left(cd^2+ae^2\right)} - \frac{e^3\int \frac{1}{d+ex^2}\,dx}{d\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)} + \frac{\left(c\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2a\left(cd^2+ae^2\right)} - \frac{\left(c\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2a\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)} - \frac{\left(c^{5/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)} - \frac{\left(c^{5/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}}{-\frac{\sqrt{a}}{\sqrt{c}}+\sqrt{2}}}{4\sqrt{2}a^{5/4}\left(cd^2+ae\right.}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)} - \frac{c^{5/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)} + \frac{c^{5/4}\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\mathrm{lo}}{4\sqrt{}}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)} + \frac{c^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}\left(cd^2+ae^2\right)} - \frac{c^{3/4}\left(\sqrt{c}d+\sqrt{a}e\right)\tan^{-1}}{2\sqrt{2}a^{5/4}\left(cd^2+\right.}$$

Mathematica [A] time = 0.25288, size = 389, normalized size = 1.12

$$-\sqrt{d}\left(8a^{5/4}e^2+\sqrt{2}c^{5/4}d^2x\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\sqrt{2}c^{5/4}d^2x\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-\sqrt{2}\sqrt{a}c^{3/4}dex$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a^(5/4)*e^(5/2)*x*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - Sqrt[d]*(8*a^(1/4)*c*d^
2 + 8*a^(5/4)*e^2 - 2*Sqrt[2]*c^(3/4)*d*(Sqrt[c]*d + Sqrt[a]*e)*x*ArcTan[1
- (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*c^(3/4)*d*(Sqrt[c]*d + Sqrt[a]*e
)*x*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*c^(5/4)*d^2*x*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*Sqrt[a]*c^(3/4)*d
*e*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*c^(5/
4)*d^2*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*S
qrt[a]*c^(3/4)*d*e*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
))/(8*a^(5/4)*d^(3/2)*(c*d^2 + a*e^2)*x)
```

Maple [A] time = 0.01, size = 390, normalized size = 1.1

$$-\frac{ec\sqrt{2}}{\left(4\,ae^2+4\,cd^2\right)a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}+1\right)-\frac{ec\sqrt{2}}{\left(4\,ae^2+4\,cd^2\right)a}\sqrt[4]{\frac{a}{c}}\arctan\left(\sqrt{2}x\frac{1}{\sqrt[4]{\frac{a}{c}}}-1\right)-\frac{ec\sqrt{2}}{\left(8\,ae^2+8\,cd^2\right)a}\sqrt[4]{\frac{a}{c}}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/4*c/(a*e^2+c*d^2)/a*e*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)
*x+1)-1/4*c/(a*e^2+c*d^2)/a*e*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^
(1/4)*x-1)-1/8*c/(a*e^2+c*d^2)/a*e*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1
/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))-1
/8*c/(a*e^2+c*d^2)/a*d/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2
)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))-1/4*c/(a*e^2+
c*d^2)/a*d/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)-1/4*c/(a
*e^2+c*d^2)/a*d/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)-1/a
/d/x-1/d*e^3/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 13.5046, size = 8235, normalized size = 23.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*e^2*x*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d))
+ (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 +
4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c
^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^
2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^
5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6
*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2
*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^
3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e
^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) - (a*c*d^3 + a^
2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqr
t(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e
^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*
c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c
*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4
*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^
4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*
d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4
*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))
/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + (a*c*d^3 + a^2*d*e^2)*x*sqrt
(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2
*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*
d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4
*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2
*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*
c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2
*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e
^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^
3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 +
 2*a^3*c*d^2*e^2 + a^4*e^4))) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e -
(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2
+ a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8
*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^
3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*
d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c
^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)
))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*
d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a
^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^
2 + a^4*e^4))) - 4*c*d^2 - 4*a*e^2)/((a*c*d^3 + a^2*d*e^2)*x), -1/4*(4*a*e^
2*x*sqrt(e/d)*arctan(x*sqrt(e/d)) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*
e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*
e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4
*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(
-(c^3*d^2 - a*c^2*e^2)*x + (a^2*c^2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^
5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a
^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*
e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(
c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 +
 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4))) + (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e + (a^2*c^2*d^
4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e
^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6
+ a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c
```

```
^2*e^2)*x - (a^2*c^2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a
^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*
a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2
*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c
^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*
e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4
))) - (a*c*d^3 + a^2*d*e^2)*x*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2
*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^
8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a
^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x + (a^
2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(
-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2
 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2
*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^
2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d
^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + (a*c*d^3
+ a^2*d*e^2)*x*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)
*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d
^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*
a^3*c*d^2*e^2 + a^4*e^4))*log(-(c^3*d^2 - a*c^2*e^2)*x - (a^2*c^2*d^2*e - a
^3*c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a
*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^
4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^
3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5
*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^
8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + 4*c*d^2 + 4*a*e^2)/((a*c
*d^3 + a^2*d*e^2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12155, size = 470, normalized size = 1.35

$$-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*((a*c^3)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1
/2*((a*c^3)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/4
```

```
*((a*c^3)^(1/4)*a*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/4*((a*c^3)^(1/4)*a
*c*e - (a*c^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(
2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - arctan(x*e^(1/2)/sqrt(d))*e^(5/2)/((c
*d^3 + a*d*e^2)*sqrt(d)) - 1/(a*d*x)
```

3.243 $\int \frac{1}{x^4\left(d+ex^2\right)\left(a+cx^4\right)}\,dx$

Optimal. Leaf size=360

$$\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}\right.}{2\sqrt{2}a^{7/}}$$

```
[Out] -1/(3*a*d*x^3) + e/(a*d^2*x) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/
2)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt[c
]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4
)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c
^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.300999, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1288, 205, 1168, 1162, 617, 204, 1165, 628}

$$\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}\right.}{2\sqrt{2}a^{7/}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/(3*a*d*x^3) + e/(a*d^2*x) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/
2)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*
c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt[c
]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4
)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c
^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
```

```
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(\frac{1}{adx^4}-\frac{e}{ad^2x^2}+\frac{e^4}{d^2\left(cd^2+ae^2\right)\left(d+ex^2\right)}-\frac{c^2\left(d-ex^2\right)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}-\frac{c^2\int \frac{d-ex^2}{a+cx^4}\,dx}{a\left(cd^2+ae^2\right)}+\frac{e^4\int \frac{1}{d+ex^2}\,dx}{d^2\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)}-\frac{\left(c\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2a\left(cd^2+ae^2\right)}-\frac{\left(c\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\right)\int \frac{\sqrt{a}\sqrt{c-c}}{a+cx^4}}{2a\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)}-\frac{\left(c\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4a\left(cd^2+ae^2\right)}-\frac{\left(c\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{}{\frac{\sqrt{a}}{\sqrt{c}}}}{4a\left(cd^2+\right.}$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)}+\frac{c^{5/4}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x^2\right)}{4\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}-\frac{c^{5}}{}$$

$$= -\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)}+\frac{c^{5/4}\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{7/4}\left(cd^2+ae^2\right)}-\frac{c^{5/4}\left(\sqrt{c}d-\sqrt{}\right.}{2\sqrt{2}a}$$

Mathematica [A] time = 0.428307, size = 367, normalized size = 1.02

$$\frac{3\sqrt{2}c^{5/4}d^{5/2}x^3\left(a^{3/4}e+\sqrt[4]{a}\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)-3\sqrt{2}c^{5/4}d^{5/2}x^3\left(a^{3/4}e+\sqrt[4]{a}\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a*d^(3/2)*(c*d^2 + a*e^2) + 24*a*Sqrt[d]*e*(c*d^2 + a*e^2)*x^2 + 24*a^2
*e^(7/2)*x^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 6*Sqrt[2]*a^(1/4)*c^(5/4)*d^(5/2
)*(Sqrt[c]*d - Sqrt[a]*e)*x^3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*S
qrt[2]*a^(1/4)*c^(5/4)*d^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*x^3*ArcTan[1 + (S
qrt[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*c^(5/4)*d^(5/2)*(a^(1/4)*Sqrt[c]*d +
 a^(3/4)*e)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - 3*
Sqrt[2]*c^(5/4)*d^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*x^3*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*a^2*d^(5/2)*(c*d^2 + a*e^2)*x^
3)
```

Maple [A] time = 0.012, size = 406, normalized size = 1.1

$$-\frac{c^2d\sqrt{2}}{\left(8\,ae^2+8\,cd^2\right)a^2}\sqrt[4]{\frac{a}{c}}\ln\left(\left(x^2+\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)\left(x^2-\sqrt[4]{\frac{a}{c}}x\sqrt{2}+\sqrt{\frac{a}{c}}\right)^{-1}\right)-\frac{c^2d\sqrt{2}}{\left(4\,ae^2+4\,cd^2\right)a^2}\sqrt[4]{\frac{a}{c}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)/(c*x^4+a),x)
```

```
[Out] -1/8*c^2/(a*e^2+c*d^2)/a^2*d*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*
2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))-1/4*c^2
/(a*e^2+c*d^2)/a^2*d*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1
)-1/4*c^2/(a*e^2+c*d^2)/a^2*d*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^
(1/4)*x-1)+1/8*c/(a*e^2+c*d^2)/a*e/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1
/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))+1
/4*c/(a*e^2+c*d^2)/a*e/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x
+1)+1/4*c/(a*e^2+c*d^2)/a*e/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1
/4)*x-1)-1/3/a/d/x^3+e/a/d^2/x+1/d^2*e^4/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e
*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 47.0868, size = 8462, normalized size = 23.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*a*e^3*x^3*sqrt(-e/d)*log((e*x^2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d
)) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c
*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^
4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8
)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x
 + (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^
5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3
*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d
*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2
*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 +
4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)))
 - 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d
^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))
/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x -
(a^2*c^4*d^3 - a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*
sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^
6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e
+ (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^
2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a
^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) + 3
*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e
^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8
+ 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^
2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqr
t(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e
^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 +
 a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10
*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a
*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2
+ a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4
*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*
c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c
^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-
(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2
+ 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3
*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^
2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*
d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 4*c*d^3
 - 4*a*d*e^2 + 12*(c*d^2*e + a*e^3)*x^2)/((a*c*d^4 + a^2*d^2*e^2)*x^3), 1/1
2*(12*a*e^3*x^3*sqrt(e/d)*arctan(x*sqrt(e/d)) + 3*(a*c*d^4 + a^2*d^2*e^2)*x
^3*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d
^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^
9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e
^2 + a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*d^3 - a^3*c^3*d*e^2
+ (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*
e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4
*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*
e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8
```

```
 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*
sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4
- 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c
^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2
+ a^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 + (
a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2
 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^
10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2
 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 +
4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3
*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) + 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqr
t((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2
*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*
d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a
^5*e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x + (a^2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6
*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 +
a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*
c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 +
a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a
^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^
2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4 + a^2*d^2*e^2)*x^3*sqrt((
2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*
c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4
*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*
e^4))*log(-(c^5*d^2 - a*c^4*e^2)*x - (a^2*c^4*d^3 - a^3*c^3*d*e^2 - (a^6*c^
2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2
*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d
^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5
*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*
c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d
^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 4*c*d^3 - 4*a*d*e^2 + 12*(c*d^2*e + a*e
^3)*x^2)/((a*c*d^4 + a^2*d^2*e^2)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14216, size = 491, normalized size = 1.36

$$-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2c^2d^2+\sqrt{2}a^3ce^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1
/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/4
*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/4*((a*c^3)^(1/4)*c
^2*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(
2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/((c
*d^4 + a*d^2*e^2)*sqrt(d)) + 1/3*(3*x^2*e - d)/(a*d^2*x^3)
```

3.244 $\int \frac{x^9}{(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=169

$$\frac{a(ae+cdx^2)}{4c^2(a+cx^4)(ae^2+cd^2)}+\frac{ae(ae^2+2cd^2)\log(a+cx^4)}{4c^2(ae^2+cd^2)^2}-\frac{\sqrt{a}d(ae^2+3cd^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}(ae^2+cd^2)^2}+\frac{d^4\log(d+ex^2)}{2e(ae^2+cd^2)^2}$$

```
[Out] (a*(a*e + c*d*x^2))/(4*c^2*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[a]*d*(3*c*d
^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2 + a*e^2)^2) +
(d^4*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)^2) + (a*e*(2*c*d^2 + a*e^2)*Log[a
 + c*x^4])/(4*c^2*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.366727, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 1647, 1629, 635, 205, 260}

$$\frac{a(ae+cdx^2)}{4c^2(a+cx^4)(ae^2+cd^2)}+\frac{ae(ae^2+2cd^2)\log(a+cx^4)}{4c^2(ae^2+cd^2)^2}-\frac{\sqrt{a}d(ae^2+3cd^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}(ae^2+cd^2)^2}+\frac{d^4\log(d+ex^2)}{2e(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*(a*e + c*d*x^2))/(4*c^2*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[a]*d*(3*c*d
^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2 + a*e^2)^2) +
(d^4*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)^2) + (a*e*(2*c*d^2 + a*e^2)*Log[a
 + c*x^4])/(4*c^2*(c*d^2 + a*e^2)^2)
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(d+ex)(a+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2d^2}{cd^2+ae^2}-\frac{a^2dex}{cd^2+ae^2}-2ax^2}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4ac}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \left(-\frac{2acd^4}{(cd^2+ae^2)^2(d+ex)} + \frac{a^2(d(3cd^2+ae^2)-2e(2cd^2+ae^2)x)}{(cd^2+ae^2)^2(a+cx^2)}\right) dx, x, x^2\right.}{4ac}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} + \frac{d^4\log(d+ex^2)}{2e(cd^2+ae^2)^2} - \frac{a\,\text{Subst}\left(\int \frac{d(3cd^2+ae^2)-2e(2cd^2+ae^2)x}{a+cx^2}\,dx, x, x^2\right)}{4c(cd^2+ae^2)^2}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} + \frac{d^4\log(d+ex^2)}{2e(cd^2+ae^2)^2} + \frac{(ae(2cd^2+ae^2))\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2c(cd^2+ae^2)^2} -$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{a}d(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}(cd^2+ae^2)^2} + \frac{d^4\log(d+ex^2)}{2e(cd^2+ae^2)^2} + \frac{ae(2cd^2+}{4c^2(}$$

Mathematica [A] time = 0.21659, size = 135, normalized size = 0.8

$$\frac{\frac{a(ae^2+cd^2)(ae+cdx^2)}{c^2(a+cx^4)} + \frac{ae(ae^2+2cd^2)\log(a+cx^4)}{c^2} - \frac{\sqrt{a}d(ae^2+3cd^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{c^{3/2}} + \frac{2d^4\log(d+ex^2)}{e}}{4(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((a*(c*d^2 + a*e^2)*(a*e + c*d*x^2))/(c^2*(a + c*x^4)) - (Sqrt[a]*d*(3*c*d^
2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2) + (2*d^4*Log[d + e*x^2])/
e + (a*e*(2*c*d^2 + a*e^2)*Log[a + c*x^4])/c^2)/(4*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.033, size = 305, normalized size = 1.8

$$\frac{da^2x^2e^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)c}+\frac{ax^2d^3}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}+\frac{a^3e^3}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)c^2}+\frac{a^2ed^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)c}+\frac{a^2}{4(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*a^2/(a*e^2+c*d^2)^2/(c*x^4+a)*d/c*x^2*e^2+1/4*a/(a*e^2+c*d^2)^2/(c*x^4+
a)*x^2*d^3+1/4*a^3/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3/c^2+1/4*a^2/(a*e^2+c*d^2)^
2/(c*x^4+a)*e/c*d^2+1/4*a^2/(a*e^2+c*d^2)^2/c^2*ln(c*x^4+a)*e^3+1/2*a/(a*e^
2+c*d^2)^2/c*ln(c*x^4+a)*e*d^2-1/4*a^2/(a*e^2+c*d^2)^2/c/(a*c)^(1/2)*arctan
(c*x^2/(a*c)^(1/2))*d*e^2-3/4*a/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(c*x^2/(a
*c)^(1/2))*d^3+1/2*d^4*ln(e*x^2+d)/e/(a*e^2+c*d^2)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 81.2695, size = 1106, normalized size = 6.54

$$\left[\frac{2\,a^2cd^2e^2+2\,a^3e^4+2\left(ac^2d^3e+a^2cde^3\right)x^2+\left(3\,ac^2d^3e+a^2cde^3+\left(3\,c^3d^3e+ac^2de^3\right)x^4\right)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4-2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)+2}{8\left(ac^4d^4e+2\,a^2c^3d^2e^3+a^3c^2e^5+\left(c^5d^4e+2\,a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a^2*c*d^2*e^2 + 2*a^3*e^4 + 2*(a*c^2*d^3*e + a^2*c*d*e^3)*x^2 + (3*
a*c^2*d^3*e + a^2*c*d*e^3 + (3*c^3*d^3*e + a*c^2*d*e^3)*x^4)*sqrt(-a/c)*log
((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + 2*(2*a^2*c*d^2*e^2 + a^3*e
^4 + (2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*log(c*x^4 + a) + 4*(c^3*d^4*x^4 + a
*c^2*d^4)*log(e*x^2 + d))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3 + a^3*c^2*e^5 +
(c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^4), 1/4*(a^2*c*d^2*e^2 + a^3*
e^4 + (a*c^2*d^3*e + a^2*c*d*e^3)*x^2 - (3*a*c^2*d^3*e + a^2*c*d*e^3 + (3*c
^3*d^3*e + a*c^2*d*e^3)*x^4)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) + (2*a^2*c
*d^2*e^2 + a^3*e^4 + (2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*log(c*x^4 + a) + 2*
(c^3*d^4*x^4 + a*c^2*d^4)*log(e*x^2 + d))/(a*c^4*d^4*e + 2*a^2*c^3*d^2*e^3
+ a^3*c^2*e^5 + (c^5*d^4*e + 2*a*c^4*d^2*e^3 + a^2*c^3*e^5)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09794, size = 339, normalized size = 2.01

$$\frac{d^4\log\left(\left|x^2e+d\right|\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)}+\frac{\left(2\,acd^2e+a^2e^3\right)\log\left(cx^4+a\right)}{4\left(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4\right)}-\frac{\left(3\,acd^3+a^2de^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\sqrt{ac}}-\frac{2\,acd^2x^4e-acd^3x^2+a^2}{4\left(c^3d^4+2\,ac^2d^2e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(2*
a*c*d^2*e + a^2*e^3)*log(c*x^4 + a)/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^
4) - 1/4*(3*a*c*d^3 + a^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4)*sqrt(a*c)) - 1/4*(2*a*c*d^2*x^4*e - a*c*d^3*x^2 + a^
2*x^4*e^3 - a^2*d*x^2*e^2 + a^2*d^2*e)/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*
e^4)*(c*x^4 + a))
```

3.245 $\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{a}e\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(ae^2+cd^2\right)^2}+\frac{a\left(d-ex^2\right)}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{d^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}$$

```
[Out] (a*(d - e*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[a]*e*(3*c*d^2 + a
*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2 + a*e^2)^2) - (d^3*L
og[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d^3*Log[a + c*x^4])/(4*(c*d^2 + a*e
^2)^2)
```

Rubi [A] time = 0.246675, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 1647, 801, 635, 205, 260}

$$\frac{\sqrt{a}e\left(ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(ae^2+cd^2\right)^2}+\frac{a\left(d-ex^2\right)}{4c\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{d^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*(d - e*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[a]*e*(3*c*d^2 + a
*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2 + a*e^2)^2) - (d^3*L
og[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d^3*Log[a + c*x^4])/(4*(c*d^2 + a*e
^2)^2)
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)(a+cx^2)^2}\,dx,x,x^2\right)$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \frac{-\frac{a^2de}{cd^2+ae^2}-\frac{a(2cd^2+ae^2)x}{cd^2+ae^2}}{(d+ex)(a+cx^2)}\,dx,x,x^2\right)}{4ac}$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \left(\frac{2acd^3e}{(cd^2+ae^2)^2(d+ex)}-\frac{a(3acd^2e+a^2e^3+2c^2d^3x)}{(cd^2+ae^2)^2(a+cx^2)}\right)dx,x,x^2\right)}{4ac}$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{\text{Subst}\left(\int \frac{3acd^2e+a^2e^3+2c^2d^3x}{a+cx^2}\,dx,x,x^2\right)}{4c(cd^2+ae^2)^2}$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{(cd^3)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)}{2(cd^2+ae^2)^2} + \frac{(ae(3cd^2+\ldots}{\ldots}$$

$$= \frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{\sqrt{a}e(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4c^{3/2}(cd^2+ae^2)^2} - \frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{d^3\log(a+c\ldots}{4(cd^2+ae\ldots}$$

Mathematica [A] time = 0.115127, size = 142, normalized size = 0.95

$$\frac{\sqrt{c}\left(a(d-ex^2)(ae^2+cd^2)-2cd^3(a+cx^4)\log(d+ex^2)+cd^3(a+cx^4)\log(a+cx^4)\right)+\sqrt{a}e(a+cx^4)(ae^2+3cd^2)\tan\ldots}{4c^{3/2}(a+cx^4)(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (Sqrt[a]*e*(3*c*d^2 + a*e^2)*(a + c*x^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + Sq
rt[c]*(a*(c*d^2 + a*e^2)*(d - e*x^2) - 2*c*d^3*(a + c*x^4)*Log[d + e*x^2] +
 c*d^3*(a + c*x^4)*Log[a + c*x^4]))/(4*c^(3/2)*(c*d^2 + a*e^2)^2*(a + c*x^4
))
```

Maple [A] time = 0.017, size = 260, normalized size = 1.7

$$-\frac{a^2e^3x^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)c}-\frac{aex^2d^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}+\frac{a^2de^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)c}+\frac{ad^3}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}+\frac{d^3\ln}{4\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*e^3/c*x^2-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*
a*e*x^2*d^2+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*d/c*e^2+1/4/(a*e^2+c*d^2)^2/(
c*x^4+a)*a*d^3+1/4*d^3*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4/(a*e^2+c*d^2)^2/c/(a
*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))*a^2*e^3+3/4/(a*e^2+c*d^2)^2/(a*c)^(1/2)
*arctan(c*x^2/(a*c)^(1/2))*a*d^2*e-1/2*d^3*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 39.6776, size = 922, normalized size = 6.15

$$\left[\frac{2\,acd^3+2\,a^2de^2-2\left(acd^2e+a^2e^3\right)x^2+\left(3\,acd^2e+a^2e^3+\left(3\,c^2d^2e+ace^3\right)x^4\right)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)+2\left(c^2d^3x^4+\right.}{8\left(ac^3d^4+2\,a^2c^2d^2e^2+a^3ce^4+\left(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4\right)x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*c*d^3 + 2*a^2*d*e^2 - 2*(a*c*d^2*e + a^2*e^3)*x^2 + (3*a*c*d^2*e
+ a^2*e^3 + (3*c^2*d^2*e + a*c*e^3)*x^4)*sqrt(-a/c)*log((c*x^4 + 2*c*x^2*sq
rt(-a/c) - a)/(c*x^4 + a)) + 2*(c^2*d^3*x^4 + a*c*d^3)*log(c*x^4 + a) - 4*(
c^2*d^3*x^4 + a*c*d^3)*log(e*x^2 + d))/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3
*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4), 1/4*(a*c*d^3 + a^2
*d*e^2 - (a*c*d^2*e + a^2*e^3)*x^2 + (3*a*c*d^2*e + a^2*e^3 + (3*c^2*d^2*e
+ a*c*e^3)*x^4)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) + (c^2*d^3*x^4 + a*c*d^
3)*log(c*x^4 + a) - 2*(c^2*d^3*x^4 + a*c*d^3)*log(e*x^2 + d))/(a*c^3*d^4 +
2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x
^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10852, size = 301, normalized size = 2.01

$$-\frac{d^3 e \log\left(\left|x^2 e + d\right|\right)}{2\left(c^2 d^4 e + 2\,acd^2 e^3 + a^2 e^5\right)} + \frac{d^3 \log\left(cx^4 + a\right)}{4\left(c^2 d^4 + 2\,acd^2 e^2 + a^2 e^4\right)} + \frac{\left(3\,acd^2 e + a^2 e^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^3 d^4 + 2\,ac^2 d^2 e^2 + a^2 ce^4\right)\sqrt{ac}} - \frac{c^2 d^3 x^4 + acd^2 x^2 e + a^2 x^2}{4\left(c^3 d^4 + 2\,ac^2 d^2 e^2 + a^2 c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*e*log(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*
d^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(3*a*c*d^2*e +
 a^2*e^3)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*
sqrt(a*c)) - 1/4*(c^2*d^3*x^4 + a*c*d^2*x^2*e + a^2*x^2*e^3 - a^2*d*e^2)/((
c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a))
```

3.246 $\int \frac{x^5}{(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=153

$$-\frac{ae+cdx^2}{4c(a+cx^4)(ae^2+cd^2)}+\frac{d^2e\log(d+ex^2)}{2(ae^2+cd^2)^2}-\frac{d^2e\log(a+cx^4)}{4(ae^2+cd^2)^2}+\frac{d(cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}(ae^2+cd^2)^2}$$

```
[Out] -(a*e + c*d*x^2)/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (d*(c*d^2 - a*e^2)*Arc
Tan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) + (d^2*e*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (d^2*e*Log[a + c*x^4])/(4*(c*d^2 +
a*e^2)^2)
```

Rubi [A] time = 0.247646, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 1647, 801, 635, 205, 260}

$$-\frac{ae+cdx^2}{4c(a+cx^4)(ae^2+cd^2)}+\frac{d^2e\log(d+ex^2)}{2(ae^2+cd^2)^2}-\frac{d^2e\log(a+cx^4)}{4(ae^2+cd^2)^2}+\frac{d(cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(a*e + c*d*x^2)/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (d*(c*d^2 - a*e^2)*Arc
Tan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) + (d^2*e*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (d^2*e*Log[a + c*x^4])/(4*(c*d^2 +
a*e^2)^2)
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^5}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{ae+cdx^2}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{acd^2}{cd^2+ae^2}+\frac{acdex}{cd^2+ae^2}}{(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)}{4ac}$$

$$= -\frac{ae+cdx^2}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \left(-\frac{2acd^2e^2}{\left(cd^2+ae^2\right)^2(d+ex)}+\frac{acd\left(-cd^2+ae^2+2cdex\right)}{\left(cd^2+ae^2\right)^2\left(a+cx^2\right)}\right)dx, x, x^2\right)}{4ac}$$

$$= -\frac{ae+cdx^2}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^2e\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} - \frac{d\,\text{Subst}\left(\int \frac{-cd^2+ae^2+2cdex}{a+cx^2}\,dx, x, x^2\right)}{4\left(cd^2+ae^2\right)^2}$$

$$= -\frac{ae+cdx^2}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^2e\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} - \frac{\left(cd^2e\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)^2} + \frac{\left(d\left(cd^2 -\right.\right.}{}$$

$$= -\frac{ae+cdx^2}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(cd^2+ae^2\right)^2} + \frac{d^2e\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} - \frac{d^2e\log\left(a+c\right.}{4\left(cd^2+ae^2\right)}$$

Mathematica [A] time = 0.157612, size = 120, normalized size = 0.78

$$\frac{-\frac{\left(ae^2+cd^2\right)\left(ae+cdx^2\right)}{c\left(a+cx^4\right)}+\frac{d\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}-d^2e\log\left(a+cx^4\right)+2d^2e\log\left(d+ex^2\right)}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-(((c*d^2 + a*e^2)*(a*e + c*d*x^2))/(c*(a + c*x^4))) + (d*(c*d^2 - a*e^2)*
ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + 2*d^2*e*Log[d + e*x^2] -
 d^2*e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.016, size = 252, normalized size = 1.7

$$-\frac{x^2e^2da}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}-\frac{cx^2d^3}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}-\frac{a^2e^3}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)c}-\frac{aed^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}-\frac{ed^2\ln}{4\left(ae\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*e^2*d*a-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^
2*c*d^3-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*e^3/c-1/4/(a*e^2+c*d^2)^2/(c*x^4+
a)*a*e*d^2-1/4*d^2*e*ln(c*x^4+a)/(a*e^2+c*d^2)^2-1/4/(a*e^2+c*d^2)^2*d/(a*c
)^(1/2)*arctan(c*x^2/(a*c)^(1/2))*a*e^2+1/4/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arc
tan(c*x^2/(a*c)^(1/2))*c*d^3+1/2*d^2*e*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 16.3171, size = 987, normalized size = 6.45

$$\left[-\frac{2\,a^2cd^2e+2\,a^3e^3+2\left(ac^2d^3+a^2cde^2\right)x^2-\left(acd^3-a^2de^2+\left(c^2d^3-acde^2\right)x^4\right)\sqrt{-ac}\log\left(\frac{cx^4+2\sqrt{-ac}x^2-a}{cx^4+a}\right)+2\left(ac^2d^2ex\right.}{8\left(a^2c^3d^4+2\,a^3c^2d^2e^2+a^4ce^4+\left(ac^4d^4+2\,a^2c^3d^2e^2+a^3c^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*a^2*c*d^2*e + 2*a^3*e^3 + 2*(a*c^2*d^3 + a^2*c*d*e^2)*x^2 - (a*c*d
^3 - a^2*d*e^2 + (c^2*d^3 - a*c*d*e^2)*x^4)*sqrt(-a*c)*log((c*x^4 + 2*sqrt(
-a*c)*x^2 - a)/(c*x^4 + a)) + 2*(a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(c*x^4 +
 a) - 4*(a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^
3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x
^4), -1/4*(a^2*c*d^2*e + a^3*e^3 + (a*c^2*d^3 + a^2*c*d*e^2)*x^2 + (a*c*d^3
 - a^2*d*e^2 + (c^2*d^3 - a*c*d*e^2)*x^4)*sqrt(a*c)*arctan(sqrt(a*c)/(c*x^2
)) + (a*c^2*d^2*e*x^4 + a^2*c*d^2*e)*log(c*x^4 + a) - 2*(a*c^2*d^2*e*x^4 +
a^2*c*d^2*e)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4 +
 (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08849, size = 297, normalized size = 1.94

$$-\frac{d^2e\log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{d^2e^2\log\left(\left|x^2e+d\right|\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)}+\frac{\left(cd^3-ade^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{ac}}+\frac{c^2d^2x^4e-c^2d^3x^2-acdx^2}{4\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*d^2*e*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*d^2*e^2
*log(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(c*d^3 - a
*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a
*c)) + 1/4*(c^2*d^2*x^4*e - c^2*d^3*x^2 - a*c*d*x^2*e^2 - a^2*e^3)/((c^3*d^
4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a))
```

3.247 $\int \frac{x^3}{(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=148

$$-\frac{d-ex^2}{4\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{de^2\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{de^2\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}-\frac{e\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)^2}$$

```
[Out] -(d - e*x^2)/(4*(c*d^2 + a*e^2)*(a + c*x^4)) - (e*(c*d^2 - a*e^2)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) - (d*e^2*Log[d
+ e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d*e^2*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
^2)
```

Rubi [A] time = 0.186862, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 823, 801, 635, 205, 260}

$$-\frac{d-ex^2}{4\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{de^2\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}+\frac{de^2\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}-\frac{e\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(d - e*x^2)/(4*(c*d^2 + a*e^2)*(a + c*x^4)) - (e*(c*d^2 - a*e^2)*ArcTan[(S
qrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) - (d*e^2*Log[d
+ e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d*e^2*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
^2)
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^3}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$
$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{acde-ace^2x}{(d+ex)\left(a+cx^2\right)}\,dx, x, x^2\right)}{4ac\left(cd^2+ae^2\right)}$$
$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{2acde^3}{\left(cd^2+ae^2\right)(d+ex)} - \frac{ace\left(-cd^2+ae^2+2cdex\right)}{\left(cd^2+ae^2\right)\left(a+cx^2\right)}\right)dx, x, x^2\right)}{4ac\left(cd^2+ae^2\right)}$$
$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{de^2\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} + \frac{e\,\text{Subst}\left(\int \frac{-cd^2+ae^2+2cdex}{a+cx^2}\,dx, x, x^2\right)}{4\left(cd^2+ae^2\right)^2}$$
$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{de^2\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} + \frac{\left(cde^2\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)^2} - \frac{\left(e\left(cd^2-\right.\right.}{}$$
$$= -\frac{d-ex^2}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e\left(cd^2-ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\left(cd^2+ae^2\right)^2} - \frac{de^2\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} + \frac{de^2\log\left(a+cx^4\right.}{4\left(cd^2+ae^2\right)^2}$$

Mathematica [A] time = 0.148467, size = 114, normalized size = 0.77

$$\frac{\frac{\left(ex^2-d\right)\left(ae^2+cd^2\right)}{a+cx^4} + \frac{e\left(ae^2-cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}} + de^2\log\left(a+cx^4\right) - 2de^2\log\left(d+ex^2\right)}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(-d + e*x^2))/(a + c*x^4) + (e*(-(c*d^2) + a*e^2)*ArcTan[
(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - 2*d*e^2*Log[d + e*x^2] + d*e^2*
Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.016, size = 247, normalized size = 1.7

$$\frac{x^2e^3a}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}+\frac{ex^2d^2c}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}-\frac{e^2da}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}-\frac{cd^3}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}+\frac{de^2\ln\left(c\right.}{4\left(ae^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*e^3*a+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^2*e
*d^2*c-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*e^2*d*a-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*
c*d^3+1/4*d*e^2*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4/(a*e^2+c*d^2)^2/(a*c)^(1/2)
*arctan(c*x^2/(a*c)^(1/2))*a*e^3-1/4/(a*e^2+c*d^2)^2*e/(a*c)^(1/2)*arctan(c
*x^2/(a*c)^(1/2))*c*d^2-1/2*d*e^2*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 16.4727, size = 992, normalized size = 6.7

$$\left[-\frac{2\,ac^2d^3+2\,a^2cde^2-2\left(ac^2d^2e+a^2ce^3\right)x^2-\left(acd^2e-a^2e^3+\left(c^2d^2e-ace^3\right)x^4\right)\sqrt{-ac}\log\left(\frac{cx^4-2\sqrt{-ac}x^2-a}{cx^4+a}\right)-2\left(ac^2de^2x\right.}{8\left(a^2c^3d^4+2\,a^3c^2d^2e^2+a^4ce^4+\left(ac^4d^4+2\,a^2c^3d^2e^2+a^3c^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*a*c^2*d^3 + 2*a^2*c*d*e^2 - 2*(a*c^2*d^2*e + a^2*c*e^3)*x^2 - (a*c
*d^2*e - a^2*e^3 + (c^2*d^2*e - a*c*e^3)*x^4)*sqrt(-a*c)*log((c*x^4 - 2*sqr
t(-a*c)*x^2 - a)/(c*x^4 + a)) - 2*(a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(c*x^4
 + a) + 4*(a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*
a^3*c^2*d^2*e^2 + a^4*c*e^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)
*x^4), -1/4*(a*c^2*d^3 + a^2*c*d*e^2 - (a*c^2*d^2*e + a^2*c*e^3)*x^2 - (a*c
*d^2*e - a^2*e^3 + (c^2*d^2*e - a*c*e^3)*x^4)*sqrt(a*c)*arctan(sqrt(a*c)/(c
*x^2)) - (a*c^2*d*e^2*x^4 + a^2*c*d*e^2)*log(c*x^4 + a) + 2*(a*c^2*d*e^2*x^
4 + a^2*c*d*e^2)*log(e*x^2 + d))/(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e
^4 + (a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10727, size = 254, normalized size = 1.72

$$\frac{de^2\log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}-\frac{de^3\log\left(\left|x^2e+d\right|\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)}-\frac{\left(cd^2e-ae^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{ac}}-\frac{cd^3-\left(cd^2e+ae^3\right)x^2+ade^2}{4\left(cx^4+a\right)\left(cd^2+ae^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*d*e^2*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d*e^3*lo
g(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) - 1/4*(c*d^2*e - a*
e^3)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)
) - 1/4*(c*d^3 - (c*d^2*e + a*e^3)*x^2 + a*d*e^2)/((c*x^4 + a)*(c*d^2 + a*e
^2)^2)
```

3.248 $\int \frac{x}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{c}d\left(3ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)^2}+\frac{ae+cdx^2}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{e^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}$$

```
[Out] (a*e + c*d*x^2)/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[c]*d*(c*d^2 + 3*a
*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2)^2) + (e^3*L
og[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (e^3*Log[a + c*x^4])/(4*(c*d^2 + a*e
^2)^2)
```

Rubi [A] time = 0.180638, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1248, 741, 801, 635, 205, 260}

$$\frac{\sqrt{c}d\left(3ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)^2}+\frac{ae+cdx^2}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{e^3\log\left(d+ex^2\right)}{2\left(ae^2+cd^2\right)^2}-\frac{e^3\log\left(a+cx^4\right)}{4\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*e + c*d*x^2)/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[c]*d*(c*d^2 + 3*a
*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2)^2) + (e^3*L
og[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (e^3*Log[a + c*x^4])/(4*(c*d^2 + a*e
^2)^2)
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)(a+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \frac{-cd^2-2ae^2-cdex}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4a(cd^2+ae^2)}$$

$$= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \left(-\frac{2ae^4}{(cd^2+ae^2)(d+ex)} - \frac{c(cd^3+3ade^2-2ae^3x)}{(cd^2+ae^2)(a+cx^2)}\right)dx, x, x^2\right)}{4a(cd^2+ae^2)}$$

$$= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{c\,\text{Subst}\left(\int \frac{cd^3+3ade^2-2ae^3x}{a+cx^2}\,dx, x, x^2\right)}{4a(cd^2+ae^2)^2}$$

$$= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^3\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{(ce^3)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)^2} + \frac{(cd(cd^2+3a\ldots}{\ldots}$$

$$= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} + \frac{\sqrt{c}d(cd^2+3ae^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}(cd^2+ae^2)^2} + \frac{e^3\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{e^3\log(a+c\ldots}{4(cd^2+ae^{\ldots}}$$

Mathematica [A] time = 0.138577, size = 117, normalized size = 0.77

$$\frac{\frac{\sqrt{c}d(3ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{a^{3/2}} + \frac{(ae^2+cd^2)(ae+cdx^2)}{a(a+cx^4)} - e^3\log(a+cx^4) + 2e^3\log(d+ex^2)}{4(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(a*e + c*d*x^2))/(a*(a + c*x^4)) + (Sqrt[c]*d*(c*d^2 + 3*
a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/a^(3/2) + 2*e^3*Log[d + e*x^2] - e^3*
Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.016, size = 255, normalized size = 1.7

$$\frac{x^2cde^2}{4(ae^2+cd^2)^2(cx^4+a)} + \frac{c^2d^3x^2}{4(ae^2+cd^2)^2(cx^4+a)a} + \frac{e^3a}{4(ae^2+cd^2)^2(cx^4+a)} + \frac{ed^2c}{4(ae^2+cd^2)^2(cx^4+a)} - \frac{e^3\ln(cx^4\ldots}{4(ae^2+c\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*d*x^2*e^2+1/4*c^2/(a*e^2+c*d^2)^2/(c*x^4+a)
*d^3/a*x^2+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3*a+1/4*c/(a*e^2+c*d^2)^2/(c*x^4
+a)*e*d^2-1/4*e^3*ln(c*x^4+a)/(a*e^2+c*d^2)^2+3/4*c/(a*e^2+c*d^2)^2/(a*c)^(
1/2)*arctan(c*x^2/(a*c)^(1/2))*e^2*d+1/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*
arctan(c*x^2/(a*c)^(1/2))*d^3+1/2*e^3*ln(e*x^2+d)/(a*e^2+c*d^2)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 39.4054, size = 925, normalized size = 6.13

$$\left[\frac{2\,acd^2e + 2\,a^2e^3 + 2\left(c^2d^3 + acde^2\right)x^2 + \left(acd^3 + 3\,a^2de^2 + \left(c^2d^3 + 3\,acde^2\right)x^4\right)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right) - 2\left(ace^3x^4 + \right.}{8\left(a^2c^2d^4 + 2\,a^3cd^2e^2 + a^4e^4 + \left(ac^3d^4 + 2\,a^2c^2d^2e^2 + a^3ce^4\right)x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*c*d^2*e + 2*a^2*e^3 + 2*(c^2*d^3 + a*c*d*e^2)*x^2 + (a*c*d^3 + 3*
a^2*d*e^2 + (c^2*d^3 + 3*a*c*d*e^2)*x^4)*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sq
rt(-c/a) - a)/(c*x^4 + a)) - 2*(a*c*e^3*x^4 + a^2*e^3)*log(c*x^4 + a) + 4*(
a*c*e^3*x^4 + a^2*e^3)*log(e*x^2 + d))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4
*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4), 1/4*(a*c*d^2*e + a
^2*e^3 + (c^2*d^3 + a*c*d*e^2)*x^2 - (a*c*d^3 + 3*a^2*d*e^2 + (c^2*d^3 + 3*
a*c*d*e^2)*x^4)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) - (a*c*e^3*x^4 + a^2*
e^3)*log(c*x^4 + a) + 2*(a*c*e^3*x^4 + a^2*e^3)*log(e*x^2 + d))/(a^2*c^2*d^
4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)
*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09404, size = 269, normalized size = 1.78

$$-\frac{e^3 \log\left(cx^4 + a\right)}{4\left(c^2d^4 + 2\,acd^2e^2 + a^2e^4\right)} + \frac{e^4 \log\left(\left|x^2e + d\right|\right)}{2\left(c^2d^4e + 2\,acd^2e^3 + a^2e^5\right)} + \frac{\left(c^2d^3 + 3\,acde^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(ac^2d^4 + 2\,a^2cd^2e^2 + a^3e^4\right)\sqrt{ac}} + \frac{acd^2e + \left(c^2d^3 + acde^2\right)x^2}{4\left(cx^4 + a\right)\left(cd^2 + ae^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*e^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*e^4*log(a
bs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(c^2*d^3 + 3*a*c
*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sq
rt(a*c)) + 1/4*(a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x^2 + a^2*e^3)/((c*x^4 +
a)*(c*d^2 + a*e^2)^2*a)
```

3.249 $\int \frac{1}{x(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=209

$$-\frac{cd\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}-\frac{\sqrt{c}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)}+\frac{\log(x)}{a^2d}+\frac{c\left(d-ex^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e^4\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)^2}-\frac{\sqrt{c}e^3\tan}{2\sqrt{a}\left(ae\right.}$$

```
[Out] (c*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[c]*e^3*ArcTan[(Sq
rt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)^2) - (Sqrt[c]*e*ArcTan[(Sqr
t[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2)) + Log[x]/(a^2*d) - (e^4*Log
[d + e*x^2])/(2*d*(c*d^2 + a*e^2)^2) - (c*d*(c*d^2 + 2*a*e^2)*Log[a + c*x^4
])/(4*a^2*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.239187, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 894, 639, 205, 635, 260}

$$-\frac{cd\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}-\frac{\sqrt{c}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)}+\frac{\log(x)}{a^2d}+\frac{c\left(d-ex^2\right)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e^4\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)^2}-\frac{\sqrt{c}e^3\tan}{2\sqrt{a}\left(ae\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[c]*e^3*ArcTan[(Sq
rt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)^2) - (Sqrt[c]*e*ArcTan[(Sqr
t[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2)) + Log[x]/(a^2*d) - (e^4*Log
[d + e*x^2])/(2*d*(c*d^2 + a*e^2)^2) - (c*d*(c*d^2 + 2*a*e^2)*Log[a + c*x^4
])/(4*a^2*(c*d^2 + a*e^2)^2)
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a^2dx} - \frac{e^5}{d\left(cd^2+ae^2\right)^2(d+ex)} - \frac{c(ae+cdx)}{a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} + \frac{c\left(-a^2e^3-cd\left(c\right.\right.}{a^2\left(cd^2+ae^2\right)}\right.\right.$$

$$= \frac{\log(x)}{a^2d} - \frac{e^4\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)^2} + \frac{c\,\text{Subst}\left(\int \frac{-a^2e^3-cd\left(cd^2+2ae^2\right)x}{a+cx^2}\,dx, x, x^2\right)}{2a^2\left(cd^2+ae^2\right)^2} - \frac{c\,\text{Subst}\left(\int \frac{ae+cdx}{\left(a+cx^2\right)^2}\,d\right.}{2a\left(cd^2+ae^2\right.}$$

$$= \frac{c\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{\log(x)}{a^2d} - \frac{e^4\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)^2} - \frac{\left(ce^3\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2\left(cd^2+ae^2\right)^2} - \frac{(c}{}$$

$$= \frac{c\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\sqrt{c}e^3\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{a}\left(cd^2+ae^2\right)^2} - \frac{\sqrt{c}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^2+ae^2\right)} + \frac{\log(x)}{a^2d} - \frac{e^4\log\left(d+e\right.}{2d\left(cd^2+ae\right.}$$

Mathematica [A] time = 0.195079, size = 241, normalized size = 1.15

$$\frac{-2a^2e^4\left(a+cx^4\right)\log\left(d+ex^2\right)+acd\left(d-ex^2\right)\left(ae^2+cd^2\right)+4\log(x)\left(a+cx^4\right)\left(ae^2+cd^2\right)^2-cd^2\left(a+cx^4\right)\left(2ae^2+cd^2\right)\text{lo}}{4a^2d\left(a+cx^4\right)\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*c*d*(c*d^2 + a*e^2)*(d - e*x^2) + Sqrt[a]*Sqrt[c]*d*e*(c*d^2 + 3*a*e^2)*
(a + c*x^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*Sqrt[c]*d*e*(
c*d^2 + 3*a*e^2)*(a + c*x^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*(c
*d^2 + a*e^2)^2*(a + c*x^4)*Log[x] - 2*a^2*e^4*(a + c*x^4)*Log[d + e*x^2] -
 c*d^2*(c*d^2 + 2*a*e^2)*(a + c*x^4)*Log[a + c*x^4])/(4*a^2*d*(c*d^2 + a*e^
2)^2*(a + c*x^4))
```

Maple [A] time = 0.022, size = 309, normalized size = 1.5

$$-\frac{e^3cx^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)} - \frac{c^2x^2ed^2}{4\left(ae^2+cd^2\right)^2a\left(cx^4+a\right)} + \frac{cde^2}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)} + \frac{c^2d^3}{4\left(ae^2+cd^2\right)^2a\left(cx^4+a\right)} - \frac{c\ln\left(cx\right.}{2\left(ae^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3*x^2-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a
)*x^2*e*d^2+1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*d*e^2+1/4*c^2/(a*e^2+c*d^2)^2/a
/(c*x^4+a)*d^3-1/2*c/(a*e^2+c*d^2)^2/a*ln(c*x^4+a)*e^2*d-1/4*c^2/(a*e^2+c*d
^2)^2/a^2*ln(c*x^4+a)*d^3-3/4*c/(a*e^2+c*d^2)^2/(a*c)^(1/2)*arctan(c*x^2/(a
*c)^(1/2))*e^3-1/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/
2))*e*d^2+ln(x)/a^2/d-1/2*e^4*ln(e*x^2+d)/d/(a*e^2+c*d^2)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07542, size = 377, normalized size = 1.8

$$-\frac{\left(c^2d^3+2\,acde^2\right)\log\left(cx^4+a\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)}-\frac{e^5\log\left(\left|x^2e+d\right|\right)}{2\left(c^2d^5e+2\,acd^3e^3+a^2de^5\right)}-\frac{\left(c^2d^2e+3\,ace^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}}+\frac{c^3d^3x^4+2\,ac^2dx}{4\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(c^2*d^3 + 2*a*c*d*e^2)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
+ a^4*e^4) - 1/2*e^5*log(abs(x^2*e + d))/(c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d
*e^5) - 1/4*(c^2*d^2*e + 3*a*c*e^3)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2
*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) + 1/4*(c^3*d^3*x^4 + 2*a*c^2*d*x^4*e^2
 - a*c^2*d^2*x^2*e + 2*a*c^2*d^3 - a^2*c*x^2*e^3 + 3*a^2*c*d*e^2)/((a^2*c^2
*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*(c*x^4 + a)) + 1/2*log(x^2)/(a^2*d)
```

3.250 $\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=236

$$-\frac{c^{3/2}d\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)^2}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(ae^2+cd^2\right)}-\frac{c\left(ae+cdx^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{ce\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}-\frac{e}{}$$

```
[Out] -1/(2*a^2*d*x^2) - (c*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(5/2)*(c*d^2 + a*e^2)) - (
c^(3/2)*d*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(5/2)*(c*d^
2 + a*e^2)^2) - (e*Log[x])/(a^2*d^2) + (e^5*Log[d + e*x^2])/(2*d^2*(c*d^2 +
 a*e^2)^2) + (c*e*(c*d^2 + 2*a*e^2)*Log[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2)^
2)
```

Rubi [A] time = 0.261094, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 894, 639, 205, 635, 260}

$$-\frac{c^{3/2}d\left(2ae^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}\left(ae^2+cd^2\right)^2}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(ae^2+cd^2\right)}-\frac{c\left(ae+cdx^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{ce\left(2ae^2+cd^2\right)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}-\frac{e}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/(2*a^2*d*x^2) - (c*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(5/2)*(c*d^2 + a*e^2)) - (
c^(3/2)*d*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(5/2)*(c*d^
2 + a*e^2)^2) - (e*Log[x])/(a^2*d^2) + (e^5*Log[d + e*x^2])/(2*d^2*(c*d^2 +
 a*e^2)^2) + (c*e*(c*d^2 + 2*a*e^2)*Log[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2)^
2)
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{1}{a^2dx^2} - \frac{e}{a^2d^2x} + \frac{e^6}{d^2\left(cd^2+ae^2\right)^2(d+ex)} - \frac{c^2(d-ex)}{a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2} - \frac{c^2 (}{a^2 (}\right.\right.$$

$$= -\frac{1}{2a^2dx^2} - \frac{e\log(x)}{a^2d^2} + \frac{e^5\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)^2} - \frac{c^2\operatorname{Subst}\left(\int \frac{d-ex}{\left(a+cx^2\right)^2}\,dx, x, x^2\right)}{2a\left(cd^2+ae^2\right)} - \frac{\left(c^2\left(cd^2+2ae\right.\right.}{2}$$

$$= -\frac{1}{2a^2dx^2} - \frac{c\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{e\log(x)}{a^2d^2} + \frac{e^5\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)^2} - \frac{\left(c^2d\right)\operatorname{Subst}\left(\int \frac{1}{a+c}\right.}{4a^2\left(cd^2+\right.}$$

$$= -\frac{1}{2a^2dx^2} - \frac{c\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(cd^2+ae^2\right)} - \frac{c^{3/2}d\left(cd^2+2ae^2\right)\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{}}\right.}{2a^{5/2}\left(cd^2+ae^2\right)^2}$$

Mathematica [A] time = 0.450885, size = 248, normalized size = 1.05

$$\frac{1}{4}\left(\frac{c^{3/2}d\left(5ae^2+3cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/2}\left(ae^2+cd^2\right)^2} + \frac{c^{3/2}d\left(5ae^2+3cd^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)}{a^{5/2}\left(ae^2+cd^2\right)^2} - \frac{c\left(ae+cdx^2\right)}{a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)} + \frac{c\left(2ae^3+\right.}{a^2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-2/(a^2*d*x^2) - (c*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4)) + (
c^(3/2)*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(
5/2)*(c*d^2 + a*e^2)^2) + (c^(3/2)*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) - (4*e*Log[x])/(a^2*d^2)
 + (2*e^5*Log[d + e*x^2])/(c*d^3 + a*d*e^2)^2 + (c*(c*d^2*e + 2*a*e^3)*Log[
a + c*x^4])/(a^2*(c*d^2 + a*e^2)^2))/4
```

Maple [A] time = 0.025, size = 332, normalized size = 1.4

$$-\frac{c^2x^2e^2d}{4\left(ae^2+cd^2\right)^2a\left(cx^4+a\right)}-\frac{c^3x^2d^3}{4\left(ae^2+cd^2\right)^2a^2\left(cx^4+a\right)}-\frac{e^3c}{4\left(ae^2+cd^2\right)^2\left(cx^4+a\right)}-\frac{ed^2c^2}{4\left(ae^2+cd^2\right)^2a\left(cx^4+a\right)}+\frac{c}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^2*e^2*d-1/4*c^3/(a*e^2+c*d^2)^2/a^2/
(c*x^4+a)*x^2*d^3-1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3-1/4*c^2/(a*e^2+c*d^2)
^2/a/(c*x^4+a)*e*d^2+1/2*c/(a*e^2+c*d^2)^2/a*ln(c*x^4+a)*e^3+1/4*c^2/(a*e^2
+c*d^2)^2/a^2*ln(c*x^4+a)*e*d^2-5/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1/2)*arcta
n(c*x^2/(a*c)^(1/2))*e^2*d-3/4*c^3/(a*e^2+c*d^2)^2/a^2/(a*c)^(1/2)*arctan(c
*x^2/(a*c)^(1/2))*d^3-1/2/a^2/d/x^2-e*ln(x)/a^2/d^2+1/2*e^5*ln(e*x^2+d)/d^2
/(a*e^2+c*d^2)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10956, size = 464, normalized size = 1.97

$$\frac{\left(c^2d^2e+2\,ace^3\right)\log\left(cx^4+a\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)}+\frac{e^6\log\left(\left|x^2e+d\right|\right)}{2\left(c^2d^6e+2\,acd^4e^3+a^2d^2e^5\right)}-\frac{\left(3\,c^3d^3+5\,ac^2de^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)\sqrt{ac}}-\frac{9\,c^3d^5x^4+15\,ac^2d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(c^2*d^2*e + 2*a*c*e^3)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
 a^4*e^4) + 1/2*e^6*log(abs(x^2*e + d))/(c^2*d^6*e + 2*a*c*d^4*e^3 + a^2*d^
2*e^5) - 1/4*(3*c^3*d^3 + 5*a*c^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2*
d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/12*(9*c^3*d^5*x^4 + 15*a*c^
2*d^3*x^4*e^2 - 2*a^2*c*x^6*e^5 + 3*a*c^2*d^4*x^2*e + 6*a^2*c*d*x^4*e^4 + 6
*a*c^2*d^5 + 3*a^2*c*d^2*x^2*e^3 + 12*a^2*c*d^3*e^2 - 2*a^3*x^2*e^5 + 6*a^3
*d*e^4)/((a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d^2*e^4)*(c*x^6 + a*x^2)) - 1
/2*e*log(x^2)/(a^2*d^2)
```

3.251 $\int \frac{1}{x^5(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=265

$$-\frac{c^2(d-ex^2)}{4a^2(a+cx^4)(ae^2+cd^2)}+\frac{c^2d(3ae^2+2cd^2)\log(a+cx^4)}{4a^3(ae^2+cd^2)^2}+\frac{c^{3/2}e(2ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}(ae^2+cd^2)^2}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}(ae^2+cd^2)}-$$

```
[Out] -1/(4*a^2*d*x^4) + e/(2*a^2*d^2*x^2) - (c^2*(d - e*x^2))/(4*a^2*(c*d^2 + a*
e^2)*(a + c*x^4)) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(5/2)*(c
*d^2 + a*e^2)) + (c^(3/2)*e*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*a^(5/2)*(c*d^2 + a*e^2)^2) - ((2*c*d^2 - a*e^2)*Log[x])/(a^3*d^3) - (e
^6*Log[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)^2) + (c^2*d*(2*c*d^2 + 3*a*e^2)*L
og[a + c*x^4])/(4*a^3*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.326811, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1252, 894, 639, 205, 635, 260}

$$-\frac{c^2(d-ex^2)}{4a^2(a+cx^4)(ae^2+cd^2)}+\frac{c^2d(3ae^2+2cd^2)\log(a+cx^4)}{4a^3(ae^2+cd^2)^2}+\frac{c^{3/2}e(2ae^2+cd^2)\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2a^{5/2}(ae^2+cd^2)^2}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}(ae^2+cd^2)}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/(4*a^2*d*x^4) + e/(2*a^2*d^2*x^2) - (c^2*(d - e*x^2))/(4*a^2*(c*d^2 + a*
e^2)*(a + c*x^4)) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(5/2)*(c
*d^2 + a*e^2)) + (c^(3/2)*e*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]]
)/(2*a^(5/2)*(c*d^2 + a*e^2)^2) - ((2*c*d^2 - a*e^2)*Log[x])/(a^3*d^3) - (e
^6*Log[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)^2) + (c^2*d*(2*c*d^2 + 3*a*e^2)*L
og[a + c*x^4])/(4*a^3*(c*d^2 + a*e^2)^2)
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^2}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{1}{a^2dx^3}-\frac{e}{a^2d^2x^2}+\frac{-2cd^2+ae^2}{a^3d^3x}-\frac{e^7}{d^3\left(cd^2+ae^2\right)^2(d+ex)}+\frac{c^2(ae+\cdots}{a^2\left(cd^2+ae^2\right)\cdots}\right.\right.$$

$$= -\frac{1}{4a^2dx^4}+\frac{e}{2a^2d^2x^2}-\frac{\left(2cd^2-ae^2\right)\log(x)}{a^3d^3}-\frac{e^6\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)^2}+\frac{c^2\operatorname{Subst}\left(\int\frac{ae\left(cd^2+2ae^2\right)+\cdots}{a+\cdots}\right.}{2a^3\left(cd^2\cdots\right.}$$

$$= -\frac{1}{4a^2dx^4}+\frac{e}{2a^2d^2x^2}-\frac{c^2\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{\left(2cd^2-ae^2\right)\log(x)}{a^3d^3}-\frac{e^6\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)^{\cdots}}$$

$$= -\frac{1}{4a^2dx^4}+\frac{e}{2a^2d^2x^2}-\frac{c^2\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(cd^2+ae^2\right)}+\frac{c^{3/2}e\left(cd^2+2ae^2\right)\cdots}{2a^{5/2}\left(cd^2\cdots\right.}$$

Mathematica [A] time = 0.426983, size = 278, normalized size = 1.05

$$\frac{1}{4}\left(\frac{c^2\left(ex^2-d\right)}{a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{c^2\left(3ade^2+2cd^3\right)\log\left(a+cx^4\right)}{a^3\left(ae^2+cd^2\right)^2}-\frac{c^{3/2}e\left(5ae^2+3cd^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{a^{5/2}\left(ae^2+cd^2\right)^2}-\frac{c^{3/2}e\left(5ae^2+3c\cdots\right.}{a^{5/2}\left(\cdots\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-(1/(a^2*d*x^4)) + (2*e)/(a^2*d^2*x^2) + (c^2*(-d + e*x^2))/(a^2*(c*d^2 +
a*e^2)*(a + c*x^4)) - (c^(3/2)*e*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) - (c^(3/2)*e*(3*c*d^2 + 5*a*
e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) +
 (4*(-2*c*d^2 + a*e^2)*Log[x])/(a^3*d^3) - (2*e^6*Log[d + e*x^2])/(d^3*(c*d
^2 + a*e^2)^2) + (c^2*(2*c*d^3 + 3*a*d*e^2)*Log[a + c*x^4])/(a^3*(c*d^2 + a
*e^2)^2))/4
```

Maple [A] time = 0.025, size = 363, normalized size = 1.4

$$\frac{e^3c^2x^2}{4\left(ae^2+cd^2\right)^2 a\left(cx^4+a\right)}+\frac{c^3x^2ed^2}{4\left(ae^2+cd^2\right)^2 a^2\left(cx^4+a\right)}-\frac{e^2dc^2}{4\left(ae^2+cd^2\right)^2 a\left(cx^4+a\right)}-\frac{c^3d^3}{4\left(ae^2+cd^2\right)^2 a^2\left(cx^4+a\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*e^3*x^2+1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*
x^4+a)*x^2*e*d^2-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*d*e^2-1/4*c^3/(a*e^2+c
*d^2)^2/a^2/(c*x^4+a)*d^3+3/4*c^2/(a*e^2+c*d^2)^2/a^2*ln(c*x^4+a)*e^2*d+1/2
*c^3/(a*e^2+c*d^2)^2/a^3*ln(c*x^4+a)*d^3+5/4*c^2/(a*e^2+c*d^2)^2/a/(a*c)^(1
/2)*arctan(c*x^2/(a*c)^(1/2))*e^3+3/4*c^3/(a*e^2+c*d^2)^2/a^2/(a*c)^(1/2)*a
rctan(c*x^2/(a*c)^(1/2))*e*d^2-1/4/a^2/d/x^4+1/a^2/d^3*ln(x)*e^2-2/a^3/d*ln
(x)*c+1/2*e/a^2/d^2/x^2-1/2*e^6*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09672, size = 473, normalized size = 1.78

$$\frac{\left(2\,c^3d^3+3\,ac^2de^2\right)\log\left(cx^4+a\right)}{4\left(a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4\right)}-\frac{e^7\log\left(\left|x^2e+d\right|\right)}{2\left(c^2d^7e+2\,acd^5e^3+a^2d^3e^5\right)}+\frac{\left(3\,c^3d^2e+5\,ac^2e^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)\sqrt{ac}}-\frac{2\,c^4d^3x^4+3\,ac^3d}{4\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*c^3*d^3 + 3*a*c^2*d*e^2)*log(c*x^4 + a)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e
^2 + a^5*e^4) - 1/2*e^7*log(abs(x^2*e + d))/(c^2*d^7*e + 2*a*c*d^5*e^3 + a^
2*d^3*e^5) + 1/4*(3*c^3*d^2*e + 5*a*c^2*e^3)*arctan(c*x^2/sqrt(a*c))/((a^2*
c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/4*(2*c^4*d^3*x^4 + 3*a*
c^3*d*x^4*e^2 - a*c^3*d^2*x^2*e + 3*a*c^3*d^3 - a^2*c^2*x^2*e^3 + 4*a^2*c^2
*d*e^2)/((a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*(c*x^4 + a)) - 1/2*(2*c*
d^2 - a*e^2)*log(x^2)/(a^3*d^3) + 1/4*(6*c*d^2*x^4 - 3*a*x^4*e^2 + 2*a*d*x^
2*e - a*d^2)/(a^3*d^3*x^4)
```

3.252 $\int \frac{x^8}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=712

$$\frac{\sqrt[4]{a}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{a}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{a}\left(3\sqrt{a}e \cdots\right.}{\cdots}$$

```
[Out] (d*x)/(4*c*(c*d^2 + a*e^2)) - (x^3*(a*e + c*d*x^2))/(4*c*(c*d^2 + a*e^2)*(a
 + c*x^4)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)
^2) + (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) + (a^(1/4)*(Sqrt[c]*d - 3*Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)*(c*d^2
 + a*e^2)) - (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d
 - 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)
*(c*d^2 + a*e^2)) + (a^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2)
 + (a^(1/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a^(1/4)*d^2*(Sqr
t[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*c^(7/4
)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.670693, antiderivative size = 712, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1314, 1276, 1280, 1168, 1162, 617, 204, 1165, 628, 1288, 205}

$$\frac{\sqrt[4]{a}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{a}d^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}c^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{a}\left(3\sqrt{a}e \cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (d*x)/(4*c*(c*d^2 + a*e^2)) - (x^3*(a*e + c*d*x^2))/(4*c*(c*d^2 + a*e^2)*(a
 + c*x^4)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)
^2) + (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) + (a^(1/4)*(Sqrt[c]*d - 3*Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)*(c*d^2
 + a*e^2)) - (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d
 - 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)
*(c*d^2 + a*e^2)) + (a^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2)
 + (a^(1/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a^(1/4)*d^2*(Sqr
t[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*c^(7/4
)*(c*d^2 + a*e^2))
```

Rule 1314

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> -Dist[(a*f^4)/(c*d^2 + a*e^2), Int[(f*x)^(m - 4)*(d - e*x^2)*
(a + c*x^4)^p, x], x] + Dist[(d^2*f^4)/(c*d^2 + a*e^2), Int[((f*x)^(m - 4)*
(a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && Lt
Q[p, -1] && GtQ[m, 2]
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*(a*e - c*d*x^2))/(4*a
*c*(p + 1)), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1280

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1))/(c*(m + 4*p + 3)),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^8}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{a\int \frac{x^4(d-ex^2)}{(a+cx^4)^2}\,dx}{cd^2+ae^2}+\frac{d^2\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx}{cd^2+ae^2}$$

$$= -\frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)}-\frac{\int \frac{x^2(-3ae-cdx^2)}{a+cx^4}\,dx}{4c(cd^2+ae^2)}+\frac{d^2\int\left(\frac{d^2}{(cd^2+ae^2)(d+ex^2)}-\frac{a(d-ex^2)}{(cd^2+ae^2)(a+cx^4)}\right)d\ldots}{cd^2+ae^2}$$

$$= \frac{dx}{4c(cd^2+ae^2)}-\frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)}-\frac{(ad^2)\int\frac{d-ex^2}{a+cx^4}\,dx}{(cd^2+ae^2)^2}+\frac{d^4\int\frac{1}{d+ex^2}\,dx}{(cd^2+ae^2)^2}+\frac{\int\frac{-acd+3a\ldots}{a+cx\ldots}}{4c^2(cd^2\ldots}$$

$$= \frac{dx}{4c(cd^2+ae^2)}-\frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(cd^2+ae^2)^2}-\frac{\left(ad^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx\ldots}{a+cx^4}}{2c(cd^2+ae^2)^2}$$

$$= \frac{dx}{4c(cd^2+ae^2)}-\frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(cd^2+ae^2)^2}-\frac{\left(ad^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{\ldots}}{\sqrt[4]{\ldots}}}}{4c(cd^2+ae^2)^2}$$

$$= \frac{dx}{4c(cd^2+ae^2)}-\frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(cd^2+ae^2)^2}+\frac{\sqrt[4]{a}d^2(\sqrt{c}d+\sqrt{a}e)\log(\ldots}{4\sqrt{2}c^{3/4}(c\ldots}$$

$$= \frac{dx}{4c(cd^2+ae^2)}-\frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}(cd^2+ae^2)^2}+\frac{a^{3/4}d^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\tan^{-1}(1\ldots}{2\sqrt{2}c^{3/4}(cd^2+ae\ldots}$$

Mathematica [A] time = 0.318675, size = 431, normalized size = 0.61

$$\frac{\frac{\sqrt{2}\sqrt[4]{a}\left(3a^{3/2}e^3+7\sqrt{a}cd^2e+a\sqrt{c}de^2+5c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{7/4}}-\frac{\sqrt{2}\sqrt[4]{a}\left(3a^{3/2}e^3+7\sqrt{a}cd^2e+a\sqrt{c}de^2+5c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{c^{7/4}}-\frac{2\sqrt{2}\ldots}{\ldots}}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*a*(c*d^2 + a*e^2)*x*(d - e*x^2))/(c*(a + c*x^4)) + (32*d^(7/2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*Sqrt[2]*a^(1/4)*(-5*c^(3/2)*d^3 + 7*Sqrt[
a]*c*d^2*e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/c^(7/4) + (2*Sqrt[2]*a^(1/4)*(-5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*
e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/c^(7/4) + (Sqrt[2]*a^(1/4)*(5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e + a*Sqrt[c
]*d*e^2 + 3*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/c^(7/4) - (Sqrt[2]*a^(1/4)*(5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e + a*Sqr
t[c]*d*e^2 + 3*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/c^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.014, size = 873, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*a^2/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3/c*x^3-1/4*a/(a*e^2+c*d^2)^2/(c*x^4+a
)*e*x^3*d^2+1/4*a^2/(a*e^2+c*d^2)^2/(c*x^4+a)*d/c*x*e^2+1/4*a/(a*e^2+c*d^2)
^2/(c*x^4+a)*d^3*x-1/16*a/(a*e^2+c*d^2)^2/c*(1/c*a)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(1/c*a)^(1/4)*x+1)*d*e^2-5/16/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^3-1/16*a/(a*e^2+c*d^2)^2/c*(1/c*a)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d*e^2-5/16/(a*e^2+c*d^2)^2*(1/c*
a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^3-1/32*a/(a*e^2+c*d^2)
^2/c*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(
x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d*e^2-5/32/(a*e^2+c*d^2)^2*(1/c
*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*
a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^3+3/32*a^2/(a*e^2+c*d^2)^2/c^2/(1/c*a)
^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^
(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*e^3+7/32*a/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)*
2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x
*2^(1/2)+(1/c*a)^(1/2)))*d^2*e+3/16*a^2/(a*e^2+c*d^2)^2/c^2/(1/c*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*e^3+7/16*a/(a*e^2+c*d^2)^2/c/(1/c*
a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2*e+3/16*a^2/(a*e^2+c*
d^2)^2/c^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^3+7/16
*a/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1
)*d^2*e+d^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 96.6749, size = 20569, normalized size = 28.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(a*c*d^2*e + a^2*e^3)*x^3 - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*
c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e
 + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*
d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2
*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d
^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 +
 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5
*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))
/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c
^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594
*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2
*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 + (7*c^10*d^10*e + 31*a
*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9
 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*
c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e
^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 +
56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*
c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e +
44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4
*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^
5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*
e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28
*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^
10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c
^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*
e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^
2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d
*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 +
 a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d
^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 +
 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^
3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d
^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 +
 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 -
750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*
x - (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3
*e^6 - 9*a^4*c^2*d*e^8 + (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6
*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625
*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^
6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16
 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4
*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*
e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^
5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^
4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*
e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81
*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c
^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*
e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*
a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) - (a*c^3*d^4 + 2*a^2*c
^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqr
t((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6
*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^
6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6
 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*
```

a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11
*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14
+ a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^
4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*
c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (125*c^6*d^9 - 170*a*c^5*
d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c
^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 1
9*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^
10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8
+ 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2
*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d
^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((
70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^
2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d
^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 +
2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c
^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^
8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a
^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d
^2*e^6 + a^4*c^3*e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4
*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*
d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*
a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^
2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738
*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13
*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^
10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 +
4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log
(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e
^6 - 81*a^4*e^8)*x - (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4
 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c^10*d^10*e + 31*a*c^9*d^8*e^3
 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5
*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4
+ 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7
*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*
d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12
 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3
*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3
*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 -
 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^
6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^
12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10
+ 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a
*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) - 8*(
c^2*d^3*x^4 + a*c*d^3)*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2
 + d)) - 4*(a*c*d^3 + a^2*d*e^2)*x)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*
e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4), -1/16*(4*(a*c*d^2*e +
 a^2*e^3)*x^3 - 16*(c^2*d^3*x^4 + a*c*d^3)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d
) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2
 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5
+ (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*
c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^
4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a
^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^1
2*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^
12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^
2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a
*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (
125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6

- 9*a^4*c^2*d*e^8 + (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5
+ 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^
6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6
 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*
a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11
*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14
+ a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (
c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3
*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 +
 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*
e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d
^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12
+ 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c
^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^
2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70
*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2
+ 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^1
2 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 23
83*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^1
4*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*
e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8
*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2
*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d
^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (125*c^6*d^9 - 170*a*c^5*d^7*e
^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 + (7*c^10*d
^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4
*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^
2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738
*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13
*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^
10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*
c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6
*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 -
 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*
a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d
^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8
 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^
7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^
6 + a^4*c^3*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4
+ 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e
^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c
^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 5
29*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*
c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12
*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 +
28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c
^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(62
5*c^4*d^8 - 750*a*c^3*d^6*e^2 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 -
81*a^4*e^8)*x + (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86
*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54
*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11
)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 274
8*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12
)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*
e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*
a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3
+ 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*
d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*
a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d

```
^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^
4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*
a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*
d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) + (a*c^3*d
^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e
^4)*x^4)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 +
 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqr
t(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4
*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^
15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 +
 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c
^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^
4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-(625*c^4*d^8 - 750*a*c^3*d^6*e^2
 - 1376*a^2*c^2*d^4*e^4 - 594*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (125*c^6*d^9
- 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*
d*e^8 - (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7
*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 195
0*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*
c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*
e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 5
6*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^
16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*
a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-
(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^
3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*
d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70
*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*
d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 +
 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) - 4*(a*c*d^3 + a^2*d*e^2)*x)/(a*c^3*d^4
 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4
)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13348, size = 784, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) - 1/8*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - 7*(a*c^3)^
(3/4)*c*d^2*e - 3*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + sqrt(2
```

```
)*a^2*c^4*e^4) - 1/8*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 -
 7*(a*c^3)^(3/4)*c*d^2*e - 3*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e
^2 + sqrt(2)*a^2*c^4*e^4) - 1/16*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a
*c^2*d*e^2 + 7*(a*c^3)^(3/4)*c*d^2*e + 3*(a*c^3)^(3/4)*a*e^3)*log(x^2 + sqr
t(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2
+ sqrt(2)*a^2*c^4*e^4) + 1/16*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^
2*d*e^2 + 7*(a*c^3)^(3/4)*c*d^2*e + 3*(a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2
)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + s
qrt(2)*a^2*c^4*e^4) - 1/4*(a*x^3*e - a*d*x)/((c*x^4 + a)*(c^2*d^2 + a*c*e^2
))
```

3.253 $\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=687

$$-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\right.}{8\sqrt{2}\sqrt[4]{a}c^{5/4}\left(ae^2+c\right.}$$

```
[Out] -(x*(a*e + c*d*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) - (d^(5/2)*Sqrt[e]*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (d^2*(Sqrt[c]*d + Sqrt[a]*e
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^
2 + a*e^2)^2) + ((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(8*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) + (d^2*(Sqrt[c]*d + Sqrt
[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*
(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/(8*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) + (d^2*(Sqrt[c]*d -
 Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt
[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4)*
(c*d^2 + a*e^2)) - (d^2*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2)
+ ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.601917, antiderivative size = 687, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1314, 1276, 1168, 1162, 617, 204, 1165, 628, 1288, 205}

$$-\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}\sqrt[4]{a}c^{5/4}\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\right.}{8\sqrt{2}\sqrt[4]{a}c^{5/4}\left(ae^2+c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```


```
[Out] -(x*(a*e + c*d*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) - (d^(5/2)*Sqrt[e]*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (d^2*(Sqrt[c]*d + Sqrt[a]*e
)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^
2 + a*e^2)^2) + ((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(8*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) + (d^2*(Sqrt[c]*d + Sqrt
[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*
(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/(8*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) + (d^2*(Sqrt[c]*d -
 Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt
[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4)*
(c*d^2 + a*e^2)) - (d^2*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2)
+ ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c
]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2))
```


Rule 1314

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> -Dist[(a*f^4)/(c*d^2 + a*e^2), Int[(f*x)^(m - 4)*(d - e*x^2)*
```

```
(a + c*x^4)^p, x], x] + Dist[(d^2*f^4)/(c*d^2 + a*e^2), Int[((f*x)^(m - 4)*
(a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && Lt
Q[p, -1] && GtQ[m, 2]
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*(a*e - c*d*x^2))/(4*a
*c*(p + 1)), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1288

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + c*x^4), x],
```

```
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = -\frac{a\int \frac{x^2\left(d-ex^2\right)}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2} + \frac{d^2\int \frac{x^2}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx}{cd^2+ae^2}$$

$$= -\frac{x\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\int \frac{-ae+cdx^2}{a+cx^4}\,dx}{4c\left(cd^2+ae^2\right)} + \frac{d^2\int \left(-\frac{de}{\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{ae+cdx^2}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx}{cd^2+ae^2}$$

$$= -\frac{x\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{d^2\int \frac{ae+cdx^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2} - \frac{\left(d^3e\right)\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2} - \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8c\left(cd^2+ae^2\right)} + \cdots$$

$$= -\frac{x\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} - \frac{\left(d^2\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2} + \frac{\left(d^2\left(d+\cdots\right.\right.}{2\left(\cdots\right.}$$

$$= -\frac{x\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} - \frac{\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x\cdots\right.}{16\sqrt{2}\sqrt[4]{a}c^{5/4}\left(cd^2+ae^2\right)}$$

$$= -\frac{x\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} + \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\sqrt[4]{a}c^{3/4}\left(cd^2+ae^2\right)} - \frac{\left(d-\frac{\sqrt{a}e}{\sqrt{c}}\right)\cdots}{8\sqrt{2}\sqrt[4]{\cdots}}$$

$$= -\frac{x\left(ae+cdx^2\right)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{d^{5/2}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}d^2\left(d+\frac{\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\left(cd^2+ae^2\right)^2} + \frac{\left(d-\cdots\right.}{8\cdots}$$

Mathematica [A] time = 0.378776, size = 428, normalized size = 0.62

$$-\frac{\frac{\sqrt{2}\left(a^{3/2}e^3+5\sqrt{a}cd^2e+a\sqrt{c}de^2-3c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{a}c^{5/4}} - \frac{\sqrt{2}\left(a^{3/2}e^3+5\sqrt{a}cd^2e+a\sqrt{c}de^2-3c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{\sqrt[4]{a}c^{5/4}} + \frac{2\sqrt{2}\left(a^{3/2}e^3+5\sqrt{\cdots}\right.}{\cdots}}{32\left(a\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(c*(a + c*x^4)) + 32*d^(5/2)*Sqrt[e
]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (2*Sqrt[2]*(3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2
*e - a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
)/(a^(1/4)*c^(5/4)) - (2*Sqrt[2]*(3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e - a*Sqr
t[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(1/4)
*c^(5/4)) + (Sqrt[2]*(-3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*e^2
+ a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(
```

```
1/4)*c^(5/4)) - (Sqrt[2]*(-3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d*
e^2 + a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/
(a^(1/4)*c^(5/4)))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.014, size = 852, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^3*e^2*d*a-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^
3*c*d^3-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*a^2*e^3/c*x-1/4/(a*e^2+c*d^2)^2/(c*x^
4+a)*e*a*d^2*x+1/16/(a*e^2+c*d^2)^2/c*(1/c*a)^(1/4)*a*2^(1/2)*arctan(2^(1/2
)/(1/c*a)^(1/4)*x+1)*e^3+5/16/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2*e+1/16/(a*e^2+c*d^2)^2/c*(1/c*a)^(1/4)*a*2^(
1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^3+5/16/(a*e^2+c*d^2)^2*(1/c*a)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^2*e+1/32/(a*e^2+c*d^2)^2/c*(
1/c*a)^(1/4)*a*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-
(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*e^3+5/32/(a*e^2+c*d^2)^2*(1/c*a)^(1
/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/
4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^2*e-1/32/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)*2^(
1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^
(1/2)+(1/c*a)^(1/2)))*a*d*e^2+3/32/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*2^(1/2)*ln
((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(
1/c*a)^(1/2)))*d^3-1/16/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(1/c*a)^(1/4)*x+1)*a*d*e^2+3/16/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^3-1/16/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*a*d*e^2+3/16/(a*e^2+c*d^2)^2/(1/c*
a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^3-d^3*e/(a*e^2+c*d^2)^
2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 59.3622, size = 20034, normalized size = 29.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*
c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e
```

```
- 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*
e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*
e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*
a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11
*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10
 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*
a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-
(81*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 -
a^4*e^8)*x + (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 1
4*a^4*c^2*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3
*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-
(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e
^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a
^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9
*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 +
 a^9*c^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d
^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)
*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^
3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^1
6 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*
a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2
*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*
a^3*c^3*d^2*e^6 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e
^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4
*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4
 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2
 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5
*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^
12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 +
28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c
^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81
*c^4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4
*e^8)*x - (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a
^4*c^2*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^
7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81
*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6
+ 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*
c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^
8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^
9*c^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8
+ 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sq
rt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d
^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 +
 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5
*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^
14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3
*c^3*d^2*e^6 + a^4*c^2*e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4
+ (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*
c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 +
4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 +
799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*
d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*
e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*
a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*
d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^
4*d^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^
8)*x + (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*
c^2*d^2*e^7 - a^5*c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d
^7*e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^
```

6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 1
43*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^1
2*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e
^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c
^5*e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4
*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(
-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*
e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*
a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^
9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14
+ a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^
3*d^2*e^6 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (
c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d
^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a
^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799
*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2
*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4
 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7
*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6
*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d
^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*
x - (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2
*d^2*e^7 - a^5*c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*
e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d
^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*
a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d
^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8
+ 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*
e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*
c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(8
1*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6
 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2
*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d
^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a
^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d
^2*e^6 + a^4*c^2*e^8))) - 8*(c^2*d^2*x^4 + a*c*d^2)*sqrt(-d*e)*log((e*x^2 -
 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 4*(a*c*d^2*e + a^2*e^3)*x)/(a*c^3*d^4 +
 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*
x^4), -1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 + 16*(c^2*d^2*x^4 + a*c*d^2)*sqrt(
d*e)*arctan(sqrt(d*e)*x/d) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (
c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d
^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a
^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799
*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2
*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4
 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7
*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6
*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d
^8 - 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*
x + (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2
*d^2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*
e^4 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d
^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*
a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d
^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8
+ 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*
e^16)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*
c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(8
1*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6

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+ 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2
*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d
^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a
^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d
^2*e^6 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4
*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*
e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*
c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^
2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^
10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 +
56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^
7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^
2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d^8
- 270*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x -
 (45*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^
2*e^7 - a^5*c*e^9 - (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4
 + 6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12
 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4
*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14
*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 5
6*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^1
6)))*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5
*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c
^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 +
143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^
12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*
e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*
c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*
e^6 + a^4*c^2*e^8))) + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^
4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3
 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3
*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c
^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10
+ a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*
a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d
^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 +
 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d^8 - 2
70*a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x + (4
5*a*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e
^7 - a^5*c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 +
6*a^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12 -
558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^
2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^
2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a
^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16))
)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^
6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*
d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143
*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*
d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8
 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5
*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6
 + a^4*c^2*e^8))) - (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4 + (c^4*d^4 +
 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 -
2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^
2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*
d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a
^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4
*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*
```

```
e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*
a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-(81*c^4*d^8 - 270*
a*c^3*d^6*e^2 - 112*a^2*c^2*d^4*e^4 - 18*a^3*c*d^2*e^6 - a^4*e^8)*x - (45*a
*c^5*d^8*e - 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7
- a^5*c*e^9 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a
^4*c^6*d^5*e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12 - 558
*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d
^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 +
 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*
c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*s
qrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^8 + 4*a*c^5*d^6*e
^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^1
2 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^
4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^1
4*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 +
56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^
16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 +
a^4*c^2*e^8))) + 4*(a*c*d^2*e + a^2*e^3)*x)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2
+ a^3*c*e^4 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out
```

Giac [A] time = 1.13715, size = 803, normalized size = 1.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) + 1/8*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + 3*(a*c^3)^(3/4)*c*d^3 + (a*c^3)^(1
/4)*a^2*c*e^3 - (a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + sq
rt(2)*a^3*c^3*e^4) + 1/8*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + 3*(a*c^3)^(3/4)*c*d
^3 + (a*c^3)^(1/4)*a^2*c*e^3 - (a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c
^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/16*(5*(a*c^3)^(1/4)*a*c^2*d^2*e - 3*(
a*c^3)^(3/4)*c*d^3 + (a*c^3)^(1/4)*a^2*c*e^3 + (a*c^3)^(3/4)*a*d*e^2)*log(x
^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*
c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/16*(5*(a*c^3)^(1/4)*a*c^2*d^2*e - 3*
(a*c^3)^(3/4)*c*d^3 + (a*c^3)^(1/4)*a^2*c*e^3 + (a*c^3)^(3/4)*a*d*e^2)*log(
x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2
*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/4*(c*d*x^3 + a*x*e)/((c*x^4 + a)*(c
^2*d^2 + a*c*e^2))
```

3.254 $\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=685

$$\frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}+\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1\right.}{8\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+\right.}$$

```
[Out] -(x*(d - e*x^2))/(4*(c*d^2 + a*e^2)*(a + c*x^4)) + (d^(3/2)*e^(3/2)*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]
*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*
e^2)^2) + ((3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/(8*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d^2*(Sqrt[c]*d -
Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d
^2 + a*e^2)^2) - ((3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d^2*(Sqrt[
c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4)*
(c*d^2 + a*e^2)) + (c^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
- ((3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.609404, antiderivative size = 685, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1314, 1179, 1168, 1162, 617, 204, 1165, 628, 1171, 205}

$$\frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{a}e+3\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+cd^2\right)}+\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1\right.}{8\sqrt{2}a^{3/4}c^{3/4}\left(ae^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(x*(d - e*x^2))/(4*(c*d^2 + a*e^2)*(a + c*x^4)) + (d^(3/2)*e^(3/2)*ArcTan[
(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]
*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*
e^2)^2) + ((3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/(8*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d^2*(Sqrt[c]*d -
Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d
^2 + a*e^2)^2) - ((3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/
a^(1/4)])/(8*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d^2*(Sqrt[
c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4)*
(c*d^2 + a*e^2)) + (c^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2)
- ((3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2))
```

Rule 1314

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> -Dist[(a*f^4)/(c*d^2 + a*e^2), Int[(f*x)^(m - 4)*(d - e*x^2)*
```

```
(a + c*x^4)^p, x], x] + Dist[(d^2*f^4)/(c*d^2 + a*e^2), Int[((f*x)^(m - 4)*
(a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && Lt
Q[p, -1] && GtQ[m, 2]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = -\frac{a\int \frac{d-ex^2}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2}+\frac{d^2\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)}\,dx}{cd^2+ae^2}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{\int \frac{-3d+ex^2}{a+cx^4}\,dx}{4\left(cd^2+ae^2\right)}+\frac{d^2\int \left(\frac{e^2}{\left(cd^2+ae^2\right)\left(d+ex^2\right)}+\frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx}{cd^2+ae^2}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{\left(cd^2\right)\int \frac{d-ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2}+\frac{\left(d^2e^2\right)\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2}-\frac{\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{8c\left(cd^2+ae^2\right)}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2}+\frac{\left(d^2\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2}+\frac{\left(d^2\left(\frac{\sqrt{c}d}{\sqrt{a}}\right.\right.}{2(}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2}+\frac{\left(3\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}\right.}{16\sqrt{2}a^{3/4}c^{3/4}\left(cd^2+ae^2\right)}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2}+\frac{\left(3\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{3/4}c^{3/4}\left(cd^2+ae^2\right)}-\frac{\left(3\sqrt{c}\right.}{8}$$

$$= -\frac{x\left(d-ex^2\right)}{4\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\left(cd^2+ae^2\right)^2}-\frac{\sqrt[4]{c}d^2\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}+\cdots$$

Mathematica [A] time = 0.285044, size = 423, normalized size = 0.62

$$\frac{\frac{\sqrt{2}\left(a^{3/2}e^3-3\sqrt{a}cd^2e+3a\sqrt{c}de^2-c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}c^{3/4}}+\frac{\sqrt{2}\left(-a^{3/2}e^3+3\sqrt{a}cd^2e-3a\sqrt{c}de^2+c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{3/4}c^{3/4}}-\frac{2\sqrt{2}\left(a^{3/2}e^3-3\sqrt{a}\cdots\right.}{}}{32\left(ae^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^2 + a*e^2)*(-(d*x) + e*x^3))/(a + c*x^4) + 32*d^(3/2)*e^(3/2)*ArcT
an[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a
*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(
3/4)*c^(3/4)) + (2*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d
*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(3/4)*c^(3/
4)) + (Sqrt[2]*(-(c^(3/2)*d^3) - 3*Sqrt[a]*c*d^2*e + 3*a*Sqrt[c]*d*e^2 + a^
(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(3/4)
*c^(3/4)) + (Sqrt[2]*(c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 -
 a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(3
```

```
/4)*c^(3/4)))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.016, size = 848, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^3*e^3*a+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x^3*e
*d^2*c-1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x*e^2*d*a-1/4/(a*e^2+c*d^2)^2/(c*x^4+a
)*x*c*d^3-3/16/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)
^(1/4)*x-1)*d*e^2+1/16/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(1/
2)/(1/c*a)^(1/4)*x-1)*c*d^3-3/32/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(1/2)*ln((
x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/
c*a)^(1/2)))*d*e^2+1/32/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/
c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1
/2)))*c*d^3-3/16/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*
a)^(1/4)*x+1)*d*e^2+1/16/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)/a*2^(1/2)*arctan(2^(
1/2)/(1/c*a)^(1/4)*x+1)*c*d^3+1/16/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*a*e^3-3/16/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^2*e+1/32/(a*e^2+c*d^2)^2/c/(1/c
*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*
a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*a*e^3-3/32/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)
*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*
x*2^(1/2)+(1/c*a)^(1/2)))*d^2*e+1/16/(a*e^2+c*d^2)^2/c/(1/c*a)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*a*e^3-3/16/(a*e^2+c*d^2)^2/(1/c*a)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2*e+d^2*e^2/(a*e^2+c*d^2)^2/(
d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 59.1055, size = 19637, normalized size = 28.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(c*d^2*e + a*e^3)*x^3 - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (
c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*
e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*
```

```
a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*
c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10
 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 5
6*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*
d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^
6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 -
 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^2*c^4
*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3
*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 -
 a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*
a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*
e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4
 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*
c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 2
0*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^
4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2
+ 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*
c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^
12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 2
8*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*
a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + (a
*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e
^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*
a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-
(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 +
 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4
*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8
*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^
11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c
^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^
6 - a^4*e^8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*
a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 +
14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sq
rt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e
^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8
*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7
*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14
+ a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^
5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e
^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3
*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^
16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a
^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2
*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^
4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c
*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255
*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2
*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^
4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9
*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c
^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*
d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^
2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 - (
3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*
e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 +
 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c
*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^1
```

```
2*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28
*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*
e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c
^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10
*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30
*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c
^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^1
0 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8
 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))
 + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^
2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8
 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*s
qrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*
e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 +
8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^
7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14
 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*
a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d
^2*e^6 - a^4*e^8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4
- 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 - (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e
^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^1
1)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*
d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^1
6 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^
7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*
e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 -
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^
3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^
11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 +
 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^
4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^
4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + 8*(c*d*e*x^4 + a*d*e)*sqrt(-d*e)
*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 4*(c*d^3 + a*d*e^2)*x)/(a*
c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^
4)*x^4), 1/16*(4*(c*d^2*e + a*e^3)*x^3 + 16*(c*d*e*x^4 + a*d*e)*sqrt(d*e)*a
rctan(sqrt(d*e)*x/d) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 +
2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^
2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^
2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^
4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^1
2)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*
d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 +
 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*
a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*
d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 +
 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*
e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d
^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^
8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6
*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*
c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^
12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*
e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*
a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*
c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10
 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 5
6*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*
d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^
```

```
6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + (a*c^2*d^4 +
 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*s
qrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^
6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12
- 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c
^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14
*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*
a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^1
6)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6
 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^
8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^
3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5
*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d
^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a
^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*
d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 +
 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3
*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*
a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-
(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 +
 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4
*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8
*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^
11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c
^2*d^2*e^6 + a^5*c*e^8))) - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d
^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 +
 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c
^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d
^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^
6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6
*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e
^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2
 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a
*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a^4*e^8)*x + (a*c^5*d^9 - 18*a^2*c^4*d^7*
e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 - (3*a^3*c^7*
d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*
c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c
^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10
+ a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56
*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d
^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c
*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255
*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2
*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^
4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9
*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c
^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + (a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x
^4)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8 + 4*a^2*c
^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*
d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*
a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10
*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8
+ 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^
3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^
2*e^6 + a^5*c*e^8))*log(-(c^4*d^8 - 14*a*c^3*d^6*e^2 + 14*a^3*c*d^2*e^6 - a
^4*e^8)*x - (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c
^2*d^3*e^6 + 3*a^5*c*d*e^8 - (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^
```

```
5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(
c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 +
255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*
c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*
e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^1
1*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d^8
 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*s
qrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*
e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 +
8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^
7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14
 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*
a^4*c^2*d^2*e^6 + a^5*c*e^8))) - 4*(c*d^3 + a*d*e^2)*x)/(a*c^2*d^4 + 2*a^2*
c*d^2*e^2 + a^3*e^4 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out
```

Giac [A] time = 1.15073, size = 791, normalized size = 1.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^
4) + 1/8*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(
3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)
^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + sqrt(
2)*a^3*c^3*e^4) + 1/8*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2
- 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^
2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/16*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4
)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 + sq
rt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2
*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/16*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)
*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 - sqr
t(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*
e^2 + sqrt(2)*a^3*c^3*e^4) + 1/4*(x^3*e - d*x)/((c*x^4 + a)*(c*d^2 + a*e^2)
)
```

3.255 $\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=685

$$\frac{\sqrt[4]{c}de\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}de\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\left(\sqrt{c}d-3\sqrt{a}e\right)}{16}$$

```
[Out] (x*(a*e + c*d*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[d]*e^(5/2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (c^(1/4)*d*e*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d*e*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(
1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) - ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.560447, antiderivative size = 685, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1316, 1179, 1168, 1162, 617, 204, 1165, 628, 1171, 205}

$$\frac{\sqrt[4]{c}de\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}de\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\left(\sqrt{c}d-3\sqrt{a}e\right)}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (x*(a*e + c*d*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[d]*e^(5/2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (c^(1/4)*d*e*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d*e*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(
1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) - ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2))
```

Rule 1316

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[f^2/(c*d^2 + a*e^2), Int[(f*x)^(m - 2)*(a*e + c*d*x^2)*(
```

```
a + c*x^4)^p, x], x] - Dist[(d*e*f^2)/(c*d^2 + a*e^2), Int[((f*x)^(m - 2)*(
a + c*x^4)^(p + 1))/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && LtQ
[p, -1] && GtQ[m, 0]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\int \frac{ae+cdx^2}{(a+cx^4)^2}\,dx}{cd^2+ae^2} - \frac{(de)\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx}{cd^2+ae^2}$$

$$= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\int \frac{-3ae-cdx^2}{a+cx^4}\,dx}{4a(cd^2+ae^2)} - \frac{(de)\int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)}\right)dx}{cd^2+ae^2}$$

$$= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{(cde)\int \frac{d-ex^2}{a+cx^4}\,dx}{(cd^2+ae^2)^2} - \frac{(de^3)\int \frac{1}{d+ex^2}\,dx}{(cd^2+ae^2)^2} - \frac{\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a(cd^2+ae^2)} +$$

$$= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\left(d\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2(cd^2+ae^2)^2} - \frac{\left(de\left(\frac{\sqrt{c}d}{\sqrt{a}}+\right.\right.}{2(c}$$

$$= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\sqrt[4]{c}\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}x\right.}{16\sqrt{2}a^{5/4}(cd^2+ae^2)}$$

$$= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}\left(d+\frac{3\sqrt{a}e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{5/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}\left(d\right.}{}$$

$$= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\sqrt[4]{c}de\left(\sqrt{c}d-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(cd^2+ae^2)^2} - \frac{\sqrt[4]{}}{}$$

Mathematica [A] time = 0.293582, size = 428, normalized size = 0.62

$$\frac{\frac{\sqrt{2}\left(-3a^{3/2}e^3+\sqrt{a}cd^2e+5a\sqrt{c}de^2+c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{5/4}\sqrt[4]{c}} - \frac{\sqrt{2}\left(-3a^{3/2}e^3+\sqrt{a}cd^2e+5a\sqrt{c}de^2+c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{5/4}\sqrt[4]{c}} - \frac{2\sqrt{2}\left(3a^{3/2}e^3-\right.}{}}{32\left(ae\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(a*(a + c*x^4)) - 32*Sqrt[d]*e^(5/2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e +
5*a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(a^(5/4)*c^(1/4)) + (2*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e + 5*a*Sqrt[c
]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/4)*
c^(1/4)) + (Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^2 - 3*
a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(5/
4)*c^(1/4)) - (Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^2 -
 3*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^
```

```
(5/4)*c^(1/4)))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.013, size = 852, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*c*d*x^3*e^2+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*c^2
*d^3/a*x^3+1/4/(a*e^2+c*d^2)^2/(c*x^4+a)*x*e^3*a+1/4/(a*e^2+c*d^2)^2/(c*x^4
+a)*x*e*d^2*c+3/16/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/
c*a)^(1/4)*x-1)*e^3-1/16/(a*e^2+c*d^2)^2/a*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(1/c*a)^(1/4)*x-1)*c*d^2*e+3/32/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(1/2)*
ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)
+(1/c*a)^(1/2)))*e^3-1/32/(a*e^2+c*d^2)^2/a*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(
1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^
(1/2)))*c*d^2*e+3/16/(a*e^2+c*d^2)^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
1/c*a)^(1/4)*x+1)*e^3-1/16/(a*e^2+c*d^2)^2/a*(1/c*a)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(1/c*a)^(1/4)*x+1)*c*d^2*e+5/16/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d*e^2+1/16/(a*e^2+c*d^2)^2/a*c/(1/c*a)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^3+5/32/(a*e^2+c*d^2)^2/(1
/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/
c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d*e^2+1/32/(a*e^2+c*d^2)^2/a*c/(1/c*a)
^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^
(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^3+5/16/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d*e^2+1/16/(a*e^2+c*d^2)^2/a*c/(1/c*a
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^3-d*e^3/(a*e^2+c*d^2)^2
/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 69.4296, size = 19999, normalized size = 29.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(c^2*d^3 + a*c*d*e^2)*x^3 + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e
^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4
*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*
```

d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 +
143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c
*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^
12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 +
28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 +
4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c
^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^
4*e^8)*x + (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5
*c*d^2*e^7 + 27*a^6*e^9 - (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^
7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^
6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 79
9*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6
*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*
e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a
^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d
^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqr
t(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^
6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 +
8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^
5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^
14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 +
4*a^5*c*d^2*e^6 + a^6*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (
a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d
^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^
4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^
2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e
^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4
+ 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^1
1*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*
c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8
+ 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)
*x - (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2
*e^7 + 27*a^6*e^9 - (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4
+ 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12
+ 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*
c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d
^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 +
56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*
e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4
*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^
6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 79
9*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6
*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*
e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a
^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*
c*d^2*e^6 + a^6*e^8))) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*
d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3
- 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*
a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*
d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 +
81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*
a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*
d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^
6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*
a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (
a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 +
27*a^6*e^9 + (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a
^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*
a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^

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4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^
2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^1
0*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16))
)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c
^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12
 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*
c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d
^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 +
56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*
e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*
e^6 + a^6*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 +
2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*
a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*
d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^
4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6
*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^
6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^
12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2
+ 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*
d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (a^2*c^
4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^
6*e^9 + (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3
*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*
d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8
- 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28
*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*
d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt
((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6
*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*
a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^
4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^
2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^1
0*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16))
)/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 +
a^6*e^8))) + 8*(a*c*e^2*x^4 + a^2*e^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)
*x - d)/(e*x^2 + d)) + 4*(a*c*d^2*e + a^2*e^3)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4), 1/16*(4
*(c^2*d^3 + a*c*d*e^2)*x^3 - 16*(a*c*e^2*x^4 + a^2*e^2)*sqrt(d*e)*arctan(sq
rt(d*e)*x/d) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^
2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*
d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*
e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 +
540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^1
2)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^
10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 +
 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*
a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*
e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (a^2*c^4*d^
8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^
9 - (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5
*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10
*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 55
8*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7
*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*
e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*
c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2
 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^
5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^
8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 +
```

28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 - (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x + (a^2*c^4*d^8*e + 6*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 + (a^4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 +

```
4*a^5*c*d^2*e^6 + a^6*e^8))*log(-(c^4*d^8 + 18*a*c^3*d^6*e^2 + 112*a^2*c^2*
d^4*e^4 + 270*a^3*c*d^2*e^6 - 81*a^4*e^8)*x - (a^2*c^4*d^8*e + 6*a^3*c^3*d^
6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 + (a^4*c^6*d^11
+ 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 + 21*a^8*c^2*
d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4
*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 +
 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56
*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3
*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5*e + 4*a*c*
d^3*e^3 - 30*a^2*d*e^5 - (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e
^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a
^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*
e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^
4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^
11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3
*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))) + 4*(a*c*d^
2*e + a^2*e^3)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2
*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out
```

Giac [A] time = 1.13137, size = 814, normalized size = 1.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(5/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) - 1/8*((a*c^3)^(1/4)*a*c^2*d^2*e - (a*c^3)^(3/4)*c*d^3 - 3*(a*c^3)^(1/4
)*a^2*c*e^3 - 5*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 +
sqrt(2)*a^4*c^2*e^4) - 1/8*((a*c^3)^(1/4)*a*c^2*d^2*e - (a*c^3)^(3/4)*c*d^3
 - 3*(a*c^3)^(1/4)*a^2*c*e^3 - 5*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a
^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*((a*c^3)^(1/4)*a*c^2*d^2*e + (
a*c^3)^(3/4)*c*d^3 - 3*(a*c^3)^(1/4)*a^2*c*e^3 + 5*(a*c^3)^(3/4)*a*d*e^2)*l
og(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2
)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*((a*c^3)^(1/4)*a*c^2*d^2*e
+ (a*c^3)^(3/4)*c*d^3 - 3*(a*c^3)^(1/4)*a^2*c*e^3 + 5*(a*c^3)^(3/4)*a*d*e^2
)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqr
t(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/4*(c*d*x^3 + a*x*e)/((c*x^4
 + a)*(a*c*d^2 + a^2*e^2))
```

3.256 $\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=689

$$-\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{\ldots}\right.}{}$$

```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/4)*e^2*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]
*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(
7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rubi [A] time = 0.600629, antiderivative size = 689, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1239, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt[4]{c}e^2\left(\sqrt{a}e+\sqrt{c}d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}\left(ae^2+cd^2\right)^2}-\frac{\sqrt[4]{c}\left(\sqrt{a}e+3\sqrt{\ldots}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/4)*e^2*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]
*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(
7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rule 1239

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
```

```
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \int \left(\frac{e^4}{\left(cd^2+ae^2\right)^2\left(d+ex^2\right)} + \frac{c\left(d-ex^2\right)}{\left(cd^2+ae^2\right)\left(a+cx^4\right)^2} - \frac{ce^2\left(-d+ex^2\right)}{\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}\right) dx$$

$$= -\frac{\left(ce^2\right)\int \frac{-d+ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{e^4\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{c\int \frac{d-ex^2}{\left(a+cx^4\right)^2}\,dx}{cd^2+ae^2}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} + \frac{\left(\left(\frac{\sqrt{cd}}{\sqrt{a}}-e\right)e^2\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2\left(cd^2+ae^2\right)^2} + \frac{\left(e^2\left(\frac{\sqrt{cd}}{\sqrt{a}}+e\right)\right)}{2\left(cd^2 \cdots\right.}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} + \frac{\left(\left(\frac{\sqrt{cd}}{\sqrt{a}}-e\right)e^2\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}+x^2}\,dx}{4\left(cd^2+ae^2\right)^2} + \frac{\left(\left(\frac{\sqrt{cd}}{\sqrt{a}}-e\right)\cdots\right.}{4\cdots}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}e^2\left(\sqrt{cd}+\sqrt{a}e\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{c}\cdots\right.}{4\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}e^2\left(\sqrt{cd}-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} + \frac{\sqrt[4]{c}e^2 \cdots}{\cdots}$$

$$= \frac{cx\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}e^2\left(\sqrt{cd}-\sqrt{a}e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}\left(cd^2+ae^2\right)^2} - \frac{\sqrt[4]{c}\left(3 \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.307578, size = 429, normalized size = 0.62

$$\frac{-\frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}e^3+\sqrt{a}cd^2e+7a\sqrt{c}de^2+3c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}} + \frac{\sqrt{2}\sqrt[4]{c}\left(5a^{3/2}e^3+\sqrt{a}cd^2e+7a\sqrt{c}de^2+3c^{3/2}d^3\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{7/4}} + \frac{2\sqrt{2}\sqrt[4]{\cdots}}{\cdots}}{32\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]
*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e
- 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/a^(7/4) - (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqrt[c
]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqr
t[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A] time = 0.016, size = 873, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*e^3*x^3-1/4*c^2/(a*e^2+c*d^2)^2/(c*x^4+a)*
e/a*x^3*d^2+1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*d*x*e^2+1/4*c^2/(a*e^2+c*d^2)^2
/(c*x^4+a)*d^3/a*x+7/16*c/(a*e^2+c*d^2)^2/a*(1/c*a)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(1/c*a)^(1/4)*x-1)*d*e^2+3/16*c^2/(a*e^2+c*d^2)^2/a^2*(1/c*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^3+7/32*c/(a*e^2+c*d^2)^2/a*(1/c*
a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a
)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d*e^2+3/32*c^2/(a*e^2+c*d^2)^2/a^2*(1/c*a
)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)
^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^3+7/16*c/(a*e^2+c*d^2)^2/a*(1/c*a)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d*e^2+3/16*c^2/(a*e^2+c*d^2)^2/a
^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^3-5/16/(a*e^2+
c*d^2)^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^3-1/16*c
/(a*e^2+c*d^2)^2/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*
d^2*e-5/32/(a*e^2+c*d^2)^2/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^
(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*e^3-1/32*
c/(a*e^2+c*d^2)^2/a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(
1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^2*e-5/16/(a*e^
2+c*d^2)^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*e^3-1/16
*c/(a*e^2+c*d^2)^2/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1
)*d^2*e+e^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 132.414, size = 20650, normalized size = 29.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(c^2*d^2*e + a*c*e^3)*x^3 + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*
e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e +
44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^
5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d
^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8
 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e
^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*
a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))
/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a
^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*
a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2
```

+ 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^
10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^1
0*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a
^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*
d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*
d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^1
0 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5
*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 +
 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*
c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^
4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d
^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8
 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^
16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^
6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*
a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70
*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6
*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^
5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e
^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*
e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 2
8*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4
*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5
*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625
*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4
 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*
e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*
e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748
*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*
e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^
5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e
^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3
 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 +
4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a
^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*
d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*
d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^1
0 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 +
4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a^2
*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3
*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c
^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)
*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*
c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)
/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^1
0*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*
c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*
e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a
^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^
6 - 175*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*
e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d
^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 52
9*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 +
 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*
c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^
14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (
a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7
*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748

```
*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*
e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^
5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e
^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6
*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*
c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^
6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 +
 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4
*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^
8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d
^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 +
a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c
*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^
4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*
c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 -
 (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*
e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*
e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1
950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 +
 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12
*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqr
t((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c
^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d
^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 52
9*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 +
 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*
c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^
14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*
a^6*c*d^2*e^6 + a^7*e^8))) - 8*(a*c*e^3*x^4 + a^2*e^3)*sqrt(-e/d)*log((e*x^
2 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)) - 4*(c^2*d^3 + a*c*d*e^2)*x)/(a^2*c^
2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*
e^4)*x^4), -1/16*(4*(c^2*d^2*e + a*c*e^3)*x^3 - 16*(a*c*e^3*x^4 + a^2*e^3)*
sqrt(e/d)*arctan(x*sqrt(e/d)) + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 +
(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c
^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*
d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^
2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 195
0*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 2
8*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c
^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*
c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8
))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^
2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*
a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e +
 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^
2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5
*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^
10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e
^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28
*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 4
4*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5
*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^
10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8
- 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^
2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a
^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/
(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^
7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^
```

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2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c
*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2
*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*
e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 +
625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 +
56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13
*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d
^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 +
 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c
*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198
*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 +
26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*
sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c
^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/
(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10
*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*
a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*
c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5
*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^
10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e
^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28
*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*
c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a^2*c^2*d
^4 + 2*a^3*c*d^2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4
)*x^4)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8
 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(
-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^
6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*
c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6
+ 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^1
4*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^
4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 +
1376*a^2*c^3*d^4*e^4 + 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x + (27*a^2*c^5
*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 17
5*a^6*c*d*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 +
34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 +
738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*
c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8
*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^
8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a
^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^
4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*
sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c
^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/
(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10
*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c
^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
+ a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)*sqrt((6*c^3*d^
5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2
+ 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a
*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d
^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*
d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8
 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e
^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e
^6 + a^7*e^8))*log(-(81*c^5*d^8 + 594*a*c^4*d^6*e^2 + 1376*a^2*c^3*d^4*e^4
+ 750*a^3*c^2*d^2*e^6 - 625*a^4*c*e^8)*x - (27*a^2*c^5*d^9 + 186*a^3*c^4*d^
```

```
7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 - (a^6*
c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 +
21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 +
2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^
5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^
9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d
^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c
^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^6
*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 +
738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*
c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8
*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^
8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a
^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*
d^2*e^6 + a^7*e^8))) - 4*(c^2*d^3 + a*c*d*e^2)*x)/(a^2*c^2*d^4 + 2*a^3*c*d^
2*e^2 + a^4*e^4 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)*x^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)

[Out] Timed out
```

Giac [A] time = 1.12803, size = 814, normalized size = 1.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")

[Out] 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*
c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2
)*a^4*c^2*e^4) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2
 - (a*c^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3
*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)
^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(x^2
 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*
c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c
^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(
x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a
^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/(
(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d)) - 1/4*(c*x^3*e - c*d*x)/((c*x^
4 + a)*(a*c*d^2 + a^2*e^2))
```

3.257 $\int \frac{1}{x^2(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=745

$$\frac{c^{3/4}\left(a^{3/2}e^3-\sqrt{c}d\left(2ae^2+cd^2\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)^2}-\frac{c^{3/4}\left(\sqrt{c}d-3\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)}-\frac{c}{}$$

```
[Out] -(1/(a^2*d*x)) - (c*x*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^2)^2) + (c^(3
/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(9/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(a^(3/2)*e^3 + Sqrt[c]*d*(c*d^2
+ 2*a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(c*
d^2 + a*e^2)^2) - (c^(3/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(a^(3/2)*
e^3 + Sqrt[c]*d*(c*d^2 + 2*a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4
)*(c*d^2 + a*e^2)) + (c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(
c*d^2 + a*e^2)^2) + (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) -
(c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2)
```

Rubi [A] time = 0.772443, antiderivative size = 745, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1336, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$\frac{c^{3/4}\left(a^{3/2}e^3-\sqrt{c}d\left(2ae^2+cd^2\right)\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)^2}-\frac{c^{3/4}\left(\sqrt{c}d-3\sqrt{a}e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{9/4}\left(ae^2+cd^2\right)}-\frac{c}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```


```
[Out] -(1/(a^2*d*x)) - (c*x*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^2)^2) + (c^(3
/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(9/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(a^(3/2)*e^3 + Sqrt[c]*d*(c*d^2
+ 2*a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(c*
d^2 + a*e^2)^2) - (c^(3/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(a^(3/2)*
e^3 + Sqrt[c]*d*(c*d^2 + 2*a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4
)*(c*d^2 + a*e^2)) + (c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(
c*d^2 + a*e^2)^2) + (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) -
(c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2)
```


Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \int\left(\frac{1}{a^2dx^2}-\frac{e^5}{d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}-\frac{c\left(ae+cdx^2\right)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)^2}+\frac{c\left(-a^2e^3-cd\left(cd^2+2a\right.\right.}{a^2\left(cd^2+ae^2\right)^2\left(a+\right.}\right.$$

$$=-\frac{1}{a^2dx}+\frac{c\int\frac{-a^2e^3-cd\left(cd^2+2ae^2\right)x^2}{a+cx^4}\,dx}{a^2\left(cd^2+ae^2\right)^2}-\frac{e^5\int\frac{1}{d+ex^2}\,dx}{d\left(cd^2+ae^2\right)^2}-\frac{c\int\frac{ae+cdx^2}{\left(a+cx^4\right)^2}\,dx}{a\left(cd^2+ae^2\right)}$$

$$=-\frac{1}{a^2dx}-\frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{c\int\frac{-3ae-cdx^2}{a+cx^4}\,dx}{4a^2\left(cd^2+ae^2\right)}+\frac{\left(c\left(cd^3+2ad\right.\right.}{2a}$$

$$=-\frac{1}{a^2dx}-\frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{\left(c\left(d-\frac{3\sqrt{a}e}{\sqrt{c}}\right)\right)\int\frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{8a^2\left(cd^2+ae^2\right)}-$$

$$=-\frac{1}{a^2dx}-\frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2}-\frac{c^{5/4}\left(cd^3+2ade^2-\frac{a^{3/2}e^3}{\sqrt{c}}\right)\log\left(\right.}{4\sqrt{2}a^{9/4}\left(cd^2+\right.}$$

$$=-\frac{1}{a^2dx}-\frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{c^{5/4}\left(cd^3+2ade^2+\frac{a^{3/2}e^3}{\sqrt{c}}\right)\tan^{-1}}{2\sqrt{2}a^{9/4}\left(cd^2+ae^2\right)}$$

$$=-\frac{1}{a^2dx}-\frac{cx\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{c^{3/4}\left(\sqrt{c}d+3\sqrt{a}e\right)\tan^{-1}\left(1-\right.}{8\sqrt{2}a^{9/4}\left(cd^2+ae^2\right)}$$

Mathematica [A] time = 0.382472, size = 499, normalized size = 0.67

$$\frac{1}{32}\left(\frac{\sqrt{2}c^{3/4}\left(7a^{3/2}e^3+3\sqrt{a}cd^2e-9a\sqrt{c}de^2-5c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{9/4}\left(ae^2+cd^2\right)^2}+\frac{\sqrt{2}c^{3/4}\left(-7a^{3/2}e^3-3\sqrt{a}cd^2e+9a\right.}{a}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-32/(a^2*d*x) - (8*c*x*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (32*e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^2)^2) + (2
*Sqrt[2]*c^(3/4)*(5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2 + 7
*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(9/4)*(c*d^2 + a*
e^2)^2) - (2*Sqrt[2]*c^(3/4)*(5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[
c]*d*e^2 + 7*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(9/4)
*(c*d^2 + a*e^2)^2) + (Sqrt[2]*c^(3/4)*(-5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e
- 9*a*Sqrt[c]*d*e^2 + 7*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2])/(a^(9/4)*(c*d^2 + a*e^2)^2) + (Sqrt[2]*c^(3/4)*(5*c^(3/2)
*d^3 - 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2 - 7*a^(3/2)*e^3)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(9/4)*(c*d^2 + a*e^2)^2))/32
```

Maple [A] time = 0.017, size = 911, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] -1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^3*e^2*d-1/4*c^3/(a*e^2+c*d^2)^2/a^2/
(c*x^4+a)*x^3*d^3-1/4*c/(a*e^2+c*d^2)^2/(c*x^4+a)*x*e^3-1/4*c^2/(a*e^2+c*d^
2)^2/a/(c*x^4+a)*x*e*d^2-7/16*c/(a*e^2+c*d^2)^2/a*(1/c*a)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(1/c*a)^(1/4)*x+1)*e^3-3/16*c^2/(a*e^2+c*d^2)^2/a^2*(1/c*a)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2*e-7/16*c/(a*e^2+c*d^2)^2/a
*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*e^3-3/16*c^2/(a*e^
2+c*d^2)^2/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^2*
e-7/32*c/(a*e^2+c*d^2)^2/a*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^
(1/2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*e^3-3/32*
c^2/(a*e^2+c*d^2)^2/a^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/
2)+(1/c*a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^2*e-9/32*c
/(a*e^2+c*d^2)^2/a/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1
/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d*e^2-5/32*c^2/(a
*e^2+c*d^2)^2/a^2/(1/c*a)^(1/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/
c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^3-9/16*c/(a*e^2+
c*d^2)^2/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d*e^2-5/
16*c^2/(a*e^2+c*d^2)^2/a^2/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/
4)*x+1)*d^3-9/16*c/(a*e^2+c*d^2)^2/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
1/c*a)^(1/4)*x-1)*d*e^2-5/16*c^2/(a*e^2+c*d^2)^2/a^2/(1/c*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^3-1/a^2/d/x-1/d*e^5/(a*e^2+c*d^2)^2/(d*
e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12159, size = 863, normalized size = 1.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(3*(a*c^3)^(1/4)*a*c^2*d^2*e + 5*(a*c^3)^(3/4)*c*d^3 + 7*(a*c^3)^(1/4)
*a^2*c*e^3 + 9*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/
c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + s
qrt(2)*a^5*c*e^4) - 1/8*(3*(a*c^3)^(1/4)*a*c^2*d^2*e + 5*(a*c^3)^(3/4)*c*d^
3 + 7*(a*c^3)^(1/4)*a^2*c*e^3 + 9*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*
a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(3*(a*c^3)^(1/4)*a*c^2*d^2*e -
5*(a*c^3)^(3/4)*c*d^3 + 7*(a*c^3)^(1/4)*a^2*c*e^3 - 9*(a*c^3)^(3/4)*a*d*e^2
)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sqr
t(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/16*(3*(a*c^3)^(1/4)*a*c^2*d^2
*e - 5*(a*c^3)^(3/4)*c*d^3 + 7*(a*c^3)^(1/4)*a^2*c*e^3 - 9*(a*c^3)^(3/4)*a*
d*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 +
2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - arctan(x*e^(1/2)/sqrt(d))*
e^(9/2)/((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d)) - 1/4*(5*c^2*d^2*x^
4 + 4*a*c*x^4*e^2 + a*c*d*x^2*e + 4*a*c*d^2 + 4*a^2*e^2)/((a^2*c*d^3 + a^3*
d*e^2)*(c*x^5 + a*x))
```

3.258 $\int \frac{1}{x^4\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx$

Optimal. Leaf size=751

$$-\frac{c^2x\left(d-ex^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\left(2ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{c^{5/4}\left(\sqrt{a}e+3\sqrt{c}d\right)\log(}{16\sqrt{2}a^{11}}$$

```
[Out] -1/(3*a^2*d*x^3) + e/(a^2*d^2*x) - (c^2*x*(d - e*x^2))/(4*a^2*(c*d^2 + a*e^
2)*(a + c*x^4)) + (e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*(c*d^2 +
a*e^2)^2) + (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - S
qrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqr
t[2]*a^(11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) -
(c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) + (c^(5/4)*(3*Sqrt[
c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
16*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*
d^2 + 2*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(11/4)*(
c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(11/4)*(c*d^
2 + a*e^2)^2)
```

Rubi [A] time = 0.686109, antiderivative size = 751, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1336, 205, 1179, 1168, 1162, 617, 204, 1165, 628}

$$-\frac{c^2x\left(d-ex^2\right)}{4a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{a}e+\sqrt{c}d\right)\left(2ae^2+cd^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{11/4}\left(ae^2+cd^2\right)^2}+\frac{c^{5/4}\left(\sqrt{a}e+3\sqrt{c}d\right)\log(}{16\sqrt{2}a^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/(3*a^2*d*x^3) + e/(a^2*d^2*x) - (c^2*x*(d - e*x^2))/(4*a^2*(c*d^2 + a*e^
2)*(a + c*x^4)) + (e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*(c*d^2 +
a*e^2)^2) + (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*
x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - S
qrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqr
t[2]*a^(11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTa
n[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) -
(c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1 + (Sqrt[2]*c^(1
/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) + (c^(5/4)*(3*Sqrt[
c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(
16*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*
d^2 + 2*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(11/4)*(
c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(11/4)*(c*d^
2 + a*e^2)^2)
```

Rule 1336

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x
*(d + e*x^2)*(a + c*x^4)^(p + 1))/(4*a*(p + 1)), x] + Dist[1/(4*a*(p + 1)),
 Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /;
FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2
*p]
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx=\int\left(\frac{1}{a^2dx^4}-\frac{e}{a^2d^2x^2}+\frac{e^6}{d^2\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}-\frac{c^2\left(d-ex^2\right)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)^2}-\frac{c^2\left(cd^2+\right.}{a^2\left(cd^2-\right.}\right.$$

$$=-\frac{1}{3a^2dx^3}+\frac{e}{a^2d^2x}+\frac{e^6\int\frac{1}{d+ex^2}\,dx}{d^2\left(cd^2+ae^2\right)^2}-\frac{c^2\int\frac{d-ex^2}{\left(a+cx^4\right)^2}\,dx}{a\left(cd^2+ae^2\right)}-\frac{\left(c^2\left(cd^2+2ae^2\right)\right)\int\frac{d-ex^2}{a+cx^4}\,dx}{a^2\left(cd^2+ae^2\right)^2}$$

$$=-\frac{1}{3a^2dx^3}+\frac{e}{a^2d^2x}-\frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2}+\frac{c^2\int\frac{-3d+ex^2}{a+cx^4}\,dx}{4a^2\left(cd^2+ae^2\right)}-$$

$$=-\frac{1}{3a^2dx^3}+\frac{e}{a^2d^2x}-\frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2}-\frac{\left(c\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\right)\int}{8a^2\left(cd^2+\right.}$$

$$=-\frac{1}{3a^2dx^3}+\frac{e}{a^2d^2x}-\frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2}+\frac{c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}+e\right)\left(cd\right.}{}$$

$$=-\frac{1}{3a^2dx^3}+\frac{e}{a^2d^2x}-\frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2}+\frac{c^{5/4}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)\left(cd\right.}{2\sqrt{2}}$$

$$=-\frac{1}{3a^2dx^3}+\frac{e}{a^2d^2x}-\frac{c^2x\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)^2}+\frac{c^{5/4}\left(\frac{3\sqrt{c}d}{\sqrt{a}}-e\right)\mathrm{ta}}{8\sqrt{2}a^{9/4}\left(c\right.}$$

Mathematica [A] time = 0.39944, size = 513, normalized size = 0.68

$$\frac{1}{96}\left(-\frac{24c^2x\left(d-ex^2\right)}{a^2\left(a+cx^4\right)\left(ae^2+cd^2\right)}+\frac{3\sqrt{2}c^{5/4}\left(9a^{3/2}e^3+5\sqrt{a}cd^2e+11a\sqrt{c}de^2+7c^{3/2}d^3\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{a^{11/4}\left(ae^2+cd^2\right)^2}-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-32/(a^2*d*x^3) + (96*e)/(a^2*d^2*x) - (24*c^2*x*(d - e*x^2))/(a^2*(c*d^2
+ a*e^2)*(a + c*x^4)) + (96*e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*
(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^(5/4)*(7*c^(3/2)*d^3 - 5*Sqrt[a]*c*d^2*e
+ 11*a*Sqrt[c]*d*e^2 - 9*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4
)])/(a^(11/4)*(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^(5/4)*(-7*c^(3/2)*d^3 + 5*S
qrt[a]*c*d^2*e - 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(a^(11/4)*(c*d^2 + a*e^2)^2) + (3*Sqrt[2]*c^(5/4)*(7*c^(
3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(11/4)*(c*d^2 + a*e^2)^2
) - (3*Sqrt[2]*c^(5/4)*(7*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*
e^2 + 9*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
)/(a^(11/4)*(c*d^2 + a*e^2)^2))/96
```

Maple [A] time = 0.02, size = 932, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x)
```

```
[Out] 1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x^3*e^3+1/4*c^3/(a*e^2+c*d^2)^2/a^2/(c*
x^4+a)*x^3*e*d^2-1/4*c^2/(a*e^2+c*d^2)^2/a/(c*x^4+a)*x*e^2*d-1/4*c^3/(a*e^2
+c*d^2)^2/a^2/(c*x^4+a)*x*d^3-11/16*c^2/(a*e^2+c*d^2)^2/a^2*(1/c*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d*e^2-7/16*c^3/(a*e^2+c*d^2)^2/a^3
*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^3-11/32*c^2/(a*e
^2+c*d^2)^2/a^2*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*
a)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d*e^2-7/32*c^3/(a*e^
2+c*d^2)^2/a^3*(1/c*a)^(1/4)*2^(1/2)*ln((x^2+(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a
)^(1/2))/(x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2)))*d^3-11/16*c^2/(a*e^2+
c*d^2)^2/a^2*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d*e^2-
7/16*c^3/(a*e^2+c*d^2)^2/a^3*(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(
1/4)*x+1)*d^3+9/16*c/(a*e^2+c*d^2)^2/a/(1/c*a)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(1/c*a)^(1/4)*x-1)*e^3+5/16*c^2/(a*e^2+c*d^2)^2/a^2/(1/c*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/c*a)^(1/4)*x-1)*d^2*e+9/32*c/(a*e^2+c*d^2)^2/a/(1/c*a)^(1
/4)*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/
4)*x*2^(1/2)+(1/c*a)^(1/2)))*e^3+5/32*c^2/(a*e^2+c*d^2)^2/a^2/(1/c*a)^(1/4)
*2^(1/2)*ln((x^2-(1/c*a)^(1/4)*x*2^(1/2)+(1/c*a)^(1/2))/(x^2+(1/c*a)^(1/4)*
x*2^(1/2)+(1/c*a)^(1/2)))*d^2*e+9/16*c/(a*e^2+c*d^2)^2/a/(1/c*a)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*e^3+5/16*c^2/(a*e^2+c*d^2)^2/a^2/(1/c
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c*a)^(1/4)*x+1)*d^2*e-1/3/a^2/d/x^3+e/a
^2/d^2/x+1/d^2*e^6/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11836, size = 848, normalized size = 1.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*e^2 - 5*(a*c^3)^(3
/4)*c*d^2*e - 9*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + sq
rt(2)*a^5*c*e^4) - 1/8*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*
e^2 - 5*(a*c^3)^(3/4)*c*d^2*e - 9*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^
4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*
c^3)^(1/4)*a*c^2*d*e^2 + 5*(a*c^3)^(3/4)*c*d^2*e + 9*(a*c^3)^(3/4)*a*e^3)*l
og(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2
)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/16*(7*(a*c^3)^(1/4)*c^3*d^3 + 11
*(a*c^3)^(1/4)*a*c^2*d*e^2 + 5*(a*c^3)^(3/4)*c*d^2*e + 9*(a*c^3)^(3/4)*a*e^
3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sq
rt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(1
1/2)/((c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4)*sqrt(d)) + 1/4*(c^2*x^3*e - c
^2*d*x)/((a^2*c*d^2 + a^3*e^2)*(c*x^4 + a)) + 1/3*(3*x^2*e - d)/(a^2*d^2*x^
3)
```

3.259 $\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=70

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

```
[Out] -ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Rubi [A] time = 0.0594416, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1318, 220, 1699, 203}

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] -ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) + ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Rule 1318

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{1+x^4}}\,dx - \frac{1}{2}\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$$

$$= \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{1+x^4}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

Mathematica [C] time = 0.0904195, size = 40, normalized size = 0.57

$$\sqrt[4]{-1}\left(\Pi\left(-i; i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle| -1\right) - \text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right), -1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] (-1)^(1/4)*(-EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + EllipticPi[-I, I*ArcS
inh[(-1)^(1/4)*x], -1])
```

Maple [C] time = 0.058, size = 110, normalized size = 1.6

$$\frac{\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right), i\right)}{\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}}\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{1}{\sqrt{x^4+1}} + (-1)^{\frac{3}{4}}\,\text{EllipticPi}\left(\sqrt[4]{-1}x, i, \sqrt{-i}-(-1)^{\frac{3}{4}}\right)\sqrt{1-ix^2}\sqrt{1+ix^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)/(x^4+1)^(1/2),x)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I
*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^4+1}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4+1}x^2}{x^6+x^4+x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 1)*x^2/(x^6 + x^4 + x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x^2+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] Integral(x**2/((x**2 + 1)*sqrt(x**4 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^4+1}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

3.260 $\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) - ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Rubi [A] time = 0.0607269, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1318, 220, 1699, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) - ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Rule 1318

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx = -\left(\frac{1}{2}\int \frac{1}{\sqrt{1+x^4}}\,dx\right) + \frac{1}{2}\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx$$

$$= -\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{1+x^4}} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} - \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

Mathematica [C] time = 0.0891999, size = 36, normalized size = 0.51

$$\sqrt[4]{-1}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right),-1\right) - \Pi\left(i;\sin^{-1}\left((-1)^{3/4}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] (-1)^(1/4)*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - EllipticPi[I, ArcSin[(
-1)^(3/4)*x], -1])
```

Maple [C] time = 0.017, size = 112, normalized size = 1.6

$$-\frac{\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right),i\right)}{\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}}\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{1}{\sqrt{x^4+1}} - (-1)^{\frac{3}{4}}\text{EllipticPi}\left(\sqrt[4]{-1}x,-i,\sqrt{-i}-(-1)^{\frac{3}{4}}\right)\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{}{\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(x^4+1)^(1/2),x)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+
I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{x^4+1}(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^4+1}x^2}{x^6-x^4+x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^4 + 1)*x^2/(x^6 - x^4 + x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\sqrt{x^4+1}(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^2/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

3.261 $\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=99

$$\frac{\sqrt{x^2+1}\sqrt{1-x^2}\text{EllipticF}\left(\sin^{-1}(x),-1\right)}{\sqrt{1-x^4}}-\frac{x\left(1-x^2\right)}{2\sqrt{1-x^4}}-\frac{\sqrt{x^2+1}\sqrt{1-x^2}E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{1-x^4}}$$

```
[Out] -(x*(1 - x^2))/(2*Sqrt[1 - x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[A
rcSin[x], -1])/(2*Sqrt[1 - x^4]) + (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticF[A
rcSin[x], -1])/Sqrt[1 - x^4]
```

Rubi [A] time = 0.0548223, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1256, 471, 423, 424, 248, 221}

$$-\frac{x\left(1-x^2\right)}{2\sqrt{1-x^4}}+\frac{\sqrt{x^2+1}\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{1-x^4}}-\frac{\sqrt{x^2+1}\sqrt{1-x^2}E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x^2)*Sqrt[1 - x^4]),x]
```


```
[Out] -(x*(1 - x^2))/(2*Sqrt[1 - x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[A
rcSin[x], -1])/(2*Sqrt[1 - x^4]) + (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticF[A
rcSin[x], -1])/Sqrt[1 - x^4]
```


Rule 1256

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&& !IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 248

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx &= \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{x^2}{\sqrt{1-x^2}(1+x^2)^{3/2}}\,dx}{\sqrt{1-x^4}} \\
&= -\frac{x(1-x^2)}{2\sqrt{1-x^4}} + \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx}{2\sqrt{1-x^4}} \\
&= -\frac{x(1-x^2)}{2\sqrt{1-x^4}} - \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{1-x^4}} + \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}}\,dx}{\sqrt{1-x^4}} \\
&= -\frac{x(1-x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{1-x^4}} + \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{1-x^4}}\,dx}{\sqrt{1-x^4}} \\
&= -\frac{x(1-x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{1-x^4}} + \frac{\sqrt{1-x^2}\sqrt{1+x^2}F\left(\sin^{-1}(x)\middle|-1\right)}{\sqrt{1-x^4}}
\end{aligned}$$

Mathematica [A] time = 0.106792, size = 46, normalized size = 0.46

$$\frac{1}{2}\left(2\text{EllipticF}\left(\sin^{-1}(x),-1\right)+\frac{x^3}{\sqrt{1-x^4}}-\frac{x}{\sqrt{1-x^4}}-E\left(\sin^{-1}(x)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[1 - x^4]),x]
```

```
[Out] (-(x/Sqrt[1 - x^4]) + x^3/Sqrt[1 - x^4] - EllipticE[ArcSin[x], -1] + 2*Elli
pticF[ArcSin[x], -1])/2
```

Maple [A] time = 0.017, size = 96, normalized size = 1.

$$\frac{\text{EllipticF}(x,i)}{2}\sqrt{-x^2+1}\sqrt{x^2+1}\frac{1}{\sqrt{-x^4+1}}-\frac{(-x^2+1)x}{2}\frac{1}{\sqrt{(-x^2+1)(x^2+1)}}+\frac{\text{EllipticF}(x,i)-\text{EllipticE}(x,i)}{2}\sqrt{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)/(-x^4+1)^(1/2),x)
```

```
[Out] 1/2*EllipticF(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)-1/2*(-x^2+1)
*x/((-x^2+1)*(x^2+1))^(1/2)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)
*(EllipticF(x,I)-EllipticE(x,I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-x^4+1}x^2}{x^6+x^4-x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 1)*x^2/(x^6 + x^4 - x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(x-1)(x+1)(x^2+1)}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)/(-x**4+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^4+1}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

3.262 $\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=61

$$\frac{x(x^2+1)}{2\sqrt{1-x^4}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{1-x^4}}$$

```
[Out] (x*(1 + x^2))/(2*Sqrt[1 - x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[Ar
cSin[x], -1])/(2*Sqrt[1 - x^4])
```

Rubi [A] time = 0.0406191, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1256, 471, 424}

$$\frac{x(x^2+1)}{2\sqrt{1-x^4}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*Sqrt[1 - x^4]),x]
```

```
[Out] (x*(1 + x^2))/(2*Sqrt[1 - x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[Ar
cSin[x], -1])/(2*Sqrt[1 - x^4])
```

Rule 1256

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx = \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{x^2}{(1-x^2)^{3/2}\sqrt{1+x^2}}\,dx}{\sqrt{1-x^4}}$$
$$= \frac{x(1+x^2)}{2\sqrt{1-x^4}} - \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{1-x^4}}$$
$$= \frac{x(1+x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{1-x^4}}$$

Mathematica [A] time = 0.0812855, size = 37, normalized size = 0.61

$$\frac{x^3-\sqrt{1-x^4}E\left(\sin^{-1}(x)\middle|-1\right)+x}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[1 - x^4]),x]
```

```
[Out] (x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1])/(2*Sqrt[1 - x^4])
```

Maple [B] time = 0.024, size = 143, normalized size = 2.3

$$-\frac{\mathrm{EllipticF}(x,i)}{2}\sqrt{-x^2+1}\sqrt{x^2+1}\frac{1}{\sqrt{-x^4+1}}-\frac{-x^3-x^2-x-1}{4}\frac{1}{\sqrt{(x-1)\left(-x^3-x^2-x-1\right)}}+\frac{\mathrm{EllipticF}(x,i)-\mathrm{Elliptic}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(-x^4+1)^(1/2),x)
```

```
[Out] -1/2*EllipticF(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)-1/4*(-x^3-x
^2-x-1)/((x-1)*(-x^3-x^2-x-1))^(1/2)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4
+1)^(1/2)*(EllipticF(x,I)-EllipticE(x,I))-1/4*(-x^3+x^2-x+1)/((x+1)*(-x^3+x
^2-x+1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/(sqrt(-x^4 + 1)*(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^4+1}x^2}{x^6-x^4-x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 1)*x^2/(x^6 - x^4 - x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^2\sqrt{1-x^4}-\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)/(-x**4+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(1 - x**4) - sqrt(1 - x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\sqrt{-x^4+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^2/(sqrt(-x^4 + 1)*(x^2 - 1)), x)
```

3.263 $\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=113

$$\frac{\sqrt{x^2-1}\sqrt{x^2+1}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\frac{x\left(1-x^2\right)}{2\sqrt{x^4-1}}-\frac{\sqrt{x^2+1}\sqrt{1-x^2}E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{x^4-1}}$$

```
[Out] -(x*(1 - x^2))/(2*Sqrt[-1 + x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[
ArcSin[x], -1])/(2*Sqrt[-1 + x^4]) + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*Elliptic
F[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt[2]*Sqrt[-1 + x^4])
```

Rubi [A] time = 0.0647771, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1256, 471, 423, 427, 424, 253, 222}

$$-\frac{x\left(1-x^2\right)}{2\sqrt{x^4-1}}+\frac{\sqrt{x^2-1}\sqrt{x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^4-1}}-\frac{\sqrt{x^2+1}\sqrt{1-x^2}E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x^2)*Sqrt[-1 + x^4]),x]
```

```
[Out] -(x*(1 - x^2))/(2*Sqrt[-1 + x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[
ArcSin[x], -1])/(2*Sqrt[-1 + x^4]) + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*Elliptic
F[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt[2]*Sqrt[-1 + x^4])
```

Rule 1256

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 427

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d*x^2)/c
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-(a*b), 2]}, Sim
p[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)
/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(1+x^2\right)\sqrt{-1+x^4}}\,dx &= \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{x^2}{\sqrt{-1+x^2}\left(1+x^2\right)^{3/2}}\,dx}{\sqrt{-1+x^4}}\\
&= -\frac{x\left(1-x^2\right)}{2\sqrt{-1+x^4}}-\frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{-1+x^2}}{\sqrt{1+x^2}}\,dx}{2\sqrt{-1+x^4}}\\
&= -\frac{x\left(1-x^2\right)}{2\sqrt{-1+x^4}}-\frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^2}}\,dx}{2\sqrt{-1+x^4}}+\frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}\sqrt{1+x^2}}\,dx}{\sqrt{-1+x^4}}\\
&= -\frac{x\left(1-x^2\right)}{2\sqrt{-1+x^4}}-\frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{-1+x^4}}+\int \frac{1}{\sqrt{-1+x^4}}\,dx\\
&= -\frac{x\left(1-x^2\right)}{2\sqrt{-1+x^4}}-\frac{\sqrt{1-x^2}\sqrt{1+x^2}E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{-1+x^4}}+\frac{\sqrt{-1+x^2}\sqrt{1+x^2}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1+x^4}}
\end{aligned}$$

Mathematica [A] time = 0.078912, size = 54, normalized size = 0.48

$$\frac{2\sqrt{1-x^4}\text{EllipticF}\left(\sin^{-1}(x),-1\right)+x^3-\sqrt{1-x^4}E\left(\sin^{-1}(x)\middle|-1\right)-x}{2\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[-1 + x^4]),x]
```

```
[Out] (-x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1] + 2*Sqrt[1 - x^4]*Ellipt
icF[ArcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

Maple [A] time = 0.015, size = 99, normalized size = 0.9

$$-\frac{i}{2}\text{EllipticF}\left(ix,i\right)\sqrt{x^2+1}\sqrt{-x^2+1}\frac{1}{\sqrt{x^4-1}}+\frac{x\left(x^2-1\right)}{2}\frac{1}{\sqrt{\left(x^2+1\right)\left(x^2-1\right)}}+\frac{i}{2}\left(\text{EllipticF}\left(ix,i\right)-\text{EllipticE}\left(ix,i\right)\right)\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)/(x^4-1)^(1/2),x)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*EllipticF(I*x,I)+1/2*(x^2
-1)*x/((x^2+1)*(x^2-1))^(1/2)+1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1
/2)*(EllipticF(I*x,I)-EllipticE(I*x,I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^4-1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(x^4 - 1)*(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^4-1}x^2}{x^6+x^4-x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 - 1)*x^2/(x^6 + x^4 - x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{(x-1)(x+1)\left(x^2+1\right)}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^4-1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(x^4 - 1)*(x^2 + 1)), x)
```

3.264 $\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=57

$$\frac{x\left(x^2+1\right)}{2\sqrt{x^4-1}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{x^4-1}}$$

```
[Out] (x*(1 + x^2))/(2*Sqrt[-1 + x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[A
rcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

Rubi [A] time = 0.053107, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1256, 471, 426, 424}

$$\frac{x\left(x^2+1\right)}{2\sqrt{x^4-1}}-\frac{\sqrt{1-x^2}\sqrt{x^2+1}E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*Sqrt[-1 + x^4]),x]
```

```
[Out] (x*(1 + x^2))/(2*Sqrt[-1 + x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[A
rcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

Rule 1256

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c*x^2)/e
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx = \frac{\left(\sqrt{-1-x^2}\sqrt{1-x^2}\right)\int \frac{x^2}{\sqrt{-1-x^2}(1-x^2)^{3/2}}\,dx}{\sqrt{-1+x^4}}$$
$$= \frac{x(1+x^2)}{2\sqrt{-1+x^4}} + \frac{\left(\sqrt{-1-x^2}\sqrt{1-x^2}\right)\int \frac{\sqrt{-1-x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{-1+x^4}}$$
$$= \frac{x(1+x^2)}{2\sqrt{-1+x^4}} + \frac{\left((-1-x^2)\sqrt{1-x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{-1+x^4}}$$
$$= \frac{x(1+x^2)}{2\sqrt{-1+x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}E\left(\sin^{-1}(x)\middle|-1\right)}{2\sqrt{-1+x^4}}$$

Mathematica [A] time = 0.0757512, size = 35, normalized size = 0.61

$$\frac{x^3-\sqrt{1-x^4}E\left(\sin^{-1}(x)\middle|-1\right)+x}{2\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[-1 + x^4]),x]
```

```
[Out] (x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

Maple [B] time = 0.02, size = 134, normalized size = 2.4

$$\frac{i}{2}\text{EllipticF}(ix,i)\sqrt{x^2+1}\sqrt{-x^2+1}\frac{1}{\sqrt{x^4-1}} + \frac{x^3+x^2+x+1}{4}\frac{1}{\sqrt{(x-1)(x^3+x^2+x+1)}} + \frac{i}{2}(\text{EllipticF}(ix,i) - \text{Elli}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(x^4-1)^(1/2),x)
```

```
[Out] 1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*EllipticF(I*x,I)+1/4*(x^3+
x^2+x+1)/((x-1)*(x^3+x^2+x+1))^(1/2)+1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^
4-1)^(1/2)*(EllipticF(I*x,I)-EllipticE(I*x,I))+1/4*(x^3-x^2+x-1)/((x+1)*(x^
3-x^2+x-1))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{x^4-1}(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/(sqrt(x^4 - 1)*(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{x^4-1}x^2}{x^6-x^4-x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(x^4 - 1)*x^2/(x^6 - x^4 - x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^2\sqrt{x^4-1}-\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)/(x**4-1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 - 1) - sqrt(x**4 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2}{\sqrt{x^4-1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^2/(sqrt(x^4 - 1)*(x^2 - 1)), x)
```

3.265 $\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx$

Optimal. Leaf size=74

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{2}\right)}{4\sqrt{-x^4-1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}$$

```
[Out] -ArcTanh[(Sqrt[2]*x)/Sqrt[-1 - x^4]]/(2*Sqrt[2]) + ((1 + x^2)*Sqrt[(1 + x^4
)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[-1 - x^4])
```

Rubi [A] time = 0.0635063, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1318, 220, 1699, 206}

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-x^4-1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x^2)*Sqrt[-1 - x^4]),x]
```

```
[Out] -ArcTanh[(Sqrt[2]*x)/Sqrt[-1 - x^4]]/(2*Sqrt[2]) + ((1 + x^2)*Sqrt[(1 + x^4
)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[-1 - x^4])
```

Rule 1318

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(1+x^2\right)\sqrt{-1-x^4}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{-1-x^4}}\,dx - \frac{1}{2}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{-1-x^4}}\,dx$$

$$= \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{-1-x^4}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{-1-x^4}}\right)$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}} + \frac{\left(1+x^2\right)\sqrt{\frac{1+x^4}{\left(1+x^2\right)^2}}F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{-1-x^4}}$$

Mathematica [C] time = 0.092931, size = 60, normalized size = 0.81

$$\frac{\sqrt[4]{-1}\sqrt{x^4+1}\left(\Pi\left(-i;i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)-\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right),-1\right)\right)}{\sqrt{-x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[-1 - x^4]),x]
```

```
[Out] ((-1)^(1/4)*Sqrt[1 + x^4]*(-EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + Ellipt
icPi[-I, I*ArcSinh[(-1)^(1/4)*x], -1]))/Sqrt[-1 - x^4]
```

Maple [C] time = 0.033, size = 168, normalized size = 2.3

$$\frac{\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i}{2}\sqrt{2}\right)x, i\right)}{\frac{\sqrt{2}}{2}-\frac{i}{2}\sqrt{2}}\sqrt{1+ix^2}\sqrt{1-ix^2}\frac{1}{\sqrt{-x^4-1}} - \frac{i}{2}\sqrt{-i}\sqrt{1+ix^2}\sqrt{1-ix^2}\text{EllipticPi}\left(\sqrt{-i}x, -i, \frac{\sqrt[4]{-1}}{\sqrt{-i}}\right)\frac{1}{\sqrt{-x^4-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)/(-x^4-1)^(1/2),x)
```

```
[Out] 1/(1/2*2^(1/2)-1/2*I*2^(1/2))*(1+I*x^2)^(1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/2
)*EllipticF((1/2*2^(1/2)-1/2*I*2^(1/2))*x,I)-1/2*I*(-I)^(1/2)*(1+I*x^2)^(1/
2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/2)*EllipticPi((-I)^(1/2)*x,-I,(-1)^(1/4)/(-I
)^(1/2))-1/2/(-I)^(1/2)*(1+I*x^2)^(1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/2)*Elli
pticPi((-I)^(1/2)*x,-I,(-1)^(1/4)/(-I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x^4-1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-x^4 - 1)*(x^2 + 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}\sqrt{2}\log\left(\frac{\sqrt{2}x+\sqrt{-x^4-1}}{x^2+1}\right)+\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}x-\sqrt{-x^4-1}}{x^2+1}\right)+\text{integral}\left(-\frac{\sqrt{-x^4-1}}{2\left(x^4+1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*log((sqrt(2)*x + sqrt(-x^4 - 1))/(x^2 + 1)) + 1/8*sqrt(2)*log(
-(sqrt(2)*x - sqrt(-x^4 - 1))/(x^2 + 1)) + integral(-1/2*sqrt(-x^4 - 1)/(x^
4 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left(x^2+1\right)\sqrt{-x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)/(-x**4-1)**(1/2),x)
```

```
[Out] Integral(x**2/((x**2 + 1)*sqrt(-x**4 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{-x^4-1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-x^4 - 1)*(x^2 + 1)), x)
```

3.266 $\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}(x),\frac{1}{2}\right)}{4\sqrt{-x^4-1}}$$

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-1 - x^4]]/(2*Sqrt[2]) - ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[-1 - x^4])
```

Rubi [A] time = 0.0647003, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1318, 220, 1699, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}-\frac{\left(x^2+1\right)\sqrt{\frac{x^4+1}{(x^2+1)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*Sqrt[-1 - x^4]),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-1 - x^4]]/(2*Sqrt[2]) - ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[-1 - x^4])
```

Rule 1318

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = -\left(\frac{1}{2}\int \frac{1}{\sqrt{-1-x^4}}\,dx\right) + \frac{1}{2}\int \frac{1+x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx$$
$$= -\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-1-x^4}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{-1-x^4}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}} - \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-1-x^4}}$$

Mathematica [C] time = 0.0864783, size = 56, normalized size = 0.76

$$\frac{\sqrt[4]{-1}\sqrt{x^4+1}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right),-1\right)-\Pi\left(i;\sin^{-1}\left((-1)^{3/4}x\right)\middle|-1\right)\right)}{\sqrt{-x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[-1 - x^4]),x]
```

```
[Out] ((-1)^(1/4)*Sqrt[1 + x^4]*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - Ellipti
cPi[I, ArcSin[(-1)^(3/4)*x], -1]))/Sqrt[-1 - x^4]
```

Maple [C] time = 0.014, size = 115, normalized size = 1.6

$$-\frac{\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i}{2}\sqrt{2}\right)x, i\right)}{\frac{\sqrt{2}}{2}-\frac{i}{2}\sqrt{2}}\sqrt{1+ix^2}\sqrt{1-ix^2}\frac{1}{\sqrt{-x^4-1}} + \frac{1}{\sqrt{-i}}\sqrt{1+ix^2}\sqrt{1-ix^2}\text{EllipticPi}\left(\sqrt{-i}x, i, \frac{\sqrt[4]{-1}}{\sqrt{-i}}\right)\frac{1}{\sqrt{-x^4-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(-x^4-1)^(1/2),x)
```

```
[Out] -1/(1/2*2^(1/2)-1/2*I*2^(1/2))*(1+I*x^2)^(1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/
2)*EllipticF((1/2*2^(1/2)-1/2*I*2^(1/2))*x,I)+1/(-I)^(1/2)*(1+I*x^2)^(1/2)*
(1-I*x^2)^(1/2)/(-x^4-1)^(1/2)*EllipticPi((-I)^(1/2)*x,I,(-1)^(1/4)/(-I)^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{\sqrt{-x^4-1}(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/(sqrt(-x^4 - 1)*(x^2 - 1)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}i\sqrt{2}\log\left(\frac{i\sqrt{2}x+\sqrt{-x^4-1}}{x^2-1}\right)+\frac{1}{8}i\sqrt{2}\log\left(\frac{-i\sqrt{2}x+\sqrt{-x^4-1}}{x^2-1}\right)+\text{integral}\left(\frac{\sqrt{-x^4-1}}{2\left(x^4+1\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*I*sqrt(2)*log((I*sqrt(2)*x + sqrt(-x^4 - 1))/(x^2 - 1)) + 1/8*I*sqrt(2
)*log((-I*sqrt(2)*x + sqrt(-x^4 - 1))/(x^2 - 1)) + integral(1/2*sqrt(-x^4 -
 1)/(x^4 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2}{x^2\sqrt{-x^4-1}-\sqrt{-x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)/(-x**4-1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(-x**4 - 1) - sqrt(-x**4 - 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int-\frac{x^2}{\sqrt{-x^4-1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^2/(sqrt(-x^4 - 1)*(x^2 - 1)), x)
```

3.267 $\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=243

$$\frac{c^2\sqrt{a^2+2abx^2+b^2x^4}(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}\left(a+bx^2\right)}-\frac{cx\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(bc-2ad)}{16d^2\left(a+bx^2\right)}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{6d\left(a+b\right.}$$

```
[Out] -(c*(b*c - 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*d^
2*(a + b*x^2)) - ((b*c - 2*a*d)*x^3*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(8*d*(a + b*x^2)) + (b*x^3*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(6*d*(a + b*x^2)) + (c^2*(b*c - 2*a*d)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(5/2)*(a + b*x^2))
```

Rubi [A] time = 0.130078, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1250, 459, 279, 321, 217, 206}

$$\frac{c^2\sqrt{a^2+2abx^2+b^2x^4}(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}\left(a+bx^2\right)}-\frac{cx\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(bc-2ad)}{16d^2\left(a+bx^2\right)}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{6d\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(c*(b*c - 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*d^
2*(a + b*x^2)) - ((b*c - 2*a*d)*x^3*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(8*d*(a + b*x^2)) + (b*x^3*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(6*d*(a + b*x^2)) + (c^2*(b*c - 2*a*d)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(5/2)*(a + b*x^2))
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int x^2\left(ab+b^2x^2\right)\sqrt{c+dx^2}\,dx}{ab+b^2x^2}\\
&= \frac{bx^3\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{6d\left(a+bx^2\right)} - \frac{\left(b(bc-2ad)\sqrt{a^2+2abx^2+b^2x^4}\right)\int x^2\sqrt{c}}{2d\left(ab+b^2x^2\right)}\\
&= -\frac{(bc-2ad)x^3\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\left(a+bx^2\right)} + \frac{bx^3\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2}}{6d\left(a+bx^2\right)}\\
&= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{16d^2\left(a+bx^2\right)} - \frac{(bc-2ad)x^3\sqrt{c+dx^2}\sqrt{a^2+2abx^2}}{8d\left(a+bx^2\right)}\\
&= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{16d^2\left(a+bx^2\right)} - \frac{(bc-2ad)x^3\sqrt{c+dx^2}\sqrt{a^2+2abx^2}}{8d\left(a+bx^2\right)}\\
&= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{16d^2\left(a+bx^2\right)} - \frac{(bc-2ad)x^3\sqrt{c+dx^2}\sqrt{a^2+2abx^2}}{8d\left(a+bx^2\right)}
\end{aligned}$$

Mathematica [A] time = 0.176697, size = 142, normalized size = 0.58

$$\frac{\sqrt{\left(a+bx^2\right)^2}\sqrt{c+dx^2}\left(\sqrt{d}x\sqrt{\frac{dx^2}{c}+1}\left(6ad\left(c+2dx^2\right)+b\left(-3c^2+2cdx^2+8d^2x^4\right)\right)+3c^{3/2}(bc-2ad)\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right)}{48d^{5/2}\left(a+bx^2\right)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*Sqrt[c + d*x^2]*(Sqrt[d]*x*Sqrt[1 + (d*x^2)/c]*(6*a*d*
(c + 2*d*x^2) + b*(-3*c^2 + 2*c*d*x^2 + 8*d^2*x^4)) + 3*c^(3/2)*(b*c - 2*a*
d)*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(48*d^(5/2)*(a + b*x^2)*Sqrt[1 + (d*x^2)/
c])
```

Maple [A] time = 0.012, size = 159, normalized size = 0.7

$$\frac{1}{48\,bx^2+48\,a}\sqrt{\left(bx^2+a\right)^2}\left(8\,d^{3/2}\left(dx^2+c\right)^{3/2}x^3b+12\,d^{3/2}\left(dx^2+c\right)^{3/2}xa-6\,\sqrt{d}\left(dx^2+c\right)^{3/2}xbc-6\,d^{3/2}\sqrt{dx^2+c}xac$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/48*((b*x^2+a)^2)^(1/2)*(8*d^(3/2)*(d*x^2+c)^(3/2)*x^3*b+12*d^(3/2)*(d*x^2
+c)^(3/2)*x*a-6*d^(1/2)*(d*x^2+c)^(3/2)*x*b*c-6*d^(3/2)*(d*x^2+c)^(1/2)*x*a
*c+3*d^(1/2)*(d*x^2+c)^(1/2)*x*b*c^2-6*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c^2*
d+3*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^3)/(b*x^2+a)/d^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{dx^2+c}\sqrt{\left(bx^2+a\right)^2}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)*x^2, x)
```

Fricas [A] time = 1.94447, size = 475, normalized size = 1.95

$$\left[-\frac{3\left(bc^3-2\,ac^2d\right)\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\left(8\,bd^3x^5+2\left(bcd^2+6\,ad^3\right)x^3-3\left(bc^2d-2\,acd^2\right)x\right)\sqrt{dx^2}}{96\,d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt
(d)*x - c) - 2*(8*b*d^3*x^5 + 2*(b*c*d^2 + 6*a*d^3)*x^3 - 3*(b*c^2*d - 2*a*
c*d^2)*x)*sqrt(d*x^2 + c))/d^3, -1/48*(3*(b*c^3 - 2*a*c^2*d)*sqrt(-d)*arcta
n(sqrt(-d)*x/sqrt(d*x^2 + c)) - (8*b*d^3*x^5 + 2*(b*c*d^2 + 6*a*d^3)*x^3 -
3*(b*c^2*d - 2*a*c*d^2)*x)*sqrt(d*x^2 + c))/d^3]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10844, size = 211, normalized size = 0.87

$$\frac{1}{48}\left(2\left(4\,bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{bcd^3\mathrm{sgn}\left(bx^2+a\right)+6\,ad^4\mathrm{sgn}\left(bx^2+a\right)}{d^4}\right)x^2-\frac{3\left(bc^2d^2\mathrm{sgn}\left(bx^2+a\right)-2\,acd^3\mathrm{sgn}\left(bx^2+a\right)\right)}{d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*x^2*sgn(b*x^2 + a) + (b*c*d^3*sgn(b*x^2 + a) + 6*a*d^4*sgn(b*x
^2 + a))/d^4)*x^2 - 3*(b*c^2*d^2*sgn(b*x^2 + a) - 2*a*c*d^3*sgn(b*x^2 + a))
/d^4)*sqrt(d*x^2 + c)*x - 1/16*(b*c^3*sgn(b*x^2 + a) - 2*a*c^2*d*sgn(b*x^2
+ a))*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

3.268 $\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=108

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{5/2}}{5d^2\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^2\left(a+bx^2\right)}$$

```
[Out] -((b*c - a*d)*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^2*(a
+ b*x^2)) + (b*(c + d*x^2)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^2*(a
 + b*x^2))
```

Rubi [A] time = 0.101093, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1247, 646, 43}

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{5/2}}{5d^2\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}(bc-ad)}{3d^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((b*c - a*d)*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^2*(a
+ b*x^2)) + (b*(c + d*x^2)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^2*(a
 + b*x^2))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \sqrt{c+dx}\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(ab+b^2x\right)\sqrt{c+dx}\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \left(-\frac{b(bc-ad)\sqrt{c+dx}}{d}+\frac{b^2(c+dx)^{3/2}}{d}\right)dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}$$
$$= -\frac{(bc-ad)\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^2\left(a+bx^2\right)}+\frac{b\left(c+dx^2\right)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^2\left(a+bx^2\right)}$$

Mathematica [A] time = 0.0303362, size = 56, normalized size = 0.52

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(c+dx^2\right)^{3/2}\left(5ad-2bc+3bdx^2\right)}{15d^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(c + d*x^2)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x^2))/(15*d^
2*(a + b*x^2))
```

Maple [A] time = 0.004, size = 51, normalized size = 0.5

$$\frac{3\,bx^2d+5\,ad-2\,bc}{15\,d^2\left(bx^2+a\right)}\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/15*(d*x^2+c)^(3/2)*(3*b*d*x^2+5*a*d-2*b*c)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2
+a)
```

Maxima [A] time = 0.96501, size = 68, normalized size = 0.63

$$\frac{\left(3\,bd^2x^4-2\,bc^2+5\,acd+\left(bcd+5\,ad^2\right)x^2\right)\sqrt{dx^2+c}}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*b*d^2*x^4 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x^2)*sqrt(d*x^2 +
 c)/d^2
```

Fricas [A] time = 1.80305, size = 113, normalized size = 1.05

$$\frac{\left(3\,bd^{2}x^{4}-2\,bc^{2}+5\,acd+\left(bcd+5\,ad^{2}\right)x^{2}\right)\sqrt{dx^{2}+c}}{15\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*d^2*x^4 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x^2)*sqrt(d*x^2 +
 c)/d^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09989, size = 85, normalized size = 0.79

$$\frac{5\left(dx^{2}+c\right)^{\frac{3}{2}}a\mathrm{sgn}\left(bx^{2}+a\right)+\frac{\left(3\left(dx^{2}+c\right)^{\frac{5}{2}}-5\left(dx^{2}+c\right)^{\frac{3}{2}}c\right)b\mathrm{sgn}\left(bx^{2}+a\right)}{d}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(5*(d*x^2 + c)^(3/2)*a*sgn(b*x^2 + a) + (3*(d*x^2 + c)^(5/2) - 5*(d*x^
2 + c)^(3/2)*c)*b*sgn(b*x^2 + a)/d)/d
```

3.269 $\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=178

$$-\frac{c\sqrt{a^2+2abx^2+b^2x^4}(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}\left(a+bx^2\right)}+\frac{bx\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{4d\left(a+bx^2\right)}-\frac{x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}}{8d\left(a+bx^2\right)}$$

```
[Out] -((b*c - 4*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*d*(a
+ b*x^2)) + (b*x*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*d*(a
 + b*x^2)) - (c*(b*c - 4*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt
[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)*(a + b*x^2))
```

Rubi [A] time = 0.0762214, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1148, 388, 195, 217, 206}

$$-\frac{c\sqrt{a^2+2abx^2+b^2x^4}(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}\left(a+bx^2\right)}+\frac{bx\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{4d\left(a+bx^2\right)}-\frac{x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}}{8d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```


```
[Out] -((b*c - 4*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*d*(a
+ b*x^2)) + (b*x*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*d*(a
 + b*x^2)) - (c*(b*c - 4*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt
[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)*(a + b*x^2))
```


Rule 1148

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \left(ab+b^2x^2\right)\sqrt{c+dx^2}\,dx}{ab+b^2x^2}$$

$$= \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d\left(a+bx^2\right)} - \frac{\left(b(bc-4ad)\sqrt{a^2+2abx^2+b^2x^4}\right)\int \sqrt{c+}}{4d\left(ab+b^2x^2\right)}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^{}}}{4d\left(a+bx^2\right)}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^{}}}{4d\left(a+bx^2\right)}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{8d\left(a+bx^2\right)} + \frac{bx\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^{}}}{4d\left(a+bx^2\right)}$$

Mathematica [A] time = 0.111615, size = 121, normalized size = 0.68

$$\frac{\sqrt{\left(a+bx^2\right)^2}\sqrt{c+dx^2}\left(\sqrt{d}x\sqrt{\frac{dx^2}{c}+1}\left(4ad+b\left(c+2dx^2\right)\right)-\sqrt{c}(bc-4ad)\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right)}{8d^{3/2}\left(a+bx^2\right)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*Sqrt[c + d*x^2]*(Sqrt[d]*x*Sqrt[1 + (d*x^2)/c]*(4*a*d
+ b*(c + 2*d*x^2)) - Sqrt[c]*(b*c - 4*a*d)*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(
8*d^(3/2)*(a + b*x^2)*Sqrt[1 + (d*x^2)/c])
```

Maple [A] time = 0.007, size = 119, normalized size = 0.7

$$\frac{1}{8\,bx^2+8\,a}\sqrt{\left(bx^2+a\right)^2}\left(2\,\sqrt{d}\left(dx^2+c\right)^{3/2}xb+4\,d^{3/2}\sqrt{dx^2+c}xa-\sqrt{d}\sqrt{dx^2+c}xbc+4\,\ln\left(\sqrt{d}x+\sqrt{dx^2+c}\right)acd-\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/8*((b*x^2+a)^2)^(1/2)*(2*d^(1/2)*(d*x^2+c)^(3/2)*x*b+4*d^(3/2)*(d*x^2+c)^
(1/2)*x*a-d^(1/2)*(d*x^2+c)^(1/2)*x*b*c+4*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c
*d-ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^2)/(b*x^2+a)/d^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx^2+c}\sqrt{\left(bx^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2), x)
```

Fricas [A] time = 1.82027, size = 370, normalized size = 2.08

$$\left[-\frac{\left(bc^2-4\,acd\right)\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)-2\left(2\,bd^2x^3+\left(bcd+4\,ad^2\right)x\right)\sqrt{dx^2+c}}{16\,d^2},\ \frac{\left(bc^2-4\,acd\right)\sqrt{-d}\arctan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b*c^2 - 4*a*c*d)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*
x - c) - 2*(2*b*d^2*x^3 + (b*c*d + 4*a*d^2)*x)*sqrt(d*x^2 + c))/d^2, 1/8*((
b*c^2 - 4*a*c*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b*d^2*x^3
 + (b*c*d + 4*a*d^2)*x)*sqrt(d*x^2 + c))/d^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c+dx^2}\sqrt{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2), x)
```

Giac [A] time = 1.1184, size = 147, normalized size = 0.83

$$\frac{1}{8}\left(2\,bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{bcd\mathrm{sgn}\left(bx^2+a\right)+4\,ad^2\mathrm{sgn}\left(bx^2+a\right)}{d^2}\right)\sqrt{dx^2+c}x+\frac{\left(bc^2\mathrm{sgn}\left(bx^2+a\right)-4\,acd\mathrm{sgn}\left(bx^2+a\right)\right)\mathrm{lo}}{8\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2*sgn(b*x^2 + a) + (b*c*d*sgn(b*x^2 + a) + 4*a*d^2*sgn(b*x^2 + a
))/d^2)*sqrt(d*x^2 + c)*x + 1/8*(b*c^2*sgn(b*x^2 + a) - 4*a*c*d*sgn(b*x^2 +
 a))*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(3/2)
```

3.270 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$

Optimal. Leaf size=152

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{3d\left(a+bx^2\right)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}}{a+bx^2}-\frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}$$

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b*(c + d
*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) - (a*Sqrt[c]
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a + b*x
^2)
```

Rubi [A] time = 0.0948647, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1250, 446, 80, 50, 63, 208}

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{3d\left(a+bx^2\right)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}}{a+bx^2}-\frac{a\sqrt{c}\sqrt{a^2+2abx^2+b^2x^4}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b*(c + d
*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) - (a*Sqrt[c]
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a + b*x
^2)
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx &= \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x}\,dx}{ab+b^2x^2}\\
&= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\mathrm{Subst}\left(\int \frac{(ab+b^2x)\sqrt{c+dx}}{x}\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}\\
&= \frac{b\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} + \frac{\left(ab\sqrt{a^2+2abx^2+b^2x^4}\right)\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx,x\right.}{2\left(ab+b^2x^2\right)}\\
&= \frac{a\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} + \frac{\left(abc\sqrt{a^2+\right.}{}\\
&= \frac{a\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} + \frac{\left(abc\sqrt{a^2+\right.}{}\\
&= \frac{a\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)} - \frac{a\sqrt{c}\sqrt{a^2+2}}{}
\end{aligned}$$

Mathematica [A] time = 0.0460664, size = 83, normalized size = 0.55

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(\sqrt{c+dx^2}\left(3ad+b\left(c+dx^2\right)\right)-3a\sqrt{c}d\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{3d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[c + d*x^2]*(3*a*d + b*(c + d*x^2)) - 3*a*Sqrt[c]
*d*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(3*d*(a + b*x^2))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.5

$$-\frac{1}{\left(3\,b x^{2}+3\,a\right) d}\sqrt{\left(b x^{2}+a\right)^{2}}\left(3\,\sqrt{c}\ln\left(2\,\frac{\sqrt{c}\sqrt{d x^{2}+c}+c}{x}\right) a d-b\left(d x^{2}+c\right)^{\frac{3}{2}}-3\,\sqrt{d x^{2}+c}\, a d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x)
```

```
[Out] -1/3*((b*x^2+a)^2)^(1/2)*(3*c^(1/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*a*d
-b*(d*x^2+c)^(3/2)-3*(d*x^2+c)^(1/2)*a*d)/(b*x^2+a)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d x^{2}+c}\sqrt{\left(b x^{2}+a\right)^{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)/x, x)
```

Fricas [A] time = 1.83764, size = 300, normalized size = 1.97

$$\left[\frac{3\,a\sqrt{c}d\log\left(-\frac{d x^{2}-2\,\sqrt{d x^{2}+c}\sqrt{c}+2\,c}{x^{2}}\right)+2\left(b d x^{2}+b c+3\,a d\right)\sqrt{d x^{2}+c}}{6\,d}, \frac{3\,a\sqrt{-c}d\arctan\left(\frac{\sqrt{-c}}{\sqrt{d x^{2}+c}}\right)+\left(b d x^{2}+b c+3\,a d\right)\sqrt{dx}}{3\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2
*(b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d, 1/3*(3*a*sqrt(-c)*d*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) + (b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c+d x^{2}}\sqrt{\left(a+b x^{2}\right)^{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x, x)
```

Giac [A] time = 1.1103, size = 113, normalized size = 0.74

$$\frac{ac\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)\operatorname{sgn}\left(bx^2+a\right)}{\sqrt{-c}}+\frac{\left(dx^2+c\right)^{\frac{3}{2}}bd^2\operatorname{sgn}\left(bx^2+a\right)+3\sqrt{dx^2+c}ad^3\operatorname{sgn}\left(bx^2+a\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))*sgn(b*x^2 + a)/sqrt(-c) + 1/3*((d*x^2
+ c)^(3/2)*b*d^2*sgn(b*x^2 + a) + 3*sqrt(d*x^2 + c)*a*d^3*sgn(b*x^2 + a))/d
^3
```

3.271 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$

Optimal. Leaf size=177

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{cx\left(a+bx^2\right)}+\frac{x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(2ad+bc)}{2c\left(a+bx^2\right)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}(2ad+bc)\tanh}{2\sqrt{d}\left(a+bx^2\right)}$$

```
[Out] ((b*c + 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*(a +
 b*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(c*x*(a +
b*x^2)) + ((b*c + 2*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt[d]*x
)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a + b*x^2))
```

Rubi [A] time = 0.0912602, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {1250, 453, 195, 217, 206}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{cx\left(a+bx^2\right)}+\frac{x\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(2ad+bc)}{2c\left(a+bx^2\right)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}(2ad+bc)\tanh}{2\sqrt{d}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] ((b*c + 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*(a +
 b*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(c*x*(a +
b*x^2)) + ((b*c + 2*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt[d]*x
)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a + b*x^2))
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x^2}\,dx}{ab+b^2x^2}$$

$$= -\frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\left(a+bx^2\right)} + -\frac{\left(\left(-b^2c-2abd\right)\sqrt{a^2+2abx^2+b^2x^4}\right)\int \sqrt{c}}{c\left(ab+b^2x^2\right)}$$

$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\left(a+bx^2\right)} + -$$

$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\left(a+bx^2\right)} + -$$

$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\left(a+bx^2\right)} + -\frac{(b}{}$$

Mathematica [A] time = 0.119107, size = 122, normalized size = 0.69

$$\frac{\sqrt{\left(a+bx^2\right)^2}\sqrt{c+dx^2}\left(\sqrt{c}\sqrt{d}\left(bx^2-2a\right)\sqrt{\frac{dx^2}{c}+1}+x(2ad+bc)\sinh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right)}{2\sqrt{c}\sqrt{d}x\left(a+bx^2\right)\sqrt{\frac{dx^2}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*Sqrt[c + d*x^2]*(Sqrt[c]*Sqrt[d]*(-2*a + b*x^2)*Sqrt[1
 + (d*x^2)/c] + (b*c + 2*a*d)*x*ArcSinh[(Sqrt[d]*x)/Sqrt[c]]))/(2*Sqrt[c]*S
qrt[d]*x*(a + b*x^2)*Sqrt[1 + (d*x^2)/c])
```

Maple [A] time = 0.01, size = 128, normalized size = 0.7

$$\frac{1}{\left(2\,bx^2+2\,a\right)cx}\sqrt{\left(bx^2+a\right)^2}\left(2\,d^{3/2}\sqrt{dx^2+c}x^2a+\sqrt{d}\sqrt{dx^2+c}x^2bc-2\,\sqrt{d}\left(dx^2+c\right)^{3/2}a+2\,\ln\left(\sqrt{d}x+\sqrt{dx^2+c}\right)xacd\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x)
```

```
[Out] 1/2*((b*x^2+a)^2)^(1/2)*(2*d^(3/2)*(d*x^2+c)^(1/2)*x^2*a+d^(1/2)*(d*x^2+c)^
(1/2)*x^2*b*c-2*d^(1/2)*(d*x^2+c)^(3/2)*a+2*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x
```

```
*a*c*d+ln(d^(1/2)*x+(d*x^2+c)^(1/2))*x*b*c^2)/(b*x^2+a)/c/x/d^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx^2+c}\sqrt{\left(bx^2+a\right)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)/x^2, x)
```

Fricas [A] time = 1.83253, size = 320, normalized size = 1.81

$$\left[\frac{(bc+2\,ad)\sqrt{d}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\left(bdx^2-2\,ad\right)\sqrt{dx^2+c}}{4\,dx}, -\frac{(bc+2\,ad)\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(\right.}{2\,dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) + 2*(b*d*x^2 - 2*a*d)*sqrt(d*x^2 + c))/(d*x), -1/2*((b*c + 2*a*d)*sqrt(-
d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (b*d*x^2 - 2*a*d)*sqrt(d*x^2 + c)
)/(d*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10584, size = 157, normalized size = 0.89

$$\frac{1}{2}\sqrt{dx^2+c}bx\mathrm{sgn}\left(bx^2+a\right)+\frac{2\,ac\sqrt{d}\mathrm{sgn}\left(bx^2+a\right)}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c}-\frac{\left(bc\sqrt{d}\mathrm{sgn}\left(bx^2+a\right)+2\,ad^{\frac{3}{2}}\mathrm{sgn}\left(bx^2+a\right)\right)\log\left(\left(\sqrt{d}x-\sqrt{dx^2}\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b*x*sgn(b*x^2 + a) + 2*a*c*sqrt(d)*sgn(b*x^2 + a)/((sqr
t(d)*x - sqrt(d*x^2 + c))^2 - c) - 1/4*(b*c*sqrt(d)*sgn(b*x^2 + a) + 2*a*d^
(3/2)*sgn(b*x^2 + a))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d
```

3.272 $\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$

Optimal. Leaf size=177

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{2cx^2\left(a+bx^2\right)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(ad+2bc)}{2c\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}(ad+2bc)\tanh^{-}}{2\sqrt{c}\left(a+bx^2\right)}$$

```
[Out] ((2*b*c + a*d)*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*(a + b
*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*x^2*(a
+ b*x^2)) - ((2*b*c + a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x^2))
```

Rubi [A] time = 0.118521, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1250, 446, 78, 50, 63, 208}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(c+dx^2\right)^{3/2}}{2cx^2\left(a+bx^2\right)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\sqrt{c+dx^2}(ad+2bc)}{2c\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}(ad+2bc)\tanh^{-}}{2\sqrt{c}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] ((2*b*c + a*d)*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*(a + b
*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*x^2*(a
+ b*x^2)) - ((2*b*c + a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x^2))
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x^3}\,dx}{ab+b^2x^2}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)\sqrt{c+dx}}{x^2}\,dx, x, x^2\right)}{2\left(ab+b^2x^2\right)}$$

$$= -\frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\left(a+bx^2\right)} + \frac{\left(\left(b^2c+\frac{abd}{2}\right)\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Subst}\left(\int\right.}{2c\left(ab+b^2x^2\right)}$$

$$= \frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\left(a+bx^2\right)} + \frac{\left(\left(\right.\right.}{}$$

$$= \frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\left(a+bx^2\right)} + \frac{\left(\left(\right.\right.}{}$$

$$= \frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c\left(a+bx^2\right)} - \frac{a\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\left(a+bx^2\right)} - \frac{(2b}{}$$

Mathematica [A] time = 0.0449428, size = 90, normalized size = 0.51

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(\sqrt{c}\left(a-2bx^2\right)\sqrt{c+dx^2}+x^2(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{2\sqrt{c}x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(Sqrt[c]*(a - 2*b*x^2)*Sqrt[c + d*x^2] + (2*b*c + a*d
)*x^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(2*Sqrt[c]*x^2*(a + b*x^2))
```

Maple [A] time = 0.01, size = 133, normalized size = 0.8

$$-\frac{1}{\left(2\,b x^{2}+2\,a\right) c x^{2}}\sqrt{\left(b x^{2}+a\right)^{2}}\left(\sqrt{c}\ln\left(2\,\frac{\sqrt{c}\sqrt{d x^{2}+c}+c}{x}\right)x^{2}ad+2\,c^{3/2}\ln\left(2\,\frac{\sqrt{c}\sqrt{d x^{2}+c}+c}{x}\right)x^{2}b-\sqrt{d x^{2}+c}x^{2}ad-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x)
```

```
[Out] -1/2*((b*x^2+a)^2)^(1/2)*(c^(1/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*x^2*a
*d+2*c^(3/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*x^2*b-(d*x^2+c)^(1/2)*x^2*
a*d-2*(d*x^2+c)^(1/2)*x^2*b*c+(d*x^2+c)^(3/2)*a)/(b*x^2+a)/c/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d x^{2}+c}\sqrt{\left(b x^{2}+a\right)^{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x^2 + a)^2)/x^3, x)
```

Fricas [A] time = 2.02218, size = 332, normalized size = 1.88

$$\left[\frac{(2\,bc+ad)\sqrt{c}x^{2}\log\left(-\frac{dx^{2}-2\,\sqrt{dx^{2}+c}\sqrt{c}+2\,c}{x^{2}}\right)+2\left(2\,bcx^{2}-ac\right)\sqrt{dx^{2}+c}}{4\,cx^{2}},\ \frac{(2\,bc+ad)\sqrt{-c}x^{2}\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^{2}+c}}\right)+\left(2\,bcx^{2}-a\right.}{2\,cx^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2
*c)/x^2) + 2*(2*b*c*x^2 - a*c)*sqrt(d*x^2 + c))/(c*x^2), 1/2*((2*b*c + a*d)
*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b*c*x^2 - a*c)*sqrt(d*x
^2 + c))/(c*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**3,x)
```

[Out] Timed out

Giac [A] time = 1.1049, size = 135, normalized size = 0.76

$$\frac{2\sqrt{dx^2+c}bd\operatorname{sgn}\left(bx^2+a\right)+\frac{\left(2\,bcd\operatorname{sgn}\left(bx^2+a\right)+ad^2\operatorname{sgn}\left(bx^2+a\right)\right)\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{dx^2+c}ad\operatorname{sgn}\left(bx^2+a\right)}{x^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(d*x^2 + c)*b*d*sgn(b*x^2 + a) + (2*b*c*d*sgn(b*x^2 + a) + a*d^2
*sgn(b*x^2 + a))*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - sqrt(d*x^2 + c
)*a*d*sgn(b*x^2 + a)/x^2)/d
```

3.273 $\int x^3 \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=78

$$\frac{1}{8}x^8\left(e(ae+2bd)+cd^2\right)+\frac{1}{6}dx^6(2ae+bd)+\frac{1}{4}ad^2x^4+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{12}ce^2x^{12}$$

```
[Out] (a*d^2*x^4)/4 + (d*(b*d + 2*a*e)*x^6)/6 + ((c*d^2 + e*(2*b*d + a*e))*x^8)/8
 + (e*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^12)/12
```

Rubi [A] time = 0.13454, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1251, 771}

$$\frac{1}{8}x^8\left(e(ae+2bd)+cd^2\right)+\frac{1}{6}dx^6(2ae+bd)+\frac{1}{4}ad^2x^4+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{12}ce^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^4)/4 + (d*(b*d + 2*a*e)*x^6)/6 + ((c*d^2 + e*(2*b*d + a*e))*x^8)/8
 + (e*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^12)/12
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int x^3\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)dx &= \frac{1}{2}\text{Subst}\left(\int x(d+ex)^2\left(a+bx+cx^2\right)dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(ad^2x+d(bd+2ae)x^2+\left(cd^2+e(2bd+ae)\right)x^3+e(2cd+be)x^4+c\right.\right.\\ &= \frac{1}{4}ad^2x^4+\frac{1}{6}d(bd+2ae)x^6+\frac{1}{8}\left(cd^2+e(2bd+ae)\right)x^8+\frac{1}{10}e(2cd+be)x^{10}+\frac{1}{12}c\end{aligned}$$

Mathematica [A] time = 0.0256374, size = 72, normalized size = 0.92

$$\frac{1}{120}x^4\left(15x^4\left(e(ae+2bd)+cd^2\right)+20dx^2(2ae+bd)+30ad^2+12ex^6(be+2cd)+10ce^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (x^4*(30*a*d^2 + 20*d*(b*d + 2*a*e)*x^2 + 15*(c*d^2 + e*(2*b*d + a*e))*x^4
+ 12*e*(2*c*d + b*e)*x^6 + 10*c*e^2*x^8))/120
```

Maple [A] time = 0.001, size = 73, normalized size = 0.9

$$\frac{ce^2x^{12}}{12} + \frac{(e^2b + 2\,dec)\,x^{10}}{10} + \frac{(ae^2 + 2\,deb + cd^2)\,x^8}{8} + \frac{(2\,dea + d^2b)\,x^6}{6} + \frac{ad^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/12*c*e^2*x^12+1/10*(b*e^2+2*c*d*e)*x^10+1/8*(a*e^2+2*b*d*e+c*d^2)*x^8+1/6
*(2*a*d*e+b*d^2)*x^6+1/4*a*d^2*x^4
```

Maxima [A] time = 0.955376, size = 97, normalized size = 1.24

$$\frac{1}{12}\,ce^2x^{12} + \frac{1}{10}\left(2\,cde + be^2\right)x^{10} + \frac{1}{8}\left(cd^2 + 2\,bde + ae^2\right)x^8 + \frac{1}{4}\,ad^2x^4 + \frac{1}{6}\left(bd^2 + 2\,ade\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*c*e^2*x^12 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/8*(c*d^2 + 2*b*d*e + a*e^
2)*x^8 + 1/4*a*d^2*x^4 + 1/6*(b*d^2 + 2*a*d*e)*x^6
```

Fricas [A] time = 1.50246, size = 200, normalized size = 2.56

$$\frac{1}{12}x^{12}e^2c + \frac{1}{5}x^{10}edc + \frac{1}{10}x^{10}e^2b + \frac{1}{8}x^8d^2c + \frac{1}{4}x^8edb + \frac{1}{8}x^8e^2a + \frac{1}{6}x^6d^2b + \frac{1}{3}x^6eda + \frac{1}{4}x^4d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*e^2*c + 1/5*x^10*e*d*c + 1/10*x^10*e^2*b + 1/8*x^8*d^2*c + 1/4*x^
8*e*d*b + 1/8*x^8*e^2*a + 1/6*x^6*d^2*b + 1/3*x^6*e*d*a + 1/4*x^4*d^2*a
```

Sympy [A] time = 0.074094, size = 76, normalized size = 0.97

$$\frac{ad^2x^4}{4} + \frac{ce^2x^{12}}{12} + x^{10}\left(\frac{be^2}{10} + \frac{cde}{5}\right) + x^8\left(\frac{ae^2}{8} + \frac{bde}{4} + \frac{cd^2}{8}\right) + x^6\left(\frac{ade}{3} + \frac{bd^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**4/4 + c*e**2*x**12/12 + x**10*(b*e**2/10 + c*d*e/5) + x**8*(a*e**
2/8 + b*d*e/4 + c*d**2/8) + x**6*(a*d*e/3 + b*d**2/6)
```

Giac [A] time = 1.09467, size = 107, normalized size = 1.37

$$\frac{1}{12}cx^{12}e^2 + \frac{1}{5}cdx^{10}e + \frac{1}{10}bx^{10}e^2 + \frac{1}{8}cd^2x^8 + \frac{1}{4}bdx^8e + \frac{1}{8}ax^8e^2 + \frac{1}{6}bd^2x^6 + \frac{1}{3}adx^6e + \frac{1}{4}ad^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*c*x^12*e^2 + 1/5*c*d*x^10*e + 1/10*b*x^10*e^2 + 1/8*c*d^2*x^8 + 1/4*b*
d*x^8*e + 1/8*a*x^8*e^2 + 1/6*b*d^2*x^6 + 1/3*a*d*x^6*e + 1/4*a*d^2*x^4
```

3.274 $\int x^2\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)dx$

Optimal. Leaf size=78

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

```
[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7
 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

Rubi [A] time = 0.0637748, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1261}

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7
 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)dx&=\int\left(ad^2x^2+d(bd+2ae)x^4+\left(cd^2+e(2bd+ae)\right)x^6+e(2cd+be)x^8+ce^2x^{10}\right)dx\\&=\frac{1}{3}ad^2x^3+\frac{1}{5}d(bd+2ae)x^5+\frac{1}{7}\left(cd^2+e(2bd+ae)\right)x^7+\frac{1}{9}e(2cd+be)x^9+\frac{1}{11}ce^2x^{11}\end{aligned}$$

Mathematica [A] time = 0.0157542, size = 78, normalized size = 1.

$$\frac{1}{7}x^7\left(ae^2+2bde+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + 2*b*d*e + a*e^2)*x^7)/7
 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

Maple [A] time = 0., size = 73, normalized size = 0.9

$$\frac{ce^2x^{11}}{11}+\frac{\left(e^2b+2\,dec\right)x^9}{9}+\frac{\left(ae^2+2\,deb+cd^2\right)x^7}{7}+\frac{\left(2\,dea+d^2b\right)x^5}{5}+\frac{ad^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/11*c*e^2*x^11+1/9*(b*e^2+2*c*d*e)*x^9+1/7*(a*e^2+2*b*d*e+c*d^2)*x^7+1/5*(
2*a*d*e+b*d^2)*x^5+1/3*a*d^2*x^3
```

Maxima [A] time = 0.950279, size = 97, normalized size = 1.24

$$\frac{1}{11}\,ce^2x^{11} + \frac{1}{9}\left(2\,cde + be^2\right)x^9 + \frac{1}{7}\left(cd^2 + 2\,bde + ae^2\right)x^7 + \frac{1}{3}\,ad^2x^3 + \frac{1}{5}\left(bd^2 + 2\,ade\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/11*c*e^2*x^11 + 1/9*(2*c*d*e + b*e^2)*x^9 + 1/7*(c*d^2 + 2*b*d*e + a*e^2)
*x^7 + 1/3*a*d^2*x^3 + 1/5*(b*d^2 + 2*a*d*e)*x^5
```

Fricas [A] time = 1.48897, size = 196, normalized size = 2.51

$$\frac{1}{11}x^{11}e^2c + \frac{2}{9}x^9edc + \frac{1}{9}x^9e^2b + \frac{1}{7}x^7d^2c + \frac{2}{7}x^7edb + \frac{1}{7}x^7e^2a + \frac{1}{5}x^5d^2b + \frac{2}{5}x^5eda + \frac{1}{3}x^3d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*e^2*c + 2/9*x^9*e*d*c + 1/9*x^9*e^2*b + 1/7*x^7*d^2*c + 2/7*x^7*e
*d*b + 1/7*x^7*e^2*a + 1/5*x^5*d^2*b + 2/5*x^5*e*d*a + 1/3*x^3*d^2*a
```

Sympy [A] time = 0.07768, size = 82, normalized size = 1.05

$$\frac{ad^2x^3}{3} + \frac{ce^2x^{11}}{11} + x^9\left(\frac{be^2}{9} + \frac{2cde}{9}\right) + x^7\left(\frac{ae^2}{7} + \frac{2bde}{7} + \frac{cd^2}{7}\right) + x^5\left(\frac{2ade}{5} + \frac{bd^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**3/3 + c*e**2*x**11/11 + x**9*(b*e**2/9 + 2*c*d*e/9) + x**7*(a*e**
2/7 + 2*b*d*e/7 + c*d**2/7) + x**5*(2*a*d*e/5 + b*d**2/5)
```

Giac [A] time = 1.09783, size = 107, normalized size = 1.37

$$\frac{1}{11}\,cx^{11}e^2 + \frac{2}{9}\,cdx^9e + \frac{1}{9}\,bx^9e^2 + \frac{1}{7}\,cd^2x^7 + \frac{2}{7}\,bdx^7e + \frac{1}{7}\,ax^7e^2 + \frac{1}{5}\,bd^2x^5 + \frac{2}{5}\,adx^5e + \frac{1}{3}\,ad^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/11*c*x^11*e^2 + 2/9*c*d*x^9*e + 1/9*b*x^9*e^2 + 1/7*c*d^2*x^7 + 2/7*b*d*x
^7*e + 1/7*a*x^7*e^2 + 1/5*b*d^2*x^5 + 2/5*a*d*x^5*e + 1/3*a*d^2*x^3
```

3.275 $\int x\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=75

$$\frac{\left(d+ex^2\right)^3\left(ae^2-bde+cd^2\right)}{6e^3}-\frac{\left(d+ex^2\right)^4(2cd-be)}{8e^3}+\frac{c\left(d+ex^2\right)^5}{10e^3}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x^2)^3)/(6*e^3) - ((2*c*d - b*e)*(d + e*x^2
)^4)/(8*e^3) + (c*(d + e*x^2)^5)/(10*e^3)
```

Rubi [A] time = 0.131203, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1247, 698}

$$\frac{\left(d+ex^2\right)^3\left(ae^2-bde+cd^2\right)}{6e^3}-\frac{\left(d+ex^2\right)^4(2cd-be)}{8e^3}+\frac{c\left(d+ex^2\right)^5}{10e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x^2)^3)/(6*e^3) - ((2*c*d - b*e)*(d + e*x^2
)^4)/(8*e^3) + (c*(d + e*x^2)^5)/(10*e^3)
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int x\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx &= \frac{1}{2}\text{Subst}\left(\int (d+ex)^2\left(a+bx+cx^2\right)\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\frac{\left(cd^2-bde+ae^2\right)(d+ex)^2}{e^2}+\frac{(-2cd+be)(d+ex)^3}{e^2}+\frac{c(d+ex)^4}{e^2}\right)\right.\\ &= \frac{\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)^3}{6e^3}-\frac{(2cd-be)\left(d+ex^2\right)^4}{8e^3}+\frac{c\left(d+ex^2\right)^5}{10e^3}\end{aligned}$$

Mathematica [A] time = 0.0215497, size = 72, normalized size = 0.96

$$\frac{1}{120}x^2\left(20x^4\left(e(ae+2bd)+cd^2\right)+30dx^2(2ae+bd)+60ad^2+15ex^6(be+2cd)+12ce^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (x^2*(60*a*d^2 + 30*d*(b*d + 2*a*e)*x^2 + 20*(c*d^2 + e*(2*b*d + a*e))*x^4
+ 15*e*(2*c*d + b*e)*x^6 + 12*c*e^2*x^8))/120
```

Maple [A] time = 0., size = 73, normalized size = 1.

$$\frac{ce^2x^{10}}{10}+\frac{(e^2b+2\,dec)\,x^8}{8}+\frac{(ae^2+2\,deb+cd^2)\,x^6}{6}+\frac{(2\,dea+d^2b)\,x^4}{4}+\frac{ad^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/10*c*e^2*x^10+1/8*(b*e^2+2*c*d*e)*x^8+1/6*(a*e^2+2*b*d*e+c*d^2)*x^6+1/4*(
2*a*d*e+b*d^2)*x^4+1/2*a*d^2*x^2
```

Maxima [A] time = 0.931862, size = 97, normalized size = 1.29

$$\frac{1}{10}\,ce^2x^{10}+\frac{1}{8}\left(2\,cde+be^2\right)x^8+\frac{1}{6}\left(cd^2+2\,bde+ae^2\right)x^6+\frac{1}{2}\,ad^2x^2+\frac{1}{4}\left(bd^2+2\,ade\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/10*c*e^2*x^10 + 1/8*(2*c*d*e + b*e^2)*x^8 + 1/6*(c*d^2 + 2*b*d*e + a*e^2)
*x^6 + 1/2*a*d^2*x^2 + 1/4*(b*d^2 + 2*a*d*e)*x^4
```

Fricas [A] time = 1.48674, size = 196, normalized size = 2.61

$$\frac{1}{10}x^{10}e^2c+\frac{1}{4}x^8edc+\frac{1}{8}x^8e^2b+\frac{1}{6}x^6d^2c+\frac{1}{3}x^6edb+\frac{1}{6}x^6e^2a+\frac{1}{4}x^4d^2b+\frac{1}{2}x^4eda+\frac{1}{2}x^2d^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e^2*c + 1/4*x^8*e*d*c + 1/8*x^8*e^2*b + 1/6*x^6*d^2*c + 1/3*x^6*e
*d*b + 1/6*x^6*e^2*a + 1/4*x^4*d^2*b + 1/2*x^4*e*d*a + 1/2*x^2*d^2*a
```

Sympy [A] time = 0.074472, size = 76, normalized size = 1.01

$$\frac{ad^2x^2}{2}+\frac{ce^2x^{10}}{10}+x^8\left(\frac{be^2}{8}+\frac{cde}{4}\right)+x^6\left(\frac{ae^2}{6}+\frac{bde}{3}+\frac{cd^2}{6}\right)+x^4\left(\frac{ade}{2}+\frac{bd^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**2/2 + c*e**2*x**10/10 + x**8*(b*e**2/8 + c*d*e/4) + x**6*(a*e**2/
6 + b*d*e/3 + c*d**2/6) + x**4*(a*d*e/2 + b*d**2/4)
```

Giac [A] time = 1.07203, size = 107, normalized size = 1.43

$$\frac{1}{10}cx^{10}e^2 + \frac{1}{4}cdx^8e + \frac{1}{8}bx^8e^2 + \frac{1}{6}cd^2x^6 + \frac{1}{3}bdx^6e + \frac{1}{6}ax^6e^2 + \frac{1}{4}bd^2x^4 + \frac{1}{2}adx^4e + \frac{1}{2}ad^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/10*c*x^10*e^2 + 1/4*c*d*x^8*e + 1/8*b*x^8*e^2 + 1/6*c*d^2*x^6 + 1/3*b*d*x
^6*e + 1/6*a*x^6*e^2 + 1/4*b*d^2*x^4 + 1/2*a*d*x^4*e + 1/2*a*d^2*x^2
```

3.276 $\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=73

$$\frac{1}{5}x^5 \left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rubi [A] time = 0.044485, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1153}

$$\frac{1}{5}x^5 \left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) dx &= \int \left(ad^2+d(bd+2ae)x^2+\left(cd^2+e(2bd+ae)\right)x^4+e(2cd+be)x^6+ce^2x^8\right) dx\\ &= ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5+\frac{1}{7}e(2cd+be)x^7+\frac{1}{9}ce^2x^9\end{aligned}$$

Mathematica [A] time = 0.0161226, size = 73, normalized size = 1.

$$\frac{1}{5}x^5 \left(ae^2+2bde+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Maple [A] time = 0.001, size = 70, normalized size = 1.

$$\frac{ce^2x^9}{9}+\frac{\left(e^2b+2\,dec\right)x^7}{7}+\frac{\left(ae^2+2\,deb+cd^2\right)x^5}{5}+\frac{\left(2\,dea+d^2b\right)x^3}{3}+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/9*c*e^2*x^9+1/7*(b*e^2+2*c*d*e)*x^7+1/5*(a*e^2+2*b*d*e+c*d^2)*x^5+1/3*(2*
a*d*e+b*d^2)*x^3+a*d^2*x
```

Maxima [A] time = 0.946527, size = 93, normalized size = 1.27

$$\frac{1}{9}\,ce^{2}x^{9}+\frac{1}{7}\left(2\,cde+be^{2}\right)x^{7}+\frac{1}{5}\left(cd^{2}+2\,bde+ae^{2}\right)x^{5}+ad^{2}x+\frac{1}{3}\left(bd^{2}+2\,ade\right)x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/9*c*e^2*x^9 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*x
^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

Fricas [A] time = 1.52848, size = 185, normalized size = 2.53

$$\frac{1}{9}x^{9}e^{2}c+\frac{2}{7}x^{7}edc+\frac{1}{7}x^{7}e^{2}b+\frac{1}{5}x^{5}d^{2}c+\frac{2}{5}x^{5}edb+\frac{1}{5}x^{5}e^{2}a+\frac{1}{3}x^{3}d^{2}b+\frac{2}{3}x^{3}eda+xd^{2}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*e^2*c + 2/7*x^7*e*d*c + 1/7*x^7*e^2*b + 1/5*x^5*d^2*c + 2/5*x^5*e*d
*b + 1/5*x^5*e^2*a + 1/3*x^3*d^2*b + 2/3*x^3*e*d*a + x*d^2*a
```

Sympy [A] time = 0.07416, size = 78, normalized size = 1.07

$$ad^{2}x+\frac{ce^{2}x^{9}}{9}+x^{7}\left(\frac{be^{2}}{7}+\frac{2cde}{7}\right)+x^{5}\left(\frac{ae^{2}}{5}+\frac{2bde}{5}+\frac{cd^{2}}{5}\right)+x^{3}\left(\frac{2ade}{3}+\frac{bd^{2}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*e**2/5 + 2
*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)
```

Giac [A] time = 1.11682, size = 103, normalized size = 1.41

$$\frac{1}{9}\,cx^{9}e^{2}+\frac{2}{7}\,cdx^{7}e+\frac{1}{7}\,bx^{7}e^{2}+\frac{1}{5}\,cd^{2}x^{5}+\frac{2}{5}\,bdx^{5}e+\frac{1}{5}\,ax^{5}e^{2}+\frac{1}{3}\,bd^{2}x^{3}+\frac{2}{3}\,adx^{3}e+ad^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/7*b*x^7*e^2 + 1/5*c*d^2*x^5 + 2/5*b*d*x^5
*e + 1/5*a*x^5*e^2 + 1/3*b*d^2*x^3 + 2/3*a*d*x^3*e + a*d^2*x
```

3.277 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x}\,dx$

Optimal. Leaf size=74

$$\frac{1}{4}x^4\left(e(ae+2bd)+cd^2\right)+\frac{1}{2}dx^2(2ae+bd)+ad^2\log(x)+\frac{1}{6}ex^6(be+2cd)+\frac{1}{8}ce^2x^8$$

```
[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + e*(2*b*d + a*e))*x^4)/4 + (e*(2*c*d + b
*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

Rubi [A] time = 0.0901146, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1251, 893}

$$\frac{1}{4}x^4\left(e(ae+2bd)+cd^2\right)+\frac{1}{2}dx^2(2ae+bd)+ad^2\log(x)+\frac{1}{6}ex^6(be+2cd)+\frac{1}{8}ce^2x^8$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + e*(2*b*d + a*e))*x^4)/4 + (e*(2*c*d + b
*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^2\left(a+bx^2+cx^4\right)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^2\left(a+bx+cx^2\right)}{x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(d(bd+2ae)+\frac{ad^2}{x}+\left(cd^2+e(2bd+ae)\right)x+e(2cd+be)x^2+ce^2x^3\right)\right.\\ &= \frac{1}{2}d(bd+2ae)x^2+\frac{1}{4}\left(cd^2+e(2bd+ae)\right)x^4+\frac{1}{6}e(2cd+be)x^6+\frac{1}{8}ce^2x^8+ad^2\log(x\end{aligned}$$

Mathematica [A] time = 0.0206325, size = 74, normalized size = 1.

$$\frac{1}{4}x^4\left(ae^2+2bde+cd^2\right)+\frac{1}{2}dx^2(2ae+bd)+ad^2\log(x)+\frac{1}{6}ex^6(be+2cd)+\frac{1}{8}ce^2x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + 2*b*d*e + a*e^2)*x^4)/4 + (e*(2*c*d + b
*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

Maple [A] time = 0.003, size = 77, normalized size = 1.

$$\frac{ce^2x^8}{8}+\frac{x^6be^2}{6}+\frac{x^6cde}{3}+\frac{x^4ae^2}{4}+\frac{x^4bde}{2}+\frac{x^4cd^2}{4}+x^2ade+\frac{x^2bd^2}{2}+ad^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x)
```

```
[Out] 1/8*c*e^2*x^8+1/6*x^6*b*e^2+1/3*x^6*c*d*e+1/4*x^4*a*e^2+1/2*x^4*b*d*e+1/4*x
^4*c*d^2+x^2*a*d*e+1/2*x^2*b*d^2+a*d^2*ln(x)
```

Maxima [A] time = 0.958351, size = 99, normalized size = 1.34

$$\frac{1}{8}ce^2x^8+\frac{1}{6}\left(2cde+be^2\right)x^6+\frac{1}{4}\left(cd^2+2bde+ae^2\right)x^4+\frac{1}{2}ad^2\log\left(x^2\right)+\frac{1}{2}\left(bd^2+2ade\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/8*c*e^2*x^8 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e + a*e^2)*x
^4 + 1/2*a*d^2*log(x^2) + 1/2*(b*d^2 + 2*a*d*e)*x^2
```

Fricas [A] time = 1.66027, size = 165, normalized size = 2.23

$$\frac{1}{8}ce^2x^8+\frac{1}{6}\left(2cde+be^2\right)x^6+\frac{1}{4}\left(cd^2+2bde+ae^2\right)x^4+ad^2\log(x)+\frac{1}{2}\left(bd^2+2ade\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/8*c*e^2*x^8 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e + a*e^2)*x
^4 + a*d^2*log(x) + 1/2*(b*d^2 + 2*a*d*e)*x^2
```

Sympy [A] time = 0.316436, size = 73, normalized size = 0.99

$$ad^2\log(x)+\frac{ce^2x^8}{8}+x^6\left(\frac{be^2}{6}+\frac{cde}{3}\right)+x^4\left(\frac{ae^2}{4}+\frac{bde}{2}+\frac{cd^2}{4}\right)+x^2\left(ade+\frac{bd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x,x)
```

```
[Out] a*d**2*log(x) + c*e**2*x**8/8 + x**6*(b*e**2/6 + c*d*e/3) + x**4*(a*e**2/4
+ b*d*e/2 + c*d**2/4) + x**2*(a*d*e + b*d**2/2)
```

Giac [A] time = 1.09819, size = 107, normalized size = 1.45

$$\frac{1}{8}cx^8e^2 + \frac{1}{3}cdx^6e + \frac{1}{6}bx^6e^2 + \frac{1}{4}cd^2x^4 + \frac{1}{2}bdx^4e + \frac{1}{4}ax^4e^2 + \frac{1}{2}bd^2x^2 + adx^2e + \frac{1}{2}ad^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/8*c*x^8*e^2 + 1/3*c*d*x^6*e + 1/6*b*x^6*e^2 + 1/4*c*d^2*x^4 + 1/2*b*d*x^4
*e + 1/4*a*x^4*e^2 + 1/2*b*d^2*x^2 + a*d*x^2*e + 1/2*a*d^2*log(x^2)
```

3.278 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2}\,dx$

Optimal. Leaf size=71

$$\frac{1}{3}x^3\left(e(ae+2bd)+cd^2\right)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

```
[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + e*(2*b*d + a*e))*x^3)/3 + (e*(
2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

Rubi [A] time = 0.0483196, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1261}

$$\frac{1}{3}x^3\left(e(ae+2bd)+cd^2\right)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + e*(2*b*d + a*e))*x^3)/3 + (e*(
2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2}\,dx &= \int \left(d(bd+2ae)+\frac{ad^2}{x^2}+\left(cd^2+e(2bd+ae)\right)x^2+e(2cd+be)x^4+ce^2x^6\right)dx\\ &= -\frac{ad^2}{x}+d(bd+2ae)x+\frac{1}{3}\left(cd^2+e(2bd+ae)\right)x^3+\frac{1}{5}e(2cd+be)x^5+\frac{1}{7}ce^2x^7\end{aligned}$$

Mathematica [A] time = 0.0332107, size = 71, normalized size = 1.

$$\frac{1}{3}x^3\left(ae^2+2bde+cd^2\right)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + 2*b*d*e + a*e^2)*x^3)/3 + (e*(
2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

Maple [A] time = 0.004, size = 75, normalized size = 1.1

$$\frac{ce^2x^7}{7}+\frac{x^5be^2}{5}+\frac{2\,x^5cde}{5}+\frac{x^3ae^2}{3}+\frac{2\,x^3bde}{3}+\frac{x^3cd^2}{3}+2\,deax+d^2bx-\frac{ad^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x)
```

```
[Out] 1/7*c*e^2*x^7+1/5*x^5*b*e^2+2/5*x^5*c*d*e+1/3*x^3*a*e^2+2/3*x^3*b*d*e+1/3*x
^3*c*d^2+2*d*e*a*x+d^2*b*x-a*d^2/x
```

Maxima [A] time = 0.947018, size = 93, normalized size = 1.31

$$\frac{1}{7}\,ce^2x^7+\frac{1}{5}\left(2\,cde+be^2\right)x^5+\frac{1}{3}\left(cd^2+2\,bde+ae^2\right)x^3-\frac{ad^2}{x}+\left(bd^2+2\,ade\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c*e^2*x^7 + 1/5*(2*c*d*e + b*e^2)*x^5 + 1/3*(c*d^2 + 2*b*d*e + a*e^2)*x
^3 - a*d^2/x + (b*d^2 + 2*a*d*e)*x
```

Fricas [A] time = 1.6627, size = 170, normalized size = 2.39

$$\frac{15\,ce^2x^8+21\left(2\,cde+be^2\right)x^6+35\left(cd^2+2\,bde+ae^2\right)x^4-105\,ad^2+105\left(bd^2+2\,ade\right)x^2}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*c*e^2*x^8 + 21*(2*c*d*e + b*e^2)*x^6 + 35*(c*d^2 + 2*b*d*e + a*e^
2)*x^4 - 105*a*d^2 + 105*(b*d^2 + 2*a*d*e)*x^2)/x
```

Sympy [A] time = 0.315117, size = 73, normalized size = 1.03

$$-\frac{ad^2}{x}+\frac{ce^2x^7}{7}+x^5\left(\frac{be^2}{5}+\frac{2cde}{5}\right)+x^3\left(\frac{ae^2}{3}+\frac{2bde}{3}+\frac{cd^2}{3}\right)+x\left(2ade+bd^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -a*d**2/x + c*e**2*x**7/7 + x**5*(b*e**2/5 + 2*c*d*e/5) + x**3*(a*e**2/3 +
2*b*d*e/3 + c*d**2/3) + x*(2*a*d*e + b*d**2)
```

<u>Giac</u> [A] time = 1.08033, size = 100, normalized size = 1.41

$$\frac{1}{7}\,cx^7e^2+\frac{2}{5}\,cdx^5e+\frac{1}{5}\,bx^5e^2+\frac{1}{3}\,cd^2x^3+\frac{2}{3}\,bdx^3e+\frac{1}{3}\,ax^3e^2+bd^2x+2\,adxe-\frac{ad^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e^2 + 2/5*c*d*x^5*e + 1/5*b*x^5*e^2 + 1/3*c*d^2*x^3 + 2/3*b*d*x^3
*e + 1/3*a*x^3*e^2 + b*d^2*x + 2*a*d*x*e - a*d^2/x
```

3.279 $\int \frac{\left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)}{x^3} dx$

Optimal. Leaf size=74

$$\frac{1}{2}x^2 \left(e(ae+2bd)+cd^2\right)+d\log(x)(2ae+bd)-\frac{ad^2}{2x^2}+\frac{1}{4}ex^4(be+2cd)+\frac{1}{6}ce^2x^6$$

```
[Out] -(a*d^2)/(2*x^2) + ((c*d^2 + e*(2*b*d + a*e))*x^2)/2 + (e*(2*c*d + b*e)*x^4
)/4 + (c*e^2*x^6)/6 + d*(b*d + 2*a*e)*Log[x]
```

Rubi [A] time = 0.0957302, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1251, 893}

$$\frac{1}{2}x^2 \left(e(ae+2bd)+cd^2\right)+d\log(x)(2ae+bd)-\frac{ad^2}{2x^2}+\frac{1}{4}ex^4(be+2cd)+\frac{1}{6}ce^2x^6$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -(a*d^2)/(2*x^2) + ((c*d^2 + e*(2*b*d + a*e))*x^2)/2 + (e*(2*c*d + b*e)*x^4
)/4 + (c*e^2*x^6)/6 + d*(b*d + 2*a*e)*Log[x]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right)}{x^3} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^2 \left(a+bx+cx^2\right)}{x^2} dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(cd^2\left(1+\frac{e(2bd+ae)}{cd^2}\right)+\frac{ad^2}{x^2}+\frac{d(bd+2ae)}{x}+e(2cd+be)x+ce^2x^2\right)\right)\\
&= -\frac{ad^2}{2x^2}+\frac{1}{2}\left(cd^2+e(2bd+ae)\right)x^2+\frac{1}{4}e(2cd+be)x^4+\frac{1}{6}ce^2x^6+d(bd+2ae)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0422363, size = 71, normalized size = 0.96

$$\frac{1}{12}\left(6x^2 \left(e(ae+2bd)+cd^2\right)+12d\log(x)(2ae+bd)-\frac{6ad^2}{x^2}+3ex^4(be+2cd)+2ce^2x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] ((-6*a*d^2)/x^2 + 6*(c*d^2 + e*(2*b*d + a*e))*x^2 + 3*e*(2*c*d + b*e)*x^4 +
 2*c*e^2*x^6 + 12*d*(b*d + 2*a*e)*Log[x])/12
```

Maple [A] time = 0.007, size = 76, normalized size = 1.

$$\frac{ce^2x^6}{6}+\frac{x^4be^2}{4}+\frac{x^4cde}{2}+\frac{x^2ae^2}{2}+x^2bde+\frac{x^2cd^2}{2}+2\,\ln(x)\,ade+\ln(x)\,bd^2-\frac{ad^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x)
```

```
[Out] 1/6*c*e^2*x^6+1/4*x^4*b*e^2+1/2*x^4*c*d*e+1/2*x^2*a*e^2+x^2*b*d*e+1/2*x^2*c
*d^2+2*ln(x)*a*d*e+ln(x)*b*d^2-1/2*a*d^2/x^2
```

Maxima [A] time = 0.935074, size = 99, normalized size = 1.34

$$\frac{1}{6}\,ce^2x^6+\frac{1}{4}\left(2\,cde+be^2\right)x^4+\frac{1}{2}\left(cd^2+2\,bde+ae^2\right)x^2+\frac{1}{2}\left(bd^2+2\,ade\right)\log\left(x^2\right)-\frac{ad^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c*e^2*x^6 + 1/4*(2*c*d*e + b*e^2)*x^4 + 1/2*(c*d^2 + 2*b*d*e + a*e^2)*x
^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*a*d^2/x^2
```

Fricas [A] time = 1.70462, size = 173, normalized size = 2.34

$$\frac{2\,ce^2x^8+3\left(2\,cde+be^2\right)x^6+6\left(cd^2+2\,bde+ae^2\right)x^4+12\left(bd^2+2\,ade\right)x^2\log(x)-6\,ad^2}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*c*e^2*x^8 + 3*(2*c*d*e + b*e^2)*x^6 + 6*(c*d^2 + 2*b*d*e + a*e^2)*x
^4 + 12*(b*d^2 + 2*a*d*e)*x^2*log(x) - 6*a*d^2)/x^2
```

Sympy [A] time = 0.410208, size = 71, normalized size = 0.96

$$-\frac{ad^2}{2x^2}+\frac{ce^2x^6}{6}+d\,(2ae+bd)\log(x)+x^4\left(\frac{be^2}{4}+\frac{cde}{2}\right)+x^2\left(\frac{ae^2}{2}+bde+\frac{cd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] -a*d**2/(2*x**2) + c*e**2*x**6/6 + d*(2*a*e + b*d)*log(x) + x**4*(b*e**2/4
+ c*d*e/2) + x**2*(a*e**2/2 + b*d*e + c*d**2/2)
```

Giac [A] time = 1.08428, size = 131, normalized size = 1.77

$$\frac{1}{6}cx^6e^2 + \frac{1}{2}cdx^4e + \frac{1}{4}bx^4e^2 + \frac{1}{2}cd^2x^2 + bdx^2e + \frac{1}{2}ax^2e^2 + \frac{1}{2}\left(bd^2 + 2\,ade\right)\log\left(x^2\right) - \frac{bd^2x^2 + 2\,adx^2e + ad^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c*x^6*e^2 + 1/2*c*d*x^4*e + 1/4*b*x^4*e^2 + 1/2*c*d^2*x^2 + b*d*x^2*e +
 1/2*a*x^2*e^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*(b*d^2*x^2 + 2*a*d*x^
2*e + a*d^2)/x^2
```

3.280 $\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=168

$$\frac{x^3\left(3cd^2-e(2bd-ae)\right)}{3e^4}-\frac{d^2x\left(ae^2-bde+cd^2\right)}{2e^5\left(d+ex^2\right)}-\frac{dx\left(4cd^2-e(3bd-2ae)\right)}{e^5}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(9cd^2-e(7bd-5ae)\right)}{2e^{11/2}}-x^5$$

```
[Out] -((d*(4*c*d^2 - e*(3*b*d - 2*a*e))*x)/e^5) + ((3*c*d^2 - e*(2*b*d - a*e))*x
^3)/(3*e^4) - ((2*c*d - b*e)*x^5)/(5*e^3) + (c*x^7)/(7*e^2) - (d^2*(c*d^2 -
 b*d*e + a*e^2)*x)/(2*e^5*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - e*(7*b*d - 5*a
*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(11/2))
```

Rubi [A] time = 0.234486, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1257, 1810, 205}

$$\frac{x^3\left(3cd^2-e(2bd-ae)\right)}{3e^4}-\frac{d^2x\left(ae^2-bde+cd^2\right)}{2e^5\left(d+ex^2\right)}-\frac{dx\left(4cd^2-e(3bd-2ae)\right)}{e^5}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(9cd^2-e(7bd-5ae)\right)}{2e^{11/2}}-x^5$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -((d*(4*c*d^2 - e*(3*b*d - 2*a*e))*x)/e^5) + ((3*c*d^2 - e*(2*b*d - a*e))*x
^3)/(3*e^4) - ((2*c*d - b*e)*x^5)/(5*e^3) + (c*x^7)/(7*e^2) - (d^2*(c*d^2 -
 b*d*e + a*e^2)*x)/(2*e^5*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - e*(7*b*d - 5*a
*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(11/2))
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}dx = -\frac{d^2\left(cd^2-bde+ae^2\right)x}{2e^5\left(d+ex^2\right)} - \frac{\int \frac{-d^2\left(cd^2-bde+ae^2\right)+2de\left(cd^2-bde+ae^2\right)x^2-2e^2\left(cd^2-bde+ae^2\right)x^4+2e^3(cd-be)x^6-2ce}{d+ex^2}}{2e^5}$$

$$= -\frac{d^2\left(cd^2-bde+ae^2\right)x}{2e^5\left(d+ex^2\right)} - \frac{\int \left(2d\left(4cd^2-e(3bd-2ae)\right)-2e\left(3cd^2-e(2bd-ae)\right)x^2+2e^2(2}{2e^5}$$

$$= -\frac{d\left(4cd^2-e(3bd-2ae)\right)x}{e^5} + \frac{\left(3cd^2-e(2bd-ae)\right)x^3}{3e^4} - \frac{(2cd-be)x^5}{5e^3} + \frac{cx^7}{7e^2} - \frac{d^2\left(cd^2-bde\right.}{2e^5\left(d+\right.}$$

$$= -\frac{d\left(4cd^2-e(3bd-2ae)\right)x}{e^5} + \frac{\left(3cd^2-e(2bd-ae)\right)x^3}{3e^4} - \frac{(2cd-be)x^5}{5e^3} + \frac{cx^7}{7e^2} - \frac{d^2\left(cd^2-bde\right.}{2e^5\left(d+\right.}$$

Mathematica [A] time = 0.13674, size = 165, normalized size = 0.98

$$\frac{x^3\left(ae^2-2bde+3cd^2\right)}{3e^4} - \frac{x\left(ad^2e^2-bd^3e+cd^4\right)}{2e^5\left(d+ex^2\right)} - \frac{dx\left(2ae^2-3bde+4cd^2\right)}{e^5} + \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5ae^2-7bde+9cd^2\right)}{2e^{11/2}} +$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -((d*(4*c*d^2 - 3*b*d*e + 2*a*e^2)*x)/e^5) + ((3*c*d^2 - 2*b*d*e + a*e^2)*x
^3)/(3*e^4) + ((-2*c*d + b*e)*x^5)/(5*e^3) + (c*x^7)/(7*e^2) - ((c*d^4 - b*
d^3*e + a*d^2*e^2)*x)/(2*e^5*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - 7*b*d*e + 5
*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(11/2))
```

Maple [A] time = 0.01, size = 214, normalized size = 1.3

$$\frac{cx^7}{7e^2} + \frac{x^5b}{5e^2} - \frac{2x^5cd}{5e^3} + \frac{x^3a}{3e^2} - \frac{2x^3bd}{3e^3} + \frac{x^3cd^2}{e^4} - 2\frac{adx}{e^3} + 3\frac{d^2bx}{e^4} - 4\frac{cd^3x}{e^5} - \frac{ad^2x}{2e^3\left(ex^2+d\right)} + \frac{d^3xb}{2e^4\left(ex^2+d\right)} - \frac{d^4xc}{2e^5\left(ex^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] 1/7*c*x^7/e^2+1/5/e^2*x^5*b-2/5/e^3*x^5*c*d+1/3/e^2*x^3*a-2/3/e^3*x^3*b*d+1
/e^4*x^3*c*d^2-2/e^3*d*a*x+3/e^4*d^2*b*x-4/e^5*c*d^3*x-1/2*d^2/e^3*x/(e*x^2
+d)*a+1/2*d^3/e^4*x/(e*x^2+d)*b-1/2*d^4/e^5*x/(e*x^2+d)*c+5/2*d^2/e^3/(d*e)
^(1/2)*arctan(e*x/(d*e)^(1/2))*a-7/2*d^3/e^4/(d*e)^(1/2)*arctan(e*x/(d*e)^(
1/2))*b+9/2*d^4/e^5/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

[Out] Exception raised: ValueError

Fricas [A] time = 1.9179, size = 922, normalized size = 5.49

$$\left[\frac{60\, ce^{4} x^{9} - 12\,\left(9\, cde^{3} - 7\, be^{4}\right)x^{7} + 28\,\left(9\, cd^{2} e^{2} - 7\, bde^{3} + 5\, ae^{4}\right)x^{5} - 140\,\left(9\, cd^{3} e - 7\, bd^{2} e^{2} + 5\, ade^{3}\right)x^{3} + 105\,\left(9\, cd^{4} - 7\, bd^{3}\right.}{420\,\left(e^{6} x^{2} + de^{5}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*c*e^4*x^9 - 12*(9*c*d*e^3 - 7*b*e^4)*x^7 + 28*(9*c*d^2*e^2 - 7*b
*d*e^3 + 5*a*e^4)*x^5 - 140*(9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^3 + 105
*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2 + (9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)
*x^2)*sqrt(-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) - 210*(9*c
*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*x)/(e^6*x^2 + d*e^5), 1/210*(30*c*e^4*x^9 -
 6*(9*c*d*e^3 - 7*b*e^4)*x^7 + 14*(9*c*d^2*e^2 - 7*b*d*e^3 + 5*a*e^4)*x^5 -
 70*(9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^3 + 105*(9*c*d^4 - 7*b*d^3*e +
5*a*d^2*e^2 + (9*c*d^3*e - 7*b*d^2*e^2 + 5*a*d*e^3)*x^2)*sqrt(d/e)*arctan(e
*x*sqrt(d/e)/d) - 105*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*x)/(e^6*x^2 + d*e
^5)]
```

Sympy [B] time = 1.50105, size = 313, normalized size = 1.86

$$\frac{cx^{7}}{7e^{2}} - \frac{x\left(ad^{2}e^{2} - bd^{3}e + cd^{4}\right)}{2de^{5} + 2e^{6}x^{2}} - \frac{\sqrt{-\frac{d^{3}}{e^{11}}}\left(5ae^{2} - 7bde + 9cd^{2}\right)\log\left(-\frac{e^{5}\sqrt{-\frac{d^{3}}{e^{11}}}\left(5ae^{2}-7bde+9cd^{2}\right)}{5ade^{2}-7bd^{2}e+9cd^{3}} + x\right)}{4} + \frac{\sqrt{-\frac{d^{3}}{e^{11}}}\left(5ae^{2} - 7bde + 9c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**7/(7*e**2) - x*(a*d**2*e**2 - b*d**3*e + c*d**4)/(2*d*e**5 + 2*e**6*x*
*2) - sqrt(-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*log(-e**5*sqrt(-d**
3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*c*d**
3) + x)/4 + sqrt(-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*log(e**5*sqrt
(-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*
c*d**3) + x)/4 + x**5*(b*e - 2*c*d)/(5*e**3) + x**3*(a*e**2 - 2*b*d*e + 3*c
*d**2)/(3*e**4) - x*(2*a*d*e**2 - 3*b*d**2*e + 4*c*d**3)/e**5
```

Giac [A] time = 1.09757, size = 216, normalized size = 1.29

$$\frac{\left(9\, cd^{4} - 7\, bd^{3} e + 5\, ad^{2} e^{2}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}}{2\,\sqrt{d}} + \frac{1}{105}\,\left(15\, cx^{7} e^{12} - 42\, cdx^{5} e^{11} + 21\, bx^{5} e^{12} + 105\, cd^{2} x^{3} e^{10} - 70\, bdx^{3} e^{11} - 42\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-11/2)
/sqrt(d) + 1/105*(15*c*x^7*e^12 - 42*c*d*x^5*e^11 + 21*b*x^5*e^12 + 105*c*d
^2*x^3*e^10 - 70*b*d*x^3*e^11 - 420*c*d^3*x*e^9 + 35*a*x^3*e^12 + 315*b*d^2
*x*e^10 - 210*a*d*x*e^11)*e^(-14) - 1/2*(c*d^4*x - b*d^3*x*e + a*d^2*x*e^2)
*e^(-5)/(x^2*e + d)
```

3.281 $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2} dx$

Optimal. Leaf size=135

$$\frac{dx\left(ae^2-bde+cd^2\right)}{2e^4\left(d+ex^2\right)}+\frac{x\left(3cd^2-e(2bd-ae)\right)}{e^4}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(7cd^2-e(5bd-3ae)\right)}{2e^{9/2}}-\frac{x^3(2cd-be)}{3e^3}+\frac{cx^5}{5e^2}$$

```
[Out] ((3*c*d^2 - e*(2*b*d - a*e))*x)/e^4 - ((2*c*d - b*e)*x^3)/(3*e^3) + (c*x^5)
/(5*e^2) + (d*(c*d^2 - b*d*e + a*e^2)*x)/(2*e^4*(d + e*x^2)) - (Sqrt[d]*(7*
c*d^2 - e*(5*b*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))
```

Rubi [A] time = 0.159071, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1257, 1810, 205}

$$\frac{dx\left(ae^2-bde+cd^2\right)}{2e^4\left(d+ex^2\right)}+\frac{x\left(3cd^2-e(2bd-ae)\right)}{e^4}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(7cd^2-e(5bd-3ae)\right)}{2e^{9/2}}-\frac{x^3(2cd-be)}{3e^3}+\frac{cx^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((3*c*d^2 - e*(2*b*d - a*e))*x)/e^4 - ((2*c*d - b*e)*x^3)/(3*e^3) + (c*x^5)
/(5*e^2) + (d*(c*d^2 - b*d*e + a*e^2)*x)/(2*e^4*(d + e*x^2)) - (Sqrt[d]*(7*
c*d^2 - e*(5*b*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}dx = \frac{d\left(cd^2-bde+ae^2\right)x}{2e^4\left(d+ex^2\right)} - \frac{\int \frac{d\left(cd^2-bde+ae^2\right)-2e\left(cd^2-bde+ae^2\right)x^2+2e^2(cd-be)x^4-2ce^3x^6}{d+ex^2}dx}{2e^4}$$

$$= \frac{d\left(cd^2-bde+ae^2\right)x}{2e^4\left(d+ex^2\right)} - \frac{\int \left(-2\left(3cd^2-2bde+ae^2\right)+2e(2cd-be)x^2-2ce^2x^4+\frac{7cd^3-5bd^2e+3}{d+ex^2}\right.}{2e^4}$$

$$= \frac{\left(3cd^2-e(2bd-ae)\right)x}{e^4} - \frac{(2cd-be)x^3}{3e^3} + \frac{cx^5}{5e^2} + \frac{d\left(cd^2-bde+ae^2\right)x}{2e^4\left(d+ex^2\right)} - \frac{\left(d\left(7cd^2-e(5bd-\right.\right.}{2e^4}$$

$$= \frac{\left(3cd^2-e(2bd-ae)\right)x}{e^4} - \frac{(2cd-be)x^3}{3e^3} + \frac{cx^5}{5e^2} + \frac{d\left(cd^2-bde+ae^2\right)x}{2e^4\left(d+ex^2\right)} - \frac{\sqrt{d}\left(7cd^2-e(5bd-\right.}{2e^9}$$

Mathematica [A] time = 0.0789524, size = 133, normalized size = 0.99

$$\frac{x\left(ade^2-bd^2e+cd^3\right)}{2e^4\left(d+ex^2\right)} + \frac{x\left(ae^2-2bde+3cd^2\right)}{e^4} - \frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3ae^2-5bde+7cd^2\right)}{2e^{9/2}} + \frac{x^3(be-2cd)}{3e^3} + \frac{cx^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((3*c*d^2 - 2*b*d*e + a*e^2)*x)/e^4 + ((-2*c*d + b*e)*x^3)/(3*e^3) + (c*x^5
)/(5*e^2) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)/(2*e^4*(d + e*x^2)) - (Sqrt[d]*
(7*c*d^2 - 5*b*d*e + 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))
```

Maple [A] time = 0.012, size = 176, normalized size = 1.3

$$\frac{cx^5}{5e^2} + \frac{x^3b}{3e^2} - \frac{2x^3cd}{3e^3} + \frac{ax}{e^2} - 2\frac{bdx}{e^3} + 3\frac{cd^2x}{e^4} + \frac{dxa}{2e^2\left(ex^2+d\right)} - \frac{d^2bx}{2e^3\left(ex^2+d\right)} + \frac{d^3xc}{2e^4\left(ex^2+d\right)} - \frac{3ad}{2e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] 1/5*c*x^5/e^2+1/3/e^2*x^3*b-2/3/e^3*x^3*c*d+1/e^2*a*x-2/e^3*d*b*x+3/e^4*c*d
^2*x+1/2*d/e^2*x/(e*x^2+d)*a-1/2*d^2/e^3*x/(e*x^2+d)*b+1/2*d^3/e^4*x/(e*x^2
+d)*c-3/2*d/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a+5/2*d^2/e^3/(d*e)^(1/
2)*arctan(e*x/(d*e)^(1/2))*b-7/2*d^3/e^4/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)
)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82637, size = 756, normalized size = 5.6

$$\left[\frac{12\,ce^3x^7-4\left(7\,cde^2-5\,be^3\right)x^5+20\left(7\,cd^2e-5\,bde^2+3\,ae^3\right)x^3+15\left(7\,cd^3-5\,bd^2e+3\,ade^2+\left(7\,cd^2e-5\,bde^2+3\,ae^3\right)x^2\right.}{60\left(e^5x^2+de^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c*e^3*x^7 - 4*(7*c*d*e^2 - 5*b*e^3)*x^5 + 20*(7*c*d^2*e - 5*b*d*e
^2 + 3*a*e^3)*x^3 + 15*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2 + (7*c*d^2*e - 5*b*
d*e^2 + 3*a*e^3)*x^2)*sqrt(-d/e)*log((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2
+ d)) + 30*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*x)/(e^5*x^2 + d*e^4), 1/30*(6*
c*e^3*x^7 - 2*(7*c*d*e^2 - 5*b*e^3)*x^5 + 10*(7*c*d^2*e - 5*b*d*e^2 + 3*a*e
^3)*x^3 - 15*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2 + (7*c*d^2*e - 5*b*d*e^2 + 3*
a*e^3)*x^2)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) + 15*(7*c*d^3 - 5*b*d^2*e + 3
*a*d*e^2)*x)/(e^5*x^2 + d*e^4)]
```

Sympy [A] time = 1.40457, size = 184, normalized size = 1.36

$$\frac{cx^5}{5e^2}+\frac{x\left(ade^2-bd^2e+cd^3\right)}{2de^4+2e^5x^2}+\frac{\sqrt{-\frac{d}{e^9}}\left(3ae^2-5bde+7cd^2\right)\log\left(-e^4\sqrt{-\frac{d}{e^9}}+x\right)}{4}-\frac{\sqrt{-\frac{d}{e^9}}\left(3ae^2-5bde+7cd^2\right)\log\left(e^4\sqrt{-\frac{d}{e^9}}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**5/(5*e**2) + x*(a*d*e**2 - b*d**2*e + c*d**3)/(2*d*e**4 + 2*e**5*x**2)
 + sqrt(-d/e**9)*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*log(-e**4*sqrt(-d/e**9) +
x)/4 - sqrt(-d/e**9)*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*log(e**4*sqrt(-d/e**9)
 + x)/4 + x**3*(b*e - 2*c*d)/(3*e**3) + x*(a*e**2 - 2*b*d*e + 3*c*d**2)/e**
4
```

Giac [A] time = 1.11209, size = 169, normalized size = 1.25

$$-\frac{\left(7\,cd^3-5\,bd^2e+3\,ade^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{2\,\sqrt{d}}+\frac{1}{15}\left(3\,cx^5e^8-10\,cdx^3e^7+5\,bx^3e^8+45\,cd^2xe^6-30\,bdxe^7+15\,axe^8\right)e^{(-10)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/s
qrt(d) + 1/15*(3*c*x^5*e^8 - 10*c*d*x^3*e^7 + 5*b*x^3*e^8 + 45*c*d^2*x*e^6
- 30*b*d*x*e^7 + 15*a*x*e^8)*e^(-10) + 1/2*(c*d^3*x - b*d^2*x*e + a*d*x*e^2
)*e^(-4)/(x^2*e + d)
```

3.282 $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2} dx$

Optimal. Leaf size=106

$$-\frac{x\left(ae^2-bde+cd^2\right)}{2e^3\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5cd^2-e(3bd-ae)\right)}{2\sqrt{d}e^{7/2}}-\frac{x(2cd-be)}{e^3}+\frac{cx^3}{3e^2}$$

```
[Out] -(((2*c*d - b*e)*x)/e^3) + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^2)*x)/(2
*e^3*(d + e*x^2)) + ((5*c*d^2 - e*(3*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(2*Sqrt[d]*e^(7/2))
```

Rubi [A] time = 0.105909, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1257, 1153, 205}

$$-\frac{x\left(ae^2-bde+cd^2\right)}{2e^3\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5cd^2-e(3bd-ae)\right)}{2\sqrt{d}e^{7/2}}-\frac{x(2cd-be)}{e^3}+\frac{cx^3}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -(((2*c*d - b*e)*x)/e^3) + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^2)*x)/(2
*e^3*(d + e*x^2)) + ((5*c*d^2 - e*(3*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(2*Sqrt[d]*e^(7/2))
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{\left(cd^2-bde+ae^2\right)x}{2e^3\left(d+ex^2\right)} - \frac{\int \frac{-cd^2+bde-ae^2+2e(cd-be)x^2-2ce^2x^4}{d+ex^2}\,dx}{2e^3}$$

$$= -\frac{\left(cd^2-bde+ae^2\right)x}{2e^3\left(d+ex^2\right)} - \frac{\int \left(2(2cd-be)-2cex^2+\frac{-5cd^2+3bde-ae^2}{d+ex^2}\right)dx}{2e^3}$$

$$= -\frac{(2cd-be)x}{e^3} + \frac{cx^3}{3e^2} - \frac{\left(cd^2-bde+ae^2\right)x}{2e^3\left(d+ex^2\right)} - \frac{\left(-5cd^2+e(3bd-ae)\right)\int \frac{1}{d+ex^2}\,dx}{2e^3}$$

$$= -\frac{(2cd-be)x}{e^3} + \frac{cx^3}{3e^2} - \frac{\left(cd^2-bde+ae^2\right)x}{2e^3\left(d+ex^2\right)} + \frac{\left(5cd^2-e(3bd-ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e^{7/2}}$$

Mathematica [A] time = 0.0658701, size = 102, normalized size = 0.96

$$-\frac{x\left(ae^2-bde+cd^2\right)}{2e^3\left(d+ex^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(ae^2-3bde+5cd^2\right)}{2\sqrt{d}e^{7/2}} + \frac{x(be-2cd)}{e^3} + \frac{cx^3}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((-2*c*d + b*e)*x)/e^3 + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*e
^3*(d + e*x^2)) + ((5*c*d^2 - 3*b*d*e + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(2*Sqrt[d]*e^(7/2))
```

Maple [A] time = 0.008, size = 141, normalized size = 1.3

$$\frac{cx^3}{3\,e^2} + \frac{bx}{e^2} - 2\,\frac{cdx}{e^3} - \frac{xa}{2\,e\left(ex^2+d\right)} + \frac{dxb}{2\,e^2\left(ex^2+d\right)} - \frac{xcd^2}{2\,e^3\left(ex^2+d\right)} + \frac{a}{2\,e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}} - \frac{3\,bd}{2\,e^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] 1/3*c*x^3/e^2+1/e^2*b*x-2/e^3*c*d*x-1/2/e*x/(e*x^2+d)*a+1/2/e^2*x/(e*x^2+d)
*d*b-1/2/e^3*x/(e*x^2+d)*c*d^2+1/2/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a-
3/2/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*d*b+5/2/e^3/(d*e)^(1/2)*arctan(
e*x/(d*e)^(1/2))*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85131, size = 648, normalized size = 6.11

$$\left[\frac{4\,cde^3x^5 - 4\left(5\,cd^2e^2 - 3\,bde^3\right)x^3 - 3\left(5\,cd^3 - 3\,bd^2e + ade^2 + \left(5\,cd^2e - 3\,bde^2 + ae^3\right)x^2\right)\sqrt{-de}\log\left(\frac{ex^2 - 2\,\sqrt{-de}x - d}{ex^2 + d}\right) - 6\left(\right.}{12\left(de^5x^2 + d^2e^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*c*d*e^3*x^5 - 4*(5*c*d^2*e^2 - 3*b*d*e^3)*x^3 - 3*(5*c*d^3 - 3*b*d
^2*e + a*d*e^2 + (5*c*d^2*e - 3*b*d*e^2 + a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2
 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 6*(5*c*d^3*e - 3*b*d^2*e^2 + a*d*e^3)
*x)/(d*e^5*x^2 + d^2*e^4), 1/6*(2*c*d*e^3*x^5 - 2*(5*c*d^2*e^2 - 3*b*d*e^3)
*x^3 + 3*(5*c*d^3 - 3*b*d^2*e + a*d*e^2 + (5*c*d^2*e - 3*b*d*e^2 + a*e^3)*x
^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(5*c*d^3*e - 3*b*d^2*e^2 + a*d*e^3)
*x)/(d*e^5*x^2 + d^2*e^4)]
```

Sympy [A] time = 1.21161, size = 160, normalized size = 1.51

$$\frac{cx^3}{3e^2} - \frac{x\left(ae^2 - bde + cd^2\right)}{2de^3 + 2e^4x^2} - \frac{\sqrt{-\frac{1}{de^7}}\left(ae^2 - 3bde + 5cd^2\right)\log\left(-de^3\sqrt{-\frac{1}{de^7}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{de^7}}\left(ae^2 - 3bde + 5cd^2\right)\log\left(de^3\sqrt{\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**3/(3*e**2) - x*(a*e**2 - b*d*e + c*d**2)/(2*d*e**3 + 2*e**4*x**2) - sq
rt(-1/(d*e**7))*(a*e**2 - 3*b*d*e + 5*c*d**2)*log(-d*e**3*sqrt(-1/(d*e**7))
 + x)/4 + sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e + 5*c*d**2)*log(d*e**3*sqrt(-
1/(d*e**7)) + x)/4 + x*(b*e - 2*c*d)/e**3
```

Giac [A] time = 1.10959, size = 123, normalized size = 1.16

$$\frac{\left(5\,cd^2 - 3\,bde + ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{2\,\sqrt{d}} + \frac{1}{3}\left(cx^3e^4 - 6\,cdxe^3 + 3\,bxe^4\right)e^{(-6)} - \frac{\left(cd^2x - bdxe + axe^2\right)e^{(-3)}}{2\left(x^2e + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2 - 3*b*d*e + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d)
+ 1/3*(c*x^3*e^4 - 6*c*d*x*e^3 + 3*b*x*e^4)*e^(-6) - 1/2*(c*d^2*x - b*d*x*e
 + a*x*e^2)*e^(-3)/(x^2*e + d)
```

3.283 $\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2} dx$

Optimal. Leaf size=83

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rubi [A] time = 0.0929843, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1157, 388, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(ae+bd)\right)}{2d^{3/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{2d\left(d+ex^2\right)}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((a + (d*(c*d - b*e))/e^2)*x)/(2*d*(d + e*x^2)) - ((3*c*d^2 - e
*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^2}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}$$

$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\left(3cd^2-e(bd+ae)\right)\int \frac{1}{d+ex^2}\,dx}{2de^2}$$

$$= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\left(d+ex^2\right)} - \frac{\left(3cd^2-e(bd+ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

Mathematica [A] time = 0.0544369, size = 88, normalized size = 1.06

$$\frac{x\left(ae^2-bde+cd^2\right)}{2de^2\left(d+ex^2\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-bde+3cd^2\right)}{2d^{3/2}e^{5/2}} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A] time = 0.009, size = 118, normalized size = 1.4

$$\frac{cx}{e^2} + \frac{xa}{2\,d\left(ex^2+d\right)} - \frac{xb}{2\,e\left(ex^2+d\right)} + \frac{dxc}{2\,e^2\left(ex^2+d\right)} + \frac{a}{2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}} + \frac{b}{2\,e}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}} - \frac{3\,cd}{2\,e^2}\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x)
```

```
[Out] c*x/e^2+1/2/d*x/(e*x^2+d)*a-1/2/e*x/(e*x^2+d)*b+1/2/e^2*d*x/(e*x^2+d)*c+1/2
/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a+1/2/e/(d*e)^(1/2)*arctan(e*x/(d*e)
^(1/2))*b-3/2/e^2*d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.84387, size = 541, normalized size = 6.52

$$\left[\frac{4\,cd^2e^2x^3 + \left(3\,cd^3 - bd^2e - ade^2 + \left(3\,cd^2e - bde^2 - ae^3\right)x^2\right)\sqrt{-de}\log\left(\frac{ex^2 - 2\,\sqrt{-de}x - d}{ex^2 + d}\right) + 2\,\left(3\,cd^3e - bd^2e^2 + ade^3\right)x}{4\,\left(d^2e^4x^2 + d^3e^3\right)}, \frac{2\,cd^2e^2 \ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*d^2*e^2*x^3 + (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2
 - a*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) + 2
*(3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d^2*e^4*x^2 + d^3*e^3), 1/2*(2*c*d^2
*e^2*x^3 - (3*c*d^3 - b*d^2*e - a*d*e^2 + (3*c*d^2*e - b*d*e^2 - a*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^3*e - b*d^2*e^2 + a*d*e^3)*x)/(d
^2*e^4*x^2 + d^3*e^3)]
```

Sympy [B] time = 1.0066, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x\left(ae^2 - bde + cd^2\right)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2 + bde - 3cd^2\right)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}\left(ae^2 + bde - 3cd^2\right)\log\left(d^2e^2\sqrt{-\ldots}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

Giac [A] time = 1.12464, size = 101, normalized size = 1.22

$$cxe^{(-2)} - \frac{\left(3\,cd^2 - bde - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{\left(cd^2x - bdxe + axe^2\right)e^{(-2)}}{2\,\left(x^2e + d\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

3.284 $\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^2}\,dx$

Optimal. Leaf size=89

$$-\frac{x\left(ae^2-bde+cd^2\right)}{2d^2e\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(bd-3ae)+cd^2\right)}{2d^{5/2}e^{3/2}}-\frac{a}{d^2x}$$

```
[Out] -(a/(d^2*x)) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^2*e*(d + e*x^2)) + ((c*d^2
+ e*(b*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*e^(3/2))
```

Rubi [A] time = 0.118452, antiderivative size = 86, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1259, 453, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(bd-3ae)+cd^2\right)}{2d^{5/2}e^{3/2}}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{2\left(d+ex^2\right)}-\frac{a}{d^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -(a/(d^2*x)) - ((c/e - (b*d - a*e)/d^2)*x)/(2*(d + e*x^2)) + ((c*d^2 + e*(b
*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*e^(3/2))
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx = -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2\left(d+ex^2\right)} - \frac{\int \frac{-2ade^2-e\left(cd^2+e(bd-ae)\right)x^2}{x^2\left(d+ex^2\right)}\,dx}{2d^2e^2}$$

$$= -\frac{a}{d^2x} - \frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2\left(d+ex^2\right)} + \frac{1}{2}\left(\frac{c}{e}+\frac{bd-3ae}{d^2}\right)\int \frac{1}{d+ex^2}\,dx$$

$$= -\frac{a}{d^2x} - \frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2\left(d+ex^2\right)} + \frac{\left(cd^2+e(bd-3ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}e^{3/2}}$$

Mathematica [A] time = 0.061662, size = 89, normalized size = 1.

$$-\frac{x\left(ae^2-bde+cd^2\right)}{2d^2e\left(d+ex^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-3ae^2+bde+cd^2\right)}{2d^{5/2}e^{3/2}} - \frac{a}{d^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -(a/(d^2*x)) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^2*e*(d + e*x^2)) + ((c*d^2
+ b*d*e - 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*e^(3/2))
```

Maple [A] time = 0.013, size = 121, normalized size = 1.4

$$-\frac{a}{d^2x} - \frac{exa}{2\,d^2\left(ex^2+d\right)} + \frac{xb}{2\,d\left(ex^2+d\right)} - \frac{xc}{2\,e\left(ex^2+d\right)} - \frac{3\,ae}{2\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}} + \frac{b}{2\,d}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}} + \frac{c}{2\,e}\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x)
```

```
[Out] -a/d^2/x-1/2/d^2*e*x/(e*x^2+d)*a+1/2/d*x/(e*x^2+d)*b-1/2/e*x/(e*x^2+d)*c-3/
2/d^2*e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a+1/2/d/(d*e)^(1/2)*arctan(e*x/
(d*e)^(1/2))*b+1/2/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89304, size = 555, normalized size = 6.24

$$\left[-\frac{4\,ad^2e^2 + 2\left(cd^3e - bd^2e^2 + 3\,ade^3\right)x^2 - \left(\left(cd^2e + bde^2 - 3\,ae^3\right)x^3 + \left(cd^3 + bd^2e - 3\,ade^2\right)x\right)\sqrt{-de}\log\left(\frac{ex^2+2\sqrt{-de}x-d}{ex^2+d}\right)}{4\left(d^3e^3x^3 + d^4e^2x\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*d^2*e^2 + 2*(c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*x^2 - ((c*d^2*e +
b*d*e^2 - 3*a*e^3)*x^3 + (c*d^3 + b*d^2*e - 3*a*d*e^2)*x)*sqrt(-d*e)*log((e
*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^3*e^3*x^3 + d^4*e^2*x), -1/2*(2
*a*d^2*e^2 + (c*d^3*e - b*d^2*e^2 + 3*a*d*e^3)*x^2 - ((c*d^2*e + b*d*e^2 -
3*a*e^3)*x^3 + (c*d^3 + b*d^2*e - 3*a*d*e^2)*x)*sqrt(d*e)*arctan(sqrt(d*e)*
x/d))/(d^3*e^3*x^3 + d^4*e^2*x)]
```

Sympy [A] time = 1.33698, size = 155, normalized size = 1.74

$$\frac{\sqrt{-\frac{1}{d^5e^3}}\left(3ae^2 - bde - cd^2\right)\log\left(-d^3e\sqrt{-\frac{1}{d^5e^3}} + x\right)}{4} - \frac{\sqrt{-\frac{1}{d^5e^3}}\left(3ae^2 - bde - cd^2\right)\log\left(d^3e\sqrt{-\frac{1}{d^5e^3}} + x\right)}{4} - \frac{2ade + x^2\left(3ae^2\right.}{2d^3ex + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**2,x)
```

```
[Out] sqrt(-1/(d**5*e**3))*(3*a*e**2 - b*d*e - c*d**2)*log(-d**3*e*sqrt(-1/(d**5*
e**3)) + x)/4 - sqrt(-1/(d**5*e**3))*(3*a*e**2 - b*d*e - c*d**2)*log(d**3*e
*sqrt(-1/(d**5*e**3)) + x)/4 - (2*a*d*e + x**2*(3*a*e**2 - b*d*e + c*d**2))
/(2*d**3*e*x + 2*d**2*e**2*x**3)
```

Giac [A] time = 1.09488, size = 112, normalized size = 1.26

$$\frac{\left(cd^2 + bde - 3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{2\,d^{\frac{5}{2}}} - \frac{\left(cd^2x^2 - bdx^2e + 3\,ax^2e^2 + 2\,ade\right)e^{(-1)}}{2\left(x^3e + dx\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(c*d^2 + b*d*e - 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(5/2) -
1/2*(c*d^2*x^2 - b*d*x^2*e + 3*a*x^2*e^2 + 2*a*d*e)*e^(-1)/((x^3*e + d*x)*d
^2)
```

3.285 $\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^2} dx$

Optimal. Leaf size=106

$$\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(cd^2-e(3bd-5ae)\right)}{2d^{7/2}\sqrt{e}}-\frac{bd-2ae}{d^3x}-\frac{a}{3d^2x^3}$$

```
[Out] -a/(3*d^2*x^3) - (b*d - 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^3
*(d + e*x^2)) + ((c*d^2 - e*(3*b*d - 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
2*d^(7/2)*Sqrt[e])
```

Rubi [A] time = 0.137395, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1259, 1261, 205}

$$\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(cd^2-e(3bd-5ae)\right)}{2d^{7/2}\sqrt{e}}-\frac{bd-2ae}{d^3x}-\frac{a}{3d^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
```

```
[Out] -a/(3*d^2*x^3) - (b*d - 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^3
*(d + e*x^2)) + ((c*d^2 - e*(3*b*d - 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(
2*d^(7/2)*Sqrt[e])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx = \frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)}+\frac{\int \frac{2ad^2e^2+2de^2(bd-ae)x^2+e^2\left(cd^2-bde+ae^2\right)x^4}{x^4\left(d+ex^2\right)}dx}{2d^3e^2}$$
$$= \frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)}+\frac{\int \left(\frac{2ade^2}{x^4}-\frac{2e^2(-bd+2ae)}{x^2}+\frac{e^2\left(cd^2-e(3bd-5ae)\right)}{d+ex^2}\right)dx}{2d^3e^2}$$
$$= -\frac{a}{3d^2x^3}-\frac{bd-2ae}{d^3x}+\frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)}+\frac{\left(cd^2-e(3bd-5ae)\right)\int \frac{1}{d+ex^2}dx}{2d^3}$$
$$= -\frac{a}{3d^2x^3}-\frac{bd-2ae}{d^3x}+\frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)}+\frac{\left(cd^2-e(3bd-5ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{7/2}\sqrt{e}}$$

Mathematica [A] time = 0.065207, size = 105, normalized size = 0.99

$$\frac{x\left(ae^2-bde+cd^2\right)}{2d^3\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5ae^2-3bde+cd^2\right)}{2d^{7/2}\sqrt{e}}+\frac{2ae-bd}{d^3x}-\frac{a}{3d^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
```

```
[Out] -a/(3*d^2*x^3) + (-(b*d) + 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(2*
d^3*(d + e*x^2)) + ((c*d^2 - 3*b*d*e + 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(2*d^(7/2)*Sqrt[e])
```

Maple [A] time = 0.014, size = 146, normalized size = 1.4

$$-\frac{a}{3\,d^2x^3}+2\,\frac{ae}{d^3x}-\frac{b}{d^2x}+\frac{xae^2}{2\,d^3\left(ex^2+d\right)}-\frac{exb}{2\,d^2\left(ex^2+d\right)}+\frac{xc}{2\,d\left(ex^2+d\right)}+\frac{5\,ae^2}{2\,d^3}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}-\frac{3\,be}{2\,d^2}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x)
```

```
[Out] -1/3*a/d^2/x^3+2/d^3/x*a*e-1/d^2/x*b+1/2/d^3*x/(e*x^2+d)*a*e^2-1/2/d^2*x/(e
*x^2+d)*e*b+1/2/d*x/(e*x^2+d)*c+5/2/d^3/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
*a*e^2-3/2/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*e*b+1/2/d/(d*e)^(1/2)*ar
ctan(e*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86962, size = 672, normalized size = 6.34

$$\left[-\frac{4\,ad^3e-6\left(cd^3e-3\,bd^2e^2+5\,ade^3\right)x^4+4\left(3\,bd^3e-5\,ad^2e^2\right)x^2+3\left(\left(cd^2e-3\,bde^2+5\,ae^3\right)x^5+\left(cd^3-3\,bd^2e+5\,ade^2\right)x\right.}{12\left(d^4e^2x^5+d^5ex^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*a*d^3*e - 6*(c*d^3*e - 3*b*d^2*e^2 + 5*a*d*e^3)*x^4 + 4*(3*b*d^3*
e - 5*a*d^2*e^2)*x^2 + 3*((c*d^2*e - 3*b*d*e^2 + 5*a*e^3)*x^5 + (c*d^3 - 3*
b*d^2*e + 5*a*d*e^2)*x^3)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^
2 + d)))/(d^4*e^2*x^5 + d^5*e*x^3), -1/6*(2*a*d^3*e - 3*(c*d^3*e - 3*b*d^2*
e^2 + 5*a*d*e^3)*x^4 + 2*(3*b*d^3*e - 5*a*d^2*e^2)*x^2 - 3*((c*d^2*e - 3*b*
d*e^2 + 5*a*e^3)*x^5 + (c*d^3 - 3*b*d^2*e + 5*a*d*e^2)*x^3)*sqrt(d*e)*arcta
n(sqrt(d*e)*x/d))/(d^4*e^2*x^5 + d^5*e*x^3)]
```

Sympy [A] time = 1.73317, size = 167, normalized size = 1.58

$$-\frac{\sqrt{-\frac{1}{d^7e}}\left(5ae^2-3bde+cd^2\right)\log\left(-d^4\sqrt{-\frac{1}{d^7e}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^7e}}\left(5ae^2-3bde+cd^2\right)\log\left(d^4\sqrt{-\frac{1}{d^7e}}+x\right)}{4}+\frac{-2ad^2+x^4\left(15ae^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**2,x)
```

```
[Out] -sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(-d**4*sqrt(-1/(d**7*e)
) + x)/4 + sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(d**4*sqrt(-1
/(d**7*e)) + x)/4 + (-2*a*d**2 + x**4*(15*a*e**2 - 9*b*d*e + 3*c*d**2) + x*
*2*(10*a*d*e - 6*b*d**2))/(6*d**4*x**3 + 6*d**3*e*x**5)
```

Giac [A] time = 1.08961, size = 127, normalized size = 1.2

$$\frac{\left(cd^2-3\,bde+5\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{7}{2}}}+\frac{cd^2x-bdxe+axe^2}{2\left(x^2e+d\right)d^3}-\frac{3\,bdx^2-6\,ax^2e+ad}{3\,d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(c*d^2 - 3*b*d*e + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(7/2)
+ 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)/((x^2*e + d)*d^3) - 1/3*(3*b*d*x^2 - 6*
a*x^2*e + a*d)/(d^3*x^3)
```

3.286 $\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^2} dx$

Optimal. Leaf size=136

$$-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}-\frac{cd^2-e(2bd-3ae)}{d^4x}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(5bd-7ae)\right)}{2d^{9/2}}-\frac{bd-2ae}{3d^3x^3}-\frac{a}{5d^2x^5}$$

```
[Out] -a/(5*d^2*x^5) - (b*d - 2*a*e)/(3*d^3*x^3) - (c*d^2 - e*(2*b*d - 3*a*e))/(d
^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2)) - (Sqrt[e]*(3*c*d
^2 - e*(5*b*d - 7*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(9/2))
```

Rubi [A] time = 0.252272, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1259, 1802, 205}

$$-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)}-\frac{cd^2-e(2bd-3ae)}{d^4x}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3cd^2-e(5bd-7ae)\right)}{2d^{9/2}}-\frac{bd-2ae}{3d^3x^3}-\frac{a}{5d^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
```


```
[Out] -a/(5*d^2*x^5) - (b*d - 2*a*e)/(3*d^3*x^3) - (c*d^2 - e*(2*b*d - 3*a*e))/(d
^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2)) - (Sqrt[e]*(3*c*d
^2 - e*(5*b*d - 7*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(9/2))
```


Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^2}dx = -\frac{e\left(cd^2-bde+ae^2\right)x}{2d^4\left(d+ex^2\right)} - \frac{\int \frac{-2ad^3e^2-2d^2e^2(bd-ae)x^2-2de^2\left(cd^2-bde+ae^2\right)x^4+e^3\left(cd^2-bde+ae^2\right)x^6}{x^6\left(d+ex^2\right)}dx}{2d^4e^2}$$

$$= -\frac{e\left(cd^2-bde+ae^2\right)x}{2d^4\left(d+ex^2\right)} - \frac{\int \left(-\frac{2ad^2e^2}{x^6} - \frac{2de^2(bd-2ae)}{x^4} + \frac{2e^2\left(-cd^2+e(2bd-3ae)\right)}{x^2} + \frac{e^3\left(3cd^2-e(5bd-7ae)\right)}{d+ex^2}\right)dx}{2d^4e^2}$$

$$= -\frac{a}{5d^2x^5} - \frac{bd-2ae}{3d^3x^3} - \frac{cd^2-e(2bd-3ae)}{d^4x} - \frac{e\left(cd^2-bde+ae^2\right)x}{2d^4\left(d+ex^2\right)} - \frac{\left(e\left(3cd^2-e(5bd-7ae)\right)\right)\int \frac{1}{d+ex}}{2d^4}$$

$$= -\frac{a}{5d^2x^5} - \frac{bd-2ae}{3d^3x^3} - \frac{cd^2-e(2bd-3ae)}{d^4x} - \frac{e\left(cd^2-bde+ae^2\right)x}{2d^4\left(d+ex^2\right)} - \frac{\sqrt{e}\left(3cd^2-e(5bd-7ae)\right)\tan^{-1}}{2d^{9/2}}$$

Mathematica [A] time = 0.0901613, size = 135, normalized size = 0.99

$$-\frac{ex\left(ae^2-bde+cd^2\right)}{2d^4\left(d+ex^2\right)} + \frac{-3ae^2+2bde-cd^2}{d^4x} - \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(7ae^2-5bde+3cd^2\right)}{2d^{9/2}} + \frac{2ae-bd}{3d^3x^3} - \frac{a}{5d^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
```

```
[Out] -a/(5*d^2*x^5) + (-(b*d) + 2*a*e)/(3*d^3*x^3) + (-(c*d^2) + 2*b*d*e - 3*a*e
^2)/(d^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2)) - (Sqrt[e]*
(3*c*d^2 - 5*b*d*e + 7*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(9/2))
```

Maple [A] time = 0.013, size = 183, normalized size = 1.4

$$-\frac{a}{5\,d^2x^5} + \frac{2\,ae}{3\,d^3x^3} - \frac{b}{3\,d^2x^3} - 3\,\frac{ae^2}{d^4x} + 2\,\frac{be}{d^3x} - \frac{c}{d^2x} - \frac{e^3xa}{2\,d^4\left(ex^2+d\right)} + \frac{e^2xb}{2\,d^3\left(ex^2+d\right)} - \frac{exc}{2\,d^2\left(ex^2+d\right)} - \frac{7\,e^3a}{2\,d^4}\arctan\left(ex\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x)
```

```
[Out] -1/5*a/d^2/x^5+2/3/d^3/x^3*a*e-1/3/d^2/x^3*b-3/d^4/x*a*e^2+2/d^3/x*e*b-1/d^
2/x*c-1/2*e^3/d^4*x/(e*x^2+d)*a+1/2*e^2/d^3*x/(e*x^2+d)*b-1/2*e/d^2*x/(e*x^
2+d)*c-7/2*e^3/d^4/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a+5/2*e^2/d^3/(d*e)^
(1/2)*arctan(e*x/(d*e)^(1/2))*b-3/2*e/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2
))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.90378, size = 775, normalized size = 5.7

$$\left[-\frac{30\left(3\,cd^2e-5\,bde^2+7\,ae^3\right)x^6+20\left(3\,cd^3-5\,bd^2e+7\,ade^2\right)x^4+12\,ad^3+4\left(5\,bd^3-7\,ad^2e\right)x^2-15\left(\left(3\,cd^2e-5\,bde^2\right.\right.}{60\left(d^4ex^7+d^5x^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(30*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 20*(3*c*d^3 - 5*b*d^2*e
+ 7*a*d*e^2)*x^4 + 12*a*d^3 + 4*(5*b*d^3 - 7*a*d^2*e)*x^2 - 15*((3*c*d^2*e
- 5*b*d*e^2 + 7*a*e^3)*x^7 + (3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^5)*sqrt(-e
/d)*log((e*x^2 - 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(d^4*e*x^7 + d^5*x^5),
 -1/30*(15*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 10*(3*c*d^3 - 5*b*d^2*e
+ 7*a*d*e^2)*x^4 + 6*a*d^3 + 2*(5*b*d^3 - 7*a*d^2*e)*x^2 + 15*((3*c*d^2*e -
 5*b*d*e^2 + 7*a*e^3)*x^7 + (3*c*d^3 - 5*b*d^2*e + 7*a*d*e^2)*x^5)*sqrt(e/d
)*arctan(x*sqrt(e/d)))/(d^4*e*x^7 + d^5*x^5)]
```

Sympy [B] time = 2.49606, size = 284, normalized size = 2.09

$$\frac{\sqrt{-\frac{e}{d^9}}\left(7ae^2-5bde+3cd^2\right)\log\left(-\frac{d^5\sqrt{-\frac{e}{d^9}}\left(7ae^2-5bde+3cd^2\right)}{7ae^3-5bde^2+3cd^2e}+x\right)}{4}-\frac{\sqrt{-\frac{e}{d^9}}\left(7ae^2-5bde+3cd^2\right)\log\left(\frac{d^5\sqrt{-\frac{e}{d^9}}\left(7ae^2-5bde+3cd^2\right)}{7ae^3-5bde^2+3cd^2e}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**2,x)
```

```
[Out] sqrt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*log(-d**5*sqrt(-e/d**9)*(7*a*
e**2 - 5*b*d*e + 3*c*d**2)/(7*a*e**3 - 5*b*d*e**2 + 3*c*d**2*e) + x)/4 - sq
rt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*log(d**5*sqrt(-e/d**9)*(7*a*e**
2 - 5*b*d*e + 3*c*d**2)/(7*a*e**3 - 5*b*d*e**2 + 3*c*d**2*e) + x)/4 - (6*a*
d**3 + x**6*(105*a*e**3 - 75*b*d*e**2 + 45*c*d**2*e) + x**4*(70*a*d*e**2 -
50*b*d**2*e + 30*c*d**3) + x**2*(-14*a*d**2*e + 10*b*d**3))/(30*d**5*x**5 +
 30*d**4*e*x**7)
```

Giac [A] time = 1.10171, size = 177, normalized size = 1.3

$$-\frac{\left(3\,cd^2e-5\,bde^2+7\,ae^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{9}{2}}}-\frac{cd^2xe-bdxe^2+axe^3}{2\left(x^2e+d\right)d^4}-\frac{15\,cd^2x^4-30\,bdx^4e+45\,ax^4e^2+5\,bd^2x^2-10}{15\,d^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d
^(9/2) - 1/2*(c*d^2*x*e - b*d*x*e^2 + a*x*e^3)/((x^2*e + d)*d^4) - 1/15*(15
*c*d^2*x^4 - 30*b*d*x^4*e + 45*a*x^4*e^2 + 5*b*d^2*x^2 - 10*a*d*x^2*e + 3*a
*d^2)/(d^4*x^5)
```

3.287 $\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=167

$$\frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5cd^2-e(7bd-9ae)\right)}{2d^{11/2}}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}+\frac{e\left(2cd^2-e(3bd-4ae)\right)}{d^5x}-\frac{bd-}{5d^3}$$

```
[Out] -a/(7*d^2*x^7) - (b*d - 2*a*e)/(5*d^3*x^5) - (c*d^2 - e*(2*b*d - 3*a*e))/(3
*d^4*x^3) + (e*(2*c*d^2 - e*(3*b*d - 4*a*e)))/(d^5*x) + (e^2*(c*d^2 - b*d*e
 + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(5*c*d^2 - e*(7*b*d - 9*a*e))*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(11/2))
```

Rubi [A] time = 0.33021, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1259, 1802, 205}

$$\frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5cd^2-e(7bd-9ae)\right)}{2d^{11/2}}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}+\frac{e\left(2cd^2-e(3bd-4ae)\right)}{d^5x}-\frac{bd-}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
```


```
[Out] -a/(7*d^2*x^7) - (b*d - 2*a*e)/(5*d^3*x^5) - (c*d^2 - e*(2*b*d - 3*a*e))/(3
*d^4*x^3) + (e*(2*c*d^2 - e*(3*b*d - 4*a*e)))/(d^5*x) + (e^2*(c*d^2 - b*d*e
 + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(5*c*d^2 - e*(7*b*d - 9*a*e))*A
rcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(11/2))
```


Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx = \frac{e^2\left(cd^2-bde+ae^2\right)x}{2d^5\left(d+ex^2\right)} + \frac{\int \frac{2ad^4e^2+2d^3e^2(bd-ae)x^2+2d^2e^2\left(cd^2-bde+ae^2\right)x^4-2de^3\left(cd^2-bde+ae^2\right)x^6+e^4\left(cd^2-bde+ae^2\right)x^8}{x^8\left(d+ex^2\right)}\,d}{2d^5e^2}$$

$$= \frac{e^2\left(cd^2-bde+ae^2\right)x}{2d^5\left(d+ex^2\right)} + \frac{\int \left(\frac{2ad^3e^2}{x^8} + \frac{2d^2e^2(bd-2ae)}{x^6} + \frac{2de^2\left(cd^2-e(2bd-3ae)\right)}{x^4} + \frac{2e^3\left(-2cd^2+e(3bd-4ae)\right)}{x^2} + \frac{e^4\left(5cd^2-\right.}{d}\right.}{2d^5e^2}$$

$$= -\frac{a}{7d^2x^7} - \frac{bd-2ae}{5d^3x^5} - \frac{cd^2-e(2bd-3ae)}{3d^4x^3} + \frac{e\left(2cd^2-e(3bd-4ae)\right)}{d^5x} + \frac{e^2\left(cd^2-bde+ae^2\right)x}{2d^5\left(d+ex^2\right)} + \frac{\left(e^2\right.}{}$$

$$= -\frac{a}{7d^2x^7} - \frac{bd-2ae}{5d^3x^5} - \frac{cd^2-e(2bd-3ae)}{3d^4x^3} + \frac{e\left(2cd^2-e(3bd-4ae)\right)}{d^5x} + \frac{e^2\left(cd^2-bde+ae^2\right)x}{2d^5\left(d+ex^2\right)} + \frac{e^{3/2}}{}$$

Mathematica [A] time = 0.0950441, size = 166, normalized size = 0.99

$$\frac{e^2x\left(ae^2-bde+cd^2\right)}{2d^5\left(d+ex^2\right)} + \frac{-3ae^2+2bde-cd^2}{3d^4x^3} + \frac{e\left(4ae^2-3bde+2cd^2\right)}{d^5x} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(9ae^2-7bde+5cd^2\right)}{2d^{11/2}} + \frac{2ae-bc}{5d^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
```

```
[Out] -a/(7*d^2*x^7) + (-(b*d) + 2*a*e)/(5*d^3*x^5) + (-(c*d^2) + 2*b*d*e - 3*a*e
^2)/(3*d^4*x^3) + (e*(2*c*d^2 - 3*b*d*e + 4*a*e^2))/(d^5*x) + (e^2*(c*d^2 -
 b*d*e + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(5*c*d^2 - 7*b*d*e + 9*a*
e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(11/2))
```

Maple [A] time = 0.016, size = 221, normalized size = 1.3

$$-\frac{a}{7\,d^2x^7} + \frac{2\,ae}{5\,d^3x^5} - \frac{b}{5\,d^2x^5} - \frac{ae^2}{d^4x^3} + \frac{2\,be}{3\,d^3x^3} - \frac{c}{3\,d^2x^3} + 4\,\frac{e^3a}{d^5x} - 3\,\frac{e^2b}{d^4x} + 2\,\frac{ce}{d^3x} + \frac{e^4xa}{2\,d^5\left(ex^2+d\right)} - \frac{e^3xb}{2\,d^4\left(ex^2+d\right)} + \frac{e}{2\,d^3\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x)
```

```
[Out] -1/7*a/d^2/x^7+2/5/d^3/x^5*a*e-1/5/d^2/x^5*b-1/d^4/x^3*a*e^2+2/3/d^3/x^3*e*
b-1/3/d^2/x^3*c+4*e^3/d^5/x*a-3*e^2/d^4/x*b+2*e/d^3/x*c+1/2*e^4/d^5*x/(e*x^
2+d)*a-1/2*e^3/d^4*x/(e*x^2+d)*b+1/2*e^2/d^3*x/(e*x^2+d)*c+9/2*e^4/d^5/(d*e
)^(1/2)*arctan(e*x/(d*e)^(1/2))*a-7/2*e^3/d^4/(d*e)^(1/2)*arctan(e*x/(d*e)^
(1/2))*b+5/2*e^2/d^3/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81143, size = 938, normalized size = 5.62

$$\left[\frac{210\left(5\,cd^2e^2-7\,bde^3+9\,ae^4\right)x^8+140\left(5\,cd^3e-7\,bd^2e^2+9\,ade^3\right)x^6-60\,ad^4-28\left(5\,cd^4-7\,bd^3e+9\,ad^2e^2\right)x^4-12\,(7\cdots}{420\left(d^5ex^9+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(210*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 140*(5*c*d^3*e - 7*b*
d^2*e^2 + 9*a*d*e^3)*x^6 - 60*a*d^4 - 28*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2
)*x^4 - 12*(7*b*d^4 - 9*a*d^3*e)*x^2 + 105*((5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*
e^4)*x^9 + (5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^7)*sqrt(-e/d)*log((e*x^2
 + 2*d*x*sqrt(-e/d) - d)/(e*x^2 + d)))/(d^5*e*x^9 + d^6*x^7), 1/210*(105*(5
*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 70*(5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d
*e^3)*x^6 - 30*a*d^4 - 14*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2)*x^4 - 6*(7*b*
d^4 - 9*a*d^3*e)*x^2 + 105*((5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^9 + (5*c*
d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^7)*sqrt(e/d)*arctan(x*sqrt(e/d)))/(d^5*e
*x^9 + d^6*x^7)]
```

Sympy [B] time = 3.36566, size = 328, normalized size = 1.96

$$-\frac{\sqrt{-\frac{e^3}{d^{11}}}\left(9ae^2-7bde+5cd^2\right)\log\left(-\frac{d^6\sqrt{-\frac{e^3}{d^{11}}}\left(9ae^2-7bde+5cd^2\right)}{9ae^4-7bde^3+5cd^2e^2}+x\right)}{4}+\frac{\sqrt{-\frac{e^3}{d^{11}}}\left(9ae^2-7bde+5cd^2\right)\log\left(\frac{d^6\sqrt{-\frac{e^3}{d^{11}}}\left(9ae^2-7bde+\cdots\right.}{9ae^4-7bde^3+5cd^2e\cdots}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**8/(e*x**2+d)**2,x)
```

```
[Out] -sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*log(-d**6*sqrt(-e**3/d**
11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*c*d**2*e**2)
 + x)/4 + sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*log(d**6*sqrt(-
e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*c*d*
*2*e**2) + x)/4 + (-30*a*d**4 + x**8*(945*a*e**4 - 735*b*d*e**3 + 525*c*d**
2*e**2) + x**6*(630*a*d*e**3 - 490*b*d**2*e**2 + 350*c*d**3*e) + x**4*(-126
*a*d**2*e**2 + 98*b*d**3*e - 70*c*d**4) + x**2*(54*a*d**3*e - 42*b*d**4))/(
210*d**6*x**7 + 210*d**5*e*x**9)
```

Giac [A] time = 1.09087, size = 221, normalized size = 1.32

$$\frac{\left(5\,cd^2e^2-7\,bde^3+9\,ae^4\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{11}{2}}}+\frac{cd^2xe^2-bdxe^3+axe^4}{2\left(x^2e+d\right)d^5}+\frac{210\,cd^2x^6e-315\,bdx^6e^2-35\,cd^3x^4+420\,ax^{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/
d^(11/2) + 1/2*(c*d^2*x*e^2 - b*d*x*e^3 + a*x*e^4)/((x^2*e + d)*d^5) + 1/10
5*(210*c*d^2*x^6*e - 315*b*d*x^6*e^2 - 35*c*d^3*x^4 + 420*a*x^6*e^3 + 70*b*
d^2*x^4*e - 105*a*d*x^4*e^2 - 21*b*d^3*x^2 + 42*a*d^2*x^2*e - 15*a*d^3)/(d^
5*x^7)
```

3.288 $\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3} dx$

Optimal. Leaf size=173

$$\frac{dx\left(17cd^2-e(13bd-9ae)\right)}{8e^5\left(d+ex^2\right)}-\frac{d^2x\left(ae^2-bde+cd^2\right)}{4e^5\left(d+ex^2\right)^2}+\frac{x\left(6cd^2-e(3bd-ae)\right)}{e^5}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(15ae^2-35bde+63cd\right.}{8e^{11/2}}$$

```
[Out] ((6*c*d^2 - e*(3*b*d - a*e))*x)/e^5 - ((3*c*d - b*e)*x^3)/(3*e^4) + (c*x^5)
/(5*e^3) - (d^2*(c*d^2 - b*d*e + a*e^2)*x)/(4*e^5*(d + e*x^2)^2) + (d*(17*c
*d^2 - e*(13*b*d - 9*a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^2 - 35
*b*d*e + 15*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

Rubi [A] time = 0.321216, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1257, 1814, 1810, 205}

$$\frac{dx\left(17cd^2-e(13bd-9ae)\right)}{8e^5\left(d+ex^2\right)}-\frac{d^2x\left(ae^2-bde+cd^2\right)}{4e^5\left(d+ex^2\right)^2}+\frac{x\left(6cd^2-e(3bd-ae)\right)}{e^5}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(15ae^2-35bde+63cd\right.}{8e^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((6*c*d^2 - e*(3*b*d - a*e))*x)/e^5 - ((3*c*d - b*e)*x^3)/(3*e^4) + (c*x^5)
/(5*e^3) - (d^2*(c*d^2 - b*d*e + a*e^2)*x)/(4*e^5*(d + e*x^2)^2) + (d*(17*c
*d^2 - e*(13*b*d - 9*a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^2 - 35
*b*d*e + 15*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^3}dx = -\frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2}-\frac{\int \frac{-d^2\left(cd^2-bde+ae^2\right)+4de\left(cd^2-bde+ae^2\right)x^2-4e^2\left(cd^2-bde+ae^2\right)x^4+4e^3(cd-be)x^6-4ce^4x^8}{\left(d+ex^2\right)^2}}{4e^5}$$

$$= -\frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2}+\frac{d\left(17cd^2-e(13bd-9ae)\right)x}{8e^5\left(d+ex^2\right)}+\frac{\int \frac{-d^2\left(15cd^2-e(11bd-7ae)\right)+8de\left(3cd^2-e(2bd-ae\right.}{d+ex^2}}{8de^5}$$

$$= -\frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2}+\frac{d\left(17cd^2-e(13bd-9ae)\right)x}{8e^5\left(d+ex^2\right)}+\frac{\int \left(8d\left(6cd^2-e(3bd-ae)\right)-8de(3cd\right.}{}$$

$$= \frac{\left(6cd^2-e(3bd-ae)\right)x}{e^5}-\frac{(3cd-be)x^3}{3e^4}+\frac{cx^5}{5e^3}-\frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2}+\frac{d\left(17cd^2-e(13bd-9\right.}{8e^5\left(d+ex^2\right)}$$

$$= \frac{\left(6cd^2-e(3bd-ae)\right)x}{e^5}-\frac{(3cd-be)x^3}{3e^4}+\frac{cx^5}{5e^3}-\frac{d^2\left(cd^2-bde+ae^2\right)x}{4e^5\left(d+ex^2\right)^2}+\frac{d\left(17cd^2-e(13bd-9\right.}{8e^5\left(d+ex^2\right)}$$

Mathematica [A] time = 0.111269, size = 170, normalized size = 0.98

$$\frac{x\left(de(9ae-13bd)+17cd^3\right)}{8e^5\left(d+ex^2\right)}-\frac{x\left(d^2e(ae-bd)+cd^4\right)}{4e^5\left(d+ex^2\right)^2}+\frac{x\left(e(ae-3bd)+6cd^2\right)}{e^5}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(5e(3ae-7bd)+63cd^2\right)}{8e^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((6*c*d^2 + e*(-3*b*d + a*e))*x)/e^5 + ((-3*c*d + b*e)*x^3)/(3*e^4) + (c*x^
5)/(5*e^3) - ((c*d^4 + d^2*e*(-(b*d) + a*e))*x)/(4*e^5*(d + e*x^2)^2) + ((1
7*c*d^3 + d*e*(-13*b*d + 9*a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^
2 + 5*e*(-7*b*d + 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

Maple [A] time = 0.011, size = 239, normalized size = 1.4

$$\frac{cx^5}{5e^3}+\frac{x^3b}{3e^3}-\frac{x^3cd}{e^4}+\frac{ax}{e^3}-3\frac{bdx}{e^4}+6\frac{cd^2x}{e^5}+\frac{9dx^3a}{8e^2\left(ex^2+d\right)^2}-\frac{13d^2x^3b}{8e^3\left(ex^2+d\right)^2}+\frac{17d^3x^3c}{8e^4\left(ex^2+d\right)^2}+\frac{7ad^2x}{8e^3\left(ex^2+d\right)^2}-\frac{11}{8e^4(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] 1/5*c*x^5/e^3+1/3/e^3*x^3*b-1/e^4*x^3*c*d+1/e^3*a*x-3/e^4*d*b*x+6/e^5*c*d^2
*x+9/8*d/e^2/(e*x^2+d)^2*x^3*a-13/8*d^2/e^3/(e*x^2+d)^2*x^3*b+17/8*d^3/e^4/
(e*x^2+d)^2*x^3*c+7/8*d^2/e^3/(e*x^2+d)^2*a*x-11/8*d^3/e^4/(e*x^2+d)^2*b*x+
15/8*d^4/e^5/(e*x^2+d)^2*c*x-15/8*d/e^3/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

```
*a+35/8*d^2/e^4/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*b-63/8*d^3/e^5/(d*e)^(1
/2)*arctan(e*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86095, size = 1125, normalized size = 6.5

$$\left[\frac{48\,ce^4x^9 - 16\left(9\,cde^3 - 5\,be^4\right)x^7 + 16\left(63\,cd^2e^2 - 35\,bde^3 + 15\,ae^4\right)x^5 + 50\left(63\,cd^3e - 35\,bd^2e^2 + 15\,ade^3\right)x^3 + 15\left(63\,c\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*c*e^4*x^9 - 16*(9*c*d*e^3 - 5*b*e^4)*x^7 + 16*(63*c*d^2*e^2 - 35
*b*d*e^3 + 15*a*e^4)*x^5 + 50*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^3
+ 15*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2 + (63*c*d^2*e^2 - 35*b*d*e^3 + 1
5*a*e^4)*x^4 + 2*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^2)*sqrt(-d/e)*l
og((e*x^2 - 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)) + 30*(63*c*d^4 - 35*b*d^3*e
+ 15*a*d^2*e^2)*x)/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e^5), 1/120*(24*c*e^4*x^9 -
 8*(9*c*d*e^3 - 5*b*e^4)*x^7 + 8*(63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^5
 + 25*(63*c*d^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^3 - 15*(63*c*d^4 - 35*b*d^
3*e + 15*a*d^2*e^2 + (63*c*d^2*e^2 - 35*b*d*e^3 + 15*a*e^4)*x^4 + 2*(63*c*d
^3*e - 35*b*d^2*e^2 + 15*a*d*e^3)*x^2)*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d) +
15*(63*c*d^4 - 35*b*d^3*e + 15*a*d^2*e^2)*x)/(e^7*x^4 + 2*d*e^6*x^2 + d^2*e
^5)]
```

Sympy [A] time = 4.37259, size = 233, normalized size = 1.35

$$\frac{cx^5}{5e^3} + \frac{\sqrt{-\frac{d}{e^{11}}}\left(15ae^2 - 35bde + 63cd^2\right)\log\left(-e^5\sqrt{-\frac{d}{e^{11}}} + x\right)}{16} - \frac{\sqrt{-\frac{d}{e^{11}}}\left(15ae^2 - 35bde + 63cd^2\right)\log\left(e^5\sqrt{-\frac{d}{e^{11}}} + x\right)}{16} + \frac{x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x**5/(5*e**3) + sqrt(-d/e**11)*(15*a*e**2 - 35*b*d*e + 63*c*d**2)*log(-e*
*5*sqrt(-d/e**11) + x)/16 - sqrt(-d/e**11)*(15*a*e**2 - 35*b*d*e + 63*c*d**
2)*log(e**5*sqrt(-d/e**11) + x)/16 + (x**3*(9*a*d*e**3 - 13*b*d**2*e**2 + 1
7*c*d**3*e) + x*(7*a*d**2*e**2 - 11*b*d**3*e + 15*c*d**4))/(8*d**2*e**5 + 1
```

```
6*d*e**6*x**2 + 8*e**7*x**4) + x**3*(b*e - 3*c*d)/(3*e**4) + x*(a*e**2 - 3*
b*d*e + 6*c*d**2)/e**5
```

Giac [A] time = 1.09991, size = 216, normalized size = 1.25

$$-\frac{\left(63\,cd^3-35\,bd^2e+15\,ade^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}}{8\,\sqrt{d}}+\frac{1}{15}\left(3\,cx^5e^{12}-15\,cdx^3e^{11}+5\,bx^3e^{12}+90\,cd^2xe^{10}-45\,bdxe^{11}+15\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/8*(63*c*d^3 - 35*b*d^2*e + 15*a*d*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-11/
2)/sqrt(d) + 1/15*(3*c*x^5*e^12 - 15*c*d*x^3*e^11 + 5*b*x^3*e^12 + 90*c*d^2
*x*e^10 - 45*b*d*x*e^11 + 15*a*x*e^12)*e^(-15) + 1/8*(17*c*d^3*x^3*e - 13*b
*d^2*x^3*e^2 + 15*c*d^4*x + 9*a*d*x^3*e^3 - 11*b*d^3*x*e + 7*a*d^2*x*e^2)*e
^(-5)/(x^2*e + d)^2
```

3.289 $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3} dx$

Optimal. Leaf size=143

$$-\frac{x\left(13cd^2-e(9bd-5ae)\right)}{8e^4\left(d+ex^2\right)}+\frac{dx\left(ae^2-bde+cd^2\right)}{4e^4\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(35cd^2-3e(5bd-ae)\right)}{8\sqrt{d}e^{9/2}}-\frac{x(3cd-be)}{e^4}+\frac{cx^3}{3e^3}$$

```
[Out] -(((3*c*d - b*e)*x)/e^4) + (c*x^3)/(3*e^3) + (d*(c*d^2 - b*d*e + a*e^2)*x)/
(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - e*(9*b*d - 5*a*e))*x)/(8*e^4*(d + e*x^
2)) + ((35*c*d^2 - 3*e*(5*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[
d]*e^(9/2))
```

Rubi [A] time = 0.2102, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1257, 1814, 1153, 205}

$$-\frac{x\left(13cd^2-e(9bd-5ae)\right)}{8e^4\left(d+ex^2\right)}+\frac{dx\left(ae^2-bde+cd^2\right)}{4e^4\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(35cd^2-3e(5bd-ae)\right)}{8\sqrt{d}e^{9/2}}-\frac{x(3cd-be)}{e^4}+\frac{cx^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] -(((3*c*d - b*e)*x)/e^4) + (c*x^3)/(3*e^3) + (d*(c*d^2 - b*d*e + a*e^2)*x)/
(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - e*(9*b*d - 5*a*e))*x)/(8*e^4*(d + e*x^
2)) + ((35*c*d^2 - 3*e*(5*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[
d]*e^(9/2))
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
```

```
+ a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^3}\,dx = \frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2}-\frac{\int \frac{d\left(cd^2-bde+ae^2\right)-4e\left(cd^2-bde+ae^2\right)x^2+4e^2(cd-be)x^4-4ce^3x^6}{\left(d+ex^2\right)^2}\,dx}{4e^4}$$

$$= \frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2}-\frac{\left(13cd^2-e(9bd-5ae)\right)x}{8e^4\left(d+ex^2\right)}+\frac{\int \frac{d\left(11cd^2-e(7bd-3ae)\right)-8de(2cd-be)x^2+8cde^2x^4}{d+ex^2}\,dx}{8de^4}$$

$$= \frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2}-\frac{\left(13cd^2-e(9bd-5ae)\right)x}{8e^4\left(d+ex^2\right)}+\frac{\int \left(-8d(3cd-be)+8cdex^2+\frac{35cd^3-15bd^2e+3}{d+ex^2}\right.}{8de^4}$$

$$= -\frac{(3cd-be)x}{e^4}+\frac{cx^3}{3e^3}+\frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2}-\frac{\left(13cd^2-e(9bd-5ae)\right)x}{8e^4\left(d+ex^2\right)}+\frac{\left(35cd^2-3e(5bd-a\right.}{8e^4}$$

$$= -\frac{(3cd-be)x}{e^4}+\frac{cx^3}{3e^3}+\frac{d\left(cd^2-bde+ae^2\right)x}{4e^4\left(d+ex^2\right)^2}-\frac{\left(13cd^2-e(9bd-5ae)\right)x}{8e^4\left(d+ex^2\right)}+\frac{\left(35cd^2-3e(5bd-a\right.}{8\sqrt{d}e^{9}}$$

Mathematica [A] time = 0.0873107, size = 141, normalized size = 0.99

$$-\frac{x\left(5ae^2-9bde+13cd^2\right)}{8e^4\left(d+ex^2\right)}+\frac{x\left(ade^2-bd^2e+cd^3\right)}{4e^4\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3ae^2-15bde+35cd^2\right)}{8\sqrt{d}e^{9/2}}+\frac{x(be-3cd)}{e^4}+\frac{cx^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((-3*c*d + b*e)*x)/e^4 + (c*x^3)/(3*e^3) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)/
(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 9*b*d*e + 5*a*e^2)*x)/(8*e^4*(d + e*x^
2)) + ((35*c*d^2 - 15*b*d*e + 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt
[d]*e^(9/2))
```

Maple [A] time = 0.012, size = 202, normalized size = 1.4

$$\frac{cx^3}{3e^3}+\frac{bx}{e^3}-3\frac{cdx}{e^4}-\frac{5x^3a}{8e\left(ex^2+d\right)^2}+\frac{9x^3bd}{8e^2\left(ex^2+d\right)^2}-\frac{13x^3cd^2}{8e^3\left(ex^2+d\right)^2}-\frac{3adx}{8e^2\left(ex^2+d\right)^2}+\frac{7d^2bx}{8e^3\left(ex^2+d\right)^2}-\frac{11cd^3x}{8e^4\left(ex^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] 1/3*c*x^3/e^3+1/e^3*b*x-3/e^4*c*d*x-5/8/e/(e*x^2+d)^2*x^3*a+9/8/e^2/(e*x^2+
d)^2*x^3*b*d-13/8/e^3/(e*x^2+d)^2*x^3*c*d^2-3/8/e^2/(e*x^2+d)^2*d*a*x+7/8/e
```

```
^3/(e*x^2+d)^2*d^2*b*x-11/8/e^4/(e*x^2+d)^2*c*d^3*x+3/8/e^2/(d*e)^(1/2)*arc
tan(e*x/(d*e)^(1/2))*a-15/8/e^3/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*d*b+35/
8/e^4/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85582, size = 1010, normalized size = 7.06

$$\left[\frac{16\,cde^4x^7 - 16\left(7\,cd^2e^3 - 3\,bde^4\right)x^5 - 10\left(35\,cd^3e^2 - 15\,bd^2e^3 + 3\,ade^4\right)x^3 - 3\left(35\,cd^4 - 15\,bd^3e + 3\,ad^2e^2 + \left(35\,cd^2e^2 - \right.\right.}{48\left(de^7x^4 + 2\,d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")

[Out] [1/48*(16*c*d*e^4*x^7 - 16*(7*c*d^2*e^3 - 3*b*d*e^4)*x^5 - 10*(35*c*d^3*e^2
 - 15*b*d^2*e^3 + 3*a*d*e^4)*x^3 - 3*(35*c*d^4 - 15*b*d^3*e + 3*a*d^2*e^2 +
 (35*c*d^2*e^2 - 15*b*d*e^3 + 3*a*e^4)*x^4 + 2*(35*c*d^3*e - 15*b*d^2*e^2 +
 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) -
 6*(35*c*d^4*e - 15*b*d^3*e^2 + 3*a*d^2*e^3)*x)/(d*e^7*x^4 + 2*d^2*e^6*x^2
+ d^3*e^5), 1/24*(8*c*d*e^4*x^7 - 8*(7*c*d^2*e^3 - 3*b*d*e^4)*x^5 - 5*(35*c
*d^3*e^2 - 15*b*d^2*e^3 + 3*a*d*e^4)*x^3 + 3*(35*c*d^4 - 15*b*d^3*e + 3*a*d
^2*e^2 + (35*c*d^2*e^2 - 15*b*d*e^3 + 3*a*e^4)*x^4 + 2*(35*c*d^3*e - 15*b*d
^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - 3*(35*c*d^4*e -
15*b*d^3*e^2 + 3*a*d^2*e^3)*x)/(d*e^7*x^4 + 2*d^2*e^6*x^2 + d^3*e^5)]
```

Sympy [A] time = 3.93966, size = 211, normalized size = 1.48

$$\frac{cx^3}{3e^3} - \frac{\sqrt{-\frac{1}{de^9}}\left(3ae^2 - 15bde + 35cd^2\right)\log\left(-de^4\sqrt{-\frac{1}{de^9}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{de^9}}\left(3ae^2 - 15bde + 35cd^2\right)\log\left(de^4\sqrt{-\frac{1}{de^9}} + x\right)}{16} - \frac{x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)

[Out] c*x**3/(3*e**3) - sqrt(-1/(d*e**9))*(3*a*e**2 - 15*b*d*e + 35*c*d**2)*log(-
d*e**4*sqrt(-1/(d*e**9)) + x)/16 + sqrt(-1/(d*e**9))*(3*a*e**2 - 15*b*d*e +
 35*c*d**2)*log(d*e**4*sqrt(-1/(d*e**9)) + x)/16 - (x**3*(5*a*e**3 - 9*b*d*
e**2 + 13*c*d**2*e) + x*(3*a*d*e**2 - 7*b*d**2*e + 11*c*d**3))/(8*d**2*e**4
 + 16*d*e**5*x**2 + 8*e**6*x**4) + x*(b*e - 3*c*d)/e**4
```

Giac [A] time = 1.07332, size = 169, normalized size = 1.18

$$\frac{\left(35\,cd^2-15\,bde+3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\sqrt{d}}+\frac{1}{3}\left(cx^3e^6-9\,cdxe^5+3\,bxe^6\right)e^{(-9)}-\frac{\left(13\,cd^2x^3e-9\,bdx^3e^2+11\,cd^3x+5\,ax^3\right.}{8\left(x^2e+d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*c*d^2 - 15*b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt
(d) + 1/3*(c*x^3*e^6 - 9*c*d*x*e^5 + 3*b*x*e^6)*e^(-9) - 1/8*(13*c*d^2*x^3*
e - 9*b*d*x^3*e^2 + 11*c*d^3*x + 5*a*x^3*e^3 - 7*b*d^2*x*e + 3*a*d*x*e^2)*e
^(-4)/(x^2*e + d)^2
```

3.290 $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=124

$$\frac{x(9cd^2-e(5bd-ae))}{8de^3(d+ex^2)}-\frac{x(ae^2-bde+cd^2)}{4e^3(d+ex^2)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(15cd^2-e(ae+3bd))}{8d^{3/2}e^{7/2}}+\frac{cx}{e^3}$$

```
[Out] (c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + ((9*c*d^2 -
 e*(5*b*d - a*e))*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2 - e*(3*b*d + a*e))*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/2))
```

Rubi [A] time = 0.137904, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1257, 1157, 388, 205}

$$\frac{x(9cd^2-e(5bd-ae))}{8de^3(d+ex^2)}-\frac{x(ae^2-bde+cd^2)}{4e^3(d+ex^2)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(15cd^2-e(ae+3bd))}{8d^{3/2}e^{7/2}}+\frac{cx}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] (c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + ((9*c*d^2 -
 e*(5*b*d - a*e))*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2 - e*(3*b*d + a*e))*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/2))
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^3}\,dx &= -\frac{\left(cd^2-bde+ae^2\right)x}{4e^3\left(d+ex^2\right)^2}-\frac{\int \frac{-cd^2+bde-ae^2+4e(cd-be)x^2-4ce^2x^4}{\left(d+ex^2\right)^2}\,dx}{4e^3}\\
&= -\frac{\left(cd^2-bde+ae^2\right)x}{4e^3\left(d+ex^2\right)^2}+\frac{\left(9cd^2-e(5bd-ae)\right)x}{8de^3\left(d+ex^2\right)}+\frac{\int \frac{-7cd^2+e(3bd+ae)+8cdex^2}{d+ex^2}\,dx}{8de^3}\\
&= \frac{cx}{e^3}-\frac{\left(cd^2-bde+ae^2\right)x}{4e^3\left(d+ex^2\right)^2}+\frac{\left(9cd^2-e(5bd-ae)\right)x}{8de^3\left(d+ex^2\right)}-\frac{\left(15cd^2-e(3bd+ae)\right)\int \frac{1}{d+ex^2}\,dx}{8de^3}\\
&= \frac{cx}{e^3}-\frac{\left(cd^2-bde+ae^2\right)x}{4e^3\left(d+ex^2\right)^2}+\frac{\left(9cd^2-e(5bd-ae)\right)x}{8de^3\left(d+ex^2\right)}-\frac{\left(15cd^2-e(3bd+ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.10437, size = 122, normalized size = 0.98

$$\frac{x\left(ae^2-5bde+9cd^2\right)}{8de^3\left(d+ex^2\right)}-\frac{x\left(ae^2-bde+cd^2\right)}{4e^3\left(d+ex^2\right)^2}-\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(-ae^2-3bde+15cd^2\right)}{8d^{3/2}e^{7/2}}+\frac{cx}{e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] (c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + ((9*c*d^2 -
 5*b*d*e + a*e^2)*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2 - 3*b*d*e - a*e^2)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/2))
```

Maple [A] time = 0.01, size = 179, normalized size = 1.4

$$\frac{cx}{e^3}+\frac{x^3a}{8\left(ex^2+d\right)^2d}-\frac{5\,x^3b}{8\,e\left(ex^2+d\right)^2}+\frac{9\,x^3cd}{8\,e^2\left(ex^2+d\right)^2}-\frac{ax}{8\,e\left(ex^2+d\right)^2}-\frac{3\,bdx}{8\,e^2\left(ex^2+d\right)^2}+\frac{7\,cd^2x}{8\,e^3\left(ex^2+d\right)^2}+\frac{a}{8\,de}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] c*x/e^3+1/8/(e*x^2+d)^2/d*x^3*a-5/8/e/(e*x^2+d)^2*x^3*b+9/8/e^2/(e*x^2+d)^2
*x^3*c*d-1/8/e/(e*x^2+d)^2*a*x-3/8/e^2/(e*x^2+d)^2*d*b*x+7/8/e^3/(e*x^2+d)^
2*c*d^2*x+1/8/e/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a+3/8/e^2/(d*e)^(1/2)
*arctan(e*x/(d*e)^(1/2))*b-15/8/e^3*d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7991, size = 873, normalized size = 7.04

$$\left[\frac{16\,cd^2e^3x^5 + 2\left(25\,cd^3e^2 - 5\,bd^2e^3 + ade^4\right)x^3 + \left(15\,cd^4 - 3\,bd^3e - ad^2e^2 + \left(15\,cd^2e^2 - 3\,bde^3 - ae^4\right)x^4 + 2\left(15\,cd^3e - 3\right.\right.}{16\left(d^2e^6x^4 + 2\,d^3e^5x^2 + d^4e^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*c*d^2*e^3*x^5 + 2*(25*c*d^3*e^2 - 5*b*d^2*e^3 + a*d*e^4)*x^3 + (1
5*c*d^4 - 3*b*d^3*e - a*d^2*e^2 + (15*c*d^2*e^2 - 3*b*d*e^3 - a*e^4)*x^4 +
2*(15*c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(
-d*e)*x - d)/(e*x^2 + d)) + 2*(15*c*d^4*e - 3*b*d^3*e^2 - a*d^2*e^3)*x)/(d^
2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4), 1/8*(8*c*d^2*e^3*x^5 + (25*c*d^3*e^2
- 5*b*d^2*e^3 + a*d*e^4)*x^3 - (15*c*d^4 - 3*b*d^3*e - a*d^2*e^2 + (15*c*d^
2*e^2 - 3*b*d*e^3 - a*e^4)*x^4 + 2*(15*c*d^3*e - 3*b*d^2*e^2 - a*d*e^3)*x^2
)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (15*c*d^4*e - 3*b*d^3*e^2 - a*d^2*e^3)*
x)/(d^2*e^6*x^4 + 2*d^3*e^5*x^2 + d^4*e^4)]
```

Sympy [A] time = 3.09714, size = 201, normalized size = 1.62

$$\frac{cx}{e^3} - \frac{\sqrt{-\frac{1}{d^3e^7}}\left(ae^2 + 3bde - 15cd^2\right)\log\left(-d^2e^3\sqrt{-\frac{1}{d^3e^7}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{d^3e^7}}\left(ae^2 + 3bde - 15cd^2\right)\log\left(d^2e^3\sqrt{-\frac{1}{d^3e^7}} + x\right)}{16} + \frac{x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x/e**3 - sqrt(-1/(d**3*e**7))*(a*e**2 + 3*b*d*e - 15*c*d**2)*log(-d**2*e*
*3*sqrt(-1/(d**3*e**7)) + x)/16 + sqrt(-1/(d**3*e**7))*(a*e**2 + 3*b*d*e -
15*c*d**2)*log(d**2*e**3*sqrt(-1/(d**3*e**7)) + x)/16 + (x**3*(a*e**3 - 5*b
*d*e**2 + 9*c*d**2*e) + x*(-a*d*e**2 - 3*b*d**2*e + 7*c*d**3))/(8*d**3*e**3
 + 16*d**2*e**4*x**2 + 8*d*e**5*x**4)
```

Giac [A] time = 1.08884, size = 144, normalized size = 1.16

$$cxe^{(-3)} - \frac{\left(15\,cd^2 - 3\,bde - ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{8\,d^{\frac{3}{2}}} + \frac{\left(9\,cd^2x^3e - 5\,bdx^3e^2 + 7\,cd^3x + ax^3e^3 - 3\,bd^2xe - adxe^2\right)e^{(-3)}}{8\left(x^2e + d\right)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] c*x*e^(-3) - 1/8*(15*c*d^2 - 3*b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(
-7/2)/d^(3/2) + 1/8*(9*c*d^2*x^3*e - 5*b*d*x^3*e^2 + 7*c*d^3*x + a*x^3*e^3
- 3*b*d^2*x*e - a*d*x*e^2)*e^(-3)/((x^2*e + d)^2*d)
```

3.291 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=115

$$-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(4*d*(d + e*x^2)^2) - ((5*c*d^2 - e*(b*d + 3*
a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rubi [A] time = 0.116052, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1157, 385, 205}

$$-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}+\frac{x\left(a+\frac{d(cd-be)}{e^2}\right)}{4d\left(d+ex^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((a + (d*(c*d - b*e))/e^2)*x)/(4*d*(d + e*x^2)^2) - ((5*c*d^2 - e*(b*d + 3*
a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^3}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2} - \frac{\int \frac{-3a+\frac{d(cd-be)}{e^2}-\frac{4cdx^2}{e}}{\left(d+ex^2\right)^2}\,dx}{4d}$$
$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2} - \frac{\left(5cd^2-e(bd+3ae)\right)x}{8d^2e^2\left(d+ex^2\right)} - \frac{\left(-\frac{4cd^2}{e}+e\left(-3a+\frac{d(cd-be)}{e^2}\right)\right)\int \frac{1}{d+ex^2}\,dx}{8d^2e}$$
$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2} - \frac{\left(5cd^2-e(bd+3ae)\right)x}{8d^2e^2\left(d+ex^2\right)} + \frac{\left(3cd^2+e(bd+3ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Mathematica [A] time = 0.0970242, size = 110, normalized size = 0.96

$$\frac{x\left(e\left(ae\left(5d+3ex^2\right)+bd\left(ex^2-d\right)\right)-cd^2\left(3d+5ex^2\right)\right)}{8d^2e^2\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] (x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3*e*x^2))))
/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Maple [A] time = 0.008, size = 131, normalized size = 1.1

$$\frac{1}{\left(ex^2+d\right)^2}\left(\frac{\left(3\,ae^2+deb-5\,cd^2\right)x^3}{8\,d^2e}+\frac{\left(5\,ae^2-deb-3\,cd^2\right)x}{8\,e^2d}\right)+\frac{3\,a}{8\,d^2}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}+\frac{b}{8\,de}\arctan\left(ex\frac{1}{\sqrt{de}}\right)\frac{1}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x)
```

```
[Out] (1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2)/e^2/d*x)
/(e*x^2+d)^2+3/8/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a+1/8/d/e/(d*e)^(1
/2)*arctan(e*x/(d*e)^(1/2))*b+3/8/e^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.83935, size = 813, normalized size = 7.07

$$\left[-\frac{2\left(5\,cd^3e^2 - bd^2e^3 - 3\,ade^4\right)x^3 + \left(3\,cd^4 + bd^3e + 3\,ad^2e^2 + \left(3\,cd^2e^2 + bde^3 + 3\,ae^4\right)x^4 + 2\left(3\,cd^3e + bd^2e^2 + 3\,ade^3\right)x^2\right.}{16\left(d^3e^5x^4 + 2\,d^4e^4x^2 + d^5e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 + (3*c*d^4 + b*d^3*e +
3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3*e + b*d^2*
e^2 + 3*a*d*e^3)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 +
d)) + 2*(3*c*d^4*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x
^2 + d^5*e^3), -1/8*((5*c*d^3*e^2 - b*d^2*e^3 - 3*a*d*e^4)*x^3 - (3*c*d^4 +
 b*d^3*e + 3*a*d^2*e^2 + (3*c*d^2*e^2 + b*d*e^3 + 3*a*e^4)*x^4 + 2*(3*c*d^3
*e + b*d^2*e^2 + 3*a*d*e^3)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (3*c*d^4
*e + b*d^3*e^2 - 5*a*d^2*e^3)*x)/(d^3*e^5*x^4 + 2*d^4*e^4*x^2 + d^5*e^3)]
```

Sympy [A] time = 1.77191, size = 196, normalized size = 1.7

$$-\frac{\sqrt{-\frac{1}{d^5e^5}}\left(3ae^2 + bde + 3cd^2\right)\log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}} + x\right)}{16} + \frac{\sqrt{-\frac{1}{d^5e^5}}\left(3ae^2 + bde + 3cd^2\right)\log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}} + x\right)}{16} + \frac{x^3\left(3ae^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**2*sqrt(-1/
(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*l
og(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (x**3*(3*a*e**3 + b*d*e**2 - 5*
c*d**2*e) + x*(5*a*d*e**2 - b*d**2*e - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e*
*3*x**2 + 8*d**2*e**4*x**4)
```

Giac [A] time = 1.10543, size = 136, normalized size = 1.18

$$\frac{\left(3\,cd^2 + bde + 3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}} - \frac{\left(5\,cd^2x^3e - bdx^3e^2 + 3\,cd^3x - 3\,ax^3e^3 + bd^2xe - 5\,adxe^2\right)e^{(-2)}}{8\left(x^2e + d\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2)
- 1/8*(5*c*d^2*x^3*e - b*d*x^3*e^2 + 3*c*d^3*x - 3*a*x^3*e^3 + b*d^2*x*e -
5*a*d*x*e^2)*e^(-2)/((x^2*e + d)^2*d^2)
```

3.292 $\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=127

$$-\frac{x\left(ae^2-bde+cd^2\right)}{4d^2e\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3e(bd-5ae)+cd^2\right)}{8d^{7/2}e^{3/2}}+\frac{x\left(e(3bd-7ae)+cd^2\right)}{8d^3e\left(d+ex^2\right)}-\frac{a}{d^3x}$$

```
[Out] -(a/(d^3*x)) - ((c*d^2 - b*d*e + a*e^2)*x)/(4*d^2*e*(d + e*x^2)^2) + ((c*d^
2 + e*(3*b*d - 7*a*e))*x)/(8*d^3*e*(d + e*x^2)) + ((c*d^2 + 3*e*(b*d - 5*a*
e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)*e^(3/2))
```

Rubi [A] time = 0.204063, antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1259, 456, 453, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3e(bd-5ae)+cd^2\right)}{8d^{7/2}e^{3/2}}+\frac{x\left(e(3bd-7ae)+cd^2\right)}{8d^3e\left(d+ex^2\right)}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{4\left(d+ex^2\right)^2}-\frac{a}{d^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

```
[Out] -(a/(d^3*x)) - ((c/e - (b*d - a*e)/d^2)*x)/(4*(d + e*x^2)^2) + ((c*d^2 + e*
(3*b*d - 7*a*e))*x)/(8*d^3*e*(d + e*x^2)) + ((c*d^2 + 3*e*(b*d - 5*a*e))*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)*e^(3/2))
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^3}\,dx &= -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}-\frac{\int \frac{-4ade^2-e\left(cd^2+3e(bd-ae)\right)x^2}{x^2\left(d+ex^2\right)^2}\,dx}{4d^2e^2}\\
&= -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}+\frac{\left(cd^2+e(3bd-7ae)\right)x}{8d^3e\left(d+ex^2\right)}+\frac{\int \frac{8ae^2+e\left(cd+e\left(3b-\frac{7ae}{d}\right)\right)x^2}{x^2\left(d+ex^2\right)}\,dx}{8d^2e^2}\\
&= -\frac{a}{d^3x}-\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}+\frac{\left(cd^2+e(3bd-7ae)\right)x}{8d^3e\left(d+ex^2\right)}+\frac{\left(cd^2+3e(bd-5ae)\right)\int \frac{1}{d+ex^2}\,dx}{8d^3e}\\
&= -\frac{a}{d^3x}-\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}+\frac{\left(cd^2+e(3bd-7ae)\right)x}{8d^3e\left(d+ex^2\right)}+\frac{\left(cd^2+3e(bd-5ae)\right)\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}e^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.140273, size = 124, normalized size = 0.98

$$\frac{\frac{\sqrt{d}\left(dx^2\left(be\left(5d+3ex^2\right)+cd\left(ex^2-d\right)\right)-ae\left(8d^2+25dex^2+15e^2x^4\right)\right)}{ex\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(3e(bd-5ae)+cd^2\right)}{e^{3/2}}}{8d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

```
[Out] ((Sqrt[d]*(-(a*e*(8*d^2 + 25*d*e*x^2 + 15*e^2*x^4)) + d*x^2*(c*d*(-d + e*x^
2) + b*e*(5*d + 3*e*x^2))))/(e*x*(d + e*x^2)^2) + ((c*d^2 + 3*e*(b*d - 5*a*
e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2))/(8*d^(7/2))
```

Maple [A] time = 0.012, size = 182, normalized size = 1.4

$$-\frac{a}{d^3x}-\frac{7x^3ae^2}{8d^3\left(ex^2+d\right)^2}+\frac{3x^3be}{8d^2\left(ex^2+d\right)^2}+\frac{x^3c}{8d\left(ex^2+d\right)^2}-\frac{9aex}{8d^2\left(ex^2+d\right)^2}+\frac{5bx}{8d\left(ex^2+d\right)^2}-\frac{xc}{8\left(ex^2+d\right)^2e}-\frac{15ae}{8d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x)
```

```
[Out] -a/d^3/x-7/8/d^3/(e*x^2+d)^2*x^3*a*e^2+3/8/d^2/(e*x^2+d)^2*x^3*b*e+1/8/d/(e
*x^2+d)^2*x^3*c-9/8/d^2/(e*x^2+d)^2*e*a*x+5/8/d/(e*x^2+d)^2*b*x-1/8/(e*x^2+
d)^2/e*x*c-15/8/d^3*e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*a+3/8/d^2/(d*e)^(
1/2)*arctan(e*x/(d*e)^(1/2))*b+1/8/d/e/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*
```

c

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.77073, size = 887, normalized size = 6.98

$$\left[-\frac{16\,ad^3e^2-2\left(cd^3e^2+3\,bd^2e^3-15\,ade^4\right)x^4+2\left(cd^4e-5\,bd^3e^2+25\,ad^2e^3\right)x^2-\left(\left(cd^2e^2+3\,bde^3-15\,ae^4\right)x^5+2\left(cd^3e+\right.\right.}{16\left(d^4e^4x^5+2\,d^5e^3x^3+d^6e^2x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a*d^3*e^2 - 2*(c*d^3*e^2 + 3*b*d^2*e^3 - 15*a*d*e^4)*x^4 + 2*(c*
d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*x^2 - ((c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^
4)*x^5 + 2*(c*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*x^3 + (c*d^4 + 3*b*d^3*e -
15*a*d^2*e^2)*x)*sqrt(-d*e)*log((e*x^2 + 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/
(d^4*e^4*x^5 + 2*d^5*e^3*x^3 + d^6*e^2*x), -1/8*(8*a*d^3*e^2 - (c*d^3*e^2 +
 3*b*d^2*e^3 - 15*a*d*e^4)*x^4 + (c*d^4*e - 5*b*d^3*e^2 + 25*a*d^2*e^3)*x^2
 - ((c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*x^5 + 2*(c*d^3*e + 3*b*d^2*e^2 - 15*
a*d*e^3)*x^3 + (c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*x)*sqrt(d*e)*arctan(sqrt(
d*e)*x/d))/(d^4*e^4*x^5 + 2*d^5*e^3*x^3 + d^6*e^2*x)]
```

Sympy [A] time = 2.42076, size = 202, normalized size = 1.59

$$\frac{\sqrt{-\frac{1}{d^7e^3}}\left(15ae^2-3bde-cd^2\right)\log\left(-d^4e\sqrt{-\frac{1}{d^7e^3}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{d^7e^3}}\left(15ae^2-3bde-cd^2\right)\log\left(d^4e\sqrt{-\frac{1}{d^7e^3}}+x\right)}{16}-\frac{8ad^2e+x^4\left(15\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**3,x)
```

```
[Out] sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*log(-d**4*e*sqrt(-1/(d*
*7*e**3)) + x)/16 - sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*log
(d**4*e*sqrt(-1/(d**7*e**3)) + x)/16 - (8*a*d**2*e + x**4*(15*a*e**3 - 3*b*
d*e**2 - c*d**2*e) + x**2*(25*a*d*e**2 - 5*b*d**2*e + c*d**3))/(8*d**5*e*x
+ 16*d**4*e**2*x**3 + 8*d**3*e**3*x**5)
```

Giac [A] time = 1.09502, size = 149, normalized size = 1.17

$$\frac{\left(cd^{2}+3\,bde-15\,ae^{2}\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{8\,d^{\frac{7}{2}}}-\frac{a}{d^{3}x}+\frac{\left(cd^{2}x^{3}e+3\,bdx^{3}e^{2}-cd^{3}x-7\,ax^{3}e^{3}+5\,bd^{2}xe-9\,adxe^{2}\right)e^{(-1)}}{8\left(x^{2}e+d\right)^{2}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(c*d^2 + 3*b*d*e - 15*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(7/2)
 - a/(d^3*x) + 1/8*(c*d^2*x^3*e + 3*b*d*x^3*e^2 - c*d^3*x - 7*a*x^3*e^3 + 5
*b*d^2*x*e - 9*a*d*x*e^2)*e^(-1)/((x^2*e + d)^2*d^3)
```

3.293 $\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx$

Optimal. Leaf size=142

$$\frac{x\left(ae^2-bde+cd^2\right)}{4d^3\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(35ae^2-15bde+3cd^2\right)}{8d^{9/2}\sqrt{e}}+\frac{x\left(3cd^2-e(7bd-11ae)\right)}{8d^4\left(d+ex^2\right)}-\frac{bd-3ae}{d^4x}-\frac{a}{3d^3x^3}$$

```
[Out] -a/(3*d^3*x^3) - (b*d - 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(4*d^3
*(d + e*x^2)^2) + ((3*c*d^2 - e*(7*b*d - 11*a*e))*x)/(8*d^4*(d + e*x^2)) +
((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(9/2)*Sq
rt[e])
```

Rubi [A] time = 0.217412, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1259, 1261, 205}

$$\frac{x\left(ae^2-bde+cd^2\right)}{4d^3\left(d+ex^2\right)^2}+\frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(35ae^2-15bde+3cd^2\right)}{8d^{9/2}\sqrt{e}}+\frac{x\left(3cd^2-e(7bd-11ae)\right)}{8d^4\left(d+ex^2\right)}-\frac{bd-3ae}{d^4x}-\frac{a}{3d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

```
[Out] -a/(3*d^3*x^3) - (b*d - 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(4*d^3
*(d + e*x^2)^2) + ((3*c*d^2 - e*(7*b*d - 11*a*e))*x)/(8*d^4*(d + e*x^2)) +
((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(9/2)*Sq
rt[e])
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^3}\,dx = \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\int \frac{4ad^2e^2+4de^2(bd-ae)x^2+3e^2\left(cd^2-bde+ae^2\right)x^4}{x^4\left(d+ex^2\right)^2}dx}{4d^3e^2}$$

$$= \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\left(3cd^2-e(7bd-11ae)\right)x}{8d^4\left(d+ex^2\right)} + \frac{\int \frac{8ad^4e^4+8d^3e^4(bd-2ae)x^2+d^2e^4\left(3cd^2-e(7bd-11ae)\right)x^4}{x^4\left(d+ex^2\right)} dx}{8d^6e^4}$$

$$= \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\left(3cd^2-e(7bd-11ae)\right)x}{8d^4\left(d+ex^2\right)} + \frac{\int \left(\frac{8ad^3e^4}{x^4} + \frac{8d^2e^4(bd-3ae)}{x^2} + \frac{d^2e^4\left(3cd^2-15bde+35ae^2\right)}{d+ex^2}\right.}{8d^6e^4}$$

$$= -\frac{a}{3d^3x^3} - \frac{bd-3ae}{d^4x} + \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\left(3cd^2-e(7bd-11ae)\right)x}{8d^4\left(d+ex^2\right)} + \frac{\left(3cd^2-15bde+35ae^2\right)}{8d^4}$$

$$= -\frac{a}{3d^3x^3} - \frac{bd-3ae}{d^4x} + \frac{\left(cd^2-bde+ae^2\right)x}{4d^3\left(d+ex^2\right)^2} + \frac{\left(3cd^2-e(7bd-11ae)\right)x}{8d^4\left(d+ex^2\right)} + \frac{\left(3cd^2-15bde+35ae^2\right)}{8d^{9/2}\sqrt{e}}$$

Mathematica [A] time = 0.0858429, size = 141, normalized size = 0.99

$$\frac{x\left(11ae^2-7bde+3cd^2\right)}{8d^4\left(d+ex^2\right)} + \frac{x\left(ae^2-bde+cd^2\right)}{4d^3\left(d+ex^2\right)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(35ae^2-15bde+3cd^2\right)}{8d^{9/2}\sqrt{e}} + \frac{3ae-bd}{d^4x} - \frac{a}{3d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

```
[Out] -a/(3*d^3*x^3) + (-(b*d) + 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(4*
d^3*(d + e*x^2)^2) + ((3*c*d^2 - 7*b*d*e + 11*a*e^2)*x)/(8*d^4*(d + e*x^2))
 + ((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(9/2)
*Sqrt[e])
```

Maple [A] time = 0.014, size = 207, normalized size = 1.5

$$-\frac{a}{3\,d^3x^3} + 3\,\frac{ae}{d^4x} - \frac{b}{d^3x} + \frac{11\,x^3ae^3}{8\,d^4\left(ex^2+d\right)^2} - \frac{7\,x^3be^2}{8\,d^3\left(ex^2+d\right)^2} + \frac{3\,x^3ce}{8\,d^2\left(ex^2+d\right)^2} + \frac{13\,ae^2x}{8\,d^3\left(ex^2+d\right)^2} - \frac{9\,bex}{8\,d^2\left(ex^2+d\right)^2} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x)
```

```
[Out] -1/3*a/d^3/x^3+3/d^4/x*a*e-1/d^3/x*b+11/8/d^4/(e*x^2+d)^2*x^3*a*e^3-7/8/d^3
/(e*x^2+d)^2*x^3*b*e^2+3/8/d^2/(e*x^2+d)^2*x^3*c*e+13/8/d^3/(e*x^2+d)^2*e^2
*a*x-9/8/d^2/(e*x^2+d)^2*b*e*x+5/8/d/(e*x^2+d)^2*c*x+35/8/d^4/(d*e)^(1/2)*a
rctan(e*x/(d*e)^(1/2))*a*e^2-15/8/d^3/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*e
*b+3/8/d^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91339, size = 1031, normalized size = 7.26

$$\left[\frac{6\left(3\,cd^3e^2-15\,bd^2e^3+35\,ade^4\right)x^6-16\,ad^4e+10\left(3\,cd^4e-15\,bd^3e^2+35\,ad^2e^3\right)x^4-16\left(3\,bd^4e-7\,ad^3e^2\right)x^2-3\left(\left(3\,cd^2e^2\right.\right.}{48\left(d^5e^3x^7+2\,d^6\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(6*(3*c*d^3*e^2 - 15*b*d^2*e^3 + 35*a*d*e^4)*x^6 - 16*a*d^4*e + 10*(3
*c*d^4*e - 15*b*d^3*e^2 + 35*a*d^2*e^3)*x^4 - 16*(3*b*d^4*e - 7*a*d^3*e^2)*
x^2 - 3*((3*c*d^2*e^2 - 15*b*d*e^3 + 35*a*e^4)*x^7 + 2*(3*c*d^3*e - 15*b*d^
2*e^2 + 35*a*d*e^3)*x^5 + (3*c*d^4 - 15*b*d^3*e + 35*a*d^2*e^2)*x^3)*sqrt(-
d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)))/(d^5*e^3*x^7 + 2*d^6*e^
2*x^5 + d^7*e*x^3), 1/24*(3*(3*c*d^3*e^2 - 15*b*d^2*e^3 + 35*a*d*e^4)*x^6 -
 8*a*d^4*e + 5*(3*c*d^4*e - 15*b*d^3*e^2 + 35*a*d^2*e^3)*x^4 - 8*(3*b*d^4*e
 - 7*a*d^3*e^2)*x^2 + 3*((3*c*d^2*e^2 - 15*b*d*e^3 + 35*a*e^4)*x^7 + 2*(3*c
*d^3*e - 15*b*d^2*e^2 + 35*a*d*e^3)*x^5 + (3*c*d^4 - 15*b*d^3*e + 35*a*d^2*
e^2)*x^3)*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(d^5*e^3*x^7 + 2*d^6*e^2*x^5 + d
^7*e*x^3)]
```

Sympy [A] time = 3.40683, size = 214, normalized size = 1.51

$$-\frac{\sqrt{-\frac{1}{d^9e}}\left(35ae^2-15bde+3cd^2\right)\log\left(-d^5\sqrt{-\frac{1}{d^9e}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^9e}}\left(35ae^2-15bde+3cd^2\right)\log\left(d^5\sqrt{-\frac{1}{d^9e}}+x\right)}{16}+\frac{-8ad^3+x^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**9*e))*(35*a*e**2 - 15*b*d*e + 3*c*d**2)*log(-d**5*sqrt(-1/(d**
9*e)) + x)/16 + sqrt(-1/(d**9*e))*(35*a*e**2 - 15*b*d*e + 3*c*d**2)*log(d**
5*sqrt(-1/(d**9*e)) + x)/16 + (-8*a*d**3 + x**6*(105*a*e**3 - 45*b*d*e**2 +
 9*c*d**2*e) + x**4*(175*a*d*e**2 - 75*b*d**2*e + 15*c*d**3) + x**2*(56*a*d
**2*e - 24*b*d**3))/(24*d**6*x**3 + 48*d**5*e*x**5 + 24*d**4*e**2*x**7)
```

Giac [A] time = 1.11525, size = 173, normalized size = 1.22

$$\frac{\left(3\,cd^2-15\,bde+35\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,d^{\frac{9}{2}}}+\frac{3\,cd^2x^3e-7\,bdx^3e^2+5\,cd^3x+11\,ax^3e^3-9\,bd^2xe+13\,adxe^2}{8\left(x^2e+d\right)^2d^4}-\frac{3\,bdx^2-9}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 - 15*b*d*e + 35*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(9
/2) + 1/8*(3*c*d^2*x^3*e - 7*b*d*x^3*e^2 + 5*c*d^3*x + 11*a*x^3*e^3 - 9*b*d
^2*x*e + 13*a*d*x*e^2)/((x^2*e + d)^2*d^4) - 1/3*(3*b*d*x^2 - 9*a*x^2*e + a
*d)/(d^4*x^3)
```

3.294 $\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=171

$$-\frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2}-\frac{6ae^2-3bde+cd^2}{d^5x}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(63ae^2-35bde+15cd^2\right)}{8d^{11/2}}-\frac{ex\left(7cd^2-e(11bd-15ae)\right)}{8d^5\left(d+ex^2\right)}-\frac{bd}{3}$$

```
[Out] -a/(5*d^3*x^5) - (b*d - 3*a*e)/(3*d^4*x^3) - (c*d^2 - 3*b*d*e + 6*a*e^2)/(d
^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2)^2) - (e*(7*c*d^2 -
 e*(11*b*d - 15*a*e))*x)/(8*d^5*(d + e*x^2)) - (Sqrt[e]*(15*c*d^2 - 35*b*d*
e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(11/2))
```

Rubi [A] time = 0.373194, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1259, 1805, 1802, 205}

$$-\frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2}-\frac{6ae^2-3bde+cd^2}{d^5x}-\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(63ae^2-35bde+15cd^2\right)}{8d^{11/2}}-\frac{ex\left(7cd^2-e(11bd-15ae)\right)}{8d^5\left(d+ex^2\right)}-\frac{bd}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

```
[Out] -a/(5*d^3*x^5) - (b*d - 3*a*e)/(3*d^4*x^3) - (c*d^2 - 3*b*d*e + 6*a*e^2)/(d
^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2)^2) - (e*(7*c*d^2 -
 e*(11*b*d - 15*a*e))*x)/(8*d^5*(d + e*x^2)) - (Sqrt[e]*(15*c*d^2 - 35*b*d*
e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(11/2))
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^3}\,dx = -\frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{\int \frac{-4ad^3e^2-4d^2e^2(bd-ae)x^2-4de^2\left(cd^2-bde+ae^2\right)x^4+3e^3\left(cd^2-bde+ae^2\right)x^6}{x^6\left(d+ex^2\right)^2}\,dx}{4d^4e^2}$$

$$= -\frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd-15ae)\right)x}{8d^5\left(d+ex^2\right)} + \frac{\int \frac{8ad^3e^2+8d^2e^2(bd-2ae)x^2+8de^2\left(cd^2-e(2bd-3ae)\right)x}{x^6\left(d+ex^2\right)}}{8d^5e^2}$$

$$= -\frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd-15ae)\right)x}{8d^5\left(d+ex^2\right)} + \frac{\int \left(\frac{8ad^2e^2}{x^6}+\frac{8de^2(bd-3ae)}{x^4}+\frac{8e^2\left(cd^2-3bde+6ae^2\right)}{x^2}\right.}{8d^5e^2}$$

$$= -\frac{a}{5d^3x^5} - \frac{bd-3ae}{3d^4x^3} - \frac{cd^2-3bde+6ae^2}{d^5x} - \frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd-15ae)\right)x}{8d^5\left(d+ex^2\right)} -$$

$$= -\frac{a}{5d^3x^5} - \frac{bd-3ae}{3d^4x^3} - \frac{cd^2-3bde+6ae^2}{d^5x} - \frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd-15ae)\right)x}{8d^5\left(d+ex^2\right)} -$$

Mathematica [A] time = 0.11355, size = 173, normalized size = 1.01

$$-\frac{x\left(15ae^3-11bde^2+7cd^2e\right)}{8d^5\left(d+ex^2\right)} - \frac{ex\left(ae^2-bde+cd^2\right)}{4d^4\left(d+ex^2\right)^2} + \frac{-6ae^2+3bde-cd^2}{d^5x} - \frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\left(63ae^2-35bde+15cd^2\right)}{8d^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

```
[Out] -a/(5*d^3*x^5) + (-(b*d) + 3*a*e)/(3*d^4*x^3) + (-(c*d^2) + 3*b*d*e - 6*a*e
^2)/(d^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2)^2) - ((7*c*d
^2*e - 11*b*d*e^2 + 15*a*e^3)*x)/(8*d^5*(d + e*x^2)) - (Sqrt[e]*(15*c*d^2 -
 35*b*d*e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(11/2))
```

Maple [A] time = 0.016, size = 245, normalized size = 1.4

$$-\frac{a}{5\,d^3x^5} + \frac{ae}{d^4x^3} - \frac{b}{3\,d^3x^3} - 6\,\frac{ae^2}{d^5x} + 3\,\frac{be}{d^4x} - \frac{c}{d^3x} - \frac{15\,e^4x^3a}{8\,d^5\left(ex^2+d\right)^2} + \frac{11\,e^3x^3b}{8\,d^4\left(ex^2+d\right)^2} - \frac{7\,e^2x^3c}{8\,d^3\left(ex^2+d\right)^2} - \frac{17\,e^3ax}{8\,d^4\left(ex^2+d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x)
```

```
[Out] -1/5*a/d^3/x^5+1/d^4/x^3*a*e-1/3/d^3/x^3*b-6/d^5/x*a*e^2+3/d^4/x*e*b-1/d^3/
x*c-15/8*e^4/d^5/(e*x^2+d)^2*x^3*a+11/8*e^3/d^4/(e*x^2+d)^2*x^3*b-7/8*e^2/d
```

```
^3/(e*x^2+d)^2*x^3*c-17/8*e^3/d^4/(e*x^2+d)^2*a*x+13/8*e^2/d^3/(e*x^2+d)^2*
b*x-9/8*e/d^2/(e*x^2+d)^2*c*x-63/8*e^3/d^5/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/
2))*a+35/8*e^2/d^4/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))*b-15/8*e/d^3/(d*e)^(
1/2)*arctan(e*x/(d*e)^(1/2))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.85279, size = 1143, normalized size = 6.68

$$\left[-\frac{30\left(15\,cd^2e^2-35\,bde^3+63\,ae^4\right)x^8+50\left(15\,cd^3e-35\,bd^2e^2+63\,ade^3\right)x^6+48\,ad^4+16\left(15\,cd^4-35\,bd^3e+63\,ad^2e^2\right)x^4}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(30*(15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 50*(15*c*d^3*e - 3
5*b*d^2*e^2 + 63*a*d*e^3)*x^6 + 48*a*d^4 + 16*(15*c*d^4 - 35*b*d^3*e + 63*a
*d^2*e^2)*x^4 + 16*(5*b*d^4 - 9*a*d^3*e)*x^2 - 15*((15*c*d^2*e^2 - 35*b*d*e
^3 + 63*a*e^4)*x^9 + 2*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^7 + (15*c
*d^4 - 35*b*d^3*e + 63*a*d^2*e^2)*x^5)*sqrt(-e/d)*log((e*x^2 - 2*d*x*sqrt(-
e/d) - d)/(e*x^2 + d)))/(d^5*e^2*x^9 + 2*d^6*e*x^7 + d^7*x^5), -1/120*(15*(
15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 25*(15*c*d^3*e - 35*b*d^2*e^2 +
 63*a*d*e^3)*x^6 + 24*a*d^4 + 8*(15*c*d^4 - 35*b*d^3*e + 63*a*d^2*e^2)*x^4
+ 8*(5*b*d^4 - 9*a*d^3*e)*x^2 + 15*((15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*
x^9 + 2*(15*c*d^3*e - 35*b*d^2*e^2 + 63*a*d*e^3)*x^7 + (15*c*d^4 - 35*b*d^3
*e + 63*a*d^2*e^2)*x^5)*sqrt(e/d)*arctan(x*sqrt(e/d)))/(d^5*e^2*x^9 + 2*d^6
*e*x^7 + d^7*x^5)]
```

Sympy [B] time = 4.76643, size = 330, normalized size = 1.93

$$\frac{\sqrt{-\frac{e}{d^{11}}}\left(63ae^2-35bde+15cd^2\right)\log\left(-\frac{d^6\sqrt{-\frac{e}{d^{11}}}\left(63ae^2-35bde+15cd^2\right)}{63ae^3-35bde^2+15cd^2e}+x\right)}{16}-\frac{\sqrt{-\frac{e}{d^{11}}}\left(63ae^2-35bde+15cd^2\right)\log\left(\frac{d^6\sqrt{-\frac{e}{d^{11}}}\left(63ae^2\cdots\right.}{63ae^3-35bd\cdots}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**3,x)
```

```
[Out] sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(-d**6*sqrt(-e/d**11)*
(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*e**2 + 15*c*d**2*e)
```

```
+ x)/16 - sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(d**6*sqrt(-
e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*e**2 + 15*c
*d**2*e) + x)/16 - (24*a*d**4 + x**8*(945*a*e**4 - 525*b*d*e**3 + 225*c*d**
2*e**2) + x**6*(1575*a*d*e**3 - 875*b*d**2*e**2 + 375*c*d**3*e) + x**4*(504
*a*d**2*e**2 - 280*b*d**3*e + 120*c*d**4) + x**2*(-72*a*d**3*e + 40*b*d**4)
)/(120*d**7*x**5 + 240*d**6*e*x**7 + 120*d**5*e**2*x**9)
```

Giac [A] time = 1.12647, size = 221, normalized size = 1.29

$$-\frac{\left(15\,cd^2e-35\,bde^2+63\,ae^3\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,d^{\frac{11}{2}}}-\frac{7\,cd^2x^3e^2-11\,bdx^3e^3+9\,cd^3xe+15\,ax^3e^4-13\,bd^2xe^2+17\,adxe^3}{8\left(x^2e+d\right)^2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*c*d^2*e - 35*b*d*e^2 + 63*a*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2
)/d^(11/2) - 1/8*(7*c*d^2*x^3*e^2 - 11*b*d*x^3*e^3 + 9*c*d^3*x*e + 15*a*x^3
*e^4 - 13*b*d^2*x*e^2 + 17*a*d*x*e^3)/((x^2*e + d)^2*d^5) - 1/15*(15*c*d^2*
x^4 - 45*b*d*x^4*e + 90*a*x^4*e^2 + 5*b*d^2*x^2 - 15*a*d*x^2*e + 3*a*d^2)/(
d^5*x^5)
```

3.295 $\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=230

$$-\frac{\left(a^2ce-ab^2e-2abcd+b^3d\right)\log\left(a+bx^2+cx^4\right)}{4c^3\left(ae^2-bde+cd^2\right)}-\frac{\left(3a^2bce+2a^2c^2d-4ab^2cd-ab^3e+b^4d\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{d^4\log\ldots}{2e^3\left(ae\ldots\right.}$$

```
[Out] -((c*d + b*e)*x^2)/(2*c^2*e^2) + x^4/(4*c*e) - ((b^4*d - 4*a*b^2*c*d + 2*a^
2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^4*Log[d + e*x^2])/(2
*e^3*(c*d^2 - b*d*e + a*e^2)) - ((b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e)*Lo
g[a + b*x^2 + c*x^4])/(4*c^3*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.487482, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 1628, 634, 618, 206, 628}

$$-\frac{\left(a^2ce-ab^2e-2abcd+b^3d\right)\log\left(a+bx^2+cx^4\right)}{4c^3\left(ae^2-bde+cd^2\right)}-\frac{\left(3a^2bce+2a^2c^2d-4ab^2cd-ab^3e+b^4d\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{d^4\log\ldots}{2e^3\left(ae\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((c*d + b*e)*x^2)/(2*c^2*e^2) + x^4/(4*c*e) - ((b^4*d - 4*a*b^2*c*d + 2*a^
2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^4*Log[d + e*x^2])/(2
*e^3*(c*d^2 - b*d*e + a*e^2)) - ((b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e)*Lo
g[a + b*x^2 + c*x^4])/(4*c^3*(c*d^2 - b*d*e + a*e^2))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{-cd-be}{c^2e^2} + \frac{x}{ce} + \frac{d^4}{e^2(cd^2-bde+ae^2)(d+ex)} + \frac{-a(b^2d-acd-abe)}{c^2(cd^2-bd\ldots}\right.\right.$$

$$= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{-a(b^2d-acd-abe)-(b^3d-2abcd-ab\ldots}{a+bx+cx^2}\right.}{2c^2(cd^2-bde+ae^2\ldots}$$

$$= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)} - \frac{(b^3d-2abcd-ab^2e+a^2ce)\,\text{Subst}(\ldots}{4c^3(cd^2-bde+ae\ldots}$$

$$= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)} - \frac{(b^3d-2abcd-ab^2e+a^2ce)\log(a+\ldots}{4c^3(cd^2-bde+ae^2)}$$

$$= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} - \frac{(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \ldots$$

Mathematica [A] time = 0.25112, size = 228, normalized size = 0.99

$$\frac{1}{4}\left(\frac{(-a^2ce+ab^2e+2abcd+b^3(-d))\log(a+bx^2+cx^4)}{c^3(e(ae-bd)+cd^2)} - \frac{2(3a^2bce+2a^2c^2d-4ab^2cd-ab^3e+b^4d)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{c^3\sqrt{4ac-b^2}\,(e(bd-ae)-cd^2)}\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*(c*d + b*e)*x^2)/(c^2*e^2) + x^4/(c*e) - (2*(b^4*d - 4*a*b^2*c*d + 2*a
^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])
/(c^3*Sqrt[-b^2 + 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) + (2*d^4*Log[d + e*x^2
])/(e^3*(c*d^2 + e*(-(b*d) + a*e))) + ((-(b^3*d) + 2*a*b*c*d + a*b^2*e - a^
2*c*e)*Log[a + b*x^2 + c*x^4])/(c^3*(c*d^2 + e*(-(b*d) + a*e))))/4
```

Maple [B] time = 0.014, size = 538, normalized size = 2.3

$$\frac{x^4}{4ce} - \frac{bx^2}{2c^2e} - \frac{dx^2}{2e^2c} - \frac{\ln(cx^4+bx^2+a)\,a^2e}{(4ae^2-4deb+4cd^2)c^2} + \frac{\ln(cx^4+bx^2+a)\,ab^2e}{(4ae^2-4deb+4cd^2)c^3} + \frac{\ln(cx^4+bx^2+a)\,abd}{(2ae^2-2deb+2cd^2)c^2} - \frac{\ln(cx^4+\ldots}{(4ae^2-4d\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*x^4/c/e-1/2/c^2/e*x^2*b-1/2*d*x^2/c/e^2-1/4/(a*e^2-b*d*e+c*d^2)/c^2*ln(
c*x^4+b*x^2+a)*a^2*e+1/4/(a*e^2-b*d*e+c*d^2)/c^3*ln(c*x^4+b*x^2+a)*a*b^2*e+
1/2/(a*e^2-b*d*e+c*d^2)/c^2*ln(c*x^4+b*x^2+a)*a*b*d-1/4/(a*e^2-b*d*e+c*d^2)
/c^3*ln(c*x^4+b*x^2+a)*b^3*d+3/2/(a*e^2-b*d*e+c*d^2)/c^2/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*b*e+1/(a*e^2-b*d*e+c*d^2)/c/(4*a*
c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*d-2/(a*e^2-b*d*e+c*d
^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2*d-1/2
/(a*e^2-b*d*e+c*d^2)/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(
1/2))*b^3*a*e+1/2/(a*e^2-b*d*e+c*d^2)/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*b^4*d+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2-b*d*e+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16437, size = 319, normalized size = 1.39

$$\frac{d^4 \log\left(\left|x^2 e + d\right|\right)}{2\left(cd^2e^3 - bde^4 + ae^5\right)} - \frac{\left(b^3d - 2\,abcd - ab^2e + a^2ce\right)\log\left(cx^4 + bx^2 + a\right)}{4\left(c^4d^2 - bc^3de + ac^3e^2\right)} + \frac{\left(b^4d - 4\,ab^2cd + 2\,a^2c^2d - ab^3e + 3\,a^2bce\right)\text{ar}}{2\left(c^4d^2 - bc^3de + ac^3e^2\right)\sqrt{-b^2 +}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(x^2*e + d))/(c*d^2*e^3 - b*d*e^4 + a*e^5) - 1/4*(b^3*d - 2*
a*b*c*d - a*b^2*e + a^2*c*e)*log(c*x^4 + b*x^2 + a)/(c^4*d^2 - b*c^3*d*e +
a*c^3*e^2) + 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e
)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^4*d^2 - b*c^3*d*e + a*c^3*e^
2)*sqrt(-b^2 + 4*a*c)) + 1/4*(c*x^4*e - 2*c*d*x^2 - 2*b*x^2*e)*e^(-2)/c^2
```

3.296 $\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=189

$$\frac{\left(2a^2ce-ab^2e-3abcd+b^3d\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{\left(-abe-acd+b^2d\right)\log\left(a+bx^2+cx^4\right)}{4c^2\left(ae^2-bde+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2-bde+cd^2\right)}+\frac{x^2}{2ce}$$

```
[Out] x^2/(2*c*e) + ((b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) -
 (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a
*b*e)*Log[a + b*x^2 + c*x^4])/(4*c^2*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.329083, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 1628, 634, 618, 206, 628}

$$\frac{\left(2a^2ce-ab^2e-3abcd+b^3d\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{\left(-abe-acd+b^2d\right)\log\left(a+bx^2+cx^4\right)}{4c^2\left(ae^2-bde+cd^2\right)}-\frac{d^3\log\left(d+ex^2\right)}{2e^2\left(ae^2-bde+cd^2\right)}+\frac{x^2}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x^2/(2*c*e) + ((b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) -
 (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a
*b*e)*Log[a + b*x^2 + c*x^4])/(4*c^2*(c*d^2 - b*d*e + a*e^2))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ce} - \frac{d^3}{e(cd^2-bde+ae^2)(d+ex)} + \frac{a(bd-ae)+(b^2d-acd-abe)x}{c(cd^2-bde+ae^2)(a+bx+cx^2)}\right.\right.$$

$$= \frac{x^2}{2ce} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{a(bd-ae)+(b^2d-acd-abe)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c(cd^2-bde+ae^2)}$$

$$= \frac{x^2}{2ce} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2-bde+ae^2)} + \frac{(b^2d-acd-abe)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2(cd^2-bde+ae^2)} - \frac{(b^3 \ldots}{}$$

$$= \frac{x^2}{2ce} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2-bde+ae^2)} + \frac{(b^2d-acd-abe)\log(a+bx^2+cx^4)}{4c^2(cd^2-bde+ae^2)} + \frac{(b^3d-3abc \ldots}{}$$

$$= \frac{x^2}{2ce} + \frac{(b^3d-3abcd-ab^2e+2a^2ce)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}(cd^2-bde+ae^2)} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2-bde+ae^2)} + \frac{(b^2 \ldots}{}$$

Mathematica [A] time = 0.178524, size = 186, normalized size = 0.98

$$\frac{2e^2(2a^2ce-ab^2e-3abcd+b^3d)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)+\sqrt{4ac-b^2}\left(e\left(e\left(abe+acd+b^2(-d)\right)\log(a+bx^2+cx^4)-2cx^2(a \ldots\right.\right.}{4c^2e^2\sqrt{4ac-b^2}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*e^2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTan[(b + 2*c*x^2)/Sqrt[
-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(2*c^2*d^3*Log[d + e*x^2] + e*(-2*c*(c*
d^2 - b*d*e + a*e^2)*x^2 + e*(-(b^2*d) + a*c*d + a*b*e)*Log[a + b*x^2 + c*x
^4])))/(4*c^2*Sqrt[-b^2 + 4*a*c]*e^2*(-(c*d^2) + e*(b*d - a*e)))
```

Maple [B] time = 0.01, size = 408, normalized size = 2.2

$$\frac{x^2}{2ce} - \frac{\ln(cx^4+bx^2+a)abe}{(4ae^2-4deb+4cd^2)c^2} - \frac{\ln(cx^4+bx^2+a)ad}{(4ae^2-4deb+4cd^2)c} + \frac{\ln(cx^4+bx^2+a)b^2d}{(4ae^2-4deb+4cd^2)c^2} - \frac{a^2e}{(ae^2-deb+cd^2)c}\arctan\Big(\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*x^2/c/e-1/4/(a*e^2-b*d*e+c*d^2)/c^2*ln(c*x^4+b*x^2+a)*a*b*e-1/4/(a*e^2-
b*d*e+c*d^2)/c*ln(c*x^4+b*x^2+a)*a*d+1/4/(a*e^2-b*d*e+c*d^2)/c^2*ln(c*x^4+b
*x^2+a)*b^2*d-1/(a*e^2-b*d*e+c*d^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*a^2*e+3/2/(a*e^2-b*d*e+c*d^2)/c/(4*a*c-b^2)^(1/2)*arctan
((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*d+1/2/(a*e^2-b*d*e+c*d^2)/c^2/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*a*e-1/2/(a*e^2-b*d*e+c*d
^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*d-1/2*d
^3*ln(e*x^2+d)/e^2/(a*e^2-b*d*e+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1716, size = 262, normalized size = 1.39

$$-\frac{d^{3} \log\left(\left|x^{2} e + d\right|\right)}{2 \left(c d^{2} e^{2} - b d e^{3} + a e^{4}\right)} + \frac{x^{2} e^{(-1)}}{2 c} + \frac{\left(b^{2} d - a c d - a b e\right) \log\left(c x^{4} + b x^{2} + a\right)}{4 \left(c^{3} d^{2} - b c^{2} d e + a c^{2} e^{2}\right)} - \frac{\left(b^{3} d - 3 a b c d - a b^{2} e + 2 a^{2} c e\right) \arctan\left(\frac{2 c x^{2}}{\sqrt{-b^{2}}}\right.}{2 \left(c^{3} d^{2} - b c^{2} d e + a c^{2} e^{2}\right) \sqrt{-b^{2} + 4 a c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*d^3*log(abs(x^2*e + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4) + 1/2*x^2*e^(-1)
/c + 1/4*(b^2*d - a*c*d - a*b*e)*log(c*x^4 + b*x^2 + a)/(c^3*d^2 - b*c^2*d*
e + a*c^2*e^2) - 1/2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*arctan((2*c*
x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*sqrt(-b^2 +
 4*a*c))
```

3.297 $\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=158

$$-\frac{(-abe-2acd+b^2d)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{d^2\log(d+ex^2)}{2e\,(ae^2-bde+cd^2)}-\frac{(bd-ae)\log(a+bx^2+cx^4)}{4c\,(ae^2-bde+cd^2)}$$

```
[Out] -((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*
Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^2*Log[d + e*x^2])/(2*e*(c*d
^2 - b*d*e + a*e^2)) - ((b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*c*(c*d^2 - b
*d*e + a*e^2))
```

Rubi [A] time = 0.2606, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 1628, 634, 618, 206, 628}

$$-\frac{(-abe-2acd+b^2d)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{d^2\log(d+ex^2)}{2e\,(ae^2-bde+cd^2)}-\frac{(bd-ae)\log(a+bx^2+cx^4)}{4c\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*
Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^2*Log[d + e*x^2])/(2*e*(c*d
^2 - b*d*e + a*e^2)) - ((b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*c*(c*d^2 - b
*d*e + a*e^2))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{d^2}{(cd^2-bde+ae^2)(d+ex)} + \frac{-ad-(bd-ae)x}{(cd^2-bde+ae^2)(a+bx+cx^2)}\right) dx, x,$$

$$= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{-ad-(bd-ae)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)}$$

$$= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c(cd^2-bde+ae^2)} + \frac{(b^2d-2acd-abe)\,\text{S}}{4c\,(cd}$$

$$= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\log(a+bx^2+cx^4)}{4c(cd^2-bde+ae^2)} - \frac{(b^2d-2acd-abe)\,\text{Subst}\left(\int\right.}{2c\,(cd^2-bd}$$

$$= -\frac{(b^2d-2acd-abe)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\log(a+b}{4c\,(cd^2-bde+}$$

Mathematica [A] time = 0.107997, size = 139, normalized size = 0.88

$$-\frac{\sqrt{4ac-b^2}\left(e(bd-ae)\log(a+bx^2+cx^4)-2cd^2\log(d+ex^2)\right)+2e\left(abe+2acd+b^2(-d)\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{4ce\sqrt{4ac-b^2}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(2*e*(-(b^2*d) + 2*a*c*d + a*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]
 + Sqrt[-b^2 + 4*a*c]*(-2*c*d^2*Log[d + e*x^2] + e*(b*d - a*e)*Log[a + b*x^
2 + c*x^4]))/(4*c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [A] time = 0.007, size = 289, normalized size = 1.8

$$\frac{\ln(cx^4+bx^2+a)\,ae}{(4\,ae^2-4\,deb+4\,cd^2)\,c} - \frac{\ln(cx^4+bx^2+a)\,bd}{(4\,ae^2-4\,deb+4\,cd^2)\,c} - \frac{ad}{ae^2-deb+cd^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}} - \frac{}{(2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/(a*e^2-b*d*e+c*d^2)/c*ln(c*x^4+b*x^2+a)*a*e-1/4/(a*e^2-b*d*e+c*d^2)/c*l
n(c*x^4+b*x^2+a)*b*d-1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))*a*d-1/2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b/c*a*e+1/2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2/c*d+1/2*d^2*ln(e*x^2+d)/e
/(a*e^2-b*d*e+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17576, size = 212, normalized size = 1.34

$$\frac{d^2 \log\left(\left|x^2 e + d\right|\right)}{2\left(cd^2 e - bde^2 + ae^3\right)} - \frac{(bd - ae)\log\left(cx^4 + bx^2 + a\right)}{4\left(c^2 d^2 - bcde + ace^2\right)} + \frac{\left(b^2 d - 2\,acd - abe\right)\arctan\left(\frac{2\,cx^2 + b}{\sqrt{-b^2 + 4\,ac}}\right)}{2\left(c^2 d^2 - bcde + ace^2\right)\sqrt{-b^2 + 4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^2*log(abs(x^2*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) - 1/4*(b*d - a*e)*l
og(c*x^4 + b*x^2 + a)/(c^2*d^2 - b*c*d*e + a*c*e^2) + 1/2*(b^2*d - 2*a*c*d
- a*b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 - b*c*d*e + a*c
*e^2)*sqrt(-b^2 + 4*a*c))
```

3.298 $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=132

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2-bde+cd^2\right)}+\frac{d\log\left(a+bx^2+cx^4\right)}{4\left(ae^2-bde+cd^2\right)}$$

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)) - (d*Log[d + e*x^2])/(2*(c*d^2 - b*d*e + a*e^2)
) + (d*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.157051, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 800, 634, 618, 206, 628}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{d\log\left(d+ex^2\right)}{2\left(ae^2-bde+cd^2\right)}+\frac{d\log\left(a+bx^2+cx^4\right)}{4\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)) - (d*Log[d + e*x^2])/(2*(c*d^2 - b*d*e + a*e^2)
) + (d*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*d*e + a*e^2))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{de}{(cd^2-bde+ae^2)(d+ex)} + \frac{ae+cdx}{(cd^2-bde+ae^2)(a+bx+cx^2)}\right) dx, x\right.$$
$$= -\frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{ae+cdx}{a+bx+cx^2}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)}$$
$$= -\frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{d\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4(cd^2-bde+ae^2)} - \frac{(bd-2ae)\,\text{Subst}\left(\int \frac{1}{a+bx+c\ldots}\right.}{4(cd^2-bde+ae\ldots}$$
$$= -\frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{d\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)} + \frac{(bd-2ae)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x,\right.}{2(cd^2-bde+ae^2)}$$
$$= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} - \frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{d\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)}$$

Mathematica [A] time = 0.07163, size = 114, normalized size = 0.86

$$\frac{d\sqrt{4ac-b^2}\left(2\log(d+ex^2)-\log(a+bx^2+cx^4)\right)+2(bd-2ae)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{4\sqrt{4ac-b^2}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*(b*d - 2*a*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a
*c]*d*(2*Log[d + e*x^2] - Log[a + b*x^2 + c*x^4]))/(4*Sqrt[-b^2 + 4*a*c]*(-
(c*d^2) + e*(b*d - a*e)))
```

Maple [A] time = 0.008, size = 176, normalized size = 1.3

$$\frac{d\ln(cx^4+bx^2+a)}{4ae^2-4deb+4cd^2} + \frac{ae}{ae^2-deb+cd^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}} - \frac{bd}{2ae^2-2deb+2cd^2}\arctan\left((2\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*d*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e+c*d^2)+1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*e-1/2/(a*e^2-b*d*e+c*d^2)/
(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*d-1/2*d*ln(e*x^2+
d)/(a*e^2-b*d*e+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 112.082, size = 716, normalized size = 5.42

$$\left[\frac{\left(b^2-4\,ac\right)d\log\left(cx^4+bx^2+a\right)-2\left(b^2-4\,ac\right)d\log\left(ex^2+d\right)-\sqrt{b^2-4\,ac}(bd-2\,ae)\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-\left(2\,cx^2+b\right)\sqrt{b^2-}}{cx^4+bx^2+a}\right.}{4\left(\left(b^2c-4\,ac^2\right)d^2-\left(b^3-4\,abc\right)de+\left(ab^2-4\,a^2c\right)e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2 - 4*a*c)*d*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*d*log(e*x^2
+ d) - sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^2*c - 4*a
*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), 1/4*((b^2 - 4*a*c
)*d*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*d*log(e*x^2 + d) + 2*sqrt(-b^2
 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13567, size = 180, normalized size = 1.36

$$-\frac{de\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e-bde^2+ae^3\right)}+\frac{d\log\left(cx^4+bx^2+a\right)}{4\left(cd^2-bde+ae^2\right)}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(cd^2-bde+ae^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*d*e*log(abs(x^2*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/4*d*log(c*x^4
+ b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) - 1/2*(b*d - 2*a*e)*arctan((2*c*x^2 +
b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

3.299 $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=133

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{e\log\left(d+ex^2\right)}{2\left(ae^2-bde+cd^2\right)}-\frac{e\log\left(a+bx^2+cx^4\right)}{4\left(ae^2-bde+cd^2\right)}$$

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt[b^2 - 4*a
*c]*(c*d^2 - b*d*e + a*e^2)) + (e*Log[d + e*x^2])/(2*(c*d^2 - b*d*e + a*e^2
)) - (e*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.123315, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1247, 705, 31, 634, 618, 206, 628}

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{e\log\left(d+ex^2\right)}{2\left(ae^2-bde+cd^2\right)}-\frac{e\log\left(a+bx^2+cx^4\right)}{4\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt[b^2 - 4*a
*c]*(c*d^2 - b*d*e + a*e^2)) + (e*Log[d + e*x^2])/(2*(c*d^2 - b*d*e + a*e^2
)) - (e*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*d*e + a*e^2))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{cd-be-cex}{a+bx+cx^2}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)} + \frac{e^2\,\text{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)}\\
&= \frac{e\log(d+ex^2)}{2(cd^2-bde+ae^2)} - \frac{e\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4(cd^2-bde+ae^2)} + \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\right.}{4(cd^2-bde+ae^2)}\\
&= \frac{e\log(d+ex^2)}{2(cd^2-bde+ae^2)} - \frac{e\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b\right.}{2(cd^2-bde+ae^2)}\\
&= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{e\log(d+ex^2)}{2(cd^2-bde+ae^2)} - \frac{e\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)}
\end{aligned}$$

Mathematica [A] time = 0.0688062, size = 112, normalized size = 0.84

$$\frac{e\sqrt{4ac-b^2}\left(\log(a+bx^2+cx^4)-2\log(d+ex^2)\right)+(2be-4cd)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{4\sqrt{4ac-b^2}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-4*c*d + 2*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*
a*c]*e*(-2*Log[d + e*x^2] + Log[a + b*x^2 + c*x^4]))/(4*Sqrt[-b^2 + 4*a*c]*
(-(c*d^2) + e*(b*d - a*e)))
```

Maple [A] time = 0.008, size = 176, normalized size = 1.3

$$-\frac{e\ln(cx^4+bx^2+a)}{4ae^2-4deb+4cd^2} - \frac{be}{2ae^2-2deb+2cd^2}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}} + \frac{cd}{ae^2-deb+cd^2}\arctan\left((\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*e*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e+c*d^2)-1/2/(a*e^2-b*d*e+c*d^2)/(4*a*c
-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*e+1/(a*e^2-b*d*e+c*d^2)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*d+1/2*e*ln(e*x^2
+d)/(a*e^2-b*d*e+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 78.3602, size = 718, normalized size = 5.4

$$\left[-\frac{\left(b^2-4\,ac\right)e\log\left(cx^4+bx^2+a\right)-2\left(b^2-4\,ac\right)e\log\left(ex^2+d\right)+\sqrt{b^2-4\,ac}(2\,cd-be)\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2}}{cx^4+bx^2+a}\right.}{4\left(\left(b^2c-4\,ac^2\right)d^2-\left(b^3-4\,abc\right)de+\left(ab^2-4\,a^2c\right)e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 4*a*c)*e*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*e*log(e*x^2
 + d) + sqrt(b^2 - 4*a*c)*(2*c*d - b*e)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^2*c - 4*
a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), -1/4*((b^2 - 4*a
*c)*e*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*e*log(e*x^2 + d) + 2*sqrt(-b
^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^
2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1654, size = 181, normalized size = 1.36

$$-\frac{e\log\left(cx^4+bx^2+a\right)}{4\left(cd^2-bde+ae^2\right)}+\frac{e^2\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e-bde^2+ae^3\right)}+\frac{(2\,cd-be)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(cd^2-bde+ae^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*e*log(c*x^4 + b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) + 1/2*e^2*log(abs(x^2
*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/2*(2*c*d - b*e)*arctan((2*c*x^2 +
b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

3.300 $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=167

$$\frac{(2ace+b^2(-e)+bcd)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2-bde+cd^2\right)}-\frac{(cd-be)\log\left(a+bx^2+cx^4\right)}{4a\left(ae^2-bde+cd^2\right)}+\frac{\log(x)}{ad}$$

```
[Out] ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*S
qrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + Log[x]/(a*d) - (e^2*Log[d + e*x
^2])/(2*d*(c*d^2 - b*d*e + a*e^2)) - ((c*d - b*e)*Log[a + b*x^2 + c*x^4])/(
4*a*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.305797, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 893, 634, 618, 206, 628}

$$\frac{(2ace+b^2(-e)+bcd)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2-bde+cd^2\right)}-\frac{(cd-be)\log\left(a+bx^2+cx^4\right)}{4a\left(ae^2-bde+cd^2\right)}+\frac{\log(x)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*S
qrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + Log[x]/(a*d) - (e^2*Log[d + e*x
^2])/(2*d*(c*d^2 - b*d*e + a*e^2)) - ((c*d - b*e)*Log[a + b*x^2 + c*x^4])/(
4*a*(c*d^2 - b*d*e + a*e^2))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{1}{adx}-\frac{e^3}{d\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{-bcd+b^2e-ace-c(cd-be}{a\left(cd^2-bde+ae^2\right)\left(a+bx+c\right.}\right.\right.\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}+\frac{\text{Subst}\left(\int \frac{-bcd+b^2e-ace-c(cd-be)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a\left(cd^2-bde+ae^2\right)}\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}-\frac{(cd-be)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a\left(cd^2-bde+ae^2\right)}-\frac{\left(bcd-b^2\right.}{}\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}-\frac{(cd-be)\log\left(a+bx^2+cx^4\right)}{4a\left(cd^2-bde+ae^2\right)}+\frac{\left(bcd-b^2e+2ace\right.}{}\\
&= \frac{\left(bcd-b^2e+2ace\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}+\frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}-\frac{(cd-be)\,}{4a\left(c\right.}\end{aligned}$$

Mathematica [A] time = 0.324116, size = 242, normalized size = 1.45

$$\frac{4\log(x)\sqrt{b^2-4ac}\left(e(ae-bd)+cd^2\right)-2ae^2\sqrt{b^2-4ac}\log\left(d+ex^2\right)-d\left(cd\sqrt{b^2-4ac}-be\sqrt{b^2-4ac}+2ace+b^2(-e)+\right.}{4ad\sqrt{b^2-4ac}\,(}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*Log[x] - d*(b*c*d + c*Sqrt[
b^2 - 4*a*c]*d - b^2*e + 2*a*c*e - b*Sqrt[b^2 - 4*a*c]*e)*Log[b - Sqrt[b^2
- 4*a*c] + 2*c*x^2] + d*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e +
b*Sqrt[b^2 - 4*a*c]*e)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2] - 2*a*Sqrt[b^2
- 4*a*c]*e^2*Log[d + e*x^2])/(4*a*Sqrt[b^2 - 4*a*c]*d*(c*d^2 + e*(-(b*d) +
a*e)))
```

Maple [A] time = 0.012, size = 298, normalized size = 1.8

$$\frac{\ln\left(cx^4+bx^2+a\right)be}{\left(4\,ae^2-4\,deb+4\,cd^2\right)a}-\frac{c\ln\left(cx^4+bx^2+a\right)d}{\left(4\,ae^2-4\,deb+4\,cd^2\right)a}-\frac{ec}{ae^2-deb+cd^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{}{(2\,ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/(a*e^2-b*d*e+c*d^2)/a*ln(c*x^4+b*x^2+a)*b*e-1/4/(a*e^2-b*d*e+c*d^2)/a*c
*ln(c*x^4+b*x^2+a)*d-1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))*c*e+1/2/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*e-1/2/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b
^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c*d+ln(x)/a/d-1/2*e^2*ln(
e*x^2+d)/d/(a*e^2-b*d*e+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14009, size = 232, normalized size = 1.39

$$-\frac{(cd-be)\log\left(cx^4+bx^2+a\right)}{4\left(acd^2-abde+a^2e^2\right)}-\frac{e^3\log\left(\left|x^2e+d\right|\right)}{2\left(cd^3e-bd^2e^2+ade^3\right)}-\frac{\left(bcd-b^2e+2\,ace\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(acd^2-abde+a^2e^2\right)\sqrt{-b^2+4\,ac}}+\frac{\log\left(x^2\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(c*d - b*e)*log(c*x^4 + b*x^2 + a)/(a*c*d^2 - a*b*d*e + a^2*e^2) - 1/2
*e^3*log(abs(x^2*e + d))/(c*d^3*e - b*d^2*e^2 + a*d*e^3) - 1/2*(b*c*d - b^2
*e + 2*a*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a*c*d^2 - a*b*d*e
+ a^2*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*log(x^2)/(a*d)
```

3.301 $\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=205

$$-\frac{\left(3abce-2ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{\left(ace+b^2(-e)+bcd\right)\log\left(a+bx^2+cx^4\right)}{4a^2\left(ae^2-bde+cd^2\right)}-\frac{\log(x)(ae+bd)}{a^2d^2}+\frac{e^3}{2d^2(}$$

```
[Out] -1/(2*a*d*x^2) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)
) - ((b*d + a*e)*Log[x])/(a^2*d^2) + (e^3*Log[d + e*x^2])/(2*d^2*(c*d^2 - b
*d*e + a*e^2)) + ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^2 + c*x^4])/(4*a^2*(c
*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.470497, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 893, 634, 618, 206, 628}

$$-\frac{\left(3abce-2ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+\frac{\left(ace+b^2(-e)+bcd\right)\log\left(a+bx^2+cx^4\right)}{4a^2\left(ae^2-bde+cd^2\right)}-\frac{\log(x)(ae+bd)}{a^2d^2}+\frac{e^3}{2d^2(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*a*d*x^2) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*ArcTanh[(b + 2*
c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)
) - ((b*d + a*e)*Log[x])/(a^2*d^2) + (e^3*Log[d + e*x^2])/(2*d^2*(c*d^2 - b
*d*e + a*e^2)) + ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^2 + c*x^4])/(4*a^2*(c
*d^2 - b*d*e + a*e^2))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int\left(\frac{1}{adx^2}+\frac{-bd-ae}{a^2d^2x}+\frac{e^4}{d^2\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{b^2cd-ac^2d-b\cdots}{a^2\left(cd^2\cdots\right.}\right.\right.$$

$$= -\frac{1}{2adx^2}-\frac{(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)}+\frac{\text{Subst}\left(\int\frac{b^2cd-ac^2d-b^3e+2abc\cdots}{a+bx+\cdots}\right.}{2a^2\left(cd^2-b\cdots\right.}$$

$$= -\frac{1}{2adx^2}-\frac{(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)}+\frac{\left(bcd-b^2e+ace\right)\text{Subst}\left(\cdots\right.}{4a^2\left(cd^2-bde\cdots\right.}$$

$$= -\frac{1}{2adx^2}-\frac{(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)}+\frac{\left(bcd-b^2e+ace\right)\log\left(a+\cdots\right.}{4a^2\left(cd^2-bde+a\cdots\right.}$$

$$= -\frac{1}{2adx^2}-\frac{\left(b^2cd-2ac^2d-b^3e+3abce\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}-\frac{(bd+ae)\log(x)}{a^2d^2}+\cdots$$

Mathematica [A] time = 0.338795, size = 331, normalized size = 1.61

$$\frac{1}{4}\left(\frac{\left(b^2\left(e\sqrt{b^2-4ac}-cd\right)-bc\left(d\sqrt{b^2-4ac}+3ae\right)+ac\left(2cd-e\sqrt{b^2-4ac}\right)+b^3e\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{a^2\sqrt{b^2-4ac}\left(e(bd-ae)-cd^2\right)}+\frac{\left(b^2\cdots\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-2/(a*d*x^2) - (4*(b*d + a*e)*Log[x])/(a^2*d^2) + ((b^3*e - b*c*(Sqrt[b^2
- 4*a*c]*d + 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqr
t[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 - 4*
a*c]*(-(c*d^2) + e*(b*d - a*e))) + ((-(b^3*e) + b*c*(-(Sqrt[b^2 - 4*a*c]*d)
 + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*c*(2*c*d + Sqrt[b^2 - 4*a*c
]*e))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 - 4*a*c]*(-(c*d^2
) + e*(b*d - a*e))) + (2*e^3*Log[d + e*x^2])/(c*d^4 + d^2*e*(-(b*d) + a*e))
)/4
```

Maple [B] time = 0.016, size = 430, normalized size = 2.1

$$\frac{c\ln\left(cx^4+bx^2+a\right)e}{\left(4ae^2-4deb+4cd^2\right)a}-\frac{\ln\left(cx^4+bx^2+a\right)b^2e}{\left(4ae^2-4deb+4cd^2\right)a^2}+\frac{c\ln\left(cx^4+bx^2+a\right)bd}{\left(4ae^2-4deb+4cd^2\right)a^2}+\frac{3ecb}{\left(2ae^2-2deb+2cd^2\right)a}\arctan\left((2cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/(a*e^2-b*d*e+c*d^2)/a*c*ln(c*x^4+b*x^2+a)*e-1/4/(a*e^2-b*d*e+c*d^2)/a^2
*ln(c*x^4+b*x^2+a)*b^2*e+1/4/(a*e^2-b*d*e+c*d^2)/a^2*c*ln(c*x^4+b*x^2+a)*b*
d+3/2/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2
)^(1/2))*b*c*e-1/(a*e^2-b*d*e+c*d^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*c^2*d-1/2/(a*e^2-b*d*e+c*d^2)/a^2/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*e+1/2/(a*e^2-b*d*e+c*d^2)/a^2/(4*a*c
-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*c*d-1/2/a/d/x^2-e*ln(
x)/a/d^2-1/d/a^2*ln(x)*b+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2-b*d*e+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15765, size = 320, normalized size = 1.56

$$\frac{\left(bcd-b^2e+ace\right)\log\left(cx^4+bx^2+a\right)}{4\left(a^2cd^2-a^2bde+a^3e^2\right)}+\frac{e^4\log\left(\left|x^2e+d\right|\right)}{2\left(cd^4e-bd^3e^2+ad^2e^3\right)}+\frac{\left(b^2cd-2\,ac^2d-b^3e+3\,abce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^2cd^2-a^2bde+a^3e^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*c*d - b^2*e + a*c*e)*log(c*x^4 + b*x^2 + a)/(a^2*c*d^2 - a^2*b*d*e +
 a^3*e^2) + 1/2*e^4*log(abs(x^2*e + d))/(c*d^4*e - b*d^3*e^2 + a*d^2*e^3) +
 1/2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b
^2 + 4*a*c))/((a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(
b*d + a*e)*log(x^2)/(a^2*d^2) + 1/2*(b*d*x^2 + a*x^2*e - a*d)/(a^2*d^2*x^2)
```

3.302 $\int \frac{1}{x^5(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=268

$$-\frac{\left(2abce-ac^2d+b^2cd+b^3(-e)\right)\log\left(a+bx^2+cx^4\right)}{4a^3\left(ae^2-bde+cd^2\right)}+\frac{\left(-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd+b^4(-e)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+$$

```
[Out] -1/(4*a*d*x^4) + (b*d + a*e)/(2*a^2*d^2*x^2) + ((b^3*c*d - 3*a*b*c^2*d - b^
4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*a^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d^2 + a*b*d*e - a
*(c*d^2 - a*e^2))*Log[x])/(a^3*d^3) - (e^4*Log[d + e*x^2])/(2*d^3*(c*d^2 -
b*d*e + a*e^2)) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Log[a + b*x^2 +
c*x^4])/(4*a^3*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.596883, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1251, 893, 634, 618, 206, 628}

$$-\frac{\left(2abce-ac^2d+b^2cd+b^3(-e)\right)\log\left(a+bx^2+cx^4\right)}{4a^3\left(ae^2-bde+cd^2\right)}+\frac{\left(-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd+b^4(-e)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*a*d*x^4) + (b*d + a*e)/(2*a^2*d^2*x^2) + ((b^3*c*d - 3*a*b*c^2*d - b^
4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*a^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d^2 + a*b*d*e - a
*(c*d^2 - a*e^2))*Log[x])/(a^3*d^3) - (e^4*Log[d + e*x^2])/(2*d^3*(c*d^2 -
b*d*e + a*e^2)) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Log[a + b*x^2 +
c*x^4])/(4*a^3*(c*d^2 - b*d*e + a*e^2))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^3(d+ex)\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{adx^3} + \frac{-bd-ae}{a^2d^2x^2} + \frac{b^2d^2+abde-a\left(cd^2-ae^2\right)}{a^3d^3x} - \frac{e^5}{d^3\left(cd^2-bde+\right.}\right.\right.$$

$$= -\frac{1}{4adx^4} + \frac{bd+ae}{2a^2d^2x^2} + \frac{\left(b^2d^2+abde-a\left(cd^2-ae^2\right)\right)\log(x)}{a^3d^3} - \frac{e^4\log\left(d+ex^2\right)}{2d^3\left(cd^2-bde+ae^2\right.}$$

$$= -\frac{1}{4adx^4} + \frac{bd+ae}{2a^2d^2x^2} + \frac{\left(b^2d^2+abde-a\left(cd^2-ae^2\right)\right)\log(x)}{a^3d^3} - \frac{e^4\log\left(d+ex^2\right)}{2d^3\left(cd^2-bde+ae^2\right.}$$

$$= -\frac{1}{4adx^4} + \frac{bd+ae}{2a^2d^2x^2} + \frac{\left(b^2d^2+abde-a\left(cd^2-ae^2\right)\right)\log(x)}{a^3d^3} - \frac{e^4\log\left(d+ex^2\right)}{2d^3\left(cd^2-bde+ae^2\right.}$$

$$= -\frac{1}{4adx^4} + \frac{bd+ae}{2a^2d^2x^2} + \frac{\left(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}$$

Mathematica [A] time = 0.426105, size = 426, normalized size = 1.59

$$\frac{1}{4}\left(-\frac{\left(ac^2\left(d\sqrt{b^2-4ac}+2ae\right)+b^3\left(e\sqrt{b^2-4ac}-cd\right)-b^2c\left(d\sqrt{b^2-4ac}+4ae\right)+abc\left(3cd-2e\sqrt{b^2-4ac}\right)+b^4e\right)\log\left(\right.}{a^3\sqrt{b^2-4ac}\left(e(bd-ae)-cd^2\right)}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(1/(a*d*x^4)) + (2*(b*d + a*e))/(a^2*d^2*x^2) + (4*(b^2*d^2 + a*b*d*e + a
*(-(c*d^2) + a*e^2))*Log[x])/(a^3*d^3) - ((b^4*e + a*c^2*(Sqrt[b^2 - 4*a*c]
*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + a*b*c*(3*c*d - 2*Sqrt[b
^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 - 4*a
```

```
*c] + 2*c*x^2])/(a^3*Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) - ((-(b^
4*e) + a*c^2*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d)
+ 4*a*e) + b^3*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*
a*c]*e))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^3*Sqrt[b^2 - 4*a*c]*(-(c*
d^2) + e*(b*d - a*e))) - (2*e^4*Log[d + e*x^2])/(c*d^5 + d^3*e*(-(b*d) + a*
e)))/4
```

Maple [B] time = 0.019, size = 584, normalized size = 2.2

$$-\frac{c\ln\left(cx^4+bx^2+a\right)eb}{\left(2\,ae^2-2\,deb+2\,cd^2\right)a^2}+\frac{c^2\ln\left(cx^4+bx^2+a\right)d}{\left(4\,ae^2-4\,deb+4\,cd^2\right)a^2}+\frac{\ln\left(cx^4+bx^2+a\right)eb^3}{\left(4\,ae^2-4\,deb+4\,cd^2\right)a^3}-\frac{c\ln\left(cx^4+bx^2+a\right)db^2}{\left(4\,ae^2-4\,deb+4\,cd^2\right)a^3}+\frac{}{\left(ae^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)/a^2*c*ln(c*x^4+b*x^2+a)*e*b+1/4/(a*e^2-b*d*e+c*d^2
)/a^2*c^2*ln(c*x^4+b*x^2+a)*d+1/4/(a*e^2-b*d*e+c*d^2)/a^3*ln(c*x^4+b*x^2+a)
*e*b^3-1/4/(a*e^2-b*d*e+c*d^2)/a^3*c*ln(c*x^4+b*x^2+a)*d*b^2+1/(a*e^2-b*d*e
+c*d^2)/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*e*c^2-2/(
a*e^2-b*d*e+c*d^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/
2))*b^2*c*e+3/2/(a*e^2-b*d*e+c*d^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b
)/(4*a*c-b^2)^(1/2))*d*b*c^2+1/2/(a*e^2-b*d*e+c*d^2)/a^3/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*e-1/2/(a*e^2-b*d*e+c*d^2)/a^3/(4*
a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*c*d-1/4/a/d/x^4+1/
2*e/a/d^2/x^2+1/2/d/a^2/x^2*b+1/d^3/a*ln(x)*e^2+1/d^2/a^2*ln(x)*b*e-1/d/a^2
*ln(x)*c+1/d/a^3*ln(x)*b^2-1/2*e^4*ln(e*x^2+d)/d^3/(a*e^2-b*d*e+c*d^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18515, size = 448, normalized size = 1.67

$$-\frac{(b^2cd - ac^2d - b^3e + 2\,abce)\log(cx^4 + bx^2 + a)}{4\,(a^3cd^2 - a^3bde + a^4e^2)} - \frac{e^5\log(|x^2e + d|)}{2\,(cd^5e - bd^4e^2 + ad^3e^3)} - \frac{(b^3cd - 3\,abc^2d - b^4e + 4\,ab^2ce - 2\,a^2c}{2\,(a^3cd^2 - a^3bde + a^4e^2)\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*log(c*x^4 + b*x^2 + a)/(a^3*c*
d^2 - a^3*b*d*e + a^4*e^2) - 1/2*e^5*log(abs(x^2*e + d))/(c*d^5*e - b*d^4*e
^2 + a*d^3*e^3) - 1/2*(b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*
c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*c*d^2 - a^3*b*d*e + a
^4*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(b^2*d^2 - a*c*d^2 + a*b*d*e + a^2*e^2)*l
og(x^2)/(a^3*d^3) - 1/4*(3*b^2*d^2*x^4 - 3*a*c*d^2*x^4 + 3*a*b*d*x^4*e + 3*
a^2*x^4*e^2 - 2*a*b*d^2*x^2 - 2*a^2*d*x^2*e + a^2*d^2)/(a^3*d^3*x^4)
```

3.303 $\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=387

$$-\frac{\left(-\frac{3a^2bce+2a^2c^2d-4ab^2cd-ab^3e+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{3a^2bce+2a^2c^2d-4ab^2cd-ab^3e+b^4d}{\sqrt{b^2-4ac}}+a^2ce-a\ldots\right.}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}\ldots}}$$

```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) - ((b^3*d - 2*a*b*c*d - a*b^2*e
+ a^2*c*e - (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - ((b^3
*d - 2*a*b*c*d - a*b^2*e + a^2*c*e + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a
*b^3*e + 3*a^2*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 -
 b*d*e + a*e^2))
```

Rubi [A] time = 4.03166, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$-\frac{\left(-\frac{3a^2bce+2a^2c^2d-4ab^2cd-ab^3e+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{3a^2bce+2a^2c^2d-4ab^2cd-ab^3e+b^4d}{\sqrt{b^2-4ac}}+a^2ce-a\ldots\right.}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```


```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) - ((b^3*d - 2*a*b*c*d - a*b^2*e
+ a^2*c*e - (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - ((b^3
*d - 2*a*b*c*d - a*b^2*e + a^2*c*e + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a
*b^3*e + 3*a^2*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 -
 b*d*e + a*e^2))
```


Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^8}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{-cd-be}{c^2e^2} + \frac{x^2}{ce} + \frac{d^4}{e^2\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + \frac{-a\left(b^2d-acd-abe\right)-\left(b^3d-\right.}{c^2\left(cd^2-bde+ae^2\right)}\right.$$

$$= -\frac{(cd+be)x}{c^2e^2} + \frac{x^3}{3ce} + \frac{\int \frac{-a\left(b^2d-acd-abe\right)+\left(-b^3d+2abcd+ab^2e-a^2ce\right)x^2}{a+bx^2+cx^4}\,dx}{c^2\left(cd^2-bde+ae^2\right)} + \frac{d^4\int \frac{1}{d+ex^2}\,dx}{e^2\left(cd^2-bde+ae\right.}$$

$$= -\frac{(cd+be)x}{c^2e^2} + \frac{x^3}{3ce} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}\left(cd^2-bde+ae^2\right)} - \frac{\left(b^3d-2abcd-ab^2e+a^2ce-\frac{b^4d-4ab^2c}{}\right.}{2c^2\left(cd^2-\right.}$$

$$= -\frac{(cd+be)x}{c^2e^2} + \frac{x^3}{3ce} - \frac{\left(b^3d-2abcd-ab^2e+a^2ce-\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce}{\sqrt{b^2-4ac}}\right)\tan^{-}}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)}$$

Mathematica [A] time = 0.634654, size = 463, normalized size = 1.2

$$\frac{\left(a^2c\left(e\sqrt{b^2-4ac}-2cd\right)+b^3\left(d\sqrt{b^2-4ac}+ae\right)+ab^2\left(4cd-e\sqrt{b^2-4ac}\right)-abc\left(2d\sqrt{b^2-4ac}+3ae\right)+b^4(-d)\right)\tan^{-1}}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(e(bd-ae)-cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) + ((-(b^4*d) + b^3*(Sqrt[b^2 - 4
*a*c]*d + a*e) - a*b*c*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*(4*c*d - Sqr
t[b^2 - 4*a*c]*e) + a^2*c*(-2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^4*d + b^3*(Sq
rt[b^2 - 4*a*c]*d - a*e) + a*b*c*(-2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a^2*c*(
2*c*d + Sqrt[b^2 - 4*a*c]*e) - a*b^2*(4*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + (d^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

Maple [B] time = 0.039, size = 1449, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*x^3/c/e-1/e/c^2*b*x-d*x/c/e^2+1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*a^2*e-1/2/(a*e^2-b*d*e+c*d^2)/c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*e-1/(a
*e^2-b*d*e+c*d^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*d+1/2/(a*e^2-b*d*e+c*d^2)/c^2
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b^3*d-3/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*a^2*b*e-1/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*a^2*d+1/2/(a*e^2-b*d*e+c*d^2)/c^2/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*a*b^3*e+2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*a*b^2*d-1/2/(a*e^2-b*d*e+c*d^2)/c^2/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b^4*d-1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*e+1
/2/(a*e^2-b*d*e+c*d^2)/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*e+1/(a*e^2-b*d*e+c*d^2)
/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*a*b*d-1/2/(a*e^2-b*d*e+c*d^2)/c^2*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
b^3*d-3/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*b
*e-1/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*d+1/2/(a
*e^2-b*d*e+c*d^2)/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^3*e+2/(a*e^
2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*d-1/2/(a*e^2-b
*d*e+c*d^2)/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4*d+1/e^2*d^4/(a*e^
2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.304 $\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=323

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}+\frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}$$

```
[Out] x/(c*e) + ((b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*
e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) +
 ((b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (d^(5/2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 1.36639, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}+\frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```


```
[Out] x/(c*e) + ((b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*
e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) +
 ((b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (d^(5/2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 - b*d*e + a*e^2))
```


Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{1}{ce} - \frac{d^3}{e(cd^2-bde+ae^2)(d+ex^2)} + \frac{a(bd-ae)+(b^2d-acd-abe)x^2}{c(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx$$

$$= \frac{x}{ce} + \frac{\int \frac{a(bd-ae)+(b^2d-acd-abe)x^2}{a+bx^2+cx^4}\,dx}{c(cd^2-bde+ae^2)} - \frac{d^3\int \frac{1}{d+ex^2}\,dx}{e(cd^2-bde+ae^2)}$$

$$= \frac{x}{ce} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}(cd^2-bde+ae^2)} + \frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+c}$$

$$= \frac{x}{ce} + \frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}(cd^2-bde+ae^2)} + \frac{\left(b^2d-acd-abe\right.}{\sqrt{2}c^{3/2}}$$

Mathematica [A] time = 0.543117, size = 385, normalized size = 1.19

$$\frac{\left(-b^2\left(d\sqrt{b^2-4ac}+ae\right)+ab\left(e\sqrt{b^2-4ac}-3cd\right)+ac\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}(e(bd-ae)-cd^2)} + \frac{\left(b^2\left(d\sqrt{b^2-4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x/(c*e) + ((b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c
]*d + 2*a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^3*d + b^2*(Sqrt[b^2
- 4*a*c]*d - a*e) + a*c*(-(Sqrt[b^2 - 4*a*c]*d) + 2*a*e) - a*b*(3*c*d + Sqr
t[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*
(-(b*d) + a*e))) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 -
b*d*e + a*e^2))
```

Maple [B] time = 0.033, size = 1098, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] x/c/e+1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*e+1/2/(a*e^2-b*d*
e+c*d^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*d-1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((-b+(
```

```
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))*b^2*d+1/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*a^2*e-1/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
a*b^2*e-3/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*
b*d+1/2/(a*e^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*
d-1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*e-1/2/(a*e^2-b*d*e+c*d^2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))*a*d+1/2/(a*e^2-b*d*e+c*d^2)/c*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*d
+1/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a^2*e-1/2/(a*e
^2-b*d*e+c*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b^2*e-3/2/(a*e^2-
b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b*d+1/2/(a*e^2-b*d*e+c
*d^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(
c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3*d-1/e*d^3/(a*e^2-b*d*e+c*
d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.305 $\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=280

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(ae^2-bde+cd^2\right)}$$

```
[Out] -(((b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - ((b*d - a*e + (b^2*d - 2*a*c*d - a
*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)
) + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.894449, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```


```
[Out] -(((b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - ((b*d - a*e + (b^2*d - 2*a*c*d - a
*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)
) + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```


Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{d^2}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{-ad-(bd-ae)x^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{-ad+(-bd+ae)x^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} + \frac{d^2\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae+\frac{b^2\ldots}{\ldots}\right.}{2\ldots}$$

$$= -\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\ldots}{\sqrt{b+\ldots}}\right.}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde\ldots}$$

Mathematica [A] time = 0.344477, size = 323, normalized size = 1.15

$$\frac{\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-ae^2+bde-cd^2)} + \frac{\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}-abe-2acd\ldots\right.}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\,(\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*
e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c
]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))
 + ((b^2*d - 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - a*b*e - a*Sqrt[b^2 - 4*a*c]*
e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c
]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))
 + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Maple [B] time = 0.027, size = 764, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e+1/2/(a*e^2-b*d*e+c*d^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*b*d+1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*a*b*e+1/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))*a*d-1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*b^2*d+1/2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e-1/2/(a*e^2-b*d*e
```

```
+c*d^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*b*d+1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*a*b*e+1/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*a*d-1/2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b^2*d+d^2/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 27.4641, size = 30765, normalized size = 109.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*
d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a
*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*
a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*
e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^
2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*
d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b
^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 -
24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 +
2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4
*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d
^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*
e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-
2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x + sqrt(1/2)*((b^4 - 5*a*b
^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d
*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d
^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^
2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2
*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^
2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b
^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a
*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*
e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*
b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b
^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^
2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2
 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e
+ (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e
```

^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c

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- 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2
*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d
^2)*x + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c
)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c
^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*
d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c -
4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 -
a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(
3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c
^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 -
 a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^
4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^
5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2
 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 +
 (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 -
 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2
- 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a
^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c
)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*
c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 -
 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 -
42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*
b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4
*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2
*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2
- 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3
*c^2)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 -
3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*
c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b
^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e
^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e
- 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*
a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*
c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b
^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d
^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^
3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4
*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*
c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^
4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*((b^
4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4
*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a
^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c -
 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2
*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b
^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2
*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c
^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*
c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^
4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 -
 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7
+ (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 -
 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^
3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b
*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 -
 (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2
 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e
 + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^
4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^
```

3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*
a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*
b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 -
2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 -
2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + d*sqrt(
-d/e)*log((e*x^2 + 2*e*x*sqrt(-d/e) - d)/(e*x^2 + d)))/(c*d^2 - b*d*e + a*e
^2), 1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b
*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 -
4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c
- 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 -
a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(
3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c
^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 -
a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^
4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^
5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2
- 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^
4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*
d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*l
og(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x + sqrt(1/2)*((b^4 - 5
*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*
c)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^
4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^
2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3
*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c
+ a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)
/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 -
10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*
d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4
*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a
^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^
4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a
*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^
3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)
*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4
- 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^
3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*
(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 1
2*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5
)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b
^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3
)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^
3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a
*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) - sqrt(1/2)*(c
*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2
*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b
^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 +
(a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b
^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2
*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c
^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*
c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^
4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 -
10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7
+ (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 -
4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c -
4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a
^3*e^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*
d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 - ((b^3*c^3 -

4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*

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c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4
*a^3*c^2)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^
3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(
b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*
(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b
*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^
3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5
 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(
b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a
^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^
4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^
3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3
 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*
a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2
)*e^4))*log(-2*(2*a^2*b*d*e - a^3*e^2 - (a*b^2 - a^2*c)*d^2)*x - sqrt(1/2)*
((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2
 - 4*a^3*c)*d*e^2 + ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 -
 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4
*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4
 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2
*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)
*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b
^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^
2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^
4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c
^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*
e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d
^2 - 2*(a*b^2 - 2*a^2*c)*d*e - ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*
b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a
^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - a^4*e
^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2
*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d
^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^
3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 2
4*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2
*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*
a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^
4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e
^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) + 2*d
*sqrt(d/e)*arctan(e*x*sqrt(d/e)/d))/(c*d^2 - b*d*e + a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.306 $\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=251

$$\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

```
[Out] (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 -
b*d*e + a*e^2)) + (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(c*d^2 - b*d*e + a*e^2)
```

Rubi [A] time = 0.451104, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 -
b*d*e + a*e^2)) + (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(c*d^2 - b*d*e + a*e^2)
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^2}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \int \left(-\frac{de}{\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)}+\frac{ae+cdx^2}{\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{ae+cdx^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2}-\frac{(de)\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{cd^2-bde+ae^2}+\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\left(cd^2-bde+ae^2\right)}+\frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\cdots}{\frac{b}{2}+\cdots}}{2\left(cd^2-bde+\cdots\right.}$$

$$= \frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)}+\frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)}-\frac{\sqrt{d}\sqrt{e}\cdots}{cd^2\cdots}$$

Mathematica [A] time = 0.506967, size = 277, normalized size = 1.1

$$-\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}+2ae-bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(-ae^2+bde-cd^2\right)}-\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\left(-ae^2+bde-cd^2\right)}-\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\cdots\right.}{ae^2-bde+cd\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))) - (Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c
]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2
)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 - b*d*e + a*e^2)
```

Maple [B] time = 0.023, size = 478, normalized size = 1.9

$$-\frac{c\sqrt{2}d}{2\,ae^2-2\,deb+2\,cd^2}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}-\frac{c\sqrt{2}ae}{ae^2-deb+cd^2}\mathrm{Artanh}\left(cx\sqrt{2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d-1/(a*e^2-b*d*e+c*d^2)*c/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))*d-1/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
```

```
a*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-d*e
/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 10.0064, size = 24804, normalized size = 98.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^
2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2
*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)
*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4
*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6
*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42
*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d
^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*
a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b
^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3
- 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x
 + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3
 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 -
4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 -
3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4
)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b
^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3
*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4
 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4
*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4
*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d
^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d
^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b
*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c -
 a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24
*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2
*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 +
(a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)
*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^
3 + (a^2*b^2 - 4*a^3*c)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b
*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*
c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*
```

d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(
(b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10
*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*
e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 -
a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2
)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^
2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a
^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*l
og(-2*(c^2*d^2 - a*c*e^2)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 -
4*a^2*c)*e^3 - (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e
+ 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b
*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^
3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d
^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^
4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*
c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*
c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^
3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*
e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4
- 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2
- 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^
5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^
2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*
b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^
3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^
3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*
d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 -
2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) + sqrt(1/2)*(c*d^
2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c
^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 -
2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4
)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b
^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3
*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4
- 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4
*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3
*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 +
(a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x + sqrt(1/2)*((b^2*c
- 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 + (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(
b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 -
(b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c
^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*
e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^
2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3
)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*
b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a
^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8))
)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*
c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4
*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 +
a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^
4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b
*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4
*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c -
8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e
^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b
^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*
c)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e +

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a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2
*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*
a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d
^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^
4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*
c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*
c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^
3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4
- 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(
a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*
e^2)*x - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 + (2*(b
^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*
c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*
b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2
*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a
*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c
 - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 -
24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a
^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7
+ (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c
^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*
c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt(
(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 -
 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b
^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^
2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*
(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*
e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*
b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c
)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) + sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)
*x - d)/(e*x^2 + d)))/(c*d^2 - b*d*e + a*e^2), 1/2*(sqrt(1/2)*(c*d^2 - b*d*
e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4
- 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(
a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d
^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e
+ 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 -
 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^
4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^
3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4
*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^
4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2
 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x + sqrt(1/2)*((b^2*c - 4*a*c^
2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2
- 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2
*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^
4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b
^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a
*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^
3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^
2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*
d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-
(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*
b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c
)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)
/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 -
10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^
5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5
- a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c
^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((
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b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8
*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))
) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2
+ ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c
 - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e
^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(
b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e
^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a
^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3
*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^
4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3
*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 -
4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x -
 sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3 -
 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*
a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*
a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2
*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*
d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3
*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c
^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 -
 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b
^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a
*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2
*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4
 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c
^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a
*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a
^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b
^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a
^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d
^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3
+ (a^2*b^2 - 4*a^3*c)*e^4))) + sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c
*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^
2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*
e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b
^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a
*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^
3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^
2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*
d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*
c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2
*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log
(-2*(c^2*d^2 - a*c*e^2)*x + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4
*a^2*c)*e^3 + (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e +
4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c
^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*
b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8
 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)
*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c
- 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^
2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3
- 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e
+ a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 -
 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 -
4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)
*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*
c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^
4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*
```

```
b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*
b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^
4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2
*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) - sqrt(1/2)*(c*d^2
- b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3
)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2
 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2
*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*
d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3
*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c
^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 -
 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b
^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e
 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a
^2*b^2 - 4*a^3*c)*e^4))*log(-2*(c^2*d^2 - a*c*e^2)*x - sqrt(1/2)*((b^2*c -
4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 + (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^
3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b
^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2
)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^
4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2
- 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*
d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^
5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4
*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*
sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c
- 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a
^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^
2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*
c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c
^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(
a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8
*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8
)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2
*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)
*e^4))) - 2*sqrt(d*e)*arctan(sqrt(d*e)*x/d))/(c*d^2 - b*d*e + a*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] Exception raised: NotImplementedError
```

3.307 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=254

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2-bde+cd^2\right)}$$

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.519666, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1170, 205, 1166}

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}+\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{e^2}{\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)} + \frac{cd-be-cex^2}{\left(cd^2-bde+ae^2\right)\left(a+bx^2+cx^4\right)}\right) dx$$

$$= \frac{\int \frac{cd-be-cex^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)} - \frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2\left(cd^2-bde+ae^2\right)} - \frac{\left(c\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int}{2\left(cd^2-bc\right.}$$

$$= -\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)} - \frac{\sqrt{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)} + -$$

Mathematica [A] time = 0.250838, size = 274, normalized size = 1.08

$$\frac{\sqrt{c}\left(e\sqrt{b^2-4ac}+be-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(-ae^2+bde-cd^2\right)} + \frac{\sqrt{c}\left(e\sqrt{b^2-4ac}-be+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\left(-ae^2+bde-cd^2\right)} + \frac{e^{3/2}\tan^{-1}\left(\right.}{\sqrt{d}\left(ae^2-bde\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c]*(2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) +
(e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Maple [B] time = 0.025, size = 480, normalized size = 1.9

$$\frac{c\sqrt{2}e}{2\,ae^2-2\,deb+2\,cd^2}\mathrm{Artanh}\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}} + \frac{c\sqrt{2}be}{2\,ae^2-2\,deb+2\,cd^2}\mathrm{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/2/(a*e^2-b*d*e+c*d^2)*c
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e-1/(a*e^2-b*d*e+c*d^2)*c^2/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d-1/2/(a*e^2-b*d*e+c*d^2)*c*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))*e+1/2/(a*e^2-b*d*e+c*d^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
```

```
*b*e-1/(a*e^2-b*d*e+c*d^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+e^2/
(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.308 $\int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=298

$$-\frac{\sqrt{c}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}-\frac{e^{5/2}\tan^{-}}{d^{3/2}\left(ae^2-b\right.}$$

```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*(c*d - b*e
- (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 -
 b*d*e + a*e^2))
```

Rubi [A] time = 0.960295, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$-\frac{\sqrt{c}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{c}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde+cd^2\right)}-\frac{e^{5/2}\tan^{-}}{d^{3/2}\left(ae^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```



```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*(c*d - b*e
- (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 -
 b*d*e + a*e^2))
```


Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{1}{adx^2} - \frac{e^3}{d(cd^2-bde+ae^2)(d+ex^2)} + \frac{-bcd+b^2e-ace-c(cd-be)x^2}{a(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx$$

$$= -\frac{1}{adx} + \frac{\int \frac{-bcd+b^2e-ace-c(cd-be)x^2}{a+bx^2+cx^4}\,dx}{a(cd^2-bde+ae^2)} - \frac{e^3\int \frac{1}{d+ex^2}\,dx}{d(cd^2-bde+ae^2)}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}(cd^2-bde+ae^2)} - \frac{\left(c\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a(cd^2-bde+ae^2)} - \Big($$

$$= -\frac{1}{adx} - \frac{\sqrt{c}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}(cd^2-bde+ae^2)} - \frac{\sqrt{c}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}\,(}$$

Mathematica [A] time = 0.433557, size = 340, normalized size = 1.14

$$-\frac{\sqrt{c}\left(cd\sqrt{b^2-4ac}-be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(e(ae-bd)+cd^2)} + \frac{\sqrt{c}\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}+2ace\right.}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e -
b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-
(b*d) + a*e))) + (Sqrt[c]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e
+ b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*
(-(b*d) + a*e))) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 -
b*d*e + a*e^2))
```

Maple [B] time = 0.028, size = 817, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e+1/2/(a*e^2-b*d*e+c*d
^2)/a*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+1/(a*e^2-b*d*e+c*d^2)*c^2/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*e-1/2/(a*e^2-b*d*e+c*d^2)/a*c/(-4*a*c+b^2)^(1/2)*
```

```
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*b^2*e+1/2/(a*e^2-b*d*e+c*d^2)/a*c^2/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*b*d+1/2/(a*e^2-b*d*e+c*d^2)/a*c*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
b*e-1/2/(a*e^2-b*d*e+c*d^2)/a*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d+1/(a*e^2-b*d*e+c*d^2
)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e-1/2/(a*e^2-b*d*e+c*d^2)/a*c/(
-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e+1/2/(a*e^2-b*d*e+c*d^2)/a*c^2/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-1/a/d/x-1/d*e^3/(a*e^2-b*d*e+c*d^2)
/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.309 $\int \frac{1}{x^4(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=348

$$\frac{\sqrt{c}\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right.}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde\right.}$$

```
[Out] -1/(3*a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(b*c*d - b^2*e + a*c*e
+ (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2
 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[c]*(b*c*d - b^2*e + a*c*e - (b^
2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(
5/2)*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 1.5494, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1287, 205, 1166}

$$\frac{\sqrt{c}\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\left(ae^2-bde+cd^2\right)}+\frac{\sqrt{c}\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right.}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b}\left(ae^2-bde\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(b*c*d - b^2*e + a*c*e
+ (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2
 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[c]*(b*c*d - b^2*e + a*c*e - (b^
2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(
5/2)*(c*d^2 - b*d*e + a*e^2))
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{1}{adx^4}+\frac{-bd-ae}{a^2d^2x^2}+\frac{e^4}{d^2\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)}+\frac{b^2cd-ac^2d-b^3e+2abce}{a^2\left(cd^2-bde+ae^2\right)}\right.$$

$$= -\frac{1}{3adx^3}+\frac{bd+ae}{a^2d^2x}+\frac{\int \frac{b^2cd-ac^2d-b^3e+2abce+c\left(bcd-b^2e+ace\right)x^2}{a+bx^2+cx^4}\,dx}{a^2\left(cd^2-bde+ae^2\right)}+\frac{e^4\int \frac{1}{d+ex^2}\,dx}{d^2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{1}{3adx^3}+\frac{bd+ae}{a^2d^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2-bde+ae^2\right)}+\frac{\left(c\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3ab}{\sqrt{b^2-4ac}}\right.\right.}{2a^2\left(cd^2-bde+\right.}$$

$$= -\frac{1}{3adx^3}+\frac{bd+ae}{a^2d^2x}+\frac{\sqrt{c}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}\left(cd^2-bde+ae^2\right)}$$

Mathematica [A] time = 0.5444, size = 410, normalized size = 1.18

$$\frac{\sqrt{c}\left(b^2\left(cd-e\sqrt{b^2-4ac}\right)+bc\left(d\sqrt{b^2-4ac}+3ae\right)+ac\left(e\sqrt{b^2-4ac}-2cd\right)+b^3(-e)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\left(e(ae-bd)+cd^2\right)}+\frac{\sqrt{c}\left(-b^2\left(e\sqrt{\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(-(b^3*e) + b*c*(Sqrt[b
^2 - 4*a*c]*d + 3*a*e) + b^2*(c*d - Sqrt[b^2 - 4*a*c]*e) + a*c*(-2*c*d + Sq
rt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(
b*d) + a*e))) + (Sqrt[c]*(b^3*e + b*c*(Sqrt[b^2 - 4*a*c]*d - 3*a*e) - b^2*(
c*d + Sqrt[b^2 - 4*a*c]*e) + a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))) + (e^(7/2)*ArcTan
[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

Maple [B] time = 0.035, size = 1160, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)/a*c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/2/(a*e^2-b*d*e+c*d
^2)/a^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
```

```
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e-1/2/(a*e^2-b*d*e+c*d^2)/a^2*c^2*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b*d-3/2/(a*e^2-b*d*e+c*d^2)/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b*e+1/(a*e^2-b*d*e+c*d^2)/a*c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*d+1/2/(a*e^2-b*d*e+c*d^2)/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b^3*e-1/2/(a*e^2-b*d*e+c*d^2)/a^2*c^2/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b^2*d+1/2/(a*e^2-b*d*e+c*d^2)/a*c^2*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e-1
/2/(a*e^2-b*d*e+c*d^2)/a^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2*e+1/2/(a*e^2-b*d*e+c*d^
2)/a^2*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-3/2/(a*e^2-b*d*e+c*d^2)/a*c^2/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*b*e+1/(a*e^2-b*d*e+c*d^2)/a*c^3/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))*d+1/2/(a*e^2-b*d*e+c*d^2)/a^2*c/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b^3*e-1/2/(a*e^2-b*d*e+c*d^2)/a^2*c^2/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2))*b^2*d-1/3/a/d/x^3+e/a/d^2/x+1/d/a^2/x*b+1/d^2*e^4/(a*e^2-
b*d*e+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.310 $\int \frac{1}{\sqrt{fx}(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=866

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)e^{7/4}}{\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}+1\right)e^{7/4}}{\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}-\frac{\log\left(\sqrt{e}\sqrt{f}x+\sqrt{d}\sqrt{f}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\log\left(\sqrt{e}\right.\cdots$$

```
[Out] (c^(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f
*x])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b
 + Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqr
t[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*ArcTan[1
- (Sqrt[2]*e^(1/4)*Sqrt[f*x])/(d^(1/4)*Sqrt[f])])/(Sqrt[2]*d^(3/4)*(c*d^2 -
 b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[f*x])/
(d^(1/4)*Sqrt[f])])/(Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (c^
(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x
])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b
+ Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt
[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*Log[Sqrt[d
]*Sqrt[f] + Sqrt[e]*Sqrt[f]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt
[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*Log[Sqrt[d]*Sqrt[f]
 + Sqrt[e]*Sqrt[f]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt[2]*d^(3/
4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f])
```

Rubi [A] time = 2.50858, antiderivative size = 866, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {1269, 1424, 211, 1165, 628, 1162, 617, 204, 1422, 212, 208, 205}

$$-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}\right)e^{7/4}}{\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{e}\sqrt{fx}}{\sqrt[4]{d}\sqrt{f}}+1\right)e^{7/4}}{\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}-\frac{\log\left(\sqrt{e}\sqrt{f}x+\sqrt{d}\sqrt{f}-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}d^{3/4}\left(cd^2-bed+ae^2\right)\sqrt{f}}+\log\left(\sqrt{e}\right.\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c^(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f
*x])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b
 + Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqr
t[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*ArcTan[1
- (Sqrt[2]*e^(1/4)*Sqrt[f*x])/(d^(1/4)*Sqrt[f])])/(Sqrt[2]*d^(3/4)*(c*d^2 -
 b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[f*x])/
(d^(1/4)*Sqrt[f])])/(Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (c^
(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x
])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b
```

```
+ Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt
[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*Log[Sqrt[d
]*Sqrt[f] + Sqrt[e]*Sqrt[f]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt
[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*Log[Sqrt[d]*Sqrt[f]
 + Sqrt[e]*Sqrt[f]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt[2]*d^(3/
4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f])
```

Rule 1269

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/f, Subst[I
nt[x^(k*(m + 1) - 1)*(d + (e*x^(2*k))/f^2)^q*(a + (b*x^(2*k))/f^k + (c*x^(4
*k))/f^4)^p, x], x, (f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, p, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1424

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
 x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\left(d+\frac{ex^4}{f^2}\right)\left(a+\frac{bx^4}{f^2}+\frac{cx^8}{f^4}\right)}\,dx, x, \sqrt{fx}\right)}{f}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(\frac{e^2f^2}{\left(cd^2-bde+ae^2\right)\left(df^2+ex^4\right)} + \frac{cdf^4-bef^4-cef^2x^4}{\left(cd^2-bde+ae^2\right)\left(af^4+bf^2x^4+cx^8\right)}\right) dx, x, \sqrt{fx}\right)}{f}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \frac{cdf^4-bef^4-cef^2x^4}{af^4+bf^2x^4+cx^8}\,dx, x, \sqrt{fx}\right)}{\left(cd^2-bde+ae^2\right)f} + \frac{\left(2e^2f\right)\mathrm{Subst}\left(\int \frac{1}{df^2+ex^4}\,dx, x, \sqrt{fx}\right)}{cd^2-bde+ae^2}$$

$$= \frac{e^2\,\mathrm{Subst}\left(\int \frac{\sqrt{d}f-\sqrt{e}x^2}{df^2+ex^4}\,dx, x, \sqrt{fx}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)} + \frac{e^2\,\mathrm{Subst}\left(\int \frac{\sqrt{d}f+\sqrt{e}x^2}{df^2+ex^4}\,dx, x, \sqrt{fx}\right)}{\sqrt{d}\left(cd^2-bde+ae^2\right)} - \frac{\left(c\left(2cd\right.\right.}{}$$

$$= \frac{e^{3/2}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{d}f}{\sqrt{e}}-\frac{\sqrt{2}\sqrt[4]{d}\sqrt{fx}}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{fx}\right)}{2\sqrt{d}\left(cd^2-bde+ae^2\right)} + \frac{e^{3/2}\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt{d}f}{\sqrt{e}}+\frac{\sqrt{2}\sqrt[4]{d}\sqrt{fx}}{\sqrt[4]{e}}+x^2}\,dx, x, \right.}{2\sqrt{d}\left(cd^2-bde+ae^2\right)}$$

$$= \frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bde+ae^2\right)\sqrt{f}} - \frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-\right.\right.}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b\right.}$$

$$= \frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\sqrt{f}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\left(cd^2-bde+ae^2\right)\sqrt{f}} - \frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-\right.\right.}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b\right.}$$

Mathematica [C] time = 0.374288, size = 267, normalized size = 0.31

$$\frac{\sqrt{x}\left(\sqrt{2}e^{7/4}\left(-\log\left(-\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)+\log\left(\sqrt{2}\sqrt[4]{d}\sqrt[4]{e}\sqrt{x}+\sqrt{d}+\sqrt{e}x\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{}}{}\right.\right.\right.}{4d^{3/4}\sqrt{fx}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[x]*(Sqrt[2]*e^(7/4)*(-2*ArcTan[1 - (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)]
 + 2*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[x])/d^(1/4)] - Log[Sqrt[d] - Sqrt[2]*
d^(1/4)*e^(1/4)*Sqrt[x] + Sqrt[e]*x] + Log[Sqrt[d] + Sqrt[2]*d^(1/4)*e^(1/4
)*Sqrt[x] + Sqrt[e]*x]) - 2*d^(3/4)*RootSum[a + b*#1^4 + c*#1^8 & , (-(c*d*
Log[Sqrt[x] - #1]) + b*e*Log[Sqrt[x] - #1] + c*e*Log[Sqrt[x] - #1]*#1^4)/(b
*#1^3 + 2*c*#1^7) & ]))/(4*d^(3/4)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[f*x])
```

Maple [C] time = 0.092, size = 336, normalized size = 0.4

$$\frac{e^2\sqrt{2}}{4f\left(ae^2-deb+cd^2\right)d}\sqrt[4]{\frac{df^2}{e}}\ln\left(\left(fx+\sqrt[4]{\frac{df^2}{e}}\sqrt{fx}\sqrt{2}+\sqrt{\frac{df^2}{e}}\right)\left(fx-\sqrt[4]{\frac{df^2}{e}}\sqrt{fx}\sqrt{2}+\sqrt{\frac{df^2}{e}}\right)^{-1}\right)+\frac{e^2\sqrt{2}}{2f\left(ae^2-deb+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x)
```

```
[Out] 1/4/f*e^2/(a*e^2-b*d*e+c*d^2)*(d*f^2/e)^(1/4)/d*2^(1/2)*ln((f*x+(d*f^2/e)^(
1/4)*(f*x)^(1/2)*2^(1/2)+(d*f^2/e)^(1/2))/(f*x-(d*f^2/e)^(1/4)*(f*x)^(1/2)*
2^(1/2)+(d*f^2/e)^(1/2)))+1/2/f*e^2/(a*e^2-b*d*e+c*d^2)*(d*f^2/e)^(1/4)/d*2
^(1/2)*arctan(2^(1/2)/(d*f^2/e)^(1/4)*(f*x)^(1/2)+1)+1/2/f*e^2/(a*e^2-b*d*e
+c*d^2)*(d*f^2/e)^(1/4)/d*2^(1/2)*arctan(2^(1/2)/(d*f^2/e)^(1/4)*(f*x)^(1/2
)-1)+1/2*f/(a*e^2-b*d*e+c*d^2)*sum((-_R^4*c*e-b*e*f^2+c*d*f^2)/(2*_R^7*c+_R
^3*b*f^2)*ln((f*x)^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b*f^2+a*f^4))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)/(f*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.311 $\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal. Leaf size=272

$$-\frac{\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}e^4}+\frac{\sqrt{a+bx^2+cx^4}\left((2cd-be)(be+4cd)-2\right.}{16c^2e^3}$$

```
[Out] (((2*c*d - b*e)*(4*c*d + b*e) - 2*c*e*(2*c*d + b*e)*x^2)*Sqrt[a + b*x^2 + c
*x^4])/(16*c^2*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*c*e) - ((16*c^3*d^3 - b^
3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) - 8*c^2*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x
^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(5/2)*e^4) + (d^2*Sqrt[c*d^
2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2
- b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rubi [A] time = 0.573454, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1251, 1653, 814, 843, 621, 206, 724}

$$-\frac{\left(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}e^4}+\frac{\sqrt{a+bx^2+cx^4}\left((2cd-be)(be+4cd)-2\right.}{16c^2e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```


```
[Out] (((2*c*d - b*e)*(4*c*d + b*e) - 2*c*e*(2*c*d + b*e)*x^2)*Sqrt[a + b*x^2 + c
*x^4])/(16*c^2*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*c*e) - ((16*c^3*d^3 - b^
3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) - 8*c^2*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x
^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(5/2)*e^4) + (d^2*Sqrt[c*d^
2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2
- b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx,x,x^2\right)$$

$$= \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} + \frac{\mathrm{Subst}\left(\int \frac{\left(-\frac{3}{2}bde-\frac{3}{2}e(2cd+be)x\right)\sqrt{a+bx+cx^2}}{d+ex}\,dx,x,x^2\right)}{6ce^2}$$

$$= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} - \frac{\mathrm{Subst}\left(\int \frac{3}{4}de \cdots\right.}{\cdots}$$

$$= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} + \frac{\left(d^2\left(cd^2-bd \cdots\right.\right.}{\cdots}$$

$$= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} - \frac{\left(d^2\left(cd^2-bde \cdots\right.\right.}{\cdots}$$

$$= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} - \frac{\left(16c^3d^3-b^3e \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.426532, size = 267, normalized size = 0.98

$$\frac{2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4}\left(2ce\left(4ae-3bd+bex^2\right)-3b^2e^2+4c^2\left(6d^2-3dex^2+2e^2x^4\right)\right)+24c^2d^2\sqrt{e(ae-bd)+cd^2}\tanh^{-1}\left(\cdots\right.\right.}{96c^{5/2}e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] (-3*(16*c^3*d^3 - b^3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) + 8*c^2*d*e*(-(b*d) + a
*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]
*(e*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2*e^2 + 2*c*e*(-3*b*d + 4*a*e + b*e*x^2)
+ 4*c^2*(6*d^2 - 3*d*e*x^2 + 2*e^2*x^4)) + 24*c^2*d^2*Sqrt[c*d^2 + e*(-(b*d
) + a*e)]*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*x^2))/(2*Sqrt[c*d^2 + e*(-
(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])]))/(96*c^(5/2)*e^4)
```

Maple [B] time = 0.043, size = 1049, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x)
```

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c/e-1/8/e*b/c*x^2*(c*x^4+b*x^2+a)^(1/2)-1/16/e*b^
2/c^2*(c*x^4+b*x^2+a)^(1/2)-1/8/e*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4
+b*x^2+a)^(1/2))*a+1/32/e*b^3/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2
+a)^(1/2))-1/4/e^2*d*(c*x^4+b*x^2+a)^(1/2)*x^2-1/8/e^2*d/c*(c*x^4+b*x^2+a)^
(1/2)*b-1/4/e^2*d/c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a
+1/16/e^2*d/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^2+1/2
*d^2/e^3*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)+1/4*d^2/e^3*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(
b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/2*d^3/e^
4*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(
```

```
x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)-1/2*d^2/e^3/((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^
2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a+1/2*d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2))/(x^2+d/e))*b-1/2*d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x^2+d/e))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\sqrt{a + bx^2 + cx^4}}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}\,x^5}{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^5/(e*x^2 + d), x)
```

3.312 $\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal. Leaf size=208

$$\frac{\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}e^3}-\frac{d\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^3}-\sqrt{a+bx^2\cdots}$$

```
[Out] -((4*c*d - b*e - 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c*e^2) + ((8*c^2*d^
2 - b^2*e^2 - 4*c*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(16*c^(3/2)*e^3) - (d*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh
[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
b*x^2 + c*x^4])])/(2*e^3)
```

Rubi [A] time = 0.314967, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 814, 843, 621, 206, 724}

$$\frac{\left(-4ce(bd-ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}e^3}-\frac{d\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^3}-\sqrt{a+bx^2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] -((4*c*d - b*e - 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c*e^2) + ((8*c^2*d^
2 - b^2*e^2 - 4*c*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(16*c^(3/2)*e^3) - (d*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh
[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
b*x^2 + c*x^4])])/(2*e^3)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)$$

$$= -\frac{\left(4cd-be-2cex^2\right)\sqrt{a+bx^2+cx^4}}{8ce^2} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}d\left(4bcd-b^2e-4ace\right)-\frac{1}{2}\left(8c^2d^2-b^2e^2-4ce(bd-ae)\right)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx \right.}{8ce^2}$$

$$= -\frac{\left(4cd-be-2cex^2\right)\sqrt{a+bx^2+cx^4}}{8ce^2} - \frac{\left(d\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, \right.}{2e^3}$$

$$= -\frac{\left(4cd-be-2cex^2\right)\sqrt{a+bx^2+cx^4}}{8ce^2} + \frac{\left(d\left(cd^2-bde+ae^2\right)\right)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, \right.}{e^3}$$

$$= -\frac{\left(4cd-be-2cex^2\right)\sqrt{a+bx^2+cx^4}}{8ce^2} + \frac{\left(8c^2d^2-b^2e^2-4ce(bd-ae)\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right.}{16c^{3/2}e^3}$$

Mathematica [A] time = 0.265411, size = 205, normalized size = 0.99

$$\frac{\left(4ce(ae-bd)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)+2\sqrt{c}\left(4cd\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)+\right.}{16c^{3/2}e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] ((8*c^2*d^2 - b^2*e^2 + 4*c*e*(-(b*d) + a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*(-4*c*d + b*e + 2*c*e*x^2)*Sqr
t[a + b*x^2 + c*x^4] + 4*c*d*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(-(b*d) +
2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2
```

```
+ c*x^4])]))/(16*c^(3/2)*e^3)
```

Maple [B] time = 0.01, size = 887, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x)
```

```
[Out] 1/4/e*(c*x^4+b*x^2+a)^(1/2)*x^2+1/8/e/c*(c*x^4+b*x^2+a)^(1/2)*b+1/4/e/c^(1/
2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a-1/16/e/c^(3/2)*ln((1/2
*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^2-1/2*d/e^2*(c*(x^2+d/e)^2+(b*e-
2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)-1/4*d/e^2*ln((1/2*(b*e-2*
c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b+1/2*d^2/e^3*ln((1/2*(b*e-2*c*d)/e+c*(x^2+
d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))*c^(1/2)+1/2*d/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*
d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c
*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d
/e))*a-1/2*d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2
)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e
)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b+1/
2*d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*
e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-
2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 119.521, size = 2718, normalized size = 13.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/32*(8*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (
b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*
c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sq
rt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d
*e*x^2 + d^2)) + (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(c)*log(-8
*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c
) - 4*a*c) + 4*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c*e^2)*sqrt(c*x^4 + b*x^2 + a
```

```
))/(c^2*e^3), -1/32*(16*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d*arctan(-1/2*sqrt
(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d -
 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 +
(b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c
)*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)
*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c*e^2)*s
qrt(c*x^4 + b*x^2 + a))/(c^2*e^3), 1/16*(4*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*
d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2
*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*
x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*
x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - (8*c^2*d^2 - 4*b*c*d*e -
 (b^2 - 4*a*c)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(2*c^2*e^2*x^2 - 4*c^2*d*e + b*c
*e^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2*e^3), -1/16*(8*sqrt(-c*d^2 + b*d*e - a*
e^2)*c^2*d*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)
*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c
*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + (8*c^2*d^2
 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 +
a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(2*c^2*e^2*x^2 - 4
*c^2*d*e + b*c*e^2)*sqrt(c*x^4 + b*x^2 + a))/(c^2*e^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{a + bx^2 + cx^4}}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.313 $\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal. Leaf size=168

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^2}+\frac{\sqrt{a+bx^2+cx^4}}{2e}$$

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*e) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e
^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^2)
```

Rubi [A] time = 0.216896, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1247, 734, 843, 621, 206, 724}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^2}+\frac{\sqrt{a+bx^2+cx^4}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*e) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e
^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^2)
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{\text{Subst}\left(\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4e}\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4e^2} + \frac{\left(cd^2-bde+ae^2\right)\text{Subst}\left(\int \frac{}{(d+}\right.}{2e^2}\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2e^2} - \frac{\left(cd^2-bde+ae^2\right)\text{Subst}\left(\int\right.}{}\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^2} + \frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{bd-2ae}{2\sqrt{cd^2-bde+}}\right.}{2e^2}
\end{aligned}$$

Mathematica [A] time = 0.119311, size = 167, normalized size = 0.99

$$\frac{2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4}-\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)\right)+(be-2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] ((-2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]
+ 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4] - Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTan
h[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqr
t[a + b*x^2 + c*x^4])]))/(4*Sqrt[c]*e^2)
```

Maple [B] time = 0.004, size = 757, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x)
```

```
[Out] 1/2/e*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
+1/4/e*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d
)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/2/e^2*ln((1/2*(b*
e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))/(x^2+d/e))*a+1/2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2
-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^
2+d/e))*d*b-1/2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^
2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/
e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c*d
^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 9.72263, size = 2327, normalized size = 13.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(c*x^4 + b*x^2 + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^4
- 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c
) + 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4
*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 +
 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^
2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
+ d^2)))/(c*e^2), 1/4*(2*sqrt(c*x^4 + b*x^2 + a)*c*e + (2*c*d - b*e)*sqrt(-
c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c
*x^2 + a*c)) + sqrt(c*d^2 - b*d*e + a*e^2)*c*log(-((8*c^2*d^2 - 8*b*c*d*e +
 (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*
b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*
sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2
*d*e*x^2 + d^2)))/(c*e^2), 1/8*(4*sqrt(c*x^4 + b*x^2 + a)*c*e + 4*sqrt(-c*d
^2 + b*d*e - a*e^2)*c*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d
*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2
)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) -
 (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*
x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(c*e^2), 1/4*(2*sqrt(c*x^4 + b*x^2
 + a)*c*e + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arctan(-1/2*sqrt(c*x^4 + b*x^2
```

```
 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*
d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2
*d*e + a*b*e^2)*x^2)) + (2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^
2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.314 $\int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)}\,dx$

Optimal. Leaf size=186

$$-\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e}$$

```
[Out] -(Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d)
 + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*
e) - (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)
/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*e)
```

Rubi [A] time = 0.255888, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 895, 724, 206, 843, 621}

$$-\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

```
[Out] -(Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d)
 + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*
e) - (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)
/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*e)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\left(d+ex^2\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)}\,dx, x, x^2\right)$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{-bd+ae-cdx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d} + \frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d}$$

$$= -\frac{a\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d} + \frac{c\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e} - \frac{1}{2}\left(-b+\frac{cd}{e}+\frac{ae}{d}\right)\text{S}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{c\operatorname{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{e} - \left(b-\frac{cd}{e}-\frac{ae}{d}\right)\operatorname{Subst}\left(\int\right.$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e} - \frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}\left(\frac{b\ldots}{2\sqrt{cd^2}\ldots}\right.}{2de}$$

Mathematica [A] time = 0.157293, size = 179, normalized size = 0.96

$$-\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right) - \sqrt{c}d\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right) + \sqrt{a}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

```
[Out] -(Sqrt[a]*e*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])] - Sq
rt[c]*d*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + Sqrt[c
*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + 2*c*d*x^2 - b*e*x^2)/(2*Sqrt[c
*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*e)
```

Maple [B] time = 0.016, size = 851, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x)
```

```
[Out] 1/2/d*(c*x^4+b*x^2+a)^(1/2)+1/4/d*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))/c^(1/2)-1/2/d*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2
))/x^2)-1/2/d*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)-1/4/d*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b
*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b+1/2/e*ln((1
/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)+1/2/d/((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/(x^2+d/e))*a-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e
^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(
x^2+d/e))*b+1/2/e^2*d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*
d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+
d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{(ex^2 + d)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x(d + ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x/(e*x**2+d),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(x*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{\left(ex^2 + d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x), x)
```

3.315 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\left(d+ex^2\right)}\,dx$

Optimal. Leaf size=361

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2}+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}-\frac{be\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2}-\frac{(2cd-b}{}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*d*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])])/(4*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (Sqrt[c]*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) - (b*e*ArcTanh[(b + 2*c*x^2)
/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2) - ((2*c*d - b*e)*Arc
Tanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2) +
(Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2)
```

Rubi [A] time = 0.509155, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1251, 960, 732, 843, 621, 206, 724, 734}

$$\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2}+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}-\frac{be\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}d^2}-\frac{(2cd-b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*d*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])])/(4*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (Sqrt[c]*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) - (b*e*ArcTanh[(b + 2*c*x^2)
/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2) - ((2*c*d - b*e)*Arc
Tanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2) +
(Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*S
qrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\left(d+ex^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2}-\frac{e\sqrt{a+bx+cx^2}}{d^2x}+\frac{e^2\sqrt{a+bx+cx^2}}{d^2(d+ex)}\right)dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx,x,x^2\right)}{2d}-\frac{e\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx,x,x^2\right)}{2d^2}+\frac{e^2\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx,x,x^2\cdots\right.}{2d^2}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}+\frac{\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4d}+\frac{e\,\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4d^2}-\frac{e\,\text{Subst}\cdots}{\cdots}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}+\frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4d}+\frac{c\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d}-\frac{(ae)\,\text{Su}\cdots}{\cdots}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}-\frac{b\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{c\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d}+\frac{(\cdots}{\cdots}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}d}+\frac{\sqrt{a}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b\cdots}{2\sqrt{c}\cdots}\right.}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.31142, size = 165, normalized size = 0.46

$$
\frac{2\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)+\frac{(2ae-bd)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}}-\frac{2d\sqrt{a+bx^2+cx^4}}{x^2}}{4d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

```
[Out] ((-2*d*Sqrt[a + b*x^2 + c*x^4])/x^2 + ((-(b*d) + 2*a*e)*ArcTanh[(2*a + b*x^
2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a] + 2*Sqrt[c*d^2 - b*d*e + a
*e^2]*ArcTanh[(b*d - 2*a*e + 2*c*d*x^2 - b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a
*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(4*d^2)
```

Maple [B] time = 0.016, size = 1009, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x)
```

```
[Out] -1/2/d^2*e*(c*x^4+b*x^2+a)^(1/2)-1/4/d^2*e*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2))/c^(1/2)+1/2/d^2*e*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+
b*x^2+a)^(1/2))/x^2)-1/2/d/a/x^2*(c*x^4+b*x^2+a)^(3/2)+1/2/d*b/a*(c*x^4+b*x
^2+a)^(1/2)-1/4/d*b/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/
x^2)+1/2/d/a*c*(c*x^4+b*x^2+a)^(1/2)*x^2+1/2/d*c^(1/2)*ln((1/2*b+c*x^2)/c^(
1/2)+(c*x^4+b*x^2+a)^(1/2))+1/2*e/d^2*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/4*e/d^2*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e
))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^
(1/2))/c^(1/2)*b-1/2/d*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d
```

```
/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)-1/2*e
/d^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c
*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d
)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a+1/2/d/((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)
+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d
^2)/e^2)^(1/2))/(x^2+d/e))*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{\left(ex^2 + d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x^3), x)
```

Fricas [A] time = 3.75858, size = 2414, normalized size = 6.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a*x^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b
^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c
*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqr
t(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*
e*x^2 + d^2)) - (b*d - 2*a*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x
^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt
(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*x^2), 1/8*(4*sqrt(-c*d^2 + b*d*e - a*e^2)*a
*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c
*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 -
 a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (b*d - 2*a*e)*sq
rt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*
x^2), 1/4*((b*d - 2*a*e)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b
*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + sqrt(c*d^2 - b*d*e + a*e^
2)*a*x^2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e
+ 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c
)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d
- b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 2*sqrt(c*x^4 + b*
x^2 + a)*a*d)/(a*d^2*x^2), 1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*a*x^2*arctan
(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x
^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e +
a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) + (b*d - 2*a*e)*sqrt(-a)*x^2*
arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^
2 + a^2)) - 2*sqrt(c*x^4 + b*x^2 + a)*a*d)/(a*d^2*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**3/(e*x**2+d),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(x**3*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2+a}}{\left(ex^2+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((e*x^2 + d)*x^3), x)
```

3.316 $\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

Optimal. Leaf size=424

$$\frac{\left(263\sqrt{2}-70\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{60\ 2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}-\frac{1}{60}\left(13-6x^2\right)\sqrt{2x^4+2x^2+1}x+\frac{109}{60}$$

```
[Out] -(x*(13 - 6*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/60 + (109*x*Sqrt[1 + 2*x^2 + 2*x^
4])/(60*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sqrt[15]*ArcTan[(Sqrt[5/3]*x)/Sqrt[
1 + 2*x^2 + 2*x^4]])/16 - (109*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(
1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(60*2^
(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + ((-70 + 263*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(60*2^(3/4)*(-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + (1
5*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2
)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(16*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.561012, antiderivative size = 619, normalized size of antiderivative = 1.46, number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {1335, 1091, 1197, 1103, 1195, 1116, 1208, 1216, 1706}

$$\frac{1}{30}\left(3x^2+1\right)\sqrt{2x^4+2x^2+1}x+\frac{109\sqrt{2x^4+2x^2+1}x}{60\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{1}{4}\sqrt{2x^4+2x^2+1}x+\frac{3}{16}\sqrt{15}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{45(3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] -(x*Sqrt[1 + 2*x^2 + 2*x^4])/4 + (x*(1 + 3*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/30
 + (109*x*Sqrt[1 + 2*x^2 + 2*x^4])/(60*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sqrt
[15]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/16 - (109*(1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/
4)*x], (2 - Sqrt[2])/4])/(60*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (139*(1 - S
qrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(240*2^(1/4)*Sqrt[1 + 2*x^2 +
 2*x^4]) - ((1 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(4*2^(3/4)*
Sqrt[1 + 2*x^2 + 2*x^4]) + (45*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*
x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[
2])/4])/(112*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (15*(3 + Sqrt[2])^2*(1 + Sq
rt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 1
1*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(224*2^(1/4)*Sqrt[1 +
 2*x^2 + 2*x^4])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1116

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*(2*b*p + c*(m + 4*p - 1)*x^
2))/(c*(m + 4*p + 1)*(m + 4*p - 1)), x] - Dist[(2*p*d^2)/(c*(m + 4*p + 1)*(
m + 4*p - 1)), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \left(-\frac{3}{4}\sqrt{1+2x^2+2x^4}+\frac{1}{2}x^2\sqrt{1+2x^2+2x^4}+\frac{9\sqrt{1+2x^2+2x^4}}{4\left(3+2x^2\right)}\right)dx$$

$$= \frac{1}{2}\int x^2\sqrt{1+2x^2+2x^4}\,dx-\frac{3}{4}\int \sqrt{1+2x^2+2x^4}\,dx+\frac{9}{4}\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$$

$$= -\frac{1}{4}x\sqrt{1+2x^2+2x^4}+\frac{1}{30}x\left(1+3x^2\right)\sqrt{1+2x^2+2x^4}-\frac{1}{60}\int \frac{2-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{1}{4}\int \frac{}{\sqrt{}}$$

$$= -\frac{1}{4}x\sqrt{1+2x^2+2x^4}+\frac{1}{30}x\left(1+3x^2\right)\sqrt{1+2x^2+2x^4}-\frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{15\sqrt{2}}+\frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{2\sqrt{2}}$$

$$= -\frac{1}{4}x\sqrt{1+2x^2+2x^4}+\frac{1}{30}x\left(1+3x^2\right)\sqrt{1+2x^2+2x^4}+\frac{109x\sqrt{1+2x^2+2x^4}}{60\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{3}{16}\sqrt{15}\tan$$

Mathematica [C] time = 0.2605, size = 209, normalized size = 0.49

$$\frac{-(199-417i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)+48x^7-56x^5-80x^3-218i\sqrt{1-i}\sqrt{}}{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (-52*x - 80*x^3 - 56*x^5 + 48*x^7 - (218*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^
2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (199 - 41
7*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*Ar
cSinh[Sqrt[1 - I]*x], I] + 225*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 +
 (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(240*Sqrt
[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.069, size = 528, normalized size = 1.3

$$\frac{x^3}{10}\sqrt{2x^4+2x^2+1}-\frac{13x}{60}\sqrt{2x^4+2x^2+1}-\frac{8\,\text{EllipticF}\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{15\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{}{\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x)
```

```
[Out] 1/10*x^3*(2*x^4+2*x^2+1)^(1/2)-13/60*x*(2*x^4+2*x^2+1)^(1/2)-8/15/(-1+I)^(1
/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF
(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(13/60-13/60*I)/(-1+I)^(1/2)*(1+
(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1
+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1
/2*I*2^(1/2)))-9/4/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2
*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+9/8
*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+9/8/(-1+I)^(1/2)*(-
I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-
1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-9/8*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/
2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2
^(1/2)+1/2*I*2^(1/2))+15/8/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^
(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2
)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}x^4}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}x^4}{2\,x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] Integral(x**4*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}x^4}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

3.317 $\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

Optimal. Leaf size=417

$$-\frac{\left(17\sqrt{2}-4\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\,2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}-\frac{7\sqrt{2x^4+2x^2+1}x}{6\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{1}{6}\sqrt{2x^4+2x^2+1}x-$$

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/6 - (7*x*Sqrt[1 + 2*x^2 + 2*x^4])/(6*Sqrt[2]*(1
 + Sqrt[2]*x^2)) - (Sqrt[15]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])
/8 + (7*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(6*2^(3/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) - ((-4 + 17*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
 Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(6*2^(3/4
)*(-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - (5*(3 + Sqrt[2])*(1 + Sqrt[2]
*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqr
t[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(3/4)*(2 - 3*Sqrt[2])
*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.409872, antiderivative size = 591, normalized size of antiderivative = 1.42, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1335, 1091, 1197, 1103, 1195, 1208, 1216, 1706}

$$-\frac{7\sqrt{2x^4+2x^2+1}x}{6\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{1}{6}\sqrt{2x^4+2x^2+1}x-\frac{1}{8}\sqrt{15}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{15\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan\right.}{56\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/6 - (7*x*Sqrt[1 + 2*x^2 + 2*x^4])/(6*Sqrt[2]*(1
 + Sqrt[2]*x^2)) - (Sqrt[15]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])
/8 + (7*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(6*2^(3/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) + (3*(1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(1/4)*
Sqrt[1 + 2*x^2 + 2*x^4]) + ((1 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(6*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (15*(3 + Sqrt[2])*(1 + Sqrt[2]*x
^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4
)*x], (2 - Sqrt[2])/4])/(56*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt
[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elli
pticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(112*2^
(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \left(\frac{1}{2}\sqrt{1+2x^2+2x^4} - \frac{3\sqrt{1+2x^2+2x^4}}{2\left(3+2x^2\right)}\right)dx$$
$$= \frac{1}{2}\int \sqrt{1+2x^2+2x^4}\,dx - \frac{3}{2}\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$$
$$= \frac{1}{6}x\sqrt{1+2x^2+2x^4} + \frac{1}{6}\int \frac{2+2x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{3}{8}\int \frac{2-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{15}{4}\int \frac{}{\left(3+2x^2\right)\sqrt{}}$$
$$= \frac{1}{6}x\sqrt{1+2x^2+2x^4} - \frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{3\sqrt{2}} + \frac{3\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{2\sqrt{2}} + \frac{1}{4}\left(3\left(1-\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x}}$$
$$= \frac{1}{6}x\sqrt{1+2x^2+2x^4} - \frac{7x\sqrt{1+2x^2+2x^4}}{6\sqrt{2}\left(1+\sqrt{2}x^2\right)} - \frac{1}{8}\sqrt{15}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) + \frac{7\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1}{(}}}{6}$$

Mathematica [C] time = 0.183224, size = 204, normalized size = 0.49

$$\frac{(13-27i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)+8x^5+8x^3+14i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+}}{24\sqrt{2x^4+2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (4*x + 8*x^3 + 8*x^5 + (14*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1
 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (13 - 27*I)*Sqrt[1 - I]
*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I
]*x], I] - 15*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Ell
ipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(24*Sqrt[1 + 2*x^2 + 2*x^4
])
```

Maple [C] time = 0.007, size = 509, normalized size = 1.2

$$\frac{x}{6}\sqrt{2x^4+2x^2+1} + \frac{\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{3\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+1}} - \frac{\left(\frac{1}{6}-\frac{i}{6}\right)\left(\text{Elliptic}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x)
```

```
[Out] 1/6*x*(2*x^4+2*x^2+1)^(1/2)+1/3/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x
^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*
2^(1/2))+(-1/6+1/6*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/
(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-
EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+3/2/(-1+I)^(1/2)*(-I*x
^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I
)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-3/4*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*
(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1
/2)+1/2*I*2^(1/2))-3/4/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2
```

```
)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))
+3/4*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1
)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-5/4/(-1+I)^(1/2
)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi
(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}x^2}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}x^2}{2\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] Integral(x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}x^2}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

3.318 $\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

Optimal. Leaf size=381

$$\frac{2^{3/4}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2x^4+2x^2+1}x}{\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{1}{4}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/(Sqrt[2]*(1 + Sqrt[2]*x^2)) + (Sqrt[5/3]*ArcTan
[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/4 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2
*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt
[2])/4])/(2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(3/4)*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/((-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[
2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellipti
cPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(12*2^(3/4
)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.202387, antiderivative size = 470, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1208, 1197, 1103, 1195, 1216, 1706}

$$\frac{\sqrt{2x^4+2x^2+1}x}{\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{1}{4}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{28\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(3 + 2*x^2),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/(Sqrt[2]*(1 + Sqrt[2]*x^2)) + (Sqrt[5/3]*ArcTan
[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/4 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2
*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt
[2])/4])/(2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 - Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(4*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2
])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elliptic
F[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(28*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4
]) - (5*(3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqr
t[2])/4])/(168*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
```

```
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = -\left(\frac{1}{4}\int \frac{2-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx\right) + \frac{5}{2}\int \frac{1}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{\sqrt{2}} - \frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{14}\left(5\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}$$

$$= \frac{x\sqrt{1+2x^2+2x^4}}{\sqrt{2}\left(1+\sqrt{2}x^2\right)} + \frac{1}{4}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-1}\left(\sqrt[4]{2}\right.\right.}{2^{3/4}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] time = 0.111975, size = 127, normalized size = 0.33

$$-\frac{\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(-(3+6i)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)+(3+3i)E\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+5i\Pi\left(\frac{1}{3}+\frac{i}{3}\right.\right.}{6\sqrt{1-i}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(3 + 2*x^2),x]
```

```
[Out] -(Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*((3 + 3*I)*EllipticE[I*ArcSin
h[Sqrt[1 - I]*x], I] - (3 + 6*I)*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + (
5*I)*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(6*Sqrt[1 - I]*Sq
rt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.003, size = 341, normalized size = 0.9

$$-\frac{\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}+\frac{\frac{i}{2}\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x)
```

```
[Out] -1/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+1/2*I/(-1+I)^(1/2)*
(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*
(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+1/2/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/
2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2
^(1/2)+1/2*I*2^(1/2))-1/2*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)
^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(
1/2))+5/6/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^
2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2x^4+2x^2+1}}{2x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^4 + 2x^2 + 1}}{2x^2 + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4 + 2\,x^2 + 1}}{2\,x^2 + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

3.319 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^2\left(3+2x^2\right)}\,dx$

Optimal. Leaf size=399

$$\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{21\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{3\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{1}{6}\sqrt{}$$

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1
+ Sqrt[2]*x^2)) - (Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])
/6 - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^
2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x
^4]) + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(21*2^(1/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^
2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2
^(1/4)*x], (2 - Sqrt[2])/4])/(252*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.242102, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1311, 1281, 1197, 1103, 1195, 1216, 1706}

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{3\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\right.\right.}{21\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^2*(3 + 2*x^2)),x]
```


```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1
+ Sqrt[2]*x^2)) - (Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])
/6 - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^
2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x
^4]) + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(21*2^(1/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^
2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2
^(1/4)*x], (2 - Sqrt[2])/4])/(252*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```


Rule 1311

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[((f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
```

```
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^2\left(3+2x^2\right)}dx = \frac{1}{6}\int \frac{2+6x^2}{x^2\sqrt{1+2x^2+2x^4}}dx - \frac{5}{3}\int \frac{1}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}dx$$
$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} - \frac{1}{6}\int \frac{-6-4x^2}{\sqrt{1+2x^2+2x^4}}dx - \frac{1}{21}\left(5\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}dx + \frac{1}{21}\Big(5\Big($$
$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} - \frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\Big(2\tan}{42\sqrt[4]{2}\sqrt{1+2x^2+2x}}$$
$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)} - \frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{}}{}$$

Mathematica [C] time = 0.200288, size = 208, normalized size = 0.52

$$\frac{(9-3i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}x\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)-12x^4-12x^2-6i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+}}{18x\sqrt{2x^4+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^2*(3 + 2*x^2)),x]
```

```
[Out] (-6 - 12*x^2 - 12*x^4 - (6*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 +
(1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (9 - 3*I)*Sqrt[1 - I]
*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 -
 I]*x], I] - 5*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*
EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(18*x*Sqrt[1 + 2*x^2 +
2*x^4])
```

Maple [C] time = 0.013, size = 511, normalized size = 1.3

$$\frac{2\,\text{EllipticF}\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{3\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}} - \frac{\frac{i}{3}\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{}\right.}{\sqrt{-1+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x)
```

```
[Out] 2/3/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(
1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/3*I/(-1+I)^(1/2)
*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x
*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/3/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1
/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*
2^(1/2)+1/2*I*2^(1/2))+1/3*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1
)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^
(1/2))-5/9/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x
^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))-
1/3*(2*x^4+2*x^2+1)^(1/2)/x+2/3/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x
^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*
2^(1/2))+(-2/3+2/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/
```

```
(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-
EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{\left(2\,x^2+3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^4+3\,x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^4 + 3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^4+2x^2+1}}{x^2\left(2x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/x**2/(2*x**2+3),x)
```

```
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**2*(2*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{\left(2\,x^2+3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2), x)
```

3.320 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\left(3+2x^2\right)}\,dx$

Optimal. Leaf size=360

$$\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{63\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\right.\right.}{9\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}$$

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(9*x^3) + (Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
 2*x^2 + 2*x^4]])/9 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*2^(1/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(63*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (5*(3 + Sqrt[2])^2*(1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt
[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(378*2^(1/4)*Sqrt[1 + 2*x^2
 + 2*x^4])
```

Rubi [A] time = 0.194183, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1309, 1281, 12, 1103, 1216, 1706}

$$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}+\frac{1}{9}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{63\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^4*(3 + 2*x^2)),x]
```


```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(9*x^3) + (Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
 2*x^2 + 2*x^4]])/9 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*2^(1/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(63*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (5*(3 + Sqrt[2])^2*(1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt
[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(378*2^(1/4)*Sqrt[1 + 2*x^2
 + 2*x^4])
```


Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[((f*x)^(m + 4)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
```

```
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\left(3+2x^2\right)}\,dx = \frac{1}{9}\int \frac{3+4x^2}{x^4\sqrt{1+2x^2+2x^4}}\,dx + \frac{10}{9}\int \frac{1}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} - \frac{1}{27}\int \frac{6}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{63}\left(10\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{63}\ldots$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{1}{9}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) + \frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\ldots\right.}{63\sqrt[4]{2}\sqrt{1+2x^2+\ldots}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{1}{9}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}\ldots\right.\right.}{9\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] time = 0.180776, size = 154, normalized size = 0.43

$$-\frac{3(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}x^3\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)+6x^4+6x^2-5(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1}{27x^3\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^4*(3 + 2*x^2)),x]
```

```
[Out] -(3 + 6*x^2 + 6*x^4 + 3*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1
 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 5*(1 - I)^(3/2)*x^3*Sqr
t[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sq
rt[1 - I]*x], I])/(27*x^3*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.015, size = 448, normalized size = 1.2

$$-\frac{4\,\text{EllipticF}\left(x\sqrt{-1+i},1/2\,\sqrt{2}+i/2\sqrt{2}\right)}{9\,\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}+\frac{\frac{2\,i}{9}\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\right.}{\sqrt{-1+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x)
```

```
[Out] -4/9/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^
(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+2/9*I/(-1+I)^(1/2
)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(
x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+2/9/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(
1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2
*2^(1/2)+1/2*I*2^(1/2))-2/9*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+
1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2
^(1/2))+10/27/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+
2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2
))+(2/9-2/9*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+
2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-Ellipti
cE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-1/9*(2*x^4+2*x^2+1)^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{\left(2\,x^2+3\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^6+3\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^6 + 3*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^4+2x^2+1}}{x^4\left(2x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/x**4/(2*x**2+3),x)
```

```
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**4*(2*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{\left(2\,x^2+3\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4), x)
```

3.321 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\left(3+2x^2\right)}\,dx$

Optimal. Leaf size=546

$$-\frac{\sqrt[4]{2}\left(19-2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{2x^4+2x^2+1}}+\frac{5\sqrt[4]{2}\left(5-3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}}{189\sqrt{2x^4}}$$

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(15*x^5) + (4*Sqrt[1 + 2*x^2 + 2*x^4])/(135*x^3) -
 (4*Sqrt[1 + 2*x^2 + 2*x^4])/(45*x) + (4*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])
/(45*(1 + Sqrt[2]*x^2)) - (2*Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2
+ 2*x^4]])/27 - (4*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(45*Sqrt[1
 + 2*x^2 + 2*x^4]) + (5*2^(1/4)*(5 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 +
 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sq
rt[2])/4])/(189*Sqrt[1 + 2*x^2 + 2*x^4]) - (2^(1/4)*(19 - 2*Sqrt[2])*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(135*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqr
t[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(567*2
^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.543553, antiderivative size = 546, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {1309, 1281, 1197, 1103, 1195, 1329, 1714, 1708, 1706}

$$\frac{4\sqrt{2}\sqrt{2x^4+2x^2+1}x}{45\left(\sqrt{2}x^2+1\right)}-\frac{4\sqrt{2x^4+2x^2+1}}{45x}+\frac{4\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}-\frac{2}{27}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^6*(3 + 2*x^2)),x]
```


```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(15*x^5) + (4*Sqrt[1 + 2*x^2 + 2*x^4])/(135*x^3) -
 (4*Sqrt[1 + 2*x^2 + 2*x^4])/(45*x) + (4*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])
/(45*(1 + Sqrt[2]*x^2)) - (2*Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2
+ 2*x^4]])/27 - (4*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(45*Sqrt[1
 + 2*x^2 + 2*x^4]) + (5*2^(1/4)*(5 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 +
 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sq
rt[2])/4])/(189*Sqrt[1 + 2*x^2 + 2*x^4]) - (2^(1/4)*(19 - 2*Sqrt[2])*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(135*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqr
t[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(567*2
^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```


Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
```

```
f^4), Int[((f*x)^(m + 4)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1329

```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[(x^(m + 1)*Sqrt[a + b*x^2 + c*x^4])/(a*d*(m + 1)), x
] - Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)*Simp[a*e*(m + 1) + b*d*(m + 2) + (
b*e*(m + 2) + c*d*(m + 3))*x^2 + c*e*(m + 3)*x^4, x])/((d + e*x^2)*Sqrt[a +
 b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && ILtQ[m/2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6\left(3+2x^2\right)}\,dx = \frac{1}{9}\int \frac{3+4x^2}{x^6\sqrt{1+2x^2+2x^4}}\,dx + \frac{10}{9}\int \frac{1}{x^2\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} - \frac{10\sqrt{1+2x^2+2x^4}}{27x} - \frac{1}{45}\int \frac{4+18x^2}{x^4\sqrt{1+2x^2+2x^4}}\,dx + \frac{10}{27}\int \frac{-2+6x^2+4}{\left(3+2x^2\right)\sqrt{1+2x}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{10\sqrt{1+2x^2+2x^4}}{27x} + \frac{1}{135}\int \frac{-38+8x^2}{x^2\sqrt{1+2x^2+2x^4}}\,dx + \frac{5}{54}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{4\sqrt{1+2x^2+2x^4}}{45x} + \frac{10\sqrt{2}x\sqrt{1+2x^2+2x^4}}{27\left(1+\sqrt{2}x^2\right)} - \frac{10\sqrt[4]{2}\left(1+\right.}{}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{4\sqrt{1+2x^2+2x^4}}{45x} + \frac{10\sqrt{2}x\sqrt{1+2x^2+2x^4}}{27\left(1+\sqrt{2}x^2\right)} - \frac{2}{27}\sqrt{\frac{5}{3}}\,\mathrm{ta}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{4\sqrt{1+2x^2+2x^4}}{45x} + \frac{4\sqrt{2}x\sqrt{1+2x^2+2x^4}}{45\left(1+\sqrt{2}x^2\right)} - \frac{2}{27}\sqrt{\frac{5}{3}}\,\mathrm{tan}$$

Mathematica [C] time = 0.247131, size = 224, normalized size = 0.41

$$-\frac{-(12+24i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}x^5\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)+72x^8+48x^6+66x^4+42x^2+36i\sqrt{1}}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^6*(3 + 2*x^2)),x]
```

```
[Out] -(27 + 42*x^2 + 66*x^4 + 48*x^6 + 72*x^8 + (36*I)*Sqrt[1 - I]*x^5*Sqrt[1 +
(1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] -
```

```
(12 + 24*I)*Sqrt[1 - I]*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*El
lipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 50*(1 - I)^(3/2)*x^5*Sqrt[1 + (1 - I
)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x]
, I])/(405*x^5*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.018, size = 549, normalized size = 1.

$$\frac{8\,\mathrm{EllipticF}\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{27\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}-\frac{\frac{4i}{27}\mathrm{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\right.}{\sqrt{-1+i}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x)
```

```
[Out] 8/27/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^
(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-4/27*I/(-1+I)^(1/
2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF
(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-4/27/(-1+I)^(1/2)*(-I*x^2+x^2+1)
^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1
/2*2^(1/2)+1/2*I*2^(1/2))+(-32/135+32/135*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/
2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*
2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))
-20/81/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1
)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))-4/45
*(2*x^4+2*x^2+1)^(1/2)/x-4/45/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2
)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^
(1/2))+4/27*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+
2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/15*(2*
x^4+2*x^2+1)^(1/2)/x^5+4/135*(2*x^4+2*x^2+1)^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{\left(2\,x^2+3\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^8+3\,x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^8 + 3*x^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^4 + 2x^2 + 1}}{x^6 \left(2x^2 + 3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/x**6/(2*x**2+3),x)
```

```
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**6*(2*x**2 + 3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^4 + 2\,x^2 + 1}}{\left(2\,x^2 + 3\right)x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6), x)
```

3.322 $\int \frac{x^5\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=482

$$-\frac{\left(16bc^2e^3\left(3a^2e^2-3abde+b^2d^2\right)+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2(bd-ae)^2+3b^5e^5+256c^5d^5\right)\tan}{512c^{7/2}e^6}$$

```
[Out] ((128*c^4*d^4 + 3*b^4*e^4 - 32*c^3*d^2*e*(5*b*d - 4*a*e) + 8*b*c^2*d*e^2*(2
*b*d - 3*a*e) + 6*b^2*c*e^3*(b*d - 2*a*e) - 2*c*e*(32*c^3*d^3 - 3*b^3*e^3 -
 8*c^2*d*e*(2*b*d - 3*a*e) - 6*b*c*e^2*(b*d - 2*a*e))*x^2)*Sqrt[a + b*x^2 +
 c*x^4])/(256*c^3*e^5) + ((16*c^2*d^2 - 6*b*c*d*e - 3*b^2*e^2 - 6*c*e*(2*c*
d + b*e)*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(96*c^2*e^3) + (a + b*x^2 + c*x^4)
^(5/2)/(10*c*e) - ((256*c^5*d^5 + 3*b^5*e^5 + 6*b^3*c*e^4*(b*d - 4*a*e) - 3
84*c^4*d^3*e*(b*d - a*e) + 96*c^3*d*e^2*(b*d - a*e)^2 + 16*b*c^2*e^3*(b^2*d
^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/(512*c^(7/2)*e^6) + (d^2*(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
 b*x^2 + c*x^4])])/(2*e^6)
```

Rubi [A] time = 1.10293, antiderivative size = 482, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1251, 1653, 814, 843, 621, 206, 724}

$$-\frac{\left(16bc^2e^3\left(3a^2e^2-3abde+b^2d^2\right)+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2(bd-ae)^2+3b^5e^5+256c^5d^5\right)\tan}{512c^{7/2}e^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((128*c^4*d^4 + 3*b^4*e^4 - 32*c^3*d^2*e*(5*b*d - 4*a*e) + 8*b*c^2*d*e^2*(2
*b*d - 3*a*e) + 6*b^2*c*e^3*(b*d - 2*a*e) - 2*c*e*(32*c^3*d^3 - 3*b^3*e^3 -
 8*c^2*d*e*(2*b*d - 3*a*e) - 6*b*c*e^2*(b*d - 2*a*e))*x^2)*Sqrt[a + b*x^2 +
 c*x^4])/(256*c^3*e^5) + ((16*c^2*d^2 - 6*b*c*d*e - 3*b^2*e^2 - 6*c*e*(2*c*
d + b*e)*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(96*c^2*e^3) + (a + b*x^2 + c*x^4)
^(5/2)/(10*c*e) - ((256*c^5*d^5 + 3*b^5*e^5 + 6*b^3*c*e^4*(b*d - 4*a*e) - 3
84*c^4*d^3*e*(b*d - a*e) + 96*c^3*d*e^2*(b*d - a*e)^2 + 16*b*c^2*e^3*(b^2*d
^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/(512*c^(7/2)*e^6) + (d^2*(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
 b*x^2 + c*x^4])])/(2*e^6)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
```

```
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^5 \left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2 \left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx, x, x^2\right)$$

$$= \frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ce} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{5}{2}bde-\frac{5}{2}e(2cd+be)x\right)\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx, x, x^2\right)}{10ce^2}$$

$$= \frac{\left(16c^2d^2-6bcde-3b^2e^2-6ce(2cd+be)x^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{96c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{5/2}}{10ce} - \cdots$$

$$= \frac{\left(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2ae)-2ce\left(\cdots\right.\right.}{256c^3e^5}$$

$$= \frac{\left(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2ae)-2ce\left(\cdots\right.\right.}{256c^3e^5}$$

$$= \frac{\left(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2ae)-2ce\left(\cdots\right.\right.}{256c^3e^5}$$

$$= \frac{\left(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2ae)-2ce\left(\cdots\right.\right.}{256c^3e^5}$$

Mathematica [A] time = 1.02952, size = 545, normalized size = 1.13

$$-\frac{\frac{240d^2\left((2cd-be)\left(4ce(3ae-2bd)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)+2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4}\left(-2ce\left(4ae-5bd+bex^2\right)-b^2e^2+4c^2d\left(ex^2-2d\right)\right)+8c\left(e(ae-bd)+cd^2\right)^{3/2}\cdots\right.\right)}{c^{3/2}e^3}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] (1280*d^2*(a + b*x^2 + c*x^4)^(3/2) - (480*d*e*(b + 2*c*x^2)*(a + b*x^2 + c
*x^4)^(3/2))/c + (768*e^2*(a + b*x^2 + c*x^4)^(5/2))/c - (90*(b^2 - 4*a*c)*
d*e*(-2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/c^(5/2) + (15*b*e^2
*(-16*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2) + 3*(b^2 - 4*a*c)*((2*(b + 2*
c*x^2)*Sqrt[a + b*x^2 + c*x^4])/c + ((-b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^2)/(
2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/c^(3/2))))/c^2 - (240*d^2*((2*c*d - b*
e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4]*(-
(b^2*e^2) + 4*c^2*d*(-2*d + e*x^2) - 2*c*e*(-5*b*d + 4*a*e + b*e*x^2)) + 8*
c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*
e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])])))/(c^(3
/2)*e^3))/(7680*e^3)
```

Maple [B] time = 0.049, size = 2068, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)
```

```
[Out] 3/8*d^2/e^3*a*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+d^3
/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c
*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d
)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*b-d^4/e^5/((a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+
d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/
e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*c+d^5/e^6/((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c+7/160/e*a*b*x^2/c*(c*x^4+b*x^2+a)^(1
/2)-5/32/e^2*d*a*b/c*(c*x^4+b*x^2+a)^(1/2)+3/32/e^2*d*a*b^2/c^(3/2)*ln((1/2
*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/64/e^2*d*b^2*x^2/c*(c*x^4+b*x^2+
a)^(1/2)+1/10/e*c*x^8*(c*x^4+b*x^2+a)^(1/2)+11/80/e*b*x^6*(c*x^4+b*x^2+a)^(
1/2)-5/8*d^3/e^4*b*(c*x^4+b*x^2+a)^(1/2)+1/2*d^4/e^5*c*(c*x^4+b*x^2+a)^(1/2
)-1/2*d^5/e^6*c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+2/3*d
^2/e^3*a*(c*x^4+b*x^2+a)^(1/2)+1/5/e*a*x^4*(c*x^4+b*x^2+a)^(1/2)+3/256/e*b^
4/c^3*(c*x^4+b*x^2+a)^(1/2)-3/512/e*b^5/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c
*x^4+b*x^2+a)^(1/2))+1/10/e*a^2/c*(c*x^4+b*x^2+a)^(1/2)+1/16*d^2/e^3/c*b^2*
(c*x^4+b*x^2+a)^(1/2)-1/32*d^2/e^3*b^3/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*
x^4+b*x^2+a)^(1/2))-1/4*d^3/e^4*x^2*c*(c*x^4+b*x^2+a)^(1/2)-3/4*d^3/e^4*a*c
^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/16*d^3/e^4*b^2*ln(
(1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+3/4*d^4/e^5*b*c^(1/2)*
ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2*d^2/e^3/((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2
-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2-1/2*d^4/e^5/((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+
c*d^2)/e^2)^(1/2))/(x^2+d/e))*b^2-1/2*d^6/e^7/((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/
e^2)^(1/2))/(x^2+d/e))*c^2-3/32/e*a^2*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c
*x^4+b*x^2+a)^(1/2))-5/16/e^2*d*a*x^2*(c*x^4+b*x^2+a)^(1/2)+3/128/e^2*d*b^3
/c^2*(c*x^4+b*x^2+a)^(1/2)-3/256/e^2*d*b^4/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)
+(c*x^4+b*x^2+a)^(1/2))-3/16/e^2*d*a^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^
2+a)^(1/2))/c^(1/2)-1/8/e^2*d*c*x^6*(c*x^4+b*x^2+a)^(1/2)-3/16/e^2*d*b*x^4*
(c*x^4+b*x^2+a)^(1/2)+1/160/e*b^2*x^4/c*(c*x^4+b*x^2+a)^(1/2)-1/128/e*b^3/c
^2*x^2*(c*x^4+b*x^2+a)^(1/2)-5/64/e*a*b^2/c^2*(c*x^4+b*x^2+a)^(1/2)+3/64/e*
a*b^3/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/6*d^2/e^3*c
*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24*d^2/e^3*b*x^2*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}x^5}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^5/(e*x^2 + d), x)
```

3.323 $\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2} dx$

Optimal. Leaf size=360

$$-\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2\right)-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)+3b^3e^3+64c^3d^3\right)}{128c^2e^4}$$

```
[Out] -((64*c^3*d^3 + 3*b^3*e^3 - 16*c^2*d*e*(5*b*d - 4*a*e) + 4*b*c*e^2*(2*b*d -
 3*a*e) - 2*c*e*(16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*x^2)*Sqrt[
a + b*x^2 + c*x^4])/(128*c^2*e^4) - ((8*c*d - 3*b*e - 6*c*e*x^2)*(a + b*x^2
 + c*x^4)^(3/2))/(48*c*e^2) + ((128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d
- 3*a*e) - 192*c^3*d^2*e*(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[(b
 + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*e^5) - (d*(c
*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^5)
```

Rubi [A] time = 0.69676, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 814, 843, 621, 206, 724}

$$-\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2\left(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2\right)-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)+3b^3e^3+64c^3d^3\right)}{128c^2e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] -((64*c^3*d^3 + 3*b^3*e^3 - 16*c^2*d*e*(5*b*d - 4*a*e) + 4*b*c*e^2*(2*b*d -
 3*a*e) - 2*c*e*(16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*x^2)*Sqrt[
a + b*x^2 + c*x^4])/(128*c^2*e^4) - ((8*c*d - 3*b*e - 6*c*e*x^2)*(a + b*x^2
 + c*x^4)^(3/2))/(48*c*e^2) + ((128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d
- 3*a*e) - 192*c^3*d^2*e*(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[(b
 + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*e^5) - (d*(c
*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sq
rt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^5)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx, x, x^2\right)$$

$$= -\frac{\left(8cd-3be-6cex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{48ce^2} - \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}d\left(4ace-2b\left(4cd-\frac{3be}{2}\right)\right)-\frac{1}{2}\left(16c^2d^2-3b^2e^2-4c\right.\right.}{d+ex}\right.}{16ce^2}$$

$$= -\frac{\left(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce\left(16c^2d^2-3b^2e^2-4ce(2\right.\right.}{128c^2e^4}$$

$$= -\frac{\left(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce\left(16c^2d^2-3b^2e^2-4ce(2\right.\right.}{128c^2e^4}$$

$$= -\frac{\left(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce\left(16c^2d^2-3b^2e^2-4ce(2\right.\right.}{128c^2e^4}$$

$$= -\frac{\left(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce\left(16c^2d^2-3b^2e^2-4ce(2\right.\right.}{128c^2e^4}$$

Mathematica [A] time = 0.603905, size = 344, normalized size = 0.96

$$\frac{3\left(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)+2\sqrt{c}\left(e\sqrt{a+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] (3*(128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d - 3*a*e) - 192*c^3*d^2*e*(b*
d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[
a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4]*(-9*b^3*e^3 + 6
*b*c*e^2*(-4*b*d + 10*a*e + b*e*x^2) - 16*c^3*(12*d^3 - 6*d^2*e*x^2 + 4*d*e
^2*x^4 - 3*e^3*x^6) + 8*c^2*e*(a*e*(-32*d + 15*e*x^2) + b*(30*d^2 - 14*d*e*
x^2 + 9*e^2*x^4))) + 192*c^2*d*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(
b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[
a + b*x^2 + c*x^4])]))/(768*c^(5/2)*e^5)
```

Maple [B] time = 0.01, size = 1696, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)
```

```
[Out] 3/4*d^2/e^3*a*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/16*
d^2/e^3*b^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-3/4*d^3
/e^4*b*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/2*d/e^2/((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(
x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^
2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2+1/2*d^3/e^4/((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e
)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b^2+1/2*d^5/e^6/((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2-1/6*d/e^2*c*x^4*(c*x^4+b*x^2+a)^(1/
2)+5/32/e*a*b/c*(c*x^4+b*x^2+a)^(1/2)-3/32/e*a*b^2/c^(3/2)*ln((1/2*b+c*x^2)
/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/64/e*b^2*x^2/c*(c*x^4+b*x^2+a)^(1/2)+1/32
*d/e^2*b^3/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/4*d^2/
e^3*x^2*c*(c*x^4+b*x^2+a)^(1/2)-7/24*d/e^2*b*x^2*(c*x^4+b*x^2+a)^(1/2)-1/16
*d/e^2/c*b^2*(c*x^4+b*x^2+a)^(1/2)+5/8*d^2/e^3*b*(c*x^4+b*x^2+a)^(1/2)-1/2*
d^3/e^4*c*(c*x^4+b*x^2+a)^(1/2)+1/2*d^4/e^5*c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2
)+(c*x^4+b*x^2+a)^(1/2))-2/3*d/e^2*a*(c*x^4+b*x^2+a)^(1/2)+5/16/e*a*x^2*(c*
x^4+b*x^2+a)^(1/2)-3/128/e*b^3/c^2*(c*x^4+b*x^2+a)^(1/2)+3/256/e*b^4/c^(5/2
)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/16/e*a^2*ln((1/2*b+c*x^
2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+1/8/e*c*x^6*(c*x^4+b*x^2+a)^(1/2)
+3/16/e*b*x^4*(c*x^4+b*x^2+a)^(1/2)-3/8*d/e^2*a*b*ln((1/2*b+c*x^2)/c^(1/2)+
(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((
2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2))/(x^2+d/e))*a*b+d^3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*
d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c
*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d
/e))*a*c-d^4/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/
e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^
2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**3*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.324 $\int \frac{x\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=269

$$\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16ce^3}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh^{-}}{32c^{3/2}e^4}$$

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sq
rt[a + b*x^2 + c*x^4])/(16*c*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*e) - ((2*c
*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(3/2)*e^4) + ((c*d^2 - b*d*e
 + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b
*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rubi [A] time = 0.455549, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1247, 734, 814, 843, 621, 206, 724}

$$\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16ce^3}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh^{-}}{32c^{3/2}e^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sq
rt[a + b*x^2 + c*x^4])/(16*c*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*e) - ((2*c
*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(3/2)*e^4) + ((c*d^2 - b*d*e
 + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b
*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
```

```
2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex}\,dx, x, x^2\right)$$

$$= \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6e} - \frac{\operatorname{Subst}\left(\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)}{4e}$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2\right)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6e} +$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2\right)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6e} +$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2\right)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6e} -$$

$$= \frac{\left(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2\right)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6e} -$$

Mathematica [A] time = 0.366357, size = 255, normalized size = 0.95

$$\frac{2\sqrt{c}\left(e\sqrt{a+bx^2+cx^4}\left(2ce\left(16ae-15bd+7bex^2\right)+3b^2e^2+4c^2\left(6d^2-3dex^2+2e^2x^4\right)\right)-24c\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-}\right.}{96c^{3/2}e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] (-3*(2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b
 + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*
x^2 + c*x^4]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*x^2) + 4*c^2*(6*d
^2 - 3*d*e*x^2 + 2*e^2*x^4)) - 24*c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTan
h[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*
Sqrt[a + b*x^2 + c*x^4])]))/(96*c^(3/2)*e^4)
```

Maple [B] time = 0.006, size = 1411, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x)
```

```
[Out] -5/8/e^2*b*(c*x^4+b*x^2+a)^(1/2)*d+1/2/e^3*c*(c*x^4+b*x^2+a)^(1/2)*d^2+1/6/
e*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24/e*b*x^2*(c*x^4+b*x^2+a)^(1/2)+1/16/e/c*b
^2*(c*x^4+b*x^2+a)^(1/2)-1/2/e^4*c^(3/2)*d^3*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2))+2/3/e*a*(c*x^4+b*x^2+a)^(1/2)-1/32/e*b^3/c^(3/2)*ln((1/2*
b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4/e^2*x^2*c*(c*x^4+b*x^2+a)^(1/2)
*d+3/8/e*a*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-3/4/e^
2*d*a*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-3/16/e^2*b^2*
d*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+3/4/e^3*b*c^(1/2)
*d^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2/e/((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/
e^2)^(1/2))/(x^2+d/e))*a*b*d-1/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a
*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))
/(x^2+d/e))*a*c*d^2-1/2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*
d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c
*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d
/e))*b^2*d^2+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2
)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e
)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c*
d^3-1/2/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(
b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*
e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2*d^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.325 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x\left(d+ex^2\right)}\,dx$

Optimal. Leaf size=350

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}de^3}-\sqrt{a+bx^2+cx^4}\,(-$$

```
[Out] (a*Sqrt[a + b*x^2 + c*x^4])/(2*d) - ((4*c*d^2 - e*(5*b*d - 4*a*e) - 2*c*d*e
*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d*e^2) - (a^(3/2)*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) + (a*b*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d) + ((8*c^2*d^3 + b*e^2*(3*b
*d - 4*a*e) - 12*c*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
 + b*x^2 + c*x^4])])/(16*Sqrt[c]*d*e^3) - ((c*d^2 - b*d*e + a*e^2)^(3/2)*Ar
cTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x^2 + c*x^4])])/(2*d*e^3)
```

Rubi [A] time = 0.573223, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1251, 895, 734, 843, 621, 206, 724, 814}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\left(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}de^3}-\sqrt{a+bx^2+cx^4}\,(-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

```
[Out] (a*Sqrt[a + b*x^2 + c*x^4])/(2*d) - ((4*c*d^2 - e*(5*b*d - 4*a*e) - 2*c*d*e
*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d*e^2) - (a^(3/2)*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) + (a*b*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d) + ((8*c^2*d^3 + b*e^2*(3*b
*d - 4*a*e) - 12*c*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
 + b*x^2 + c*x^4])])/(16*Sqrt[c]*d*e^3) - ((c*d^2 - b*d*e + a*e^2)^(3/2)*Ar
cTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x^2 + c*x^4])])/(2*d*e^3)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 895

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p -
1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x\left(d+ex^2\right)}dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x(d+ex)}dx, x, x^2\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{(-bd+ae-cdx)\sqrt{a+bx+cx^2}}{d+ex}dx, x, x^2\right)}{2d} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}dx, x, x^2\right)}{2d}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{\left(4cd^2-e(5bd-4ae)-2cdex^2\right)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a\,\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}d\right.}{4d}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{\left(4cd^2-e(5bd-4ae)-2cdex^2\right)\sqrt{a+bx^2+cx^4}}{8de^2} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\right.}{2d}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{\left(4cd^2-e(5bd-4ae)-2cdex^2\right)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{4a-x^2}dx, x,\right.}{d}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{\left(4cd^2-e(5bd-4ae)-2cdex^2\right)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+c}}\right.}{2d}$$

Mathematica [A] time = 0.521335, size = 251, normalized size = 0.72

$$\frac{1}{16}\left(-\frac{8a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{d} + \frac{\left(12ce(ae-bd)+3b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}e^3} + \frac{2\left(4\left(e(ae-bd)+cd^2\right)^{3/2}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

```
[Out] ((-8*a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/d
+ ((8*c^2*d^2 + 3*b^2*e^2 + 12*c*e*(-(b*d) + a*e))*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(Sqrt[c]*e^3) + (2*(d*e*(-4*c*d + 5*b*e
 + 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4] + 4*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*
ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) +
a*e)]*Sqrt[a + b*x^2 + c*x^4])]))/(d*e^3))/16
```

Maple [B] time = 0.021, size = 1270, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x)
```

```
[Out] -3/4/e^2*d*b*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/e/((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(
x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^
2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*b+1/2/e^2*d/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+
2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b^2+1/2/e^4*d^3/((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e
```

```
^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c^2-1/2/e^2*d*c*(c*x^4+b*x^2+a)^(1/2)+1/2
/e^3*d^2*c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/4/e*x^2*
c*(c*x^4+b*x^2+a)^(1/2)+3/4/e*a*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2))+3/16/e*b^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(
1/2)+1/2/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b
*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e
-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2-1/2/d*a^
(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+5/8/e*b*(c*x^4+b*
x^2+a)^(1/2)+1/e^2*d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d
^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d
/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*
c-1/e^3*d^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(
b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*
e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{\left(ex^2+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x/(e*x**2+d),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(x*(d + e*x**2)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{\left(ex^2 + d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x), x)
```

3.326 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx$

Optimal. Leaf size=562

$$\frac{a^{3/2}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}+\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16cd^2e}-\frac{(2cd-be)\left(-4c\right.}{}$$

```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d) - (e*(b^2 + 8*a*c + 2*b*c
*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^2) + ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(
5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^
2*e) - (a + b*x^2 + c*x^4)^(3/2)/(2*d*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*d) + (a^(3/2)*e*ArcTanh[(2*a
+ b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (3*(b^2 + 4*a*c)*A
rcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*Sqrt[c]*d) +
 (b*(b^2 - 12*a*c)*e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(32*c^(3/2)*d^2) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d
 - 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*
c^(3/2)*d^2*e^2) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2
*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/
(2*d^2*e^2)
```

Rubi [A] time = 0.923926, antiderivative size = 562, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {1251, 960, 732, 814, 843, 621, 206, 724, 734}

$$\frac{a^{3/2}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}+\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16cd^2e}-\frac{(2cd-be)\left(-4c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```


```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d) - (e*(b^2 + 8*a*c + 2*b*c
*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^2) + ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(
5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^
2*e) - (a + b*x^2 + c*x^4)^(3/2)/(2*d*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*d) + (a^(3/2)*e*ArcTanh[(2*a
+ b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (3*(b^2 + 4*a*c)*A
rcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*Sqrt[c]*d) +
 (b*(b^2 - 12*a*c)*e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(32*c^(3/2)*d^2) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d
 - 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*
c^(3/2)*d^2*e^2) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2
*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/
(2*d^2*e^2)
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rubi steps

$$\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^3\left(d+ex^2\right)}dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2(d+ex)}dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\left(a+bx+cx^2\right)^{3/2}}{dx^2}-\frac{e\left(a+bx+cx^2\right)^{3/2}}{d^2x}+\frac{e^2\left(a+bx+cx^2\right)^{3/2}}{d^2(d+ex)}\right)dx,x,x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^2}dx,x,x^2\right)}{2d}-\frac{e\,\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x}dx,x,x^2\right)}{2d^2}+\frac{e^2\,\text{Subst}\left(\int \frac{(a+bx+cx}{d+ex}\right.}{2d^2}$$
$$= -\frac{\left(a+bx^2+cx^4\right)^{3/2}}{2dx^2}+\frac{3\,\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}dx,x,x^2\right)}{4d}+\frac{e\,\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\right.}{4d^2}$$
$$= \frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d}-\frac{e\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2}+\frac{\left(8c^2d^2+b^2e^2-2c\right.}{}$$
$$= \frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d}-\frac{e\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2}+\frac{\left(8c^2d^2+b^2e^2-2c\right.}{}$$
$$= \frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d}-\frac{e\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2}+\frac{\left(8c^2d^2+b^2e^2-2c\right.}{}$$
$$= \frac{3\left(3b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{8d}-\frac{e\left(b^2+8ac+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16cd^2}+\frac{\left(8c^2d^2+b^2e^2-2c\right.}{}$$

Mathematica [A] time = 0.513455, size = 240, normalized size = 0.43

$$\frac{1}{4}\left(-\frac{2\left(x^2\left(e(ae-bd)+cd^2\right)^{3/2}\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)+de\sqrt{a+bx^2+cx^4}\left(ae-cdx^2\right)\right)}{d^2e^2x^2}+\frac{\sqrt{a}(2ae-3bd)\tan}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```

```
[Out] ((Sqrt[a]*(-3*b*d + 2*a*e)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2
+ c*x^4])])/d^2 - (Sqrt[c]*(2*c*d - 3*b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]
*Sqrt[a + b*x^2 + c*x^4])])/e^2 - (2*(d*e*(a*e - c*d*x^2)*Sqrt[a + b*x^2 +
c*x^4] + (c*d^2 + e*(-(b*d) + a*e))^(3/2)*x^2*ArcTanh[(-(b*d) + 2*a*e - 2*c
*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4]
)]))/(d^2*e^2*x^2))/4
```

Maple [B] time = 0.023, size = 1207, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x)
```

```
[Out] -1/2*e/d^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b
*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e
-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2+1/d/((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^
2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+
d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*b-1/e/((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*c-1/2/e^3*d^2/((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2))/(x^2+d/e))*c^2-1/2/e^2*d*c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c
*x^4+b*x^2+a)^(1/2))+3/4/e*b*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+
a)^(1/2))-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e
^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2
+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b^2-1/2
/d*a/x^2*(c*x^4+b*x^2+a)^(1/2)-3/4/d*a^(1/2)*b*ln((2*a+b*x^2+2*a^(1/2)*(c*x
^4+b*x^2+a)^(1/2))/x^2)+1/2/d^2*e*a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*
x^2+a)^(1/2))/x^2)+1/2/e*c*(c*x^4+b*x^2+a)^(1/2)+1/e^2*d/((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*
d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b*c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{\left(ex^2 + d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2 + cx^4)^{\frac{3}{2}}}{x^3 (d + ex^2)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**3/(e*x**2+d),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(x**3*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2 + a)^{\frac{3}{2}}}{(ex^2 + d)x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((e*x^2 + d)*x^3), x)
```

3.327 $\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$

Optimal. Leaf size=463

$$-\frac{3\left(514+2717\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{140\ 2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{27}{70}\sqrt{2x^4+2x^2+1}x^3-\frac{1}{14}\left(2x^4+2\right.$$

```
[Out] (-213*x*Sqrt[1 + 2*x^2 + 2*x^4])/140 - (27*x^3*Sqrt[1 + 2*x^2 + 2*x^4])/70
- (2211*x*Sqrt[1 + 2*x^2 + 2*x^4])/(140*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (x*(1
+ 2*x^2 + 2*x^4)^(3/2))/14 + (17*Sqrt[51]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2
*x^2 + 2*x^4]])/16 + (2211*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(140*2^(3/
4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (3*(514 + 2717*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(140*2^(3/4)*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - (2
89*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^
2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(16*2^(3/4)*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.671821, antiderivative size = 875, normalized size of antiderivative = 1.89, number of steps used = 19, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {1335, 1091, 1176, 1197, 1103, 1195, 1208, 1216, 1706}

$$-\frac{1}{14}x\left(2x^4+2x^2+1\right)^{3/2}-\frac{3}{35}x\left(x^2+2\right)\sqrt{2x^4+2x^2+1}-\frac{3}{20}x\left(2x^2+9\right)\sqrt{2x^4+2x^2+1}-\frac{6\sqrt{2}x\sqrt{2x^4+2x^2+1}}{35\left(\sqrt{2}x^2+1\right)}-\frac{309x}{20\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + 2*x^2 + 2*x^4)^(3/2))/(3 - 2*x^2),x]
```

```
[Out] (-3*x*(2 + x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/35 - (3*x*(9 + 2*x^2)*Sqrt[1 + 2*x
^2 + 2*x^4])/20 - (309*x*Sqrt[1 + 2*x^2 + 2*x^4])/(20*Sqrt[2]*(1 + Sqrt[2]*
x^2)) - (6*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(35*(1 + Sqrt[2]*x^2)) - (x*(
1 + 2*x^2 + 2*x^4)^(3/2))/14 + (17*Sqrt[51]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 +
 2*x^2 + 2*x^4]])/16 + (309*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
 Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(20*2^(3/
4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (6*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(35*Sqrt[1 + 2*x^2 + 2*x^4]) + (867*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(112*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (51*(5 + Sqrt[2])*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(16*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
- (3*(3 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2
]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(70*2^(1/4)*Sqrt
[1 + 2*x^2 + 2*x^4]) - (3*(9 + 8*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(20*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + Sqrt
[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*
Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(224*2^(1/4)*Sqrt[1 + 2
*x^2 + 2*x^4])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
```

```
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}\,dx = \int \left(-\frac{1}{2}\left(1+2x^2+2x^4\right)^{3/2}+\frac{3\left(1+2x^2+2x^4\right)^{3/2}}{2\left(3-2x^2\right)}\right)dx$$

$$= -\left(\frac{1}{2}\int \left(1+2x^2+2x^4\right)^{3/2}\,dx\right)+\frac{3}{2}\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}\,dx$$

$$= -\frac{1}{14}x\left(1+2x^2+2x^4\right)^{3/2}-\frac{3}{14}\int \left(2+2x^2\right)\sqrt{1+2x^2+2x^4}\,dx-\frac{3}{8}\int \left(10+4x^2\right)\sqrt{1+2x^2-$$

$$= -\frac{3}{35}x\left(2+x^2\right)\sqrt{1+2x^2+2x^4}-\frac{3}{20}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4}-\frac{1}{14}x\left(1+2x^2+2x^4\right)^{3/2}-$$

$$= -\frac{3}{35}x\left(2+x^2\right)\sqrt{1+2x^2+2x^4}-\frac{3}{20}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4}-\frac{1}{14}x\left(1+2x^2+2x^4\right)^{3/2}+$$

$$= -\frac{3}{35}x\left(2+x^2\right)\sqrt{1+2x^2+2x^4}-\frac{3}{20}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4}-\frac{309x\sqrt{1+2x^2+2x^4}}{20\sqrt{2}\left(1+\sqrt{2}x^2\right)}-$$

Mathematica [C] time = 0.276295, size = 214, normalized size = 0.46

$$-(9669-5247i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)-160x^9-752x^7-2456x^5-2080x^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + 2*x^2 + 2*x^4)^(3/2))/(3 - 2*x^2),x]
```

```
[Out] (-892*x - 2080*x^3 - 2456*x^5 - 752*x^7 - 160*x^9 + (4422*I)*Sqrt[1 - I]*Sq
rt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x
], I] - (9669 - 5247*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*
x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 10115*(1 - I)^(3/2)*Sqrt[1 +
(1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1
- I]*x], I])/(560*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.021, size = 547, normalized size = 1.2

$$-\frac{x^5}{7}\sqrt{2x^4+2x^2+1}-\frac{37x^3}{70}\sqrt{2x^4+2x^2+1}-\frac{223x}{140}\sqrt{2x^4+2x^2+1}-\frac{9\,\mathrm{EllipticF}\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{35\sqrt{-1+i}}\sqrt{1+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x)
```

```
[Out] -1/7*x^5*(2*x^4+2*x^2+1)^(1/2)-37/70*x^3*(2*x^4+2*x^2+1)^(1/2)-223/140*x*(2
*x^4+2*x^2+1)^(1/2)-9/35/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/
2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)
)+(6/35-6/35*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4
+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-Ellipt
icE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-531/20/(-1+I)^(1/2)*(-I*x^2+
x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(
1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-309/40*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*
(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1
/2)+1/2*I*2^(1/2))-309/40/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(
1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/
2))+309/40*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2
*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+289/8/(-1
+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*El
lipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(2x^4+2x^2+1\right)^{\frac{3}{2}}x^2}{2x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(2x^6+2x^4+x^2\right)\sqrt{2x^4+2x^2+1}}{2x^2-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^6 + 2*x^4 + x^2)*sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^6\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**4+2*x**2+1)**(3/2)/(-2*x**2+3),x)
```

```
[Out] -Integral(x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**4
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**6*sqrt(2*x**4 +
 2*x**2 + 1)/(2*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}x^2}{2\,x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3), x)
```

3.328 $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}\,dx$

Optimal. Leaf size=428

$$-\frac{\left(66+383\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\ 2^{3/4}\left(2+3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{1}{10}\left(2x^2+9\right)\sqrt{2x^4+2x^2+1}x-\frac{103}{10}$$

```
[Out] -(x*(9 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/10 - (103*x*Sqrt[1 + 2*x^2 + 2*x^4
])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt[17/3]*x)/S
qrt[1 + 2*x^2 + 2*x^4]])/8 + (103*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4
)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10
*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((66 + 383*Sqrt[2])*(1 + Sqrt[2]*x^2)*S
qrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x],
 (2 - Sqrt[2])/4])/(10*2^(3/4)*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - (
289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x
^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(24*2^(3/4)*(2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.348961, antiderivative size = 602, normalized size of antiderivative = 1.41, number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1208, 1176, 1197, 1103, 1195, 1216, 1706}

$$-\frac{1}{10}\left(2x^2+9\right)\sqrt{2x^4+2x^2+1}x-\frac{103\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{17}{8}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(9+8\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(3 - 2*x^2),x]
```

```
[Out] -(x*(9 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/10 - (103*x*Sqrt[1 + 2*x^2 + 2*x^4
])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt[17/3]*x)/S
qrt[1 + 2*x^2 + 2*x^4]])/8 + (103*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4
)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10
*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sq
rt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x],
(2 - Sqrt[2])/4])/(56*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(5 + Sqrt[2])*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) -
 ((9 + 8*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x
^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1
+ 2*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^
(1/4)*x], (2 - Sqrt[2])/4])/(336*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*(a + b*x^2 + c
*x^4)^p)/(c*(4*p + 1)*(4*p + 3)), x] + Dist[(2*p)/(c*(4*p + 1)*(4*p + 3)),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}\,dx = -\left(\frac{1}{4}\int \left(10+4x^2\right)\sqrt{1+2x^2+2x^4}\,dx\right) + \frac{17}{2}\int \frac{\sqrt{1+2x^2+2x^4}}{3-2x^2}\,dx$$

$$= -\frac{1}{10}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4} - \frac{1}{120}\int \frac{192+216x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{17}{8}\int \frac{10+4x^2}{\sqrt{1+2x^2+2x^4}}\,dx +$$

$$= -\frac{1}{10}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4} + \frac{9\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{5\sqrt{2}} + \frac{17\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{2\sqrt{2}} - \frac{1}{28}\left(289\left(2-\right.\right.$$

$$= -\frac{1}{10}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4} - \frac{103x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)} + \frac{17}{8}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x}}\right.$$

Mathematica [C] time = 0.159486, size = 209, normalized size = 0.49

$$-(1371-753i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right) - 48x^7 - 264x^5 - 240x^3 + 618i\sqrt{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(3 - 2*x^2),x]
```

```
[Out] (-108*x - 240*x^3 - 264*x^5 - 48*x^7 + (618*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)
*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (1371
- 753*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[
I*ArcSinh[Sqrt[1 - I]*x], I] + 1445*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqr
t[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(12
0*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.007, size = 377, normalized size = 0.9

$$-\frac{x^3}{5}\sqrt{2x^4+2x^2+1} - \frac{9x}{10}\sqrt{2x^4+2x^2+1} - \frac{177\,\text{EllipticF}\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{10\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x)
```

```
[Out] -1/5*x^3*(2*x^4+2*x^2+1)^(1/2)-9/10*x*(2*x^4+2*x^2+1)^(1/2)-177/10/(-1+I)^(
1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellipti
cF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-103/20*I/(-1+I)^(1/2)*(-I*x^2+
x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(
1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-103/20/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I
*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2
)+1/2*I*2^(1/2))+103/20*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(
1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/
2))+289/12/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x
^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(2\,x^{4}+2\,x^{2}+1\right)^{\frac{3}{2}}}{2\,x^{2}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(2\,x^{4}+2\,x^{2}+1\right)^{\frac{3}{2}}}{2\,x^{2}-3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{2x^{4}+2x^{2}+1}}{2x^{2}-3}\,dx-\int \frac{2x^{2}\sqrt{2x^{4}+2x^{2}+1}}{2x^{2}-3}\,dx-\int \frac{2x^{4}\sqrt{2x^{4}+2x^{2}+1}}{2x^{2}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**2*sqrt
(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**4*sqrt(2*x**4 + 2*x*
*2 + 1)/(2*x**2 - 3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(2\,x^{4}+2\,x^{2}+1\right)^{\frac{3}{2}}}{2\,x^{2}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

3.329 $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^2\left(3-2x^2\right)}\,dx$

Optimal. Leaf size=722

$$-\frac{17\left(5+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{12\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{289\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{E}}{84\sqrt[4]{2}\sqrt{2x^4}}$$

```
[Out] -((1 + x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(3*x) - (17*x*Sqrt[1 + 2*x^2 + 2*x^4])
/(3*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1
+ Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*
x^4]])/12 + (17*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2
)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*Sqrt[1 + 2
*x^2 + 2*x^4]) - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 +
 2*x^2 + 2*x^4]) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]
*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*Sqrt[1
 + 2*x^2 + 2*x^4]) + (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(84*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(5 + Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(12*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*
Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*El
lipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(504*
2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.368427, antiderivative size = 722, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {1311, 1271, 12, 1139, 1103, 1195, 1208, 1197, 1216, 1706}

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{3\left(\sqrt{2}x^2+1\right)}-\frac{17\sqrt{2x^4+2x^2+1}x}{3\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{\left(x^2+1\right)\sqrt{2x^4+2x^2+1}}{3x}+\frac{17}{12}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{17\left(5+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^2*(3 - 2*x^2)),x]
```

```
[Out] -((1 + x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(3*x) - (17*x*Sqrt[1 + 2*x^2 + 2*x^4])
/(3*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1
+ Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*
x^4]])/12 + (17*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2
)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*Sqrt[1 + 2
*x^2 + 2*x^4]) - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 +
 2*x^2 + 2*x^4]) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]
*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*Sqrt[1
 + 2*x^2 + 2*x^4]) + (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(84*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(5 + Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(12*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*
Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*El
```

```
lipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(504*
2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1311

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[((f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^2\left(3-2x^2\right)}\,dx = -\left(\frac{1}{6}\int \frac{\left(-2+6x^2\right)\sqrt{1+2x^2+2x^4}}{x^2}\,dx\right)+\frac{17}{3}\int \frac{\sqrt{1+2x^2+2x^4}}{3-2x^2}\,dx$$

$$= -\frac{\left(1+x^2\right)\sqrt{1+2x^2+2x^4}}{3x}+\frac{1}{18}\int \frac{12x^2}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{17}{12}\int \frac{10+4x^2}{\sqrt{1+2x^2+2x^4}}\,dx+\frac{289}{6}$$

$$= -\frac{\left(1+x^2\right)\sqrt{1+2x^2+2x^4}}{3x}+\frac{2}{3}\int \frac{x^2}{\sqrt{1+2x^2+2x^4}}\,dx+\frac{17\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{3\sqrt{2}}-\frac{1}{42}\left(289\left(2-\right.\right.$$

$$= -\frac{\left(1+x^2\right)\sqrt{1+2x^2+2x^4}}{3x}-\frac{17x\sqrt{1+2x^2+2x^4}}{3\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{17}{12}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)+$$

$$= -\frac{\left(1+x^2\right)\sqrt{1+2x^2+2x^4}}{3x}-\frac{17x\sqrt{1+2x^2+2x^4}}{3\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)}+\frac{17}{12}\sqrt{\frac{17}{3}}\tanh^{-1}$$

Mathematica [C] time = 0.218839, size = 213, normalized size = 0.3

$$\frac{-(255-165i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,x\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)-24x^6-48x^4-36x^2+90i\sqrt{1-i}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^2*(3 - 2*x^2)),x]
```

```
[Out] (-12 - 36*x^2 - 48*x^4 - 24*x^6 + (90*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2
]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (255 - 165
*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*A
rcSinh[Sqrt[1 - I]*x], I] + 289*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[
1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(36*x
*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.013, size = 528, normalized size = 0.7

$$-\frac{x}{3}\sqrt{2x^4+2x^2+1}-\frac{59\,\mathrm{EllipticF}\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{5\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}-\frac{\frac{103\,i}{30}\mathrm{Elli}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x)
```

```
[Out] -1/3*x*(2*x^4+2*x^2+1)^(1/2)-59/5/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+
x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2
*I*2^(1/2))-103/30*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/
(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1
03/30/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)
^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+103/30*I/(-1+I)^
(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellipt
icE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+289/18/(-1+I)^(1/2)*(-I*x^2+x
^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(
1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))-1/3*(2*x^4+2*x^2+1)^(1/2)/x+16/1
5/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2
)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-14/15+14/15*I)/(-1+
I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(Ell
ipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1
/2*2^(1/2)+1/2*I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}{\left(2x^2-3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}{2x^4-3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^4 - 3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/x**2/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x) - Integral(2*x**2
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x) - Integral(2*x**4*sqrt(2*x
**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{\left(2\,x^2-3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2), x)
```

3.330 $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^4\left(3-2x^2\right)}\,dx$

Optimal. Leaf size=625

$$\frac{\sqrt[4]{2}\left(9+5\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt{2x^4+2x^2+1}}-\frac{17\left(5+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{Ellip}}{18\sqrt[4]{2}\sqrt{2x^4+2}}$$

```
[Out] (-2*Sqrt[1 + 2*x^2 + 2*x^4])/x - ((1 - 8*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(9*x
^3) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(9*(1 + Sqrt[2]*x^2)) + (17*Sqrt[
17/3]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/18 - (2^(1/4)*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*Sqrt[1 + 2*x^2 + 2*x^4]) + (289*(3 - Sqr
t[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellip
ticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(126*2^(1/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) - (17*(5 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(18*2^(1/4
)*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(1/4)*(9 + 5*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(9*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 +
11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(756*2^(1/4)*Sqrt[1
+ 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.366185, antiderivative size = 625, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {1309, 1271, 1281, 1197, 1103, 1195, 1208, 1216, 1706}

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{9\left(\sqrt{2}x^2+1\right)}-\frac{2\sqrt{2x^4+2x^2+1}}{x}-\frac{\left(1-8x^2\right)\sqrt{2x^4+2x^2+1}}{9x^3}+\frac{17}{18}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\sqrt[4]{2}\left(9+5\sqrt{\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^4*(3 - 2*x^2)),x]
```


```
[Out] (-2*Sqrt[1 + 2*x^2 + 2*x^4])/x - ((1 - 8*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(9*x
^3) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(9*(1 + Sqrt[2]*x^2)) + (17*Sqrt[
17/3]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/18 - (2^(1/4)*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*Sqrt[1 + 2*x^2 + 2*x^4]) + (289*(3 - Sqr
t[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellip
ticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(126*2^(1/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) - (17*(5 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(18*2^(1/4
)*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(1/4)*(9 + 5*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(9*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 +
11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(756*2^(1/4)*Sqrt[1
+ 2*x^2 + 2*x^4])
```


Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[((f*x)^(m + 4)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^4\left(3-2x^2\right)}\,dx = \frac{1}{9}\int \frac{\left(3+8x^2\right)\sqrt{1+2x^2+2x^4}}{x^4}\,dx + \frac{34}{9}\int \frac{\sqrt{1+2x^2+2x^4}}{3-2x^2}\,dx$$

$$= -\frac{\left(1-8x^2\right)\sqrt{1+2x^2+2x^4}}{9x^3} - \frac{1}{27}\int \frac{-54-60x^2}{x^2\sqrt{1+2x^2+2x^4}}\,dx - \frac{17}{18}\int \frac{10+4x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{289}{9}\ \ldots$$

$$= -\frac{2\sqrt{1+2x^2+2x^4}}{x} - \frac{\left(1-8x^2\right)\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{1}{27}\int \frac{60+108x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{9}\left(17\sqrt{2}\right)\int \ldots$$

$$= -\frac{2\sqrt{1+2x^2+2x^4}}{x} - \frac{\left(1-8x^2\right)\sqrt{1+2x^2+2x^4}}{9x^3} - \frac{17\sqrt{2}x\sqrt{1+2x^2+2x^4}}{9\left(1+\sqrt{2}x^2\right)} + \frac{17}{18}\sqrt{\frac{17}{3}}\tanh^{-1}\Bigg(\ldots$$

$$= -\frac{2\sqrt{1+2x^2+2x^4}}{x} - \frac{\left(1-8x^2\right)\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{9\left(1+\sqrt{2}x^2\right)} + \frac{17}{18}\sqrt{\frac{17}{3}}\tanh^{-1}\Bigg(\ldots$$

Mathematica [C] time = 0.234868, size = 219, normalized size = 0.35

$$\frac{-(195-201i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}x^3\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)-120x^6-132x^4-72x^2-6i\sqrt{1-i}\sqrt{1 \ldots}}{54x \ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^4*(3 - 2*x^2)),x]
```

```
[Out] (-6 - 72*x^2 - 132*x^4 - 120*x^6 - (6*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x
^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (195 - 2
01*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF
[I*ArcSinh[Sqrt[1 - I]*x], I] + 289*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]
*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])
```

```
/(54*x^3*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.016, size = 530, normalized size = 0.9

$$-\frac{118\,\mathrm{EllipticF}\left(x\sqrt{-1+i},1/2\,\sqrt{2}+i/2\sqrt{2}\right)}{15\,\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}-\frac{\left(\frac{12}{5}-\frac{12\,i}{5}\right)\left(\mathrm{EllipticF}\left(x\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x)
```

```
[Out] -118/15/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+
1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-12/5+12/5*I)
/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)
*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1
/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-103/45/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I
*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2
)+1/2*I*2^(1/2))-103/45*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(
1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/
2))+289/27/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x
^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
-10/9*(2*x^4+2*x^2+1)^(1/2)/x+44/15/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+
I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/
2*I*2^(1/2))+103/45*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)
/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-
1/9*(2*x^4+2*x^2+1)^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{\left(2\,x^2-3\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{2\,x^6-3\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^6 - 3*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/x**4/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x) - Integral(2*x**2
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x) - Integral(2*x**4*sqrt(2*x
**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{\left(2\,x^2-3\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4), x)
```

3.331 $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^6\left(3-2x^2\right)}\,dx$

Optimal. Leaf size=553

$$\frac{2^{3/4}\left(37+23\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{2x^4+2x^2+1}}+\frac{85\,2^{3/4}\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+}{(\sqrt{2}}}}{189\sqrt{}}$$

```
[Out] (74*Sqrt[1 + 2*x^2 + 2*x^4])/(135*x^3) - (262*Sqrt[1 + 2*x^2 + 2*x^4])/(135
*x) - ((3 + 40*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(45*x^5) + (262*Sqrt[2]*x*Sqrt
[1 + 2*x^2 + 2*x^4])/(135*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt
[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/27 - (262*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(135*Sqrt[1 + 2*x^2 + 2*x^4]) + (85*2^(3/4)*(3 - Sqrt[2])*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(189*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(3
/4)*(37 + 23*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(135*Sqrt[1 + 2
*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/
4)*x], (2 - Sqrt[2])/4])/(1134*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.487737, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {1309, 1271, 1281, 1197, 1103, 1195, 1311, 1216, 1706}

$$\frac{262\sqrt{2}\sqrt{2x^4+2x^2+1}x}{135\left(\sqrt{2}x^2+1\right)}-\frac{262\sqrt{2x^4+2x^2+1}}{135x}+\frac{74\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{\left(40x^2+3\right)\sqrt{2x^4+2x^2+1}}{45x^5}+\frac{17}{27}\sqrt{\frac{17}{3}}\tanh^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^6*(3 - 2*x^2)),x]
```


```
[Out] (74*Sqrt[1 + 2*x^2 + 2*x^4])/(135*x^3) - (262*Sqrt[1 + 2*x^2 + 2*x^4])/(135
*x) - ((3 + 40*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(45*x^5) + (262*Sqrt[2]*x*Sqrt
[1 + 2*x^2 + 2*x^4])/(135*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt
[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/27 - (262*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(135*Sqrt[1 + 2*x^2 + 2*x^4]) + (85*2^(3/4)*(3 - Sqrt[2])*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(189*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(3
/4)*(37 + 23*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(135*Sqrt[1 + 2
*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/
4)*x], (2 - Sqrt[2])/4])/(1134*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```


Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
```

```
f^4), Int[((f*x)^(m + 4)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1271

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*(d*(m
+ 4*p + 3) + e*(m + 1)*x^2))/(f*(m + 1)*(m + 4*p + 3)), x] + Dist[(2*p)/(f^
2*(m + 1)*(m + 4*p + 3)), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1311

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[((f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1))/(d + e*x^2), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
```

```
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^6\left(3-2x^2\right)}\,dx = \frac{1}{9}\int \frac{\left(3+8x^2\right)\sqrt{1+2x^2+2x^4}}{x^6}\,dx + \frac{34}{9}\int \frac{\sqrt{1+2x^2+2x^4}}{x^2\left(3-2x^2\right)}\,dx$$

$$= -\frac{\left(3+40x^2\right)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{1}{45}\int \frac{-74-68x^2}{x^4\sqrt{1+2x^2+2x^4}}\,dx - \frac{17}{27}\int \frac{-2+6x^2}{x^2\sqrt{1+2x^2+2x^4}}\,dx$$

$$= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{34\sqrt{1+2x^2+2x^4}}{27x} - \frac{\left(3+40x^2\right)\sqrt{1+2x^2+2x^4}}{45x^5} - \frac{1}{135}\int \frac{-92-1}{x^2\sqrt{1+2x}}$$

$$= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{262\sqrt{1+2x^2+2x^4}}{135x} - \frac{\left(3+40x^2\right)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{17}{27}\sqrt{\frac{17}{3}}\tanh^{-1}$$

$$= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{262\sqrt{1+2x^2+2x^4}}{135x} - \frac{\left(3+40x^2\right)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{34\sqrt{2}x\sqrt{1+2x^2}}{27\left(1+\sqrt{2}x\right.}$$

$$= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{262\sqrt{1+2x^2+2x^4}}{135x} - \frac{\left(3+40x^2\right)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{262\sqrt{2}x\sqrt{1+2x}}{135\left(1+\sqrt{2}\right.}$$

Mathematica [C] time = 0.265279, size = 224, normalized size = 0.41

$$-\frac{(543-1329i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}x^5\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)+1572x^8+1848x^6+1116x^4+1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^6*(3 - 2*x^2)),x]
```

```
[Out] -(27 + 192*x^2 + 1116*x^4 + 1848*x^6 + 1572*x^8 + (786*I)*Sqrt[1 - I]*x^5*S
qrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*
x], I] + (543 - 1329*I)*Sqrt[1 - I]*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 +
 I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 1445*(1 - I)^(3/2)*x^5*Sq
```

```
rt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[
Sqrt[1 - I]*x], I])/(405*x^5*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.016, size = 549, normalized size = 1.

$$-\frac{236\,\mathrm{EllipticF}\left(x\sqrt{-1+i},1/2\sqrt{2}+i/2\sqrt{2}\right)}{45\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}-\frac{\left(\frac{52}{15}-\frac{52i}{15}\right)\left(\mathrm{EllipticF}\left(x\sqrt{-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x)
```

```
[Out] -236/45/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+
1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-52/15+52/15*
I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/
2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^
(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-206/135/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)
*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(
1/2)+1/2*I*2^(1/2))-206/135*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+
1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2
^(1/2))+578/81/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4
+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1
/2))-262/135*(2*x^4+2*x^2+1)^(1/2)/x+184/45/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2
)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^
(1/2)+1/2*I*2^(1/2))+206/135*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2
+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*
2^(1/2))-1/15*(2*x^4+2*x^2+1)^(1/2)/x^5-46/135*(2*x^4+2*x^2+1)^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}{\left(2x^2-3\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}{2x^8-3x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^8 - 3*x^6), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/x**6/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x) - Integral(2*x**2
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x) - Integral(2*x**4*sqrt(2*x
**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}}{\left(2\,x^2-3\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6), x)
```

3.332 $\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=173

$$-\frac{(be+2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2}+\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}}+\frac{\sqrt{a+bx^2+cx^4}}{2ce}$$

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c*e) - ((2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*S
qrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)*e^2) + (d^2*ArcTanh[(b*d - 2*a
*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x
^4])])/(2*e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi [A] time = 0.31374, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 1653, 843, 621, 206, 724}

$$-\frac{(be+2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2}+\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}}+\frac{\sqrt{a+bx^2+cx^4}}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c*e) - ((2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*S
qrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)*e^2) + (d^2*ArcTanh[(b*d - 2*a
*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x
^4])])/(2*e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^5}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}bde-\frac{1}{2}e(2cd+be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2ce^2}$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2e^2} - \frac{(2cd+be)\,\text{Subst}\left(\int \frac{}{\sqrt{a+}}\right.}{4ce^2}$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)}{e^2} - \frac{(2cd+be}{}$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} - \frac{(2cd+be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2} + \frac{d^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+}}\right.}{2e^2\sqrt{cd^2-bde+ae^2}}$$

Mathematica [A] time = 0.327012, size = 167, normalized size = 0.97

$$\frac{-\frac{(be+2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{c^{3/2}}+\frac{2d^2\tanh^{-1}\left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}+\frac{2e\sqrt{a+bx^2+cx^4}}{c}}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((2*e*Sqrt[a + b*x^2 + c*x^4])/c - ((2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/c^(3/2) + (2*d^2*ArcTanh[(b*d - 2*a*e +
2*c*d*x^2 - b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]
)])/Sqrt[c*d^2 - b*d*e + a*e^2])/(4*e^2)
```

Maple [A] time = 0.02, size = 267, normalized size = 1.5

$$\frac{1}{2ce}\sqrt{cx^4+bx^2+a}-\frac{b}{4e}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}-\frac{d}{2e^2}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}-\frac{d^2}{2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1/2)/c/e-1/4/e*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))-1/2/e^2*d*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2)
)/c^(1/2)-1/2*d^2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*
d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+
d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Fricas [B] time = 130.366, size = 2921, normalized size = 16.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d^2*log(-((8*c^2*d^2 - 8*b*c*d*e +
(b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b
*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*s
qrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*
d*e*x^2 + d^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*s
qrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^
2 + b)*sqrt(c) - 4*a*c) + 4*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^4 +
b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/8*(4*sqrt(-c*d^2 + b
*d*e - a*e^2)*c^2*d^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d
*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2
)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) +
 (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(c)*log(-8*c^2
*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) -
4*a*c) + 4*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^4 + b*x^2 + a))/(c^3*
d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d^
2*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2
*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*
x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*
```

```
x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + (2*c^2*d^3 - b*c*d^2*e +
 a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)
*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(c^2*d^2*e - b*c*d*e
^2 + a*c*e^3)*sqrt(c*x^4 + b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e
^4), 1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d^2*arctan(-1/2*sqrt(c*x^4 + b
*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((
c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 -
 b^2*d*e + a*b*e^2)*x^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c
)*d*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)
/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)*sqrt(c*x^
4 + b*x^2 + a))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{cx^4+bx^2+a}(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

3.333 $\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=137

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}e}-\frac{d\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\sqrt{ae^2-bde+cd^2}}$$

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c]*e) -
(d*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x^2 + c*x^4])])/(2*e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi [A] time = 0.157354, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 843, 621, 206, 724}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}e}-\frac{d\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c]*e) -
(d*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x^2 + c*x^4])])/(2*e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2e}-\frac{d\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2e}\\ &= \frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{e}+\frac{d\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\frac{-bd+2ae-(2cd-b}{\sqrt{a+bx^2+cx^4}}\right)}{e}\\ &= \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}e}-\frac{d\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

Mathematica [A] time = 0.11596, size = 133, normalized size = 0.97

$$\frac{\frac{d\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{c}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[c] + (d*Ar
cTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*
e)]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/(2*e)
```

Maple [A] time = 0.01, size = 204, normalized size = 1.5

$$\frac{1}{2\,e}\ln\left(\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}+\frac{d}{2\,e^2}\ln\left(\left(\left(2\,\frac{ae^2-deb+cd^2}{e^2}+\frac{be-2\,cd}{e}\left(x^2+\frac{d}{e}\right)+2\,\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2/e*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+1/2*d/e^2/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x
^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2
+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^4 + bx^2 + a}(ex^2 + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Fricas [B] time = 7.3943, size = 2360, normalized size = 17.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*c*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*
d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^
2 + d^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^
2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(c^2*d^2*e -
b*c*d*e^2 + a*c*e^3), -1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*d*arctan(-1/2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b
*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^
2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)*sqrt(c)*l
og(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*s
qrt(c) - 4*a*c))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), 1/4*(sqrt(c*d^2 - b*d*e
 + a*e^2)*c*d*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b
*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 +
4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2
*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) - 2*(c*d^2 - b
*d*e + a*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -1/2*(
sqrt(-c*d^2 + b*d*e - a*e^2)*c*d*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-
c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*
e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e
^2)*x^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c^2*d^2*e - b*c*d*
e^2 + a*c*e^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(d + ex^2)\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.334 $\int \frac{x}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=86

$$\frac{\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\sqrt{ae^2-bde+cd^2}}$$

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sq
rt[a + b*x^2 + c*x^4])]/(2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi [A] time = 0.0836072, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1247, 724, 206}

$$\frac{\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sq
rt[a + b*x^2 + c*x^4])]/(2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{-bd+2ae-(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^2-bde+ae^2}}
\end{aligned}$$

Mathematica [A] time = 0.0194494, size = 87, normalized size = 1.01

$$-\frac{\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{2\sqrt{e(ae-bd)+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) +
 a*e)]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)])
```

Maple [B] time = 0.006, size = 165, normalized size = 1.9

$$-\frac{1}{2e}\ln\left(\left(2\frac{ae^2-deb+cd^2}{e^2}+\frac{be-2cd}{e}\left(x^2+\frac{d}{e}\right)+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{be-2cd}{e}\left(x^2+\frac{d}{e}\right)+\frac{ae^2-de}{e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2
*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c
*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)), x)
```

Fricas [B] time = 1.79745, size = 768, normalized size = 8.93

$$\left[\frac{\log\left(-\frac{\left(8c^2d^2-8bcde+\left(b^2+4ac\right)e^2\right)x^4-8abde+8a^2e^2+\left(b^2+4ac\right)d^2+2\left(4bcd^2+4abe^2-\left(3b^2+4ac\right)de\right)x^2+4\sqrt{cx^4+bx^2+a}\sqrt{cd^2-bde+ae^2}\left((2cd-be)x^2+bd-2a\right.}{e^2x^4+2dex^2+d^2}\right.}{4\sqrt{cd^2-bde+ae^2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*
a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*
```

```
e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*
e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2))/sqrt(c*d^2 - b*d*e + a*
e^2), 1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*
sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 -
b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e +
 a*b*e^2)*x^2))/(c*d^2 - b*d*e + a*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [A] time = 1.13953, size = 101, normalized size = 1.17

$$\frac{\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2
 + b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

3.335 $\int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=138

$$-\frac{e\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\sqrt{ae^2-bde+cd^2}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d}$$

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a]*d) -
 (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi [A] time = 0.194722, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1251, 960, 724, 206}

$$-\frac{e\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\sqrt{ae^2-bde+cd^2}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[a]*d) -
 (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} - \frac{e}{d(d+ex)\sqrt{a+bx+cx^2}}\right)dx, x, x^2\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d} - \frac{e\operatorname{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d} + \frac{e\operatorname{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be}{\sqrt{a+bx^2+cx^4}}\right.}{d}\\
&= -\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d} - \frac{e\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d\sqrt{cd^2-bde+ae^2}}
\end{aligned}$$

Mathematica [A] time = 0.140591, size = 134, normalized size = 0.97

$$-\frac{\frac{e\tanh^{-1}\left(\frac{-2ae+b\left(d-ex^2\right)+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{\sqrt{e(ae-bd)+cd^2}} + \frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{a}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[a] + (e*A
rcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*x^2))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e
)]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)])/(2*d)
```

Maple [A] time = 0.012, size = 207, normalized size = 1.5

$$-\frac{1}{2\,d}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{1}{2\,d}\ln\left(\left(2\,\frac{ae^2-deb+cd^2}{e^2}+\frac{be-2\,cd}{e}\left(x^2+\frac{d}{e}\right)+2\,\sqrt{\frac{ae^2-deb+}{e^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/d/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/2/d/((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(
x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^
2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x), x)
```

Fricas [B] time = 2.69411, size = 2381, normalized size = 17.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a*e*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*
d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^
2 + d^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b
*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c*
d^3 - a*b*d^2*e + a^2*d*e^2), -1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*a*e*arct
an(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)
*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e
+ a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)*s
qrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x
^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c*d^3 - a*b*d^2*e + a^2*d*e^2), 1/4*(sq
rt(c*d^2 - b*d*e + a*e^2)*a*e*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*
e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b
*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*
d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)
) + 2*(c*d^2 - b*d*e + a*e^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(
b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)))/(a*c*d^3 - a*b*d^2*e + a^
2*d*e^2), -1/2*(sqrt(-c*d^2 + b*d*e - a*e^2)*a*e*arctan(-1/2*sqrt(c*x^4 + b
*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((
c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 -
 b^2*d*e + a*b*e^2)*x^2)) - (c*d^2 - b*d*e + a*e^2)*sqrt(-a)*arctan(1/2*sqr
t(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)))/(a*
c*d^3 - a*b*d^2*e + a^2*d*e^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.336 $\int \frac{1}{x^3(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=218

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d}+\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\sqrt{ae^2-bde+cd^2}}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*d*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*
Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x^2)/(2*Sqrt
[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a]*d^2) + (e^2*ArcTanh[(b*d - 2*a*e
+ (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]
)])/(2*d^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rubi [A] time = 0.266096, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 960, 730, 724, 206}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d}+\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\sqrt{ae^2-bde+cd^2}}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*d*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*
Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x^2)/(2*Sqrt
[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a]*d^2) + (e^2*ArcTanh[(b*d - 2*a*e
+ (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]
)])/(2*d^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{dx^2\sqrt{a+bx+cx^2}} - \frac{e}{d^2x\sqrt{a+bx+cx^2}} + \frac{e^2}{d^2(d+ex)\sqrt{a+bx+cx^2}}\right) d$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d} - \frac{e\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d^2} + \frac{e^2\,\text{Subst}\left(\int \frac{}{(d+}\right.}{}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2adx^2} - \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4ad} + \frac{e\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+b}{\sqrt{a+bx^2}}\right.}{d^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2adx^2} + \frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d^2} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d^2\sqrt{cd^2-bde+ae^2}}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2adx^2} + \frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d} + \frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}d^2} + \frac{e^2\tanh}{}$$

Mathematica [A] time = 0.269728, size = 176, normalized size = 0.81

$$\frac{\frac{x^2(2ae+bd)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{a^{3/2}} + \frac{2e^2x^2\tanh^{-1}\left(\frac{-2ae+bd-bex^2+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}} - \frac{2d\sqrt{a+bx^2+cx^4}}{a}}{4d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-2*d*Sqrt[a + b*x^2 + c*x^4])/a + ((b*d + 2*a*e)*x^2*ArcTanh[(2*a + b*x^2
)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/a^(3/2) + (2*e^2*x^2*ArcTanh[(b*d -
 2*a*e + 2*c*d*x^2 - b*e*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2
 + c*x^4])])/Sqrt[c*d^2 - b*d*e + a*e^2])/(4*d^2*x^2)
```

Maple [A] time = 0.012, size = 276, normalized size = 1.3

$$\frac{e}{2\,d^2}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}} - \frac{1}{2\,adx^2}\sqrt{cx^4+bx^2+a} + \frac{b}{4\,d}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2/d^2*e/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-1/2*(
c*x^4+b*x^2+a)^(1/2)/a/d/x^2+1/4/d*b/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4
+b*x^2+a)^(1/2))/x^2)-1/2*e/d^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^
2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x
^2+d/e))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^3), x)
```

Fricas [A] time = 6.49643, size = 3019, normalized size = 13.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a^2*e^2*x^2*log(-((8*c^2*d^2 - 8*b*c*d*
e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*
(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 +
a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4
+ 2*d*e*x^2 + d^2)) + (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*
e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 +
 a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(a*c*d^3 - a*b*d^2*e + a^2*d*e^
2)*sqrt(c*x^4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2),
1/8*(4*sqrt(-c*d^2 + b*d*e - a*e^2)*a^2*e^2*x^2*arctan(-1/2*sqrt(c*x^4 + b*
x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c
^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 -
b^2*d*e + a*b*e^2)*x^2)) + (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)
*d^2*e)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*
x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(a*c*d^3 - a*b*d^2*e + a^2
*d*e^2)*sqrt(c*x^4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x
^2), 1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a^2*e^2*x^2*log(-((8*c^2*d^2 - 8*b*c*
d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 +
2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2
+ a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^
4 + 2*d*e*x^2 + d^2)) - (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^
2*e)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)
/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*sqrt(c*x^
4 + b*x^2 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2), 1/4*(2*sqrt(
-c*d^2 + b*d*e - a*e^2)*a^2*e^2*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqr
t(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c
*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*
```

```
b*e^2)*x^2)) - (b*c*d^3 - a*b*d*e^2 + 2*a^2*e^3 - (b^2 - 2*a*c)*d^2*e)*sqrt
(-a)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4
 + a*b*x^2 + a^2)) - 2*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*sqrt(c*x^4 + b*x^2
 + a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\left(ex^2+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(e*x^2 + d)*x^3), x)
```

3.337 $\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=418

$$\frac{\left(1-3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2x^4+2x^2+1}\,x}{2\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{3\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\tan^{-1}}{4\left(2-3\sqrt{}\right.}$$

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/(2*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (3*Sqrt[3/10]*(
3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(4*(2 - 3*Sqrt[
2])) - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(3/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) + ((1 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(3/4)*
(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + (3*(3 + Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2
])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(3/4)*(2 - 3*Sqrt[2])*Sq
rt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.184204, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1325, 1103, 1195, 1706}

$$\frac{\sqrt{2x^4+2x^2+1}\,x}{2\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{3\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{4\left(2-3\sqrt{2}\right)}+\frac{\left(1-3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\right.\right.}{2\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/(2*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (3*Sqrt[3/10]*(
3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(4*(2 - 3*Sqrt[
2])) - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(3/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) + ((1 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(3/4)*
(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + (3*(3 + Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2
])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(3/4)*(2 - 3*Sqrt[2])*Sq
rt[1 + 2*x^2 + 2*x^4])
```

Rule 1325

```
Int[(x_)^4/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, -Dist[(2*c*d - a*e*q)/(c*e*(e - d*q))
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + (-Dist[1/(e*q), Int[(1 - q*x^2)/S
qrt[a + b*x^2 + c*x^4], x], x] + Dist[d^2/(e*(e - d*q)), Int[(1 + q*x^2)/((
d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x])] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{2\sqrt{2}} + \frac{9\int \frac{1+\sqrt{2}x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx}{2\left(2-3\sqrt{2}\right)} - \frac{\left(12-2\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx}{4\left(2-3\sqrt{2}\right)}$$

$$= \frac{x\sqrt{1+2x^2+2x^4}}{2\sqrt{2}\left(1+\sqrt{2}x^2\right)} - \frac{3\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{4\left(2-3\sqrt{2}\right)} - \frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\right.}{2\,2^{3/4}\sqrt{1+}}$$

Mathematica [C] time = 0.218542, size = 127, normalized size = 0.3

$$-\frac{\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(-(1+4i)\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)+(1+i)E\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)+3i\Pi\left(\frac{1}{3}+\frac{i}{3};\right.\right.}{4\sqrt{1-i}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -(Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*((1 + I)*EllipticE[I*ArcSinh[
Sqrt[1 - I]*x], I] - (1 + 4*I)*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + (3*
I)*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(4*Sqrt[1 - I]*Sqrt
[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.02, size = 222, normalized size = 0.5

$$\frac{\left(-\frac{1}{4}+\frac{i}{4}\right)\left(\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)-\text{EllipticE}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\frac{1}{\sqrt{2\,x^4+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

```
[Out] (-1/4+1/4*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*
x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE
(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-3/4/(-1+I)^(1/2)*(1+(1-I)*x^2)^
(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/
2*2^(1/2)+1/2*I*2^(1/2))+3/4/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1
)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1
/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}x^4}{4\,x^6+10\,x^4+8\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

3.338 $\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=247

$$-\frac{(3+\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\text{EllipticF}\left(2\tan^{-1}(\sqrt[4]{2}x),\frac{1}{4}(2-\sqrt{2})\right)}{14\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{1}{4}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{(3+\sqrt{2}}{}$$

```
[Out] -(Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/4 - ((3 + Sqrt[2
])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elliptic
F[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(14*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4
]) + ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[
2])/4])/(56*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.125826, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1319, 1103, 1706}

$$-\frac{1}{4}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{(3+\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}F\left(2\tan^{-1}(\sqrt[4]{2}x)|\frac{1}{4}(2-\sqrt{2})\right)}{14\,2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{(3+\sqrt{2})^2(\sqrt{2}x}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -(Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/4 - ((3 + Sqrt[2
])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elliptic
F[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(14*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4
]) + ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[
2])/4])/(56*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1319

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, -Dist[(a*(e + d*q))/(c*d^2 - a*e^2),
Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(a*d*(e + d*q))/(c*d^2 - a*e^2
), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[
{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
 && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
```

```
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx = -\left(\frac{1}{14}\left(2+3\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx\right)+\frac{1}{14}\left(3\left(2+3\sqrt{2}\right)\right)\int \frac{1+\sqrt{2}x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+}$$

$$= -\frac{1}{4}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)-\frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\right.}{14\ 2^{3/4}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] time = 0.135825, size = 99, normalized size = 0.4

$$\frac{(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)-\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)\right)}{4\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ((1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*(EllipticF[I*Arc
Sinh[Sqrt[1 - I]*x], I] - EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I
]))/(4*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.006, size = 134, normalized size = 0.5

$$\frac{\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{2\sqrt{-1+i}}\sqrt{1+(1-i)\,x^2}\sqrt{1+(1+i)\,x^2}\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}-\frac{1}{2\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

```
[Out] 1/2/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/2/(-1+I)^(1/2)*(-
I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(
-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}x^2}{4\,x^6+10\,x^4+8\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{2\,x^4+2\,x^2+1}(2\,x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

3.339 $\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=245

$$\frac{(3+\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{15}}-\frac{(3+\sqrt{2})^2(\sqrt{2}x^2+1)\sqrt{\cdots}}{\cdots}$$

```
[Out] ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]/(2*Sqrt[15]) + ((3 + Sqrt[2])
*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[
2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(14*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
 - ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]
*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2]
)/4])/(84*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.110588, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1216, 1103, 1706}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{15}}+\frac{(3+\sqrt{2})(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{(3+\sqrt{2})^2(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+\cdots}{(\sqrt{2}x^2+1)\cdots}}}{84\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```


```
[Out] ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]/(2*Sqrt[15]) + ((3 + Sqrt[2])
*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[
2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(14*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
 - ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]
*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2]
)/4])/(84*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```


Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
```

```
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx = \frac{1}{7}\left(3+\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{7}\left(2+3\sqrt{2}\right)\int \frac{1+\sqrt{2}x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{2\sqrt{15}} + \frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{\right.\right.}{14\sqrt[4]{2}\sqrt{1+2x^2+2x^4}}$$

Mathematica [C] time = 0.0592903, size = 80, normalized size = 0.33

$$-\frac{i\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)}{3\sqrt{1-i}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ((-I/3)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I
*ArcSinh[Sqrt[1 - I]*x], I])/(Sqrt[1 - I]*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.003, size = 70, normalized size = 0.3

$$\frac{1}{3\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\,\mathrm{EllipticPi}\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

```
[Out] 1/3/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(
1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+2x^2+1}\left(2x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{4\,x^6+10\,x^4+8\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

3.340 $\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=399

$$\frac{\left(5-3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{21\,2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-$$

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1
+ Sqrt[2]*x^2)) - ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]/(3*Sqrt[15]
) - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2
]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x^
4]) + ((5 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(21*2^(3/4)*Sq
rt[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^
(1/4)*x], (2 - Sqrt[2])/4])/(126*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.347179, antiderivative size = 399, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1329, 1714, 1195, 1708, 1103, 1706}

$$\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}x^2+1\right)}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{3\sqrt{15}}+\frac{\left(5-3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\right.}{21\,2^{3/4}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```


```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1
+ Sqrt[2]*x^2)) - ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]/(3*Sqrt[15]
) - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2
]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x^
4]) + ((5 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(21*2^(3/4)*Sq
rt[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^
(1/4)*x], (2 - Sqrt[2])/4])/(126*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```


Rule 1329

```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[(x^(m + 1)*Sqrt[a + b*x^2 + c*x^4])/(a*d*(m + 1)), x
] - Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)*Simp[a*e*(m + 1) + b*d*(m + 2) + (
b*e*(m + 2) + c*d*(m + 3))*x^2 + c*e*(m + 3)*x^4, x])/((d + e*x^2)*Sqrt[a +
 b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && ILtQ[m/2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
```

```
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{1}{3}\int \frac{-2+6x^2+4x^4}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{1}{12}\int \frac{-8+12\sqrt{2}+\left(24-4\left(6-2\sqrt{2}\right)\right)x^2}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx-\frac{1}{3}\sqrt{2}\int \frac{1}{\sqrt{1}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)}-\frac{\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\tan^{-}\right.}{3\sqrt{1+2x^2+2x^4}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{3\sqrt{15}}-\frac{\sqrt[4]{2}\left(1+\sqrt{2}x^2\right)}{}$$

Mathematica [C] time = 0.233882, size = 147, normalized size = 0.37

$$-\frac{i\left(\sqrt{1-i}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(-3\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)+3E\left(i\sinh^{-1}\left(\sqrt{1-i}x\right)\middle|i\right)-(1+i)\Pi\left(\frac{1}{3}\right.\right.\right.}{9x\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ((-I/9)*((-3*I)*(1 + 2*x^2 + 2*x^4) + Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*S
qrt[1 + (1 + I)*x^2]*(3*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - 3*Elliptic
F[I*ArcSinh[Sqrt[1 - I]*x], I] - (1 + I)*EllipticPi[1/3 + I/3, I*ArcSinh[Sq
rt[1 - I]*x], I])))/(x*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.014, size = 178, normalized size = 0.5

$$-\frac{2}{9\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\,\text{EllipticPi}\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2\,x^4+2\,x^2+1}}-\frac{1}{3\,x}\sqrt{2\,x^4+2\,x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

```
[Out] -2/9/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^
(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))-1/3*(2
*x^4+2*x^2+1)^(1/2)/x+(-1/3+1/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I
)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/
2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{4\,x^8+10\,x^6+8\,x^4+3\,x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^8 + 10*x^6 + 8*x^4 + 3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2), x)
```

3.341 $\int \frac{1}{x^4(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=422

$$-\frac{\left(1+19\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\,\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{63\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}x}{3\left(\sqrt{2}x^2+1\right)}+\frac{2\sqrt{2x^4+2x^2+ \cdots}}{3x}$$

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(9*x^3) + (2*Sqrt[1 + 2*x^2 + 2*x^4])/(3*x) - (2*S
qrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1 + Sqrt[2]*x^2)) + (2*ArcTan[(Sqrt[5
/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(9*Sqrt[15]) + (2*2^(1/4)*(1 + Sqrt[2]*x^2
)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*
x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 + 19*Sqrt[2])*(1 +
Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcT
an[2^(1/4)*x], (2 - Sqrt[2])/4])/(63*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3
 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^
2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/
(189*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.517098, antiderivative size = 422, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1329, 1683, 1714, 1195, 1708, 1103, 1706}

$$-\frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}x}{3\left(\sqrt{2}x^2+1\right)}+\frac{2\sqrt{2x^4+2x^2+1}}{3x}-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}+\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{9\sqrt{15}}-\frac{\left(1+19\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\cdots}}{63\sqrt[4]{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -Sqrt[1 + 2*x^2 + 2*x^4]/(9*x^3) + (2*Sqrt[1 + 2*x^2 + 2*x^4])/(3*x) - (2*S
qrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1 + Sqrt[2]*x^2)) + (2*ArcTan[(Sqrt[5
/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(9*Sqrt[15]) + (2*2^(1/4)*(1 + Sqrt[2]*x^2
)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*
x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 + 19*Sqrt[2])*(1 +
Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcT
an[2^(1/4)*x], (2 - Sqrt[2])/4])/(63*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3
 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^
2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/
(189*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1329

```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[(x^(m + 1)*Sqrt[a + b*x^2 + c*x^4])/(a*d*(m + 1)), x
] - Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)*Simp[a*e*(m + 1) + b*d*(m + 2) + (
b*e*(m + 2) + c*d*(m + 3))*x^2 + c*e*(m + 3)*x^4, x])/((d + e*x^2)*Sqrt[a +
 b*x^2 + c*x^4]), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && ILtQ[m/2, 0]
```

Rule 1683

```
Int[((Px_)*(x_)^(m_))/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 2], C
```

```
 = Coeff[Px, x, 4]}, Simp[(A*x^(m + 1)*Sqrt[a + b*x^2 + c*x^4])/(a*d*(m + 1
)), x] + Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + b*x^2 +
 c*x^4]))*Simp[a*B*d*(m + 1) - A*(a*e*(m + 1) + b*d*(m + 2)) + (a*C*d*(m +
1) - A*(b*e*(m + 2) + c*d*(m + 3)))*x^2 - A*c*e*(m + 3)*x^4, x], x], x]] /;
 FreeQ[{a, b, c, d, e}, x] && PolyQ[Px, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && I
LtQ[m/2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{1}{9}\int \frac{-18-14x^2-4x^4}{x^2\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{1}{27}\int \frac{6+120x^2+72x^4}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{1}{108}\int \frac{24+216\sqrt{2}+\left(480-72\left(6-2\right.\right.}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{2\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)}+\frac{2\sqrt[4]{2}\left(1+\sqrt{2}x^2\right.}{}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{2\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)}+\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}}{\sqrt{1+2x^2}}\right.}{9\sqrt{15}}$$

Mathematica [C] time = 0.19626, size = 219, normalized size = 0.52

$$\frac{-(3+15i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}x^3\,\mathbf{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)+36x^6+30x^4+12x^2+18i\sqrt{1-i}\sqrt{1}}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] (-3 + 12*x^2 + 30*x^4 + 36*x^6 + (18*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^
2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (3 + 15*I
)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*A
rcSinh[Sqrt[1 - I]*x], I] + 2*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[
1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(27*x^
3*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.016, size = 260, normalized size = 0.6

$$\frac{4}{27\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\,\mathrm{EllipticPi}\left(x\sqrt{-1+i},\frac{1}{3}+\frac{i}{3},\frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}+\frac{2}{3x}\sqrt{2x^4+2x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x)
```

```
[Out] 4/27/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^
(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))+2/3*(2
*x^4+2*x^2+1)^(1/2)/x+(2/3-2/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)
*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2
*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-1/9*(2*x^4
+2*x^2+1)^(1/2)/x^3-2/9/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2
)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{4\,x^{10}+10\,x^8+8\,x^6+3\,x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^10 + 10*x^8 + 8*x^6 + 3*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(2x^2+3\right)\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+1}\left(2\,x^2+3\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4), x)
```

3.342 $\int \frac{x^7}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}} dx$

Optimal. Leaf size=236

$$\frac{x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e}-\frac{d^3\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+c}}\right.}{2e\left(ae^2-bde+cd^2\right)^{3/2}}$$

```
[Out] (a*(b^2*d - 2*a*c*d - a*b*e) + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*x^
2)/(c*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + ArcT
anh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*c^(3/2)*e) - (d^3
*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x^2 + c*x^4])])/(2*e*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi [A] time = 0.473671, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 1646, 843, 621, 206, 724}

$$\frac{x^2\left(2a^2ce-ab^2e-3abcd+b^3d\right)+a\left(-abe-2acd+b^2d\right)}{c\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e}-\frac{d^3\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+c}}\right.}{2e\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (a*(b^2*d - 2*a*c*d - a*b*e) + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*x^
2)/(c*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + ArcT
anh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*c^(3/2)*e) - (d^3
*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x^2 + c*x^4])])/(2*e*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^7}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{a\left(b^2d-2acd-abe\right)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{\frac{\left(b^2-4ac\right)d(bd-ae)}{2c\left(cd^2-bde+ae^2\right)}-\frac{\left(b^2-\right.}{}}{(d+ex)\sqrt{a+bx+cx}}\right.}{b^2-4ac}$$

$$= \frac{a\left(b^2d-2acd-abe\right)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x,\right.}{2ce}$$

$$= \frac{a\left(b^2d-2acd-abe\right)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+}{\sqrt{a+b}}\right.}{ce}$$

$$= \frac{a\left(b^2d-2acd-abe\right)+\left(b^3d-3abcd-ab^2e+2a^2ce\right)x^2}{c\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e} -$$

Mathematica [A] time = 0.841386, size = 271, normalized size = 1.15

$$\frac{1}{2}\left(\frac{2\left(a^2\left(be+2c\left(d-ex^2\right)\right)+ab\left(-bd+bex^2+3cdx^2\right)+b^3(-d)x^2\right)}{c\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}\left(e(ae-bd)+cd^2\right)} + \frac{\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{c^{3/2}e} + \frac{d^3\log\left(2\sqrt{}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*(-(b^3*d*x^2) + a*b*(-(b*d) + 3*c*d*x^2 + b*e*x^2) + a^2*(b*e + 2*c*(d
- e*x^2))))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c
*x^4]) - (d^3*Log[d + e*x^2])/(e*(c*d^2 + e*(-(b*d) + a*e))^(3/2)) + Log[b
+ 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]/(c^(3/2)*e) + (d^3*Log[-(b*d
) + 2*a*e - 2*c*d*x^2 + b*e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x^2 + c*x^4]])/(e*(c*d^2 + e*(-(b*d) + a*e))^(3/2)))/2
```

Maple [B] time = 0.066, size = 720, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/e*x^2/c/(c*x^4+b*x^2+a)^(1/2)+1/4/e*b/c^2/(c*x^4+b*x^2+a)^(1/2)+1/2/e*
b^2/c/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+1/4/e*b^3/c^2/(4*a*c-b^2)/(c*x^
4+b*x^2+a)^(1/2)+1/2/e/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/
2))+d/e^2/(c*x^4+b*x^2+a)^(1/2)*(b*x^2+2*a)/(4*a*c-b^2)+d^2/e^3*(2*c*x^2+b)
/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+2*d^3/e^3*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*
c*d)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*(c*(x^2-1/2*(-b+(-
4*a*c+b^2)^(1/2))/c)^2+(-4*a*c+b^2)^(1/2)*(x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/
c))^(1/2)-2*d^3/e^2*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(e*(-4*a*c+b^2)^(1/2
)+b*e-2*c*d)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+
(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b
*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))-2*d^3/e^3*
c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(-4*a*c+b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/2)
/c+1/2*b/c)*(c*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2-(-4*a*c+b^2)^(1/2)*(x^2
+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} \left(ex^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(cx^4+bx^2+a)^{\frac{3}{2}}\,(ex^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

3.343 $\int \frac{x^5}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=167

$$\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{x^2\left(-abe-2acd+b^2d\right)+a(bd-2ae)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

```
[Out] -((a*(b*d - 2*a*e) + (b^2*d - 2*a*c*d - a*b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 -
 b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (d^2*ArcTanh[(b*d - 2*a*e + (2*
c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(
2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi [A] time = 0.294319, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 1646, 12, 724, 206}

$$\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{x^2\left(-abe-2acd+b^2d\right)+a(bd-2ae)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] -((a*(b*d - 2*a*e) + (b^2*d - 2*a*c*d - a*b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 -
 b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (d^2*ArcTanh[(b*d - 2*a*e + (2*
c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(
2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int -\frac{\left(b^2-4ac\right)d^2}{2\left(cd^2-bde+ae^2\right)(d+ex)\sqrt{a+bx+\ldots}}\ldots\right)}{b^2-4ac}$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\ldots\right)}{2\left(cd^2-bde+ae^2\right)}$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \ldots\right)}{cd^2-bde+ae^2}$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{d^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2\left(cd^2-bde+ae^2\right)^{3/2}}$$

Mathematica [A] time = 0.652703, size = 204, normalized size = 1.22

$$\frac{1}{2}\left(\frac{2\left(-2a^2e+ab\left(d-ex^2\right)-2acdx^2+b^2dx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(e(bd-ae)-cd^2\right)} - \frac{d^2\log\left(2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex^2-2cdx^2\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}} \ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*(-2*a^2*e + b^2*d*x^2 - 2*a*c*d*x^2 + a*b*(d - e*x^2)))/((b^2 - 4*a*c)*
(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (d^2*Log[d + e*x^2])/
(c*d^2 + e*(-(b*d) + a*e))^(3/2) - (d^2*Log[-(b*d) + 2*a*e - 2*c*d*x^2 + b*
e*x^2 + 2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4]])/(c*d^2 + e*
(-(b*d) + a*e))^(3/2))/2
```

Maple [B] time = 0.011, size = 613, normalized size = 3.7

$$-\frac{bx^2}{e\left(4ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}-2\frac{a}{e\sqrt{cx^4+bx^2+a}\left(4ac-b^2\right)}-2\frac{x^2cd}{e^2\left(4ac-b^2\right)\sqrt{cx^4+bx^2+a}}-\frac{bd}{e^2\left(4ac-b^2\right)}\frac{1}{\sqrt{cx^4+b\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/e/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)*x^2*b-2/e/(c*x^4+b*x^2+a)^(1/2)/(4*a
*c-b^2)*a-2/e^2*d/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*c*x^2-1/e^2*d/(4*a*c-b^
2)/(c*x^4+b*x^2+a)^(1/2)*b-2*d^2/e^2*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(-4
*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*(c*(x^2-1/2*(-b+(-4*a*c+b^
2)^(1/2))/c)^2+(-4*a*c+b^2)^(1/2)*(x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2
)+2*d^2/e*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*
d)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d
)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/
e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))+2*d^2/e^2*c/(e*(-4*a
*c+b^2)^(1/2)+b*e-2*c*d)/(-4*a*c+b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c
)*(c*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-
4*a*c+b^2)^(1/2))/c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} \left(ex^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Fricas [B] time = 5.92126, size = 2815, normalized size = 16.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*d^2*x^4 + (b^3 - 4*a*b*c)*d^2*x^2 + (a*b^2 - 4*a^2
*c)*d^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d
^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
 + d^2)) - 4*(a*b*c*d^3 + 3*a^2*b*d*e^2 - 2*a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2
*e - (a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (b^3 - a*b*c)*d^2*e - 2*(a*b^2 -
a^2*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 -
2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2
 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*
a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c
^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4
)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 -
 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^
3 - 4*a^3*b*c)*e^4)*x^2), 1/2*(((b^2*c - 4*a*c^2)*d^2*x^4 + (b^3 - 4*a*b*c)
*d^2*x^2 + (a*b^2 - 4*a^2*c)*d^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b
```

```
*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^
2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - 2*(a*b*c*d^3 + 3*a^2*b*d*e^2 - 2*
a^3*e^3 - (a*b^2 + 2*a^2*c)*d^2*e - (a^2*b*e^3 - (b^2*c - 2*a*c^2)*d^3 + (b
^3 - a*b*c)*d^2*e - 2*(a*b^2 - a^2*c)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/
((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2
*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2
- 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e +
 (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^
3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*
c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4
 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**5/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [B] time = 1.30515, size = 536, normalized size = 3.21

$$\frac{d^2\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}-\frac{\frac{\left(b^4cd^3-6\,ab^2c^2d^3+8\,a^2c^3d^3-b^5d^2e+5\,ab^3cd^2e-4\,a^2bc^2d^2e+2\,ab^4de^2-10\,a^2b^2cde^2+8\,a^3c^2de^2-a^2b^3e^3+4\,a^3bce^3\right)}{ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4}}{32\sqrt{cx^4+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] d^2*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c
*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2
)) - 1/32*((b^4*c*d^3 - 6*a*b^2*c^2*d^3 + 8*a^2*c^3*d^3 - b^5*d^2*e + 5*a*b
^3*c*d^2*e - 4*a^2*b*c^2*d^2*e + 2*a*b^4*d*e^2 - 10*a^2*b^2*c*d*e^2 + 8*a^3
*c^2*d*e^2 - a^2*b^3*e^3 + 4*a^3*b*c*e^3)*x^2/(a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4) + (a*b^3*c*d^3 - 4*a^2*b*c^2*d^3 - a*b^4*d^2*e + 2*a^2*b^2*c*d^
2*e + 8*a^3*c^2*d^2*e + 3*a^2*b^3*d*e^2 - 12*a^3*b*c*d*e^2 - 2*a^3*b^2*e^3
+ 8*a^4*c*e^3)/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4))/sqrt(c*x^4 + b*x^2
 + a)
```

3.344 $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=159

$$\frac{cx^2(bd-2ae)+a(2cd-be)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}-\frac{de\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}$$

```
[Out] (a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)*Sqrt[a + b*x^2 + c*x^4]) - (d*e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^
2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*(c*d^2 - b*
d*e + a*e^2)^(3/2))
```

Rubi [A] time = 0.191106, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 822, 12, 724, 206}

$$\frac{cx^2(bd-2ae)+a(2cd-be)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}-\frac{de\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] (a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)*Sqrt[a + b*x^2 + c*x^4]) - (d*e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^
2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*(c*d^2 - b*
d*e + a*e^2)^(3/2))
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{(b^2-4ac)de}{2(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{(b^2-4ac)(cd^2-bde+ae^2)}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{(de)\,\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{(de)\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \ldots\right)}{cd^2-bde+ae^2}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{de\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2(cd^2-bde+ae^2)^{3/2}}$$

Mathematica [A] time = 0.207123, size = 162, normalized size = 1.02

$$\frac{a(be-2cd+2cex^2)-bcdx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(e(bd-ae)-cd^2)} + \frac{de\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{2(e(ae-bd)+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-(b*c*d*x^2) + a*(-2*c*d + b*e + 2*c*e*x^2))/((b^2 - 4*a*c)*(-(c*d^2) + e*
(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (d*e*ArcTanh[(-(b*d) + 2*a*e - 2*c*
d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])
])/(2*(c*d^2 + e*(-(b*d) + a*e))^(3/2))
```

Maple [B] time = 0.01, size = 506, normalized size = 3.2

$$\frac{2cx^2+b}{e(4ac-b^2)}\frac{1}{\sqrt{cx^4+bx^2+a}} + 2\frac{cd}{e\left(e\sqrt{-4ac+b^2}-be+2cd\right)(-4ac+b^2)\sqrt{c\left(x^2-1/2\frac{-b+\sqrt{-4ac+b^2}}{c}\right)^2+\sqrt{-4ac+\ldots}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/e*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+2*d/e*c/(e*(-4*a*c+b^2)^(
1/2)-b*e+2*c*d)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*(c*(x^2
-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2+(-4*a*c+b^2)^(1/2)*(x^2-1/2*(-b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)-2*d*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(e*(-4*a*c+b^2)
^(1/2)+b*e-2*c*d)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)
/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)
^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))-2*d/e
*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(-4*a*c+b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/2
)/c+1/2*b/c)*(c*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2-(-4*a*c+b^2)^(1/2)*(x^
2+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}\left(ex^2 + d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Fricas [B] time = 5.57832, size = 2778, normalized size = 17.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*d*e*x^4 + (b^3 - 4*a*b*c)*d*e*x^2 + (a*b^2 - 4*a^2
*c)*d*e)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d
^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
 + d^2)) + 4*(2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (a*b^2 + 2*a^2*c)*d
*e^2 + (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c + 2*a*c^2)*d^2*e)*
x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4
*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3
- 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2
*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 -
2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*
c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8
*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)
*e^4)*x^2), -1/2*(((b^2*c - 4*a*c^2)*d*e*x^4 + (b^3 - 4*a*b*c)*d*e*x^2 + (a
*b^2 - 4*a^2*c)*d*e)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 +
b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/(
(c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2
- b^2*d*e + a*b*e^2)*x^2)) - 2*(2*a*c^2*d^3 - 3*a*b*c*d^2*e - a^2*b*e^3 + (
```

```
a*b^2 + 2*a^2*c)*d*e^2 + (b*c^2*d^3 + 3*a*b*c*d*e^2 - 2*a^2*c*e^3 - (b^2*c
+ 2*a*c^2)*d^2*e)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^
4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2
*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3
- 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a
^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)
*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b
^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^
2*b^3 - 4*a^3*b*c)*e^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [B] time = 1.23176, size = 595, normalized size = 3.74

$$-\frac{d\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)e}{\left(cd^2-bde+ae^2\right)\sqrt{-cd^2+bde-ae^2}}+\frac{\frac{\left(bc^2d^3-b^2cd^2e-2\,ac^2d^2e+3\,abcde^2-2\,a^2ce^3\right)x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}}{\sqrt{cx^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -d*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))*e/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^
2)) + ((b*c^2*d^3 - b^2*c*d^2*e - 2*a*c^2*d^2*e + 3*a*b*c*d*e^2 - 2*a^2*c*e
^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*
d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c
*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (2*a*c^2*d^3 - 3*a*b*c*d^2*e + a*b^2*
d*e^2 + 2*a^2*c*d*e^2 - a^2*b*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3
*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2
- 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))/sqrt(c*x^4
+ b*x^2 + a)
```

3.345 $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=166

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

```
[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b
*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*
d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*
(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi [A] time = 0.17072, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1247, 740, 12, 724, 206}

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\left(ae^2-bde+cd^2\right)^{3/2}}-\frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b
*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*
d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*
(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{\operatorname{Subst}\left(\int -\frac{\left(b^2-4ac\right)e^2}{2(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right.}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)}\\
&= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{e^2\operatorname{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right.}{2\left(cd^2-bde+ae^2\right)}\\
&= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{e^2\operatorname{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx,x,\right.}{cd^2-bde+ae^2}\\
&= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2\left(cd^2-bde+ae^2\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.164521, size = 167, normalized size = 1.01

$$-\frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(e(ae-bd)+cd^2\right)}-\frac{e^2\tanh^{-1}\left(\frac{2ae-bd+bex^2-2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{2\left(e(ae-bd)+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 + e
*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*x^4])) - (e^2*ArcTanh[(-(b*d) + 2*a*e -
 2*c*d*x^2 + b*e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*
x^4])])/(2*(c*d^2 + e*(-(b*d) + a*e))^(3/2))
```

Maple [B] time = 0.006, size = 454, normalized size = 2.7

$$-2\,\frac{c}{\left(e\sqrt{-4\,ac+b^2}-be+2\,cd\right)\left(-4\,ac+b^2\right)}\sqrt{c\left(x^2-1/2\,\frac{-b+\sqrt{-4\,ac+b^2}}{c}\right)^2+\sqrt{-4\,ac+b^2}\left(x^2-1/2\,\frac{-b+\sqrt{-4\,ac+b^2}}{c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(-4*a*c+b^2)/(x^2-1/2*(-b+(-4*a*c+b^2
)^(1/2))/c)*(c*(x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2+(-4*a*c+b^2)^(1/2)*(x^
2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)+2*c*e/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c
*d)/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*
(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/(x^2+d/e))+2*c/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(-4*a*c+b^2)/(x^2+1/2*(b
+(-4*a*c+b^2)^(1/2))/c)*(c*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2-(-4*a*c+b^2
)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)), x)
```

Fricas [B] time = 5.24246, size = 2815, normalized size = 16.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*e^2*x^4 + (b^3 - 4*a*b*c)*e^2*x^2 + (a*b^2 - 4*a^2
*c)*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 +
4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2
+ 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d
^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
 + d^2)) - 4*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*d^2*e + (b^3 - a*b*c)*d*e^2 - (
a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e - a*b*c*e^3 + (b^2*c + 2*
a*c^2)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a*b^2*c^2 - 4*a^2*c^3)*d^4 -
2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2
 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*
a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c
^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4
)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 -
 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^
3 - 4*a^3*b*c)*e^4)*x^2), 1/2*(((b^2*c - 4*a*c^2)*e^2*x^4 + (b^3 - 4*a*b*c)
*e^2*x^2 + (a*b^2 - 4*a^2*c)*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*
sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b
*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^
2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - 2*(b*c^2*d^3 - 2*(b^2*c - a*c^2)*
d^2*e + (b^3 - a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (2*c^3*d^3 - 3*b*c^2*
d^2*e - a*b*c*e^3 + (b^2*c + 2*a*c^2)*d*e^2)*x^2)*sqrt(c*x^4 + b*x^2 + a))/
((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2
*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2
```

```
- 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e +
 (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^
3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*
c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4
 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [B] time = 1.21665, size = 613, normalized size = 3.69

$$-\frac{\frac{\left(2\,c^3d^3-3\,bc^2d^2e+b^2cde^2+2\,ac^2de^2-abce^3\right)x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{bc^2d^3-2\,b^2cd^2e+2\,ac^2d^2e}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e^3)*x^2
/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2
 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3
+ a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e +
 b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 - 4*a*c^3*
d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8
*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*
e^4))/sqrt(c*x^4 + b*x^2 + a) + arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^2/((c*d^2 - b*d*e + a*
e^2)*sqrt(-c*d^2 + b*d*e - a*e^2))
```

3.346 $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=266

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d}+\frac{e\left(2ace+b^2(-e)+cx^2(2cd-be)+bcd\right)}{d\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{-2ac+b^2+bcx^2}{ad\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{e^3\tanh^{-}}{2d}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x^2 + c*x^4]) + (e*(b
*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d*(c*d^2 - b*
d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])]/(2*a^(3/2)*d) - (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*
(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi [A] time = 0.388085, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 960, 740, 12, 724, 206}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d}+\frac{e\left(2ace+b^2(-e)+cx^2(2cd-be)+bcd\right)}{d\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{-2ac+b^2+bcx^2}{ad\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{e^3\tanh^{-}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```


```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x^2 + c*x^4]) + (e*(b
*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d*(c*d^2 - b*
d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])]/(2*a^(3/2)*d) - (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*
(c*d^2 - b*d*e + a*e^2)^(3/2))
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
```

```
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}-\frac{e}{d(d+ex)\left(a+bx+cx^2\right)^{3/2}}\right)dx,x,x^2\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)}{2d}-\frac{e\operatorname{Subst}\left(\int \frac{1}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^2\right)}{2d}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}}+\frac{e\left(bcd-b^2e+2ace+c(2cd-be)x^2\right)}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}-\cdots$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}}+\frac{e\left(bcd-b^2e+2ace+c(2cd-be)x^2\right)}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}+\cdots$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}}+\frac{e\left(bcd-b^2e+2ace+c(2cd-be)x^2\right)}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}-\cdots$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}}+\frac{e\left(bcd-b^2e+2ace+c(2cd-be)x^2\right)}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}-\cdots$$

Mathematica [A] time = 0.87, size = 236, normalized size = 0.89

$$-\frac{\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{a^{3/2}}+\frac{2d\left(bc\left(3ae+cdx^2\right)-2ac^2\left(d-ex^2\right)+b^2c\left(d-ex^2\right)+b^3(-e)\right)}{a\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}\left(e(ae-bd)+cd^2\right)}+\frac{e^3\tanh^{-1}\left(\frac{-2ae+b\left(d-ex^2\right)+2cdx^2}{2\sqrt{a+bx^2+cx^4}\sqrt{e(ae-bd)+cd^2}}\right)}{\left(e(ae-bd)+cd^2\right)^{3/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((2*d*(-(b^3*e) + b*c*(3*a*e + c*d*x^2) + b^2*c*(d - e*x^2) - 2*a*c^2*(d -
 e*x^2)))/(a*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*x
^4]) + ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/a^(3/2) +
 (e^3*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*x^2))/(2*Sqrt[c*d^2 + e*(-(b*d
) + a*e)]*Sqrt[a + b*x^2 + c*x^4])])/(c*d^2 + e*(-(b*d) + a*e))^(3/2))/(2*d
)
```

Maple [B] time = 0.02, size = 612, normalized size = 2.3

$$\frac{1}{2\,ad}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{bcx^2}{ad\left(4\,ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{b^2}{2\,ad\left(4\,ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{1}{2\,d}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/2/d/a/(c*x^4+b*x^2+a)^(1/2)-1/d*b/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*c*x
^2-1/2/d*b^2/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2/d/a^(3/2)*ln((2*a+b*x^
2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+2*e/d*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2
*c*d)/(-4*a*c+b^2)/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*(c*(x^2-1/2*(-b+(
-4*a*c+b^2)^(1/2))/c)^2+(-4*a*c+b^2)^(1/2)*(x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))
/c))^(1/2)-2*e^2/d*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(e*(-4*a*c+b^2)^(1/2)
+b*e-2*c*d)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(
b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*
e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))-2*e/d*c/(e*
(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/(-4*a*c+b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/
2*b/c)*(c*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2-(-4*a*c+b^2)^(1/2)*(x^2+1/2*
(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x), x)
```

Fricas [B] time = 22.3875, size = 9844, normalized size = 37.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")

[Out] [1/4*(((a^2*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*a^4*c)*e^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*((a*b^2*c^2 - 2*a^2*c^3)*d^4 - (2*a*b^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a*b^2*c^2 - a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a^3*c^2)*d*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^3*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d^2*e^3 + (a^5*b^2 - 4*a^6*c)*d*e^4 + ((a^2*b^2*c^3 - 4*a^3*c^4)*d^5 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*e + (a^2*b^4*c - 2*a^3*b^2*c^2 - 8*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2*e^3 + (a^4*b^2*c - 4*a^5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^5 - 2*(a^2*b^4*c - 4*a^3*b^2*c^2)*d^4*e + (a^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c^2)*d^3*e^2 - 2*(a^3*b^4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4*b^3 - 4*a^5*b*c)*d*e^4)*x^2), -1/4*(2*((a^2*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*a^4*c)*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((a*b^2*c^2 - 2*a^2*c^3)*d^4 - (2*a*b^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a*b^2*c^2 - a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a^3*c^2)*d*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^3*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d^2*e^3 + (a^5*b^2 - 4*a^6*c)*d*e^4 + ((a^2*b^2*c^3 - 4*a^3*c^4)*d^5 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*e + (a^2*b^4*c - 2*a^3*b^2*c^2 - 8*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2*e^3 + (a^4*b^2*c - 4*a^5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^5 - 2*(a^2*b^4*c - 4*a^3*b^2*c^2)*d^4*e + (a^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c^2)*d^3*e^2 - 2*(a^3*b^4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4*b^3 - 4*a^5*b*c)*d*e^4)*x^2), 1/4*(2*((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + ((a^2*b^2

```
*c - 4*a^3*c^2)*e^3*x^4 + (a^2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*a^4*
c)*e^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4
*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 +
 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^
2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2
+ d^2)) + 4*((a*b^2*c^2 - 2*a^2*c^3)*d^4 - (2*a*b^3*c - 5*a^2*b*c^2)*d^3*e
+ (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 3*a^3*b*c)*d*e^3 +
 (a*b*c^3*d^4 - 2*(a*b^2*c^2 - a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e
^2 - (a^2*b^2*c - 2*a^3*c^2)*d*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^3*b^2
*c^2 - 4*a^4*c^3)*d^5 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^4*e + (a^3*b^4 - 2*a^
4*b^2*c - 8*a^5*c^2)*d^3*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d^2*e^3 + (a^5*b^2 -
 4*a^6*c)*d*e^4 + ((a^2*b^2*c^3 - 4*a^3*c^4)*d^5 - 2*(a^2*b^3*c^2 - 4*a^3*b
*c^3)*d^4*e + (a^2*b^4*c - 2*a^3*b^2*c^2 - 8*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*
c - 4*a^4*b*c^2)*d^2*e^3 + (a^4*b^2*c - 4*a^5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c
^2 - 4*a^3*b*c^3)*d^5 - 2*(a^2*b^4*c - 4*a^3*b^2*c^2)*d^4*e + (a^2*b^5 - 2*
a^3*b^3*c - 8*a^4*b*c^2)*d^3*e^2 - 2*(a^3*b^4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4
*b^3 - 4*a^5*b*c)*d*e^4)*x^2), -1/2*(((a^2*b^2*c - 4*a^3*c^2)*e^3*x^4 + (a^
2*b^3 - 4*a^3*b*c)*e^3*x^2 + (a^3*b^2 - 4*a^4*c)*e^3)*sqrt(-c*d^2 + b*d*e -
 a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((
2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^
2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x^2)) - ((a*b^2*c^2 -
 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c -
8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^
4 + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a
*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*
c - 4*a^3*c^2)*e^4)*x^4 + ((b^3*c^2 - 4*a*b*c^3)*d^4 - 2*(b^4*c - 4*a*b^2*c
^2)*d^3*e + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*e^2 - 2*(a*b^4 - 4*a^2*b^2*
c)*d*e^3 + (a^2*b^3 - 4*a^3*b*c)*e^4)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*((a*b^2*c
^2 - 2*a^2*c^3)*d^4 - (2*a*b^3*c - 5*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*
c - 2*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 3*a^3*b*c)*d*e^3 + (a*b*c^3*d^4 - 2*(a*
b^2*c^2 - a^2*c^3)*d^3*e + (a*b^3*c - a^2*b*c^2)*d^2*e^2 - (a^2*b^2*c - 2*a
^3*c^2)*d*e^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^5
 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^4*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*
d^3*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d^2*e^3 + (a^5*b^2 - 4*a^6*c)*d*e^4 + ((a
^2*b^2*c^3 - 4*a^3*c^4)*d^5 - 2*(a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*e + (a^2*b^
4*c - 2*a^3*b^2*c^2 - 8*a^4*c^3)*d^3*e^2 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2*
e^3 + (a^4*b^2*c - 4*a^5*c^2)*d*e^4)*x^4 + ((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^5
 - 2*(a^2*b^4*c - 4*a^3*b^2*c^2)*d^4*e + (a^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c
^2)*d^3*e^2 - 2*(a^3*b^4 - 4*a^4*b^2*c)*d^2*e^3 + (a^4*b^3 - 4*a^5*b*c)*d*e
^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x), x)
```

3.347 $\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=419

$$-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2dx^2(b^2-4ac)}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2}+\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d}-\frac{e^2\left(2ace+b^2(-e)+cx^2(2\right.}{d^2(b^2-4ac)\sqrt{a+bx^2+cx}}$$

```
[Out] -((e*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*d^2*Sqrt[a + b*x^2 + c*x^4])
) + (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*x^2*Sqrt[a + b*x^2 + c*x^4])
 - (e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d^2
*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ((3*b^2 - 8*a*c)*Sqrt[a
 + b*x^2 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*d*x^2) + (3*b*ArcTanh[(2*a + b*x^2)
/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2)*d) + (e*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)*d^2) + (e^4*ArcTanh[(
b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x^2 + c*x^4])])/(2*d^2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rubi [A] time = 0.564316, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1251, 960, 740, 806, 724, 206, 12}

$$-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2dx^2(b^2-4ac)}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2}+\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d}-\frac{e^2\left(2ace+b^2(-e)+cx^2(2\right.}{d^2(b^2-4ac)\sqrt{a+bx^2+cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((e*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*d^2*Sqrt[a + b*x^2 + c*x^4])
) + (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*x^2*Sqrt[a + b*x^2 + c*x^4])
 - (e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d^2
*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ((3*b^2 - 8*a*c)*Sqrt[a
 + b*x^2 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*d*x^2) + (3*b*ArcTanh[(2*a + b*x^2)
/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2)*d) + (e*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)*d^2) + (e^4*ArcTanh[(
b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*
x^2 + c*x^4])])/(2*d^2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{dx^2\left(a+bx+cx^2\right)^{3/2}} - \frac{e}{d^2x\left(a+bx+cx^2\right)^{3/2}} + \frac{e^2}{d^2(d+ex)\left(a+\right.}\right.\right.$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)}{2d} - \frac{e\,\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)}{2d^2} + \frac{e^2\,\text{Su}}{}$$

$$= -\frac{e\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)d^2\sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)dx^2\sqrt{a+bx^2+cx^4}} - \frac{e^2\left(b\right.}{\left(b^2-4ac\right.}$$

$$= -\frac{e\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)d^2\sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)dx^2\sqrt{a+bx^2+cx^4}} - \frac{e^2\left(b\right.}{\left(b^2-4ac\right.}$$

$$= -\frac{e\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)d^2\sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)dx^2\sqrt{a+bx^2+cx^4}} - \frac{e^2\left(b\right.}{\left(b^2-4ac\right.}$$

$$= -\frac{e\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)d^2\sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)dx^2\sqrt{a+bx^2+cx^4}} - \frac{e^2\left(b\right.}{\left(b^2-4ac\right.}$$

Mathematica [A] time = 1.63858, size = 350, normalized size = 0.84

$$\frac{\frac{2d\left(a^2\left(b^2e^2+4bce\left(d-ex^2\right)-4c^2\left(d^2+dex^2+e^2x^4\right)\right)-4a^3ce^2+a\left(b^2c\left(d^2+12dex^2+e^2x^4\right)+b^3e\left(ex^2-d\right)-10bc^2dx^2\left(d-ex^2\right)-8c^3d^2x^4\right)+3b^2dx^2\left(b+cx^2\right)(cd-be)\right)}{a^2x^2\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(e(bd-ae)-cd^2\right)} + \frac{(2ae+}{}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] ((2*d*(-4*a^3*c*e^2 + 3*b^2*d*(c*d - b*e)*x^2*(b + c*x^2) + a^2*(b^2*e^2 +
4*b*c*e*(d - e*x^2) - 4*c^2*(d^2 + d*e*x^2 + e^2*x^4)) + a*(-8*c^3*d^2*x^4
- 10*b*c^2*d*x^2*(d - e*x^2) + b^3*e*(-d + e*x^2) + b^2*c*(d^2 + 12*d*e*x^2
 + e^2*x^4))))/(a^2*(b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*x^2*Sqrt[a + b
*x^2 + c*x^4]) + ((3*b*d + 2*a*e)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
 b*x^2 + c*x^4])])/a^(5/2) + (2*e^4*ArcTanh[(-2*a*e + 2*c*d*x^2 + b*(d - e*
x^2))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])])/(c*d^2 +
 e*(-(b*d) + a*e))^(3/2))/(4*d^2)
```

Maple [B] time = 0.02, size = 863, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/d^2*e/a/(c*x^4+b*x^2+a)^(1/2)+1/d^2*e*b/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^
(1/2)*c*x^2+1/2/d^2*e*b^2/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2/d^2*e/a^(
3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-1/2/d/a/x^2/(c*x^4
+b*x^2+a)^(1/2)-3/4/d*b/a^2/(c*x^4+b*x^2+a)^(1/2)+3/2/d*b^2/a^2/(4*a*c-b^2)
/(c*x^4+b*x^2+a)^(1/2)*c*x^2+3/4/d*b^3/a^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2
)+3/4/d*b/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-4/d*c
^2/a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2-2/d*c/a/(4*a*c-b^2)/(c*x^4+b*x^2
+a)^(1/2)*b-2*e^2/d^2*c/(e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(-4*a*c+b^2)/(x^2+
1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*(c*(x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c)^2+
(-4*a*c+b^2)^(1/2)*(x^2-1/2*(-b+(-4*a*c+b^2)^(1/2))/c))^(1/2)+2*e^3/d^2*c/(
e*(-4*a*c+b^2)^(1/2)-b*e+2*c*d)/(e*(-4*a*c+b^2)^(1/2)+b*e-2*c*d)/((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)
+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(
a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))+2*e^2/d^2*c/(e*(-4*a*c+b^2)^(1/2)
+b*e-2*c*d)/(-4*a*c+b^2)/(x^2+1/2*(-4*a*c+b^2)^(1/2)/c+1/2*b/c)*(c*(x^2+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)^2-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}\left(ex^2+d\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x^3), x)
```

Fricas [B] time = 55.3532, size = 13005, normalized size = 31.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*((a^3*b^2*c - 4*a^4*c^2)*e^4*x^6 + (a^3*b^3 - 4*a^4*b*c)*e^4*x^4 +
(a^4*b^2 - 4*a^5*c)*e^4*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 -
 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 - 8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)
*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 +
 b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/
(e^2*x^4 + 2*d*e*x^2 + d^2)) + ((3*(b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(3*b^4*c^2
 - 13*a*b^2*c^3 + 4*a^2*c^4)*d^4*e + (3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c^3)
*d^3*e^2 - 4*(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^3 - (a^2*b^3*c - 4
*a^3*b*c^2)*d*e^4 + 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*x^6 + (3*(b^4*c^2 - 4*a*
b^2*c^3)*d^5 - 2*(3*b^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*e + (3*b^6 - 10
*a*b^4*c - 8*a^2*b^2*c^2)*d^3*e^2 - 4*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d
^2*e^3 - (a^2*b^4 - 4*a^3*b^2*c)*d*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*e^5)*x^4 +
 (3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c
^3)*d^4*e + (3*a*b^5 - 10*a^2*b^3*c - 8*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 5
*a^3*b^2*c + 4*a^4*c^2)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + 2*(a^4*b^2
- 4*a^5*c)*e^5)*x^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c
*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((a^2*b^2*c^2 - 4
```

```
*a^3*c^3)*d^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^4*e + (a^2*b^4 - 2*a^3*b^2*c
- 8*a^4*c^2)*d^3*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d^2*e^3 + (a^4*b^2 - 4*a^5*c
)*d*e^4 + ((3*a*b^2*c^3 - 8*a^2*c^4)*d^5 - 6*(a*b^3*c^2 - 3*a^2*b*c^3)*d^4*
e + 3*(a*b^4*c - 2*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^2 - 2*(2*a^2*b^3*c - 7*a^
3*b*c^2)*d^2*e^3 + (a^3*b^2*c - 4*a^4*c^2)*d*e^4)*x^4 + ((3*a*b^3*c^2 - 10*
a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 11*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e + (3*a*b^
5 - 8*a^2*b^3*c - 10*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 4*a^3*b^2*c + a^4*c^
2)*d^2*e^3 + (a^3*b^3 - 4*a^4*b*c)*d*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(((
a^3*b^2*c^3 - 4*a^4*c^4)*d^6 - 2*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d^5*e + (a^3*b
^4*c - 2*a^4*b^2*c^2 - 8*a^5*c^3)*d^4*e^2 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^3
*e^3 + (a^5*b^2*c - 4*a^6*c^2)*d^2*e^4)*x^6 + ((a^3*b^3*c^2 - 4*a^4*b*c^3)*
d^6 - 2*(a^3*b^4*c - 4*a^4*b^2*c^2)*d^5*e + (a^3*b^5 - 2*a^4*b^3*c - 8*a^5*
b*c^2)*d^4*e^2 - 2*(a^4*b^4 - 4*a^5*b^2*c)*d^3*e^3 + (a^5*b^3 - 4*a^6*b*c)*
d^2*e^4)*x^4 + ((a^4*b^2*c^2 - 4*a^5*c^3)*d^6 - 2*(a^4*b^3*c - 4*a^5*b*c^2)
*d^5*e + (a^4*b^4 - 2*a^5*b^2*c - 8*a^6*c^2)*d^4*e^2 - 2*(a^5*b^3 - 4*a^6*b
*c)*d^3*e^3 + (a^6*b^2 - 4*a^7*c)*d^2*e^4)*x^2), 1/8*(4*((a^3*b^2*c - 4*a^4
*c^2)*e^4*x^6 + (a^3*b^3 - 4*a^4*b*c)*e^4*x^4 + (a^4*b^2 - 4*a^5*c)*e^4*x^2
)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c*
d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^2 - b*c*d*e
+ a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d*e + a*b*e^2
)*x^2)) + ((3*(b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(3*b^4*c^2 - 13*a*b^2*c^3 + 4*a
^2*c^4)*d^4*e + (3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c
 - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^3 - (a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 + 2
*(a^3*b^2*c - 4*a^4*c^2)*e^5)*x^6 + (3*(b^4*c^2 - 4*a*b^2*c^3)*d^5 - 2*(3*b
^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*e + (3*b^6 - 10*a*b^4*c - 8*a^2*b^2*
c^2)*d^3*e^2 - 4*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^3 - (a^2*b^4 - 4
*a^3*b^2*c)*d*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*e^5)*x^4 + (3*(a*b^3*c^2 - 4*a^
2*b*c^3)*d^5 - 2*(3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^4*e + (3*a*b^5
- 10*a^2*b^3*c - 8*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^
2)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + 2*(a^4*b^2 - 4*a^5*c)*e^5)*x^2)*
sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*
x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((a^2*b^2*c^2 - 4*a^3*c^3)*d^5 - 2*(a^
2*b^3*c - 4*a^3*b*c^2)*d^4*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^3*e^2
- 2*(a^3*b^3 - 4*a^4*b*c)*d^2*e^3 + (a^4*b^2 - 4*a^5*c)*d*e^4 + ((3*a*b^2*c
^3 - 8*a^2*c^4)*d^5 - 6*(a*b^3*c^2 - 3*a^2*b*c^3)*d^4*e + 3*(a*b^4*c - 2*a^
2*b^2*c^2 - 4*a^3*c^3)*d^3*e^2 - 2*(2*a^2*b^3*c - 7*a^3*b*c^2)*d^2*e^3 + (a
^3*b^2*c - 4*a^4*c^2)*d*e^4)*x^4 + ((3*a*b^3*c^2 - 10*a^2*b*c^3)*d^5 - 2*(3
*a*b^4*c - 11*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e + (3*a*b^5 - 8*a^2*b^3*c - 10*
a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 4*a^3*b^2*c + a^4*c^2)*d^2*e^3 + (a^3*b^3
 - 4*a^4*b*c)*d*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(((a^3*b^2*c^3 - 4*a^4*c
^4)*d^6 - 2*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d^5*e + (a^3*b^4*c - 2*a^4*b^2*c^2
- 8*a^5*c^3)*d^4*e^2 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^3*e^3 + (a^5*b^2*c - 4
*a^6*c^2)*d^2*e^4)*x^6 + ((a^3*b^3*c^2 - 4*a^4*b*c^3)*d^6 - 2*(a^3*b^4*c -
4*a^4*b^2*c^2)*d^5*e + (a^3*b^5 - 2*a^4*b^3*c - 8*a^5*b*c^2)*d^4*e^2 - 2*(a
^4*b^4 - 4*a^5*b^2*c)*d^3*e^3 + (a^5*b^3 - 4*a^6*b*c)*d^2*e^4)*x^4 + ((a^4*
b^2*c^2 - 4*a^5*c^3)*d^6 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^5*e + (a^4*b^4 - 2
*a^5*b^2*c - 8*a^6*c^2)*d^4*e^2 - 2*(a^5*b^3 - 4*a^6*b*c)*d^3*e^3 + (a^6*b^
2 - 4*a^7*c)*d^2*e^4)*x^2), -1/4*(((3*(b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(3*b^4*
c^2 - 13*a*b^2*c^3 + 4*a^2*c^4)*d^4*e + (3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c
^3)*d^3*e^2 - 4*(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^3 - (a^2*b^3*c
- 4*a^3*b*c^2)*d*e^4 + 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*x^6 + (3*(b^4*c^2 - 4
*a*b^2*c^3)*d^5 - 2*(3*b^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*e + (3*b^6 -
 10*a*b^4*c - 8*a^2*b^2*c^2)*d^3*e^2 - 4*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2
)*d^2*e^3 - (a^2*b^4 - 4*a^3*b^2*c)*d*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*e^5)*x^
4 + (3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^
3*c^3)*d^4*e + (3*a*b^5 - 10*a^2*b^3*c - 8*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4
- 5*a^3*b^2*c + 4*a^4*c^2)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + 2*(a^4*b
^2 - 4*a^5*c)*e^5)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
+ 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - ((a^3*b^2*c - 4*a^4*c^2)*e^4*x
```

```
^6 + (a^3*b^3 - 4*a^4*b*c)*e^4*x^4 + (a^4*b^2 - 4*a^5*c)*e^4*x^2)*sqrt(c*d^
2 - b*d*e + a*e^2)*log(-((8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^4 -
8*a*b*d*e + 8*a^2*e^2 + (b^2 + 4*a*c)*d^2 + 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b
^2 + 4*a*c)*d*e)*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c*d^2 - b*d*e + a*e^2
)*((2*c*d - b*e)*x^2 + b*d - 2*a*e))/(e^2*x^4 + 2*d*e*x^2 + d^2)) + 2*((a^2
*b^2*c^2 - 4*a^3*c^3)*d^5 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^4*e + (a^2*b^4 -
2*a^3*b^2*c - 8*a^4*c^2)*d^3*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d^2*e^3 + (a^4*b
^2 - 4*a^5*c)*d*e^4 + ((3*a*b^2*c^3 - 8*a^2*c^4)*d^5 - 6*(a*b^3*c^2 - 3*a^2
*b*c^3)*d^4*e + 3*(a*b^4*c - 2*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^2 - 2*(2*a^2*
b^3*c - 7*a^3*b*c^2)*d^2*e^3 + (a^3*b^2*c - 4*a^4*c^2)*d*e^4)*x^4 + ((3*a*b
^3*c^2 - 10*a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 11*a^2*b^2*c^2 + 2*a^3*c^3)*d^4
*e + (3*a*b^5 - 8*a^2*b^3*c - 10*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 4*a^3*b^
2*c + a^4*c^2)*d^2*e^3 + (a^3*b^3 - 4*a^4*b*c)*d*e^4)*x^2)*sqrt(c*x^4 + b*x
^2 + a))/(((a^3*b^2*c^3 - 4*a^4*c^4)*d^6 - 2*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d^
5*e + (a^3*b^4*c - 2*a^4*b^2*c^2 - 8*a^5*c^3)*d^4*e^2 - 2*(a^4*b^3*c - 4*a^
5*b*c^2)*d^3*e^3 + (a^5*b^2*c - 4*a^6*c^2)*d^2*e^4)*x^6 + ((a^3*b^3*c^2 - 4
*a^4*b*c^3)*d^6 - 2*(a^3*b^4*c - 4*a^4*b^2*c^2)*d^5*e + (a^3*b^5 - 2*a^4*b^
3*c - 8*a^5*b*c^2)*d^4*e^2 - 2*(a^4*b^4 - 4*a^5*b^2*c)*d^3*e^3 + (a^5*b^3 -
 4*a^6*b*c)*d^2*e^4)*x^4 + ((a^4*b^2*c^2 - 4*a^5*c^3)*d^6 - 2*(a^4*b^3*c -
4*a^5*b*c^2)*d^5*e + (a^4*b^4 - 2*a^5*b^2*c - 8*a^6*c^2)*d^4*e^2 - 2*(a^5*b
^3 - 4*a^6*b*c)*d^3*e^3 + (a^6*b^2 - 4*a^7*c)*d^2*e^4)*x^2), 1/4*(2*((a^3*b
^2*c - 4*a^4*c^2)*e^4*x^6 + (a^3*b^3 - 4*a^4*b*c)*e^4*x^4 + (a^4*b^2 - 4*a^
5*c)*e^4*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 +
 a)*sqrt(-c*d^2 + b*d*e - a*e^2)*((2*c*d - b*e)*x^2 + b*d - 2*a*e)/((c^2*d^
2 - b*c*d*e + a*c*e^2)*x^4 + a*c*d^2 - a*b*d*e + a^2*e^2 + (b*c*d^2 - b^2*d
*e + a*b*e^2)*x^2)) - ((3*(b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(3*b^4*c^2 - 13*a*b
^2*c^3 + 4*a^2*c^4)*d^4*e + (3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c^3)*d^3*e^2
- 4*(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e^3 - (a^2*b^3*c - 4*a^3*b*c^
2)*d*e^4 + 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*x^6 + (3*(b^4*c^2 - 4*a*b^2*c^3)*
d^5 - 2*(3*b^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*e + (3*b^6 - 10*a*b^4*c
- 8*a^2*b^2*c^2)*d^3*e^2 - 4*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*e^3 -
(a^2*b^4 - 4*a^3*b^2*c)*d*e^4 + 2*(a^3*b^3 - 4*a^4*b*c)*e^5)*x^4 + (3*(a*b^
3*c^2 - 4*a^2*b*c^3)*d^5 - 2*(3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^4*e
 + (3*a*b^5 - 10*a^2*b^3*c - 8*a^3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 5*a^3*b^2*
c + 4*a^4*c^2)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + 2*(a^4*b^2 - 4*a^5*c
)*e^5)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(
-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*((a^2*b^2*c^2 - 4*a^3*c^3)*d^5 - 2*(a^2*
b^3*c - 4*a^3*b*c^2)*d^4*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^3*e^2 -
2*(a^3*b^3 - 4*a^4*b*c)*d^2*e^3 + (a^4*b^2 - 4*a^5*c)*d*e^4 + ((3*a*b^2*c^3
 - 8*a^2*c^4)*d^5 - 6*(a*b^3*c^2 - 3*a^2*b*c^3)*d^4*e + 3*(a*b^4*c - 2*a^2*
b^2*c^2 - 4*a^3*c^3)*d^3*e^2 - 2*(2*a^2*b^3*c - 7*a^3*b*c^2)*d^2*e^3 + (a^3
*b^2*c - 4*a^4*c^2)*d*e^4)*x^4 + ((3*a*b^3*c^2 - 10*a^2*b*c^3)*d^5 - 2*(3*a
*b^4*c - 11*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e + (3*a*b^5 - 8*a^2*b^3*c - 10*a^
3*b*c^2)*d^3*e^2 - 4*(a^2*b^4 - 4*a^3*b^2*c + a^4*c^2)*d^2*e^3 + (a^3*b^3 -
 4*a^4*b*c)*d*e^4)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(((a^3*b^2*c^3 - 4*a^4*c^4
)*d^6 - 2*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d^5*e + (a^3*b^4*c - 2*a^4*b^2*c^2 -
8*a^5*c^3)*d^4*e^2 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^3*e^3 + (a^5*b^2*c - 4*a
^6*c^2)*d^2*e^4)*x^6 + ((a^3*b^3*c^2 - 4*a^4*b*c^3)*d^6 - 2*(a^3*b^4*c - 4*
a^4*b^2*c^2)*d^5*e + (a^3*b^5 - 2*a^4*b^3*c - 8*a^5*b*c^2)*d^4*e^2 - 2*(a^4
*b^4 - 4*a^5*b^2*c)*d^3*e^3 + (a^5*b^3 - 4*a^6*b*c)*d^2*e^4)*x^4 + ((a^4*b^
2*c^2 - 4*a^5*c^3)*d^6 - 2*(a^4*b^3*c - 4*a^5*b*c^2)*d^5*e + (a^4*b^4 - 2*a
^5*b^2*c - 8*a^6*c^2)*d^4*e^2 - 2*(a^5*b^3 - 4*a^6*b*c)*d^3*e^3 + (a^6*b^2
- 4*a^7*c)*d^2*e^4)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{3}\left(d+ex^{2}\right)\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}\left(ex^{2}+d\right)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(e*x^2 + d)*x^3), x)
```

3.348 $\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=449

$$\frac{(7\sqrt{2}-2)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\ 2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}+\frac{\left(1-2x^2\right)x^3}{20\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{1}{20}$$

```
[Out] (x^3*(1 - 2*x^2))/(20*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 + 2*x^4]
)/20 + (x*Sqrt[1 + 2*x^2 + 2*x^4])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (27*Sqr
t[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/80 - ((1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + ((-2 + 7*Sqrt
[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellipt
icF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(3/4)*(-2 + 3*Sqrt[2])*Sqrt
[1 + 2*x^2 + 2*x^4]) + (27*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(
1/4)*x], (2 - Sqrt[2])/4])/(80*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x
^4])
```

Rubi [A] time = 0.347427, antiderivative size = 566, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1313, 1275, 1279, 1197, 1103, 1195, 1325, 1706}

$$\frac{\left(1-2x^2\right)x^3}{20\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{1}{20}\sqrt{2x^4+2x^2+1}x-\frac{27\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{40\left(2-3\sqrt{2}\right)}-\frac{\left(7+\sqrt{2}\right)\left(\sqrt{2}x^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x^3*(1 - 2*x^2))/(20*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 + 2*x^4]
)/20 + (x*Sqrt[1 + 2*x^2 + 2*x^4])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (27*Sqr
t[3/10]*(3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(40*(2
 - 3*Sqrt[2])) - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x
^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1
+ 2*x^2 + 2*x^4]) + (9*(1 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(20*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - ((7 + Sqrt[2])*(1
+ Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*Ar
cTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(40*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (
27*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^
2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(80*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1313

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> -Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[(d^2*f
^4)/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^(m - 4)*(a + b*x^2 + c*x^4)^(p + 1)
)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
```

```
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1325

```
Int[(x_)^4/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, -Dist[(2*c*d - a*e*q)/(c*e*(e - d*q))
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + (-Dist[1/(e*q), Int[(1 - q*x^2)/S
qrt[a + b*x^2 + c*x^4], x], x] + Dist[d^2/(e*(e - d*q)), Int[(1 + q*x^2)/((
d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x])] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
```

```
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = -\left(\frac{1}{10}\int \frac{x^4(3+4x^2)}{(1+2x^2+2x^4)^{3/2}}\,dx\right)+\frac{9}{10}\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}}+\frac{1}{40}\int \frac{x^2(-6+12x^2)}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{9\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{20\sqrt{2}}+\frac{81\int \frac{1+\sqrt{\ }}{(3+2x^2)\sqrt{1}}}{20\,(2-}$$

$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}}+\frac{1}{20}x\sqrt{1+2x^2+2x^4}+\frac{9x\sqrt{1+2x^2+2x^4}}{20\sqrt{2}(1+\sqrt{2}x^2)}-\frac{27\sqrt{\frac{3}{10}}(3-\sqrt{2})\,\mathrm{t}}{40\,(2-}$$

$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}}+\frac{1}{20}x\sqrt{1+2x^2+2x^4}+\frac{9x\sqrt{1+2x^2+2x^4}}{20\sqrt{2}(1+\sqrt{2}x^2)}-\frac{27\sqrt{\frac{3}{10}}(3-\sqrt{2})\,\mathrm{t}}{40\,(2-}$$

$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}}+\frac{1}{20}x\sqrt{1+2x^2+2x^4}+\frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}(1+\sqrt{2}x^2)}-\frac{27\sqrt{\frac{3}{10}}(3-\sqrt{2})\,\mathrm{ta}}{40\,(2-}$$

Mathematica [C] time = 0.285361, size = 199, normalized size = 0.44

$$\frac{-(29-33i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)+12x^3-4i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+}}{80\sqrt{2x^4+2x^2+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (4*x + 12*x^3 - (4*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^
2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (29 - 33*I)*Sqrt[1 - I]*Sqrt[1
+ (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
 + 27*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[
1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(80*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.032, size = 603, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] 1/8*x/(2*x^4+2*x^2+1)^(1/2)-11/4/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*
x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I
*2^(1/2))+(47/32-47/32*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1
/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/
2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+3*(1/8*x^3+1/8*x)/
(2*x^4+2*x^2+1)^(1/2)+243/160*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^
2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I
*2^(1/2))-9/2*(-1/4*x^3-1/8*x)/(2*x^4+2*x^2+1)^(1/2)-243/160*I/(-1+I)^(1/2)
*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x
*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+27/16*x^3/(2*x^4+2*x^2+1)^(1/2)-81
/4*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)+81/160/(-1+I)^(1/2)*(-I*x^2+x^2+
1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2)
,1/2*2^(1/2)+1/2*I*2^(1/2))+243/160/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^
2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1
/2*I*2^(1/2))+27/40/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(
2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}x^8}{8\,x^{10}+28\,x^8+40\,x^6+32\,x^4+14\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^8/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(2x^2+3\right)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**8/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.349 $\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=423

$$-\frac{\left(\sqrt[4]{2}+2^{3/4}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{\left(1-2x^2\right)x}{20\sqrt{2x^4+2x^2+1}}$$

```
[Out] (x*(1 - 2*x^2))/(20*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 + 2*x^4])/
(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (9*Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
 2*x^2 + 2*x^4]])/40 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*S
qrt[1 + 2*x^2 + 2*x^4]) - ((2^(1/4) + 2^(3/4))*(1 + Sqrt[2]*x^2)*Sqrt[(1 +
2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqr
t[2])/4])/(8*(-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - (9*(3 + Sqrt[2])*(
1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(
12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(40*2^(3/4)*(2
- 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.256485, antiderivative size = 503, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1313, 1275, 1197, 1103, 1195, 1319, 1706}

$$\frac{\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{\left(1-2x^2\right)x}{20\sqrt{2x^4+2x^2+1}}-\frac{9}{40}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{9\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan\ldots\right.}{140\ 2^{3/4}\sqrt{2x^4+2x^2\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x*(1 - 2*x^2))/(20*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 + 2*x^4])/
(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (9*Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
 2*x^2 + 2*x^4]])/40 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*S
qrt[1 + 2*x^2 + 2*x^4]) - ((1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(40*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (9*(3 + Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(140*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (9*(3 + Sqrt
[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elli
pticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(560*2^
(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1313

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> -Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[(d^2*f
^4)/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^(m - 4)*(a + b*x^2 + c*x^4)^(p + 1)
)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1319

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, -Dist[(a*(e + d*q))/(c*d^2 - a*e^2),
Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(a*d*(e + d*q))/(c*d^2 - a*e^2
), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[
{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
 && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = -\left(\frac{1}{10}\int \frac{x^2(3+4x^2)}{(1+2x^2+2x^4)^{3/2}}\,dx\right) + \frac{9}{10}\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= \frac{x(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{40}\int \frac{-2+4x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{140}\left(9\left(2+3\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x}}$$

$$= \frac{x(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} - \frac{9}{40}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{9\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)}{140}$$

$$= \frac{x(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)} - \frac{9}{40}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{(1-}{}$$

Mathematica [C] time = 0.217584, size = 199, normalized size = 0.47

$$\frac{(8-6i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)-4x^3-2i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)}}{40\sqrt{2x^4+2x^2+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (2*x - 4*x^3 - (2*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2
]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (8 - 6*I)*Sqrt[1 - I]*Sqrt[1 + (
1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] -
9*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3
+ I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(40*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.009, size = 586, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] -2*(1/8*x^3+1/8*x)/(2*x^4+2*x^2+1)^(1/2)+7/4/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/
2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2
^(1/2)+1/2*I*2^(1/2))+(-17/16+17/16*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+
(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2
)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+3*(-1
/4*x^3-1/8*x)/(2*x^4+2*x^2+1)^(1/2)-9/8*x^3/(2*x^4+2*x^2+1)^(1/2)+27/2*(3/2
0*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)-27/80/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)
*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(
1/2)+1/2*I*2^(1/2))-81/80*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)
^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(
1/2))-81/80/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*
x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+81/80*I/(-
1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*E
llipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-9/20/(-1+I)^(1/2)*(-I*x^
```

```
2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I
)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(2x^4+2x^2+1)^{\frac{3}{2}}(2x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2x^4+2x^2+1}\,x^6}{8x^{10}+28x^8+40x^6+32x^4+14x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^6/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(2x^2+3)(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**6/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(2x^4+2x^2+1)^{\frac{3}{2}}(2x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.350 $\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=422

$$\frac{\left(2+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\,2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{\left(x^2+2\right)x}{10\sqrt{2x^4+2x^2+1}}+$$

```
[Out] -(x*(2 + x^2))/(10*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 + 2*x^4])/(
10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
2*x^2 + 2*x^4]])/20 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sq
rt[1 + 2*x^2 + 2*x^4]) + ((2 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(4*2^(3/4)*(-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + (3*(3 + Sqrt[2])*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[
(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(20*2^(3/4)*(2
 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.24018, antiderivative size = 501, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1313, 1178, 1197, 1103, 1195, 1216, 1706}

$$\frac{\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{\left(x^2+2\right)x}{10\sqrt{2x^4+2x^2+1}}+\frac{3}{20}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{9\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan\right.}{140\sqrt[4]{2}\sqrt{2x^4+2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -(x*(2 + x^2))/(10*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 + 2*x^4])/(
10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
2*x^2 + 2*x^4]])/20 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sq
rt[1 + 2*x^2 + 2*x^4]) + ((1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(20*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (9*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2
)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*
x], (2 - Sqrt[2])/4])/(140*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (3*(3 + Sqrt[
2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellip
ticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(280*2^(
1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1313

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> -Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[(d^2*f
^4)/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^(m - 4)*(a + b*x^2 + c*x^4)^(p + 1)
)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = -\left(\frac{1}{10}\int \frac{3+4x^2}{(1+2x^2+2x^4)^{3/2}}\,dx\right) + \frac{9}{10}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{x(2+x^2)}{10\sqrt{1+2x^2+2x^4}} - \frac{1}{40}\int \frac{4-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{70}\left(9\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2}}$$

$$= -\frac{x(2+x^2)}{10\sqrt{1+2x^2+2x^4}} + \frac{3}{20}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) + \frac{9\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)}{1}$$

$$= -\frac{x(2+x^2)}{10\sqrt{1+2x^2+2x^4}} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)} + \frac{3}{20}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - (1$$

Mathematica [C] time = 0.216981, size = 199, normalized size = 0.47

$$-\frac{(1-2i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)+2x^3+i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)}}{20\sqrt{2x^4+2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -(4*x + 2*x^3 + I*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*E
llipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (1 - 2*I)*Sqrt[1 - I]*Sqrt[1 + (1 -
 I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 3*(
1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I
/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(20*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.009, size = 561, normalized size = 1.3

$$-2\frac{-1/4x^3-x/8}{\sqrt{2x^4+2x^2+1}} - \frac{\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+1}} + \frac{\left(\frac{5}{8}-\frac{5i}{8}\right)(\text{El}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] -2*(-1/4*x^3-1/8*x)/(2*x^4+2*x^2+1)^(1/2)-1/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2
)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^
(1/2)+1/2*I*2^(1/2))+(5/8-5/8*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*
x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*
I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+3/4*x^3/(2*
x^4+2*x^2+1)^(1/2)-9*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)+9/40/(-1+I)^(1
/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Elliptic
F(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+27/40*I/(-1+I)^(1/2)*(-I*x^2+x^
2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/
2),1/2*2^(1/2)+1/2*I*2^(1/2))+27/40/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^
2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1
/2*I*2^(1/2))-27/40*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)
```

```
/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+
3/10/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^
(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}x^4}{8\,x^{10}+28\,x^8+40\,x^6+32\,x^4+14\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(2x^2+3)\,(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**4/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(2\,x^4+2\,x^2+1)^{\frac{3}{2}}(2\,x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.351 $\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=423

$$-\frac{\left(\sqrt[4]{2}+2^{3/4}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{5\left(\sqrt{2}x^2+1\right)}+\frac{\left(4x^2+3\right)x}{10\sqrt{2x^4+2x^2-}}$$

```
[Out] (x*(3 + 4*x^2))/(10*Sqrt[1 + 2*x^2 + 2*x^4]) - (Sqrt[2]*x*Sqrt[1 + 2*x^2 +
2*x^4])/(5*(1 + Sqrt[2]*x^2)) - (Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*
x^2 + 2*x^4]])/10 + (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(5*Sqrt[
1 + 2*x^2 + 2*x^4]) - ((2^(1/4) + 2^(3/4))*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^
2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2]
)/4])/(4*(-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3 + Sqrt[2])*(1 + Sq
rt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 1
1*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*(2 - 3*Sq
rt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.241675, antiderivative size = 503, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1315, 1178, 1197, 1103, 1195, 1216, 1706}

$$-\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}x}{5\left(\sqrt{2}x^2+1\right)}+\frac{\left(4x^2+3\right)x}{10\sqrt{2x^4+2x^2+1}}-\frac{1}{10}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(1+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F(}{20\sqrt[4]{2}\sqrt{2x^4+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x*(3 + 4*x^2))/(10*Sqrt[1 + 2*x^2 + 2*x^4]) - (Sqrt[2]*x*Sqrt[1 + 2*x^2 +
2*x^4])/(5*(1 + Sqrt[2]*x^2)) - (Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*
x^2 + 2*x^4]])/10 + (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(5*Sqrt[
1 + 2*x^2 + 2*x^4]) - (3*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(70*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)
*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x
], (2 - Sqrt[2])/4])/(20*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])^
2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticP
i[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(140*2^(1/4)
*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1315

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^(
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Dist[(d*e*f^2)/(c*d^
2 - b*d*e + a*e^2), Int[((f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1))/(d + e*
x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
p, -1] && GtQ[m, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \frac{1}{10}\int \frac{2+6x^2}{(1+2x^2+2x^4)^{3/2}}\,dx - \frac{3}{5}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= \frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}} + \frac{1}{40}\int \frac{-4-16x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{35}\left(3\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2+}}$$

$$= \frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}} - \frac{1}{10}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{3\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)}{70}$$

$$= \frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}} - \frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{5\left(1+\sqrt{2}x^2\right)} - \frac{1}{10}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right) + -$$

Mathematica [C] time = 0.198397, size = 199, normalized size = 0.47

$$\frac{-(1+3i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)+8x^3+4i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+}{20\sqrt{2x^4+2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (6*x + 8*x^3 + (4*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2
]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (1 + 3*I)*Sqrt[1 - I]*Sqrt[1 + (
1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] -
2*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3
+ I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(20*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.01, size = 536, normalized size = 1.3

$$-\frac{x^3}{2}\frac{1}{\sqrt{2x^4+2x^2+1}} + \frac{\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{2\sqrt{-1+i}}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\frac{1}{\sqrt{2x^4+2x^2+1}} - \frac{\left(\frac{1}{4}-\frac{i}{4}\right)(\text{E}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] -1/2*x^3/(2*x^4+2*x^2+1)^(1/2)+1/2/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I
)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2
*I*2^(1/2))+(-1/4+1/4*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/
2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2
))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+6*(3/20*x^3+1/20*x)
/(2*x^4+2*x^2+1)^(1/2)-3/20/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)
^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(
1/2))-9/20*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2
*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-9/20/(-1+
I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ell
ipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+9/20*I/(-1+I)^(1/2)*(-I*x^
2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)
^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/5/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*
```

```
x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,
(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}x^2}{8\,x^{10}+28\,x^8+40\,x^6+32\,x^4+14\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(2x^2+3\right)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.352 $\int \frac{1}{\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=422

$$\frac{\left(2+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\text{EllipticF}\left(2\tan^{-1}\left(\sqrt[4]{2}x\right),\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\ 2^{3/4}\left(3\sqrt{2}-2\right)\sqrt{2x^4+2x^2+1}}+\frac{3\sqrt{2x^4+2x^2+1}x}{5\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{\left(3x^2+1\right)x}{5\sqrt{2x^4+2x^2+1}}+-$$

```
[Out] -(x*(1 + 3*x^2))/(5*Sqrt[1 + 2*x^2 + 2*x^4]) + (3*x*Sqrt[1 + 2*x^2 + 2*x^4]
)/(5*Sqrt[2]*(1 + Sqrt[2]*x^2)) + ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x
^4]]/(5*Sqrt[15]) - (3*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(5*2^(3/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + ((2 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(2*2^(3/4)*(-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])*(1
+ Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12
 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(15*2^(3/4)*(2 -
3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.226269, antiderivative size = 501, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1221, 1178, 1197, 1103, 1195, 1216, 1706}

$$\frac{3\sqrt{2x^4+2x^2+1}x}{5\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{\left(3x^2+1\right)x}{5\sqrt{2x^4+2x^2+1}}+\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{5\sqrt{15}}+\frac{\left(3+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\right.}{10\ 2^{3/4}\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -(x*(1 + 3*x^2))/(5*Sqrt[1 + 2*x^2 + 2*x^4]) + (3*x*Sqrt[1 + 2*x^2 + 2*x^4]
)/(5*Sqrt[2]*(1 + Sqrt[2]*x^2)) + ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x
^4]]/(5*Sqrt[15]) - (3*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(5*2^(3/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(35*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + ((3 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)
*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x
], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3 + Sqrt[2])^
2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticP
i[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(210*2^(1/4)
*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1221

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \frac{1}{10}\int \frac{2-4x^2}{(1+2x^2+2x^4)^{3/2}}\,dx + \frac{2}{5}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{x(1+3x^2)}{5\sqrt{1+2x^2+2x^4}} + \frac{1}{40}\int \frac{16+24x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{35}\left(2\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2}\ldots}$$

$$= -\frac{x(1+3x^2)}{5\sqrt{1+2x^2+2x^4}} + \frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{5\sqrt{15}} + \frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}F\left(\ldots\right.}{35\sqrt[4]{2}\sqrt{1+2x^2\ldots}}$$

$$= -\frac{x(1+3x^2)}{5\sqrt{1+2x^2+2x^4}} + \frac{3x\sqrt{1+2x^2+2x^4}}{5\sqrt{2}\left(1+\sqrt{2}x^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{5\sqrt{15}} - \frac{3\left(1+\sqrt{2}x^2\right)\ldots}{\ldots}$$

Mathematica [C] time = 0.152022, size = 199, normalized size = 0.47

$$\frac{(6+3i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right) - 18x^3 - 9i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+\ldots}{30\sqrt{2x^4+2x^2\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (-6*x - 18*x^3 - (9*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x
^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (6 + 3*I)*Sqrt[1 - I]*Sqrt[1 +
 (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
+ 2*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/
3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(30*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.005, size = 366, normalized size = 0.9

$$-4\frac{1}{\sqrt{2x^4+2x^2+1}}\left(\frac{3x^3}{20}+x/20\right) + \frac{\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{10\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] -4*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)+1/10/(-1+I)^(1/2)*(-I*x^2+x^2+1)
^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1
/2*2^(1/2)+1/2*I*2^(1/2))+3/10*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x
^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*
I*2^(1/2))+3/10/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^
4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-3/10*I
/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2
)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+2/15/(-1+I)^(1/2)*(-I
*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-
1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{2\,x^4+2\,x^2+1}}{8\,x^{10}+28\,x^8+40\,x^6+32\,x^4+14\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 14*x^
2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2x^2+3\right)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(1/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

3.353 $\int \frac{1}{x^2(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=468

$$\frac{(3\sqrt{2}-7)(\sqrt{2}x^2+1)\sqrt{\frac{2x^4+2x^2+1}{(\sqrt{2}x^2+1)^2}}\,\text{EllipticF}\left(2\tan^{-1}(\sqrt[4]{2}x),\frac{1}{4}(2-\sqrt{2})\right)}{3\,2^{3/4}(3\sqrt{2}-2)\sqrt{2x^4+2x^2+1}}+\frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}x}{15(\sqrt{2}x^2+1)}+\frac{2(3x^2+1)x}{15\sqrt{2x^4+2x^2+}}$$

```
[Out] -x/(3*Sqrt[1 + 2*x^2 + 2*x^4]) + (2*x*(1 + 3*x^2))/(15*Sqrt[1 + 2*x^2 + 2*x
^4]) - Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (2*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4]
)/(15*(1 + Sqrt[2]*x^2)) - (2*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]
)/(15*Sqrt[15]) - (2*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(15*Sqrt
[1 + 2*x^2 + 2*x^4]) + ((-7 + 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(3*2^(3/4)*(-2 + 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - (2^(1/4)*(3 + Sq
rt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elli
pticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(45*(2
- 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.445463, antiderivative size = 644, normalized size of antiderivative = 1.38, number of steps used = 15, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {1335, 1121, 1281, 1197, 1103, 1195, 1221, 1178, 1216, 1706}

$$\frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}x}{15(\sqrt{2}x^2+1)}+\frac{2(3x^2+1)x}{15\sqrt{2x^4+2x^2+1}}-\frac{x}{3\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{15\sqrt{15}}-\frac{(3+2\sqrt{2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -x/(3*Sqrt[1 + 2*x^2 + 2*x^4]) + (2*x*(1 + 3*x^2))/(15*Sqrt[1 + 2*x^2 + 2*x
^4]) - Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (2*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4]
)/(15*(1 + Sqrt[2]*x^2)) - (2*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]
)/(15*Sqrt[15]) - (2*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(15*Sqrt
[1 + 2*x^2 + 2*x^4]) - ((1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2
*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])
/(6*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (2^(3/4)*(3 + Sqrt[2])*(1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/
4)*x], (2 - Sqrt[2])/4])/(105*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3 + 2*Sqrt[2])*(
1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*
ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(15*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) +
 ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x
^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(315*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1335

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1221

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
```

```
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \int\left(\frac{1}{3x^2\left(1+2x^2+2x^4\right)^{3/2}}-\frac{2}{3\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\right)dx$$
$$= \frac{1}{3}\int\frac{1}{x^2\left(1+2x^2+2x^4\right)^{3/2}}\,dx-\frac{2}{3}\int\frac{1}{\left(3+2x^2\right)\left(1+2x^2+2x^4\right)^{3/2}}\,dx$$
$$= -\frac{x}{3\sqrt{1+2x^2+2x^4}}-\frac{1}{15}\int\frac{2-4x^2}{\left(1+2x^2+2x^4\right)^{3/2}}\,dx+\frac{1}{12}\int\frac{4-4x^2}{x^2\sqrt{1+2x^2+2x^4}}\,$$
$$= -\frac{x}{3\sqrt{1+2x^2+2x^4}}+\frac{2x\left(1+3x^2\right)}{15\sqrt{1+2x^2+2x^4}}-\frac{\sqrt{1+2x^2+2x^4}}{3x}-\frac{1}{60}\int\frac{16+2}{\sqrt{1+2x^2}}$$
$$= -\frac{x}{3\sqrt{1+2x^2+2x^4}}+\frac{2x\left(1+3x^2\right)}{15\sqrt{1+2x^2+2x^4}}-\frac{\sqrt{1+2x^2+2x^4}}{3x}-\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2}}\right)}{15\sqrt{15}}$$
$$= -\frac{x}{3\sqrt{1+2x^2+2x^4}}+\frac{2x\left(1+3x^2\right)}{15\sqrt{1+2x^2+2x^4}}-\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{2\sqrt{2}x\sqrt{1+2x^2}}{15\left(1+\sqrt{2}x\right.}$$

Mathematica [C] time = 0.237159, size = 211, normalized size = 0.45

$$\frac{-(27-39i)\sqrt{1-ix}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt{1-i}x\right),i\right)-12i\sqrt{1-ix}\sqrt{1+(1-i)x^2}\sqrt{1+(1+}}{90x\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] ((-12*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Elliptic
E[I*ArcSinh[Sqrt[1 - I]*x], I] - (27 - 39*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)
*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 2*(15
+ 39*x^2 + 12*x^4 + 2*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I
)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(90*x*Sqrt[1 +
2*x^2 + 2*x^4])
```

Maple [C] time = 0.016, size = 553, normalized size = 1.2

$$\frac{8}{3}\left(\frac{3x^3}{20}+\frac{x}{20}\right)\frac{1}{\sqrt{2x^4+2x^2+1}}-\frac{\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right)}{15\sqrt{-1+i}}\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\frac{1}{\sqrt{2x^4+2x^2+1}}-\frac{\frac{i}{5}\text{El}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x)
```

```
[Out] 8/3*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)-1/15/(-1+I)^(1/2)*(-I*x^2+x^2+1
)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),
1/2*2^(1/2)+1/2*I*2^(1/2))-1/5*I/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x
^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*
I*2^(1/2))-1/5/(-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4
+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+1/5*I/(
-1+I)^(1/2)*(-I*x^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*
EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-4/45/(-1+I)^(1/2)*(-I*x
^2+x^2+1)^(1/2)*(I*x^2+x^2+1)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+
I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))-1/3*x/(2*x^4+2*x^2+1)^(1/2)-1
/3*(2*x^4+2*x^2+1)^(1/2)/x-1/3/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^
2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2
^(1/2))+(-1/3+1/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(
2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-E
llipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2x^4+2x^2+1\right)^{\frac{3}{2}}\left(2x^2+3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{2x^4+2x^2+1}}{8x^{12}+28x^{10}+40x^8+32x^6+14x^4+3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(8*x^12 + 28*x^10 + 40*x^8 + 32*x^6 + 14*x
^4 + 3*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(2x^2+3\right)\left(2x^4+2x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2\,x^4+2\,x^2+1\right)^{\frac{3}{2}}\left(2\,x^2+3\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2), x)
```

3.354 $\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=406

$$-\frac{\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd+b^4(-e)}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd}{\sqrt{b^2-4ac}}\right.}{}$$

```
[Out] ((b^2 - a*c)*Sqrt[d + e*x^2])/c^3 - ((c*d + b*e)*(d + e*x^2)^(3/2))/(3*c^2*
e^2) + (d + e*x^2)^(5/2)/(5*c*e^2) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*
e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 -
4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
- ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d - b^4*e
+ 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 8.59299, antiderivative size = 406, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 897, 1287, 1166, 208}

$$-\frac{\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd+b^4(-e)}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^3cd}{\sqrt{b^2-4ac}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```


```
[Out] ((b^2 - a*c)*Sqrt[d + e*x^2])/c^3 - ((c*d + b*e)*(d + e*x^2)^(3/2))/(3*c^2*
e^2) + (d + e*x^2)^(5/2)/(5*c*e^2) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*
e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 -
4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
- ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d - b^4*e
+ 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\left(b^2-ac\right)e}{c^3}-\frac{(cd+be)x^2}{c^2e}+\frac{x^4}{ce}-\frac{\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-\left(b^2cd-ac^2d-b^3e+2abce\right)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)\left(d+ex^2\right)^{3/2}}{3c^2e^2}+\frac{\left(d+ex^2\right)^{5/2}}{5ce^2}-\frac{\text{Subst}\left(\int \frac{\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)+\left(-b^2cd\right.}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be}{e^2}}\right.}{c^3e}$$

$$= \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)\left(d+ex^2\right)^{3/2}}{3c^2e^2}+\frac{\left(d+ex^2\right)^{5/2}}{5ce^2}+\frac{\left(b^2cd-ac^2d-b^3e+2abce-\frac{b^3cd-}{}\right.}{}$$

$$= \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)\left(d+ex^2\right)^{3/2}}{3c^2e^2}+\frac{\left(d+ex^2\right)^{5/2}}{5ce^2}-\frac{\left(b^2cd-ac^2d-b^3e+2abce-\frac{b^3cd-}{}\right.}{\sqrt{2}c^{7/2}\sqrt{}}$$

Mathematica [B] time = 10.8281, size = 943, normalized size = 2.32

$$\frac{c\left(105\left(-b^3+\sqrt{b^2-4ac}b^2+3acb-ac\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{c(ex^2+d)}{2cd-be+\sqrt{b^2-4ac}e}}\right)e^3+\sqrt{2}\sqrt{\frac{c(ex^2+d)}{2cd-be+\sqrt{b^2-4ac}e}}\left(105b^3e^3-35b^2\left(3\sqrt{b^2-4ac}e+c(ex^2+d)\right)e^2+7bc\left((ex^2+d)(-5cd\cdots\right.\right.\right.}{210\sqrt{2}\sqrt{b^2-4ac}e^4\left(2cd+\left(\sqrt{b^2-4ac}-b\right)e\right)^3\left(\cdots\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*(d + e*x^2)^(9/2)*(Sqrt[2]*Sqrt[(c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2
 - 4*a*c]*e)]*(105*b^3*e^3 - 35*b^2*e^2*(3*Sqrt[b^2 - 4*a*c]*e + c*(d + e*x
^2)) + 7*b*c*e*(-45*a*e^2 + (d + e*x^2)*(-5*c*d + 5*Sqrt[b^2 - 4*a*c]*e + 3
*c*(d + e*x^2))) + c*(35*a*e^2*(3*Sqrt[b^2 - 4*a*c]*e + 2*c*(d + e*x^2)) +
c*(d + e*x^2)*(7*Sqrt[b^2 - 4*a*c]*e*(5*d - 3*(d + e*x^2)) + c*(-70*d^2 + 8
4*d*(d + e*x^2) - 30*(d + e*x^2)^2)))) + 105*(-b^3 + 3*a*b*c + b^2*Sqrt[b^2
 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*e^3*ArcTanh[Sqrt[2]*Sqrt[(c*(d + e*x^2))
/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)]]))/(210*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e^4
*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)^3*(-(Sqrt[b^2 - 4*a*c]/e) - (2*c*d -
b*e)/e^2)*((c*(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e))^(9/2)) + (2
*c*d^3*(d + e*x^2)^(3/2)*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(-105*b^3*e^
3 + 35*b^2*e^2*(-3*Sqrt[b^2 - 4*a*c]*e + c*(d + e*x^2)) - 7*b*c*e*(-45*a*e^
2 + (d + e*x^2)*(-5*c*d - 5*Sqrt[b^2 - 4*a*c]*e + 3*c*(d + e*x^2))) + c*(35
*a*e^2*(3*Sqrt[b^2 - 4*a*c]*e - 2*c*(d + e*x^2)) + c*(d + e*x^2)*(7*Sqrt[b^
2 - 4*a*c]*e*(5*d - 3*(d + e*x^2)) + c*(70*d^2 - 84*d*(d + e*x^2) + 30*(d +
 e*x^2)^2)))))/(140*c^4*d^3*(d + e*x^2)) + (3*(b^3 - 3*a*b*c + b^2*Sqrt[b^2
 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*e^3*(d + e*x^2)^3*ArcTanh[Sqrt[2]*Sqrt[(
c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]])/(4*Sqrt[2]*d^3*(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)^3*((c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e))^(9/2))))/(3*Sqrt[b^2 - 4*a*c]*e^4*(Sqrt[b^2 - 4*a*c]/e - (2*c*d
- b*e)/e^2)))/e
```

Maple [C] time = 0.049, size = 496, normalized size = 1.2

$$\frac{x^2}{5ce}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{2d}{15ce^2}\left(ex^2+d\right)^{\frac{3}{2}}-\frac{b}{3c^2e}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{xa}{2c^2}\sqrt{e}-\frac{xb^2}{2c^3}\sqrt{e}-\frac{a}{2c^2}\sqrt{ex^2+d}+\frac{b^2}{2c^3}\sqrt{ex^2+d}-\frac{ad}{2c^2}\left(\sqrt{ex^2+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/5/c*x^2*(e*x^2+d)^(3/2)/e-2/15/c*d/e^2*(e*x^2+d)^(3/2)-1/3/c^2*b*(e*x^2+d
)^(3/2)/e+1/2/c^2*e^(1/2)*x*a-1/2/c^3*e^(1/2)*x*b^2-1/2/c^2*(e*x^2+d)^(1/2)
*a+1/2/c^3*(e*x^2+d)^(1/2)*b^2-1/2/c^2*d/((e*x^2+d)^(1/2)-e^(1/2)*x)*a+1/2/
c^3*d/((e*x^2+d)^(1/2)-e^(1/2)*x)*b^2-1/4/c^3*sum(((-2*a*b*c*e+a*c^2*d+b^3*
e-b^2*c*d)*_R^6+(-4*a^2*c*e^2+4*a*b^2*e^2+2*a*b*c*d*e-3*a*c^2*d^2-3*b^3*d*e
+3*b^2*c*d^2)*_R^4+d*(4*a^2*c*e^2-4*a*b^2*e^2-2*a*b*c*d*e+3*a*c^2*d^2+3*b^3
*d*e-3*b^2*c*d^2)*_R^2+2*a*b*c*d^3*e-a*c^2*d^4-b^3*d^3*e+b^2*c*d^4)/(_R^7*c
+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R
*c*d^3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^
6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}\,x^7}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^7/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**7*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.355 $\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=324

$$\frac{\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\text{t}}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(\sqrt{b^2-4\right.}}}$$

```
[Out] -((b*Sqrt[d + e*x^2])/c^2) + (d + e*x^2)^(3/2)/(3*c*e) + ((b*c*d - b^2*e +
a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]
])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((b*c*d - b^
2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 3.52866, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 897, 1287, 1166, 208}

$$\frac{\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\text{t}}{\sqrt{2}c^{5/2}\sqrt{2cd-e\left(\sqrt{b^2-4\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*Sqrt[d + e*x^2])/c^2) + (d + e*x^2)^(3/2)/(3*c*e) + ((b*c*d - b^2*e +
a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]
])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((b*c*d - b^
2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{be}{c^2}+\frac{x^2}{c}+\frac{b\left(cd^2-bde+ae^2\right)-\left(bcd-b^2e+ace\right)x^2}{c^2e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3ce}+\frac{\text{Subst}\left(\int \frac{b\left(cd^2-bde+ae^2\right)+\left(-bcd+b^2e-ace\right)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex^2}\right)}{c^2e^2}$$

$$= -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3ce}-\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{\sqrt{b^2-4ac}}{2e}-\frac{2cd-be}{2e^2}+\frac{cx}{e^2}\cdots}\right.}{2c^2e^2}$$

$$= -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{\left(d+ex^2\right)^{3/2}}{3ce}+\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\cdots\right.}{\sqrt{2}c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 7.81822, size = 591, normalized size = 1.82

$$c\left(d+ex^2\right)^{7/2}\left(\frac{e^2\left(\sqrt{2}\sqrt{\frac{c\left(d+ex^2\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\left(5be\left(3e\sqrt{b^2-4ac}+c\left(d+ex^2\right)\right)+c\left(d+ex^2\right)\left(-5e\sqrt{b^2-4ac}+4cd-6cex^2\right)+30ace^2-15b^2e^2\right)-15e^2\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\text{ta}\cdots}{\left(e\left(b-\sqrt{b^2-4ac}\right)-2cd\right)\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)^2\left(\frac{c\left(d+ex^2\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}\right)^{7/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (c*(d + e*x^2)^(7/2)*((e^2*(Sqrt[2]*Sqrt[(c*(d + e*x^2))/(2*c*d + (-b + Sqr
t[b^2 - 4*a*c])*e)]*(-15*b^2*e^2 + 30*a*c*e^2 + c*(d + e*x^2)*(4*c*d - 5*Sq
rt[b^2 - 4*a*c]*e - 6*c*e*x^2) + 5*b*e*(3*Sqrt[b^2 - 4*a*c]*e + c*(d + e*x^
2))) - 15*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*e^2*ArcTanh[Sqrt[2]*Sqrt[(c*
(d + e*x^2))/(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)]]))/((-2*c*d + (b - Sqrt[b
^2 - 4*a*c])*e)*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)^2*((c*(d + e*x^2))/(2*
c*d + (-b + Sqrt[b^2 - 4*a*c])*e))^(7/2)) - (e^2*(Sqrt[2]*Sqrt[(c*(d + e*x^
2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(-15*b^2*e^2 + 30*a*c*e^2 + c*(d +
 e*x^2)*(4*c*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*c*e*x^2) - 5*b*e*(3*Sqrt[b^2 - 4
*a*c]*e - c*(d + e*x^2))) + 15*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*e^2*ArcT
anh[Sqrt[2]*Sqrt[(c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]]))/((
-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)^3*((c*(d + e*x^2))/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e))^(7/2))))/(30*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e^4)
```

Maple [C] time = 0.026, size = 332, normalized size = 1.

$$\frac{1}{3ce}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{bx}{2c^2}\sqrt{e}-\frac{b}{2c^2}\sqrt{ex^2+d}-\frac{bd}{2c^2}\left(\sqrt{ex^2+d}-\sqrt{e}x\right)^{-1}-\frac{1}{4c^2}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+(4be-4cd)\_Z^6+\left(16ae^2-8deb+6cd^2\right)\_Z}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*(e*x^2+d)^(3/2)/c/e+1/2/c^2*b*e^(1/2)*x-1/2*b*(e*x^2+d)^(1/2)/c^2-1/2/c
^2*b*d/((e*x^2+d)^(1/2)-e^(1/2)*x)-1/4/c^2*sum(((a*c*e-b^2*e+b*c*d)*_R^6+(-
4*a*b*e^2+a*c*d*e+3*b^2*d*e-3*b*c*d^2)*_R^4+d*(4*a*b*e^2-a*c*d*e-3*b^2*d*e+
3*b*c*d^2)*_R^2-a*c*d^3*e+b^2*d^3*e-c*d^4*b)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+
8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1
/2)-e^(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c
*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ex^2+d}\,x^5}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^5/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**5*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.356 $\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=292

$$\frac{\left(-\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\mathrm{ta}}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2}\right.}}$$

```
[Out] Sqrt[d + e*x^2]/c + ((b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]*(c*d - b*
e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - ((b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d - b*e))*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e])
```

Rubi [A] time = 3.59965, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 824, 826, 1166, 208}

$$\frac{\left(-\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\sqrt{b^2-4ac}(cd-be)+2ace+b^2(-e)+bcd\right)\mathrm{ta}}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2}\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[d + e*x^2]/c + ((b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]*(c*d - b*
e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - ((b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d - b*e))*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{d+ex^2}}{c} + \frac{\text{Subst}\left(\int \frac{-ae+(cd-be)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2c}$$

$$= \frac{\sqrt{d+ex^2}}{c} + \frac{\text{Subst}\left(\int \frac{-ae^2-d(cd-be)+(cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{c}$$

$$= \frac{\sqrt{d+ex^2}}{c} - \frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}(cd-be)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{\right.}{2c\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{d+ex^2}}{c} + \frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}(cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(bcd-b^2e+\right.}{}$$

Mathematica [A] time = 0.571325, size = 308, normalized size = 1.05

$$\frac{\frac{\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{\left(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}-2ace+b^2e-bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \sqrt{}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[d + e*x^2] + ((b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e + b*Sq
rt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b
*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d +
```

```
(-b + Sqrt[b^2 - 4*a*c])*e]) + ((-(b*c*d) - c*Sqrt[b^2 - 4*a*c]*d + b^2*e
- 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2]
)/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a
*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/c
```

Maple [C] time = 0.023, size = 275, normalized size = 0.9

$$-\frac{x}{2c}\sqrt{e}+\frac{1}{2c}\sqrt{ex^2+d}+\frac{d}{2c}\left(\sqrt{ex^2+d}-\sqrt{ex}\right)^{-1}+\frac{1}{4c}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+(4be-4cd)\_Z^6+\left(16ae^2-8deb+6cd^2\right)\_Z^4+\left(4bd^2e-4cd^3\right)\_Z^2+cd^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/c*e^(1/2)*x+1/2*(e*x^2+d)^(1/2)/c+1/2/c*d/((e*x^2+d)^(1/2)-e^(1/2)*x)+
1/4/c*sum(((-b*e+c*d)*_R^6+(-4*a*e^2+3*b*d*e-3*c*d^2)*_R^4+d*(4*a*e^2-3*b*d
*e+3*c*d^2)*_R^2+b*d^3*e-c*d^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-
4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x
-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4
*b*d^2*e-4*c*d^3)*_Z^2+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}\,x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^3/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**3*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.357 $\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=202

$$\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -((Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b
^2 - 4*a*c])) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*
Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.362691, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1247, 699, 1130, 208}

$$\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b
^2 - 4*a*c])) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*
Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= e\operatorname{Subst}\left(\int \frac{x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)$$
$$= -\left(\frac{1}{2}\left(-e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{d+ex^2}\right)\right)+\frac{1}{2}\Big(e$$
$$= -\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\tanh^{-1}\Big(}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.329696, size = 179, normalized size = 0.89

$$\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)-\sqrt{e\sqrt{b^2-4ac}-be+2cd}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]]) + Sqrt[2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [C] time = 0.013, size = 177, normalized size = 0.9

$$\frac{e}{4}\sum_{\_R=\operatorname{RootOf}\left(c\_Z^8+(4be-4cd)\_Z^6+\left(16ae^2-8deb+6cd^2\right)\_Z^4+\left(4bd^2e-4cd^3\right)\_Z^2+cd^4\right)}\frac{\_R^6+\_R^4d-\_R^2d^2}{\_R^7c+3\_R^5be-3\_R^5cd+8\_R^3ae^2-4\_R^3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*e*sum((_R^6+_R^4*d-_R^2*d^2-d^3)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a
*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^(1
/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z
^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+dx}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 74.911, size = 2241, normalized size = 11.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(1/2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*
a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 + 2*sqrt(1/2
)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e + (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2
- 4*a*c^3)))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*
c^3)))/(b^2*c - 4*a*c^2)) + ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*
d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2) + 1/4*sqrt(1/2)*sqrt((2*c*d - b*e +
(b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*
e^2*x^2 + 2*b*d*e - 2*a*e^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e
+ (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e + (b
^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) + ((b^2*c
 - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x
^2) - 1/4*sqrt(1/2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2
 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*log((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 + 2*sqr
t(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*c)*e - (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2
*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 -
 4*a*c^3)))/(b^2*c - 4*a*c^2)) - ((b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*
c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/x^2) + 1/4*sqrt(1/2)*sqrt((2*c*d - b
*e - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2))*lo
g((b*e^2*x^2 + 2*b*d*e - 2*a*e^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^2 - 4*a*
c)*e - (b^3*c - 4*a*b*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))*sqrt((2*c*d - b*e
 - (b^2*c - 4*a*c^2)*sqrt(e^2/(b^2*c^2 - 4*a*c^3)))/(b^2*c - 4*a*c^2)) - ((
b^2*c - 4*a*c^2)*e*x^2 + 2*(b^2*c - 4*a*c^2)*d)*sqrt(e^2/(b^2*c^2 - 4*a*c^3
)))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.358 $\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=281

$$\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{\ldots}}{\ldots}$$

```
[Out] -((Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) + (Sqrt[c]*(b*d + Sqrt[b^2
- 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcT
anh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 1.35017, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 897, 1287, 206, 1166, 208}

$$\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) + (Sqrt[c]*(b*d + Sqrt[b^2
- 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcT
anh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{de}{a\left(d-x^2\right)}+\frac{e\left(cd^2-bde+ae^2-cdx^2\right)}{a\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{cd^2-bde+ae^2-cdx^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a} - \frac{d\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex^2}\right)}{a}$$

$$= -\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\left(c\left(bd-\sqrt{b^2-4ac}\,d-2ae\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x\right.}{2a\sqrt{b^2-4ac}}$$

$$= -\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\sqrt{c}\left(bd\right.}{}$$

Mathematica [A] time = 0.800931, size = 241, normalized size = 0.86

$$\frac{\frac{\sqrt{2}\left(\left(\sqrt{b^2-4ac}+b\right)\sqrt{e\sqrt{b^2-4ac}-be+2cd}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)+\left(\sqrt{b^2-4ac}-b\right)\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)\right)}{\sqrt{c}\sqrt{b^2-4ac}} - 4\sqrt{d}\tanh^{-1}\left(\right.}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-4*Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + (Sqrt[2]*((b + Sqrt[b^2 - 4*
a*c])*Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]] + (-b + Sqrt[b^2 - 4*
a*c])*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt
[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]]))/(Sqrt[c]*Sqrt[b^2 -
 4*a*c]))/(4*a)
```

Maple [C] time = 0.023, size = 294, normalized size = 1.1

$$\frac{1}{2a}\sqrt{ex^2+d}+\frac{x}{2a}\sqrt{e}-\frac{d}{2a}\left(\sqrt{ex^2+d}-\sqrt{e}x\right)^{-1}-\frac{1}{4a}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+(4be-4cd)\_Z^6+\left(16ae^2-8deb+6cd^2\right)\_Z^4+\left(4bd^2e-4cd^3\right)\_Z^2+cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/a*(e*x^2+d)^(1/2)+1/2/a*e^(1/2)*x-1/2/a*d/((e*x^2+d)^(1/2)-e^(1/2)*x)-1
/4/a*sum((_R^6*c*d+(-4*a*e^2+4*b*d*e-3*c*d^2)*_R^4+d*(4*a*e^2-4*b*d*e+3*c*d
^2)*_R^2-c*d^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_
R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_R),_R=RootOf(c
*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3
)*_Z^2+c*d^4))-1/a*d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}}{\left(cx^4+bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.359 $\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=382

$$-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-b\left(d\sqrt{b^2-4ac}+ae\right)-a\left(2cd-e\sqrt{b^2-\right.\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-}}$$

```
[Out] -Sqrt[d + e*x^2]/(2*a*x^2) + (e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a*Sqrt
[d]) + ((b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^2*Sqrt[d]) - (Sqrt
[c]*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqr
t[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt
[c]*(b^2*d - b*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(2*c*d - Sqrt[b^2 - 4*a*c]*e
))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e])
```

Rubi [A] time = 4.1338, antiderivative size = 370, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1251, 897, 1287, 199, 206, 1166, 208}

$$-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(2*a*x^2) + (e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a*Sqrt
[d]) + ((b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^2*Sqrt[d]) - (Sqrt
[c]*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqr
t[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt
[c]*(b^2*d - 2*a*c*d - a*b*e - Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqr
t[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
```

```
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{de^2}{a\left(d-x^2\right)^2}-\frac{e(-bd+ae)}{a^2\left(d-x^2\right)}+\frac{e\left(-b\left(cd^2-bde+ae^2\right)+c(bd-ae)x^2\right)}{a^2\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{-b\left(cd^2-bde+ae^2\right)+c(bd-ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a^2} + \frac{(de)\operatorname{Subst}\left(\int \frac{1}{\left(d-x^2\right)^2}\,dx, x, \sqrt{d+ex^2}\right)}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{2ax^2} + \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}} + \frac{e\operatorname{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex^2}\right)}{2a} - \frac{\left(c\left(b^2d-2acd\right.\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{2ax^2} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a\sqrt{d}} + \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{\sqrt{c}\left(b^2d-2acd-abe+\sqrt{b^2-}\right.}{\sqrt{2}a^2\sqrt{b^2-4a}}$$

Mathematica [A] time = 1.40653, size = 349, normalized size = 0.91

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(\frac{\left(-bd\sqrt{b^2-4ac}+ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}}+\frac{(2bd-ae)\log(\cdots}{\cdots}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(((-(b^2*d) + 2*a*c*d - b*Sq
rt[b^2 - 4*a*c]*d + a*b*e + a*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]
*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e] - ((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d +
a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]))/Sqrt[b^2 - 4*a*c] + ((-2*b*d + a*e)*Log[x])/Sqrt[d] + ((2*b*d - a*e)
*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/Sqrt[d])/(2*a^2)
```

Maple [C] time = 0.028, size = 401, normalized size = 1.1

$$-\frac{bx}{2a^2}\sqrt{e}-\frac{b}{2a^2}\sqrt{ex^2+d}+\frac{bd}{2a^2}\left(\sqrt{ex^2+d}-\sqrt{e}x\right)^{-1}+\frac{1}{4a^2}\sum_{\_R=\operatorname{RootOf}\left(c\_Z^8+(4be-4cd)\_Z^6+\left(16ae^2-8deb+6cd^2\right)\_Z^4+\left(4bd^2e-4cd^3\right)\_Z\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a^2*e^(1/2)*x*b-1/2/a^2*(e*x^2+d)^(1/2)*b+1/2/a^2*b*d/((e*x^2+d)^(1/2)
-e^(1/2)*x)+1/4/a^2*sum((c*(-a*e+b*d)*_R^6+(-4*a*b*e^2-a*c*d*e+4*b^2*d*e-3*
b*c*d^2)*_R^4+d*(4*a*b*e^2+a*c*d*e-4*b^2*d*e+3*b*c*d^2)*_R^2+a*c*d^3*e-c*d^
4*b)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_
R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*
e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^
4))+1/a^2*b*d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)-1/2/a/d/x^2*(e*x^
2+d)^(3/2)-1/2/a*e/d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)+1/2/a*e/d*
(e*x^2+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}}{\left(cx^4 + bx^2 + a\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.360 $\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=552

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2ae\right)+\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] -Sqrt[d + e*x^2]/(4*a*x^4) + (3*e*Sqrt[d + e*x^2])/(8*a*d*x^2) + ((b*d - a*
e)*Sqrt[d + e*x^2])/(2*a^2*d*x^2) - (3*e^2*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]]
)/(8*a*d^(3/2)) - (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a^2*d
^(3/2)) - ((b^2*d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^3*S
qrt[d]) + (Sqrt[c]*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b
^2 - 4*a*c]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]
*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*
(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e
) - a*b*(3*c*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*
x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*Sqrt[b^2 - 4*a
*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 4.24389, antiderivative size = 552, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1251, 897, 1287, 199, 206, 1166, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\left(-abe-acd+b^2d\right)}{a^3\sqrt{d}}+\frac{\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2ae\right)+\right.}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(4*a*x^4) + (3*e*Sqrt[d + e*x^2])/(8*a*d*x^2) + ((b*d - a*
e)*Sqrt[d + e*x^2])/(2*a^2*d*x^2) - (3*e^2*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]]
)/(8*a*d^(3/2)) - (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a^2*d
^(3/2)) - ((b^2*d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^3*S
qrt[d]) + (Sqrt[c]*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b
^2 - 4*a*c]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]
*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*
(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e
) - a*b*(3*c*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*
x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*Sqrt[b^2 - 4*a
*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
```

```
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{de^3}{a\left(d-x^2\right)^3}+\frac{e^2(-bd+ae)}{a^2\left(d-x^2\right)^2}+\frac{e\left(-b^2d+acd+abe\right)}{a^3\left(d-x^2\right)}+\frac{e\left(\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\right)x^2\right)}{a^3\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right) dx\right.}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(b^2-ac\right)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a^3} - \frac{\left(de^2\right)\text{Subst}\left(\int \frac{1}{\left(d-x^2\right)^3}\,dx, x,\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{\left(b^2d-acd-abe\right)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^3\sqrt{d}}-\frac{\left(3e^2\right)\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{3e\sqrt{d+ex^2}}{8adx^2}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2d^{3/2}}-\frac{\left(b^2d-acd\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{3e\sqrt{d+ex^2}}{8adx^2}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8ad^{3/2}}-\frac{e(bd-ae)\tanh^{-1}}{2a^2d^{3/2}}$$

Mathematica [A] time = 2.04494, size = 466, normalized size = 0.84

$$\frac{\frac{\log\left(\sqrt{d}\sqrt{d+ex^2}+d\right)\left(4abde+a\left(ae^2+8cd^2\right)-8b^2d^2\right)}{d^{3/2}}-\frac{\log(x)\left(4abde+a\left(ae^2+8cd^2\right)-8b^2d^2\right)}{d^{3/2}}-4\sqrt{2}\sqrt{c}\left(\frac{\left(b^2\left(ae-d\sqrt{b^2-4ac}\right)+ab\left(e\sqrt{b^2-4ac}+3cd\right)+ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3(-\right.}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}\right.}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((a*Sqrt[d + e*x^2]*(4*b*d*x^2 - a*(2*d + e*x^2)))/(d*x^4) - (4*Sqrt[2]*Sqr
t[c]*(((-(b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-(Sqrt[b^2 - 4*
a*c]*d) + a*e) + a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e] + ((b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*
c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTa
nh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/Sqrt[b^2 - 4*a*c] - ((-8*b^2*
d^2 + 4*a*b*d*e + a*(8*c*d^2 + a*e^2))*Log[x])/d^(3/2) + ((-8*b^2*d^2 + 4*a
*b*d*e + a*(8*c*d^2 + a*e^2))*Log[d + Sqrt[d]*Sqrt[d + e*x^2]])/d^(3/2))/(8
*a^3)
```

Maple [C] time = 0.033, size = 655, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4/a/d/x^4*(e*x^2+d)^(3/2)+1/8/a*e/d^2/x^2*(e*x^2+d)^(3/2)+1/8/a*e^2/d^(3
/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)-1/8/a*e^2/d^2*(e*x^2+d)^(1/2)-1/2
/a^2*e^(1/2)*x*c-1/2/a^2*(e*x^2+d)^(1/2)*c+1/2/a^3*e^(1/2)*x*b^2+1/2/a^3*(e
*x^2+d)^(1/2)*b^2+1/2/a^2*d/((e*x^2+d)^(1/2)-e^(1/2)*x)*c-1/2/a^3*d/((e*x^2
+d)^(1/2)-e^(1/2)*x)*b^2+1/4/a^3*sum((c*(a*b*e+a*c*d-b^2*d)*_R^6+(-4*a^2*c*
e^2+4*a*b^2*e^2+5*a*b*c*d*e-3*a*c^2*d^2-4*b^3*d*e+3*b^2*c*d^2)*_R^4+d*(4*a^
2*c*e^2-4*a*b^2*e^2-5*a*b*c*d*e+3*a*c^2*d^2+4*b^3*d*e-3*b^2*c*d^2)*_R^2-a*b
*c*d^3*e-a*c^2*d^4+b^2*c*d^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*
_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_
R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b
*d^2*e-4*c*d^3)*_Z^2+c*d^4))+1/a^2*d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2
))/x)*c-1/a^3*d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)*b^2+1/2/a^2*b/d
/x^2*(e*x^2+d)^(3/2)+1/2/a^2*b*e/d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))
/x)-1/2/a^2*b*e/d*(e*x^2+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}}{(cx^4 + bx^2 + a)x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^5), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.361 $\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=390

$$-\frac{\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\cdots}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{\cdots}\right.}}$$

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) - ((b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d
- b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b*c*d -
b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4
*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e]) + ((c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(2*c^2*Sqrt[e])
```

Rubi [A] time = 2.91949, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1291, 388, 217, 206, 1692, 377, 205}

$$-\frac{\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\cdots}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{\cdots}\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) - ((b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d
- b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b*c*d -
b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4
*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e]) + ((c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(2*c^2*Sqrt[e])
```

Rule 1291

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[((f*x)^(m - 4)*(d +
 e*x^2)^(q - 1)*Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x])/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\int \frac{cd-be+cex^2}{\sqrt{d+ex^2}}\,dx}{c^2} - \frac{\int \frac{a(cd-be)+\left(bcd-b^2e+ace\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} - \frac{\int \left(\frac{bcd-b^2e+ace+\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{bcd-b^2e+ace-\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c^2} + \frac{(cd-2be)\int}{2c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(cd-2be)\,\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)}{c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int\right.}{c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(bcd-b^2e+\right.}{c^2}$$

Mathematica [B] time = 6.41774, size = 10915, normalized size = 27.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.033, size = 290, normalized size = 0.7

$$\frac{x}{2c}\sqrt{ex^2+d}+\frac{d}{2c}\ln\left(\sqrt{e}x+\sqrt{ex^2+d}\right)\frac{1}{\sqrt{e}}+\frac{b}{c^2}\sqrt{e}\ln\left(\sqrt{ex^2+d}-\sqrt{e}x\right)+\frac{1}{2c^2}\sqrt{e}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*x*(e*x^2+d)^(1/2)/c+1/2/c*d/e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))+1/c^2
*e^(1/2)*b*ln((e*x^2+d)^(1/2)-e^(1/2)*x)+1/2/c^2*e^(1/2)*sum(((a*c*e-b^2*e+
b*c*d)*_R^2+2*(-2*a*b*e^2+a*c*d*e+b^2*d*e-b*c*d^2)*_R+e*d^2*c*a-b^2*d^2*e+b
*c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^
2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c
*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}\,x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^4/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.28065, size = 72, normalized size = 0.18

$$-\frac{(cd-2\,be)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{4\,c^2}+\frac{\sqrt{x^2e+d}x}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(c*d - 2*b*e)*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^2 + 1/2*
sqrt(x^2*e + d)*x/c
```

3.362 $\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=324

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{e}\tanh}{}$$

```
[Out] ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*
x^2])]])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]) + ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt
[d + e*x^2])]])/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]) + (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c
```

Rubi [A] time = 1.51684, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1293, 217, 206, 1692, 377, 205}

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{e}\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*
x^2])]])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]) + ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt
[d + e*x^2])]])/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]) + (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = -\frac{\int \frac{ae-(cd-be)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c}+\frac{e\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c}$$

$$= -\frac{\int \left(\frac{-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{-cd+be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c}+\frac{e\,\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{c}$$

$$= \frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}+\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c}-\frac{\left(-cd+be+\frac{bcd-b^2e}{\sqrt{b^2-}}\cdots\right.}{\cdots}$$

$$= \frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}+\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\cdots\right.}{c}$$

$$= \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2}\cdots\right.}}{\sqrt{b+\sqrt{b^2-4ac}}\cdots}\right.}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4}\cdots\right.}}$$

Mathematica [B] time = 6.17332, size = 7768, normalized size = 23.98

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.024, size = 224, normalized size = 0.7

$$-\frac{1}{c}\sqrt{e}\ln\left(\sqrt{ex^2+d}-\sqrt{e}x\right)+\frac{1}{2c}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z+cd^4\right)}\frac{(be-cd)}{\_R^3c+3\_R^2be-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -e^(1/2)/c*ln((e*x^2+d)^(1/2)-e^(1/2)*x)+1/2*e^(1/2)/c*sum(((b*e-c*d)*_R^2+
2*(2*a*e^2-b*d*e+c*d^2)*_R+b*d^2*e-c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_
R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)
^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+
(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}x^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)*x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 29.9386, size = 6734, normalized size = 20.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqr
t((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)
)*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2
*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*
a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((
c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - ((b^2*c - 4*a*c^2)*d
- (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3
)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a
*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*
c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 -
4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e
^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2 + 4*a*b*e^2 - (
b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*
x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - ((b^2*c - 4*a
*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 -
 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c
^2 - 4*a*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^
2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^
2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e
```

```
+ b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*d*e + (b*c*d^2 + 4*a*b*
e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2 - 4*a*
b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + ((b^2*
c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^
2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))
/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e -
(b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b
*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*d*e + (b*c*d^2 +
 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^2
 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) +
 ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*
e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a
*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - 2*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2
+ d)*sqrt(e)*x - d))/c, -1/4*(sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e +
(b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*
b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (b*c*d^2
+ 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((b^3*c^
2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5))
- ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)
*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*
a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a
*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 -
 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-((b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*
d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) + 2*a*c*d^2 - 2*a*b*d*e - (
b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*
((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*
a*c^5)) - ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d - (b^2
- 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*
c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) + sqrt(1/2)*c*sqrt(-(b*c*d - (b
^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b
^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*x^2*sqr
t((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*a*b*
d*e + (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^
2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c
^4 - 4*a*c^5)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-(b*c*d
- (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2
)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) - sqrt(1/2)*c*sqrt(-(b*c
*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*
e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(((b^2*c^2 - 4*a*c^3)*d*
x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 +
 2*a*b*d*e + (b*c*d^2 + 4*a*b*e^2 - (b^2 + 4*a*c)*d*e)*x^2 - 2*sqrt(1/2)*sq
rt(e*x^2 + d)*((b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)
/(b^2*c^4 - 4*a*c^5)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*x)*sqrt(-
(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e +
b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)))/x^2) + 4*sqrt(-e)*arct
an(sqrt(-e)*x/sqrt(e*x^2 + d)))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{d + ex^2}}{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.22276, size = 36, normalized size = 0.11

$$-\frac{e^{\frac{1}{2}}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*e^(1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

3.363 $\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4} dx$

Optimal. Leaf size=240

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2
- 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rubi [A] time = 0.318462, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1174, 402, 217, 206, 377, 205}

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2
- 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{\sqrt{b^2-4ac}} + \frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2\cdots\right.}\cdots}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\operatorname{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}} + \frac{\left(-2cd+\left(b+\sqrt{\cdots}\right.\right.}{\cdots}$$

$$= \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\cdots\right.}}{\sqrt{b+\sqrt{b^2-\cdots}}}\cdots\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [B] time = 5.59739, size = 2585, normalized size = 10.77

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Sq
rt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[-(Sqrt[(-b + Sqrt[b^2 - 4*a*c])
/c]/Sqrt[2]) + x] - Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*(2*c*d + (-b + Sqrt[
b^2 - 4*a*c])*e)*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[Sqrt[(-b + S
qrt[b^2 - 4*a*c])/c]/Sqrt[2] + x] - 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*
Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*
a*c])/c)]/Sqrt[2]) + x] + b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d
- b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sq
rt[2]) + x] + Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*
c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)
```

```
]/Sqrt[2]) + x] + 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e
+ Sqrt[b^2 - 4*a*c]*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] +
x] - b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x] - Sqrt[b^2 -
 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4
*a*c]*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x] + 2*c*Sqrt[
-((b + Sqrt[b^2 - 4*a*c])/c)]*d*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*L
og[2*d - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e
 + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] - b*Sqrt[-((b + Sqrt[b^2 - 4*
a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqr
t[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*
c]*e)/c]*Sqrt[d + e*x^2]] + Sqrt[b^2 - 4*a*c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c]
)/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqrt[(-
b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e
)/c]*Sqrt[d + e*x^2]] - 2*c*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*d*Sqrt[2*d -
 ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a
*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^
2]] + b*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*
a*c])*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(
4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] - Sqrt[b^2 - 4*a
*c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c]
)*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*
d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] - 2*c*Sqrt[(-b + Sqr
t[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d -
Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^
2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] + b*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*S
qrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d - Sqrt[2]*Sqrt[-((b + Sq
rt[b^2 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt
[d + e*x^2]] + Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2
*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d - Sqrt[2]*Sqrt[-((b + Sqrt[b^2
 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e
*x^2]] + 2*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^
2 - 4*a*c]*e)/c]*Log[2*d + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x +
 Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] - b*Sqrt[(-b
+ Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2
*d + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sq
rt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] - Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt
[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d + S
qrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2
 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]])/(2*c*Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b
^2 - 4*a*c])/c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*Sqrt[(2*c*d - b*e + Sqrt
[b^2 - 4*a*c]*e)/c]*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c])
```

Maple [C] time = 0.015, size = 161, normalized size = 0.7

$$-\frac{1}{2}e^{\frac{3}{2}}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z+cd^4\right)}\frac{\_R^2+2\_Rd+d^2}{\_R^3c+3\_R^2be-3\_R^2cd+8\_Rae^2-4\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*e^(3/2)*sum((_R^2+2*_R*d+d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2
-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),
_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^
2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 8.33447, size = 2026, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*
a^3*c)))/(a*b^2 - 4*a^2*c))*log(-((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 - 4
*a^3*c))*x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a^
2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^
2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2)/x^2) -
 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4
*a^3*c)))/(a*b^2 - 4*a^2*c))*log(-((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 -
4*a^3*c))*x^2 - 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a
^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b
^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2)/x^2)
+ 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 -
4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 -
4*a^3*c))*x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a
^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b
^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) + 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2)/x^2)
- 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 -
4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(((a*b^2 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 -
4*a^3*c))*x^2 - 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sqrt(e*x^2 + d)*sqrt(d^2/(a
^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b
^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) + 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.364 $\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=291

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{ax}$$

```
[Out] -(Sqrt[d + e*x^2]/(a*x)) - (c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - (c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^
2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
])
```

Rubi [A] time = 0.670755, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1295, 264, 1692, 377, 205}

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*x)) - (c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - (c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^
2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
])
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{bd-ae+cdx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a}+\frac{d\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{ax}-\frac{\int \left(\frac{cd+\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{cd-\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{ax}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a}-\frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}\right.}}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{ax}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,x,\frac{x}{\sqrt{d+ex^2}}\right)}{a}-\frac{\left(c\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{ax}-\frac{c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(\right.}}{\sqrt{b+\sqrt{}}}\right.}{a\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\right.}}$$

Mathematica [B] time = 6.33564, size = 4644, normalized size = 15.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*x)) - (-(b*d*(Log[Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/S
qrt[2] + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d - Sqrt
[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt[b^2
 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c
)*e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) -
Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])
*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2 -
4*a*c]/c]/Sqrt[2])) + (Sqrt[b^2 - 4*a*c]*d*(Log[Sqrt[-(b/c) - Sqrt[b^2 - 4*
```

```
a*c]/c]/Sqrt[2] + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2
*d - Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) -
 Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 -
 4*a*c]/c)*e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[
-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]
/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sq
rt[b^2 - 4*a*c]/c]/Sqrt[2])) + (a*e*(Log[Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]
/Sqrt[2] + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d - Sq
rt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt[b
^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]
/c)*e)/2]))/(Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) -
Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])
*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2 -
4*a*c]/c]/Sqrt[2])) + (b*d*(Log[-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
]) + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*
Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4
*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)
/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Sqrt[-(b/c) - Sqrt[b
^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[
-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/
Sqrt[2])) - (Sqrt[b^2 - 4*a*c]*d*(Log[-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/
Sqrt[2]) + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sq
rt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt[b
^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]
/c)*e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Sqrt[-(b/c) -
Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*
(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*
c]/c]/Sqrt[2])) - (a*e*(Log[-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) +
 x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*Sqrt
[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c
]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2])
)/(Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Sqrt[-(b/c) - Sqrt[b^2 - 4
*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c)
 - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
])) - (b*d*(Log[Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x]/Sqrt[d + ((
-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d - Sqrt[2]*Sqrt[-(b/c) + Sqrt[
b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt
[d + e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(2*Sqrt[2]*c*
Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sq
rt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^
2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) - (Sqr
t[b^2 - 4*a*c]*d*(Log[Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x]/Sqrt[
d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d - Sqrt[2]*Sqrt[-(b/c) +
 Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2
]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(2*Sqrt
[2]*c*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]
/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - S
qrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]))
+ (a*e*(Log[Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x]/Sqrt[d + ((-(b/
c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d - Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^2
- 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d +
 e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(Sqrt[2]*c*Sqrt[-
(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])
 - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2 - 4*
a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) + (b*d*(Log[
-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x]/Sqrt[d + ((-(b/c) + Sqrt
[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/
c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/
Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c) +
```

```
 Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt
[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]
/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) + (Sqrt[b^2 - 4*a*c
]*d*(Log[-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x]/Sqrt[d + ((-(b/
c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^2
- 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d +
 e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(2*Sqrt[2]*c*Sqrt
[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
]) + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2 -
4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) - (a*e*(Lo
g[-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x]/Sqrt[d + ((-(b/c) + Sq
rt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c
]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]
]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(Sqrt[2]*c*Sqrt[-(b/c) +
 Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt
[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]
/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])))/a
```

Maple [C] time = 0.026, size = 272, normalized size = 0.9

$$\frac{1}{a}\sqrt{e}\ln\left(\sqrt{ex^2+d}-\sqrt{e}x\right)+\frac{1}{2a}\sqrt{e}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z+cd^4\right)}\frac{\_}{\_R^3c+3\_R^2be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/a*e^(1/2)*ln((e*x^2+d)^(1/2)-e^(1/2)*x)+1/2/a*e^(1/2)*sum((_R^2*c*d+2*(-2
*a*e^2+2*b*d*e-c*d^2)*_R+c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*
_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=
RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e
-4*c*d^3)*_Z+c*d^4))-1/a/d/x*(e*x^2+d)^(3/2)+1/a*e/d*x*(e*x^2+d)^(1/2)+1/a*
e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{ex^2+d}}{\left(cx^4+bx^2+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [B] time = 19.6206, size = 4811, normalized size = 16.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b
^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^
3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*b*c
*d*e + (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c +
 a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c
- a^2*c^2)*d^2 + (4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^
2*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt
((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/
(a^6*b^2 - 4*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*
a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 -
4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a
^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) - sqrt(1/2)*a
*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 - 4*a^4*c)*sqr
t((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)
/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*b*c*d*e + (a^3*b^2*c
 - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2
*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 +
(4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 -
 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (
b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*
c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sq
rt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 - 4*a^4*c)*sqrt((a^
2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6
*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) - sqrt(1/2)*a*x*sqrt(-((b^3 -
3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 +
(b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7
*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a^2*b*c*d*e - (a^3*b^2*c - 4*a^4*c^2)*d*x
^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c
)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 + (4*a^2*b*c*e^2 +
(b^3*c - a*b*c^2)*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt
(e*x^2 + d)*((a^4*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c +
 a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5
*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b
*c)*d - (a*b^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4
- 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))
/(a^3*b^2 - 4*a^4*c)))/x^2) + sqrt(1/2)*a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b
^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c
+ a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
4*a^4*c))*log((2*a^2*b*c*d*e - (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*
e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 -
 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 + (4*a^2*b*c*e^2 + (b^3*c - a*b*c^2)
*d^2 - (5*a*b^2*c - 4*a^2*c^2)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^4
*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2
*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*b^2*c + 4*a^
3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 -
2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2
*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4
*c)))/x^2) + 4*sqrt(e*x^2 + d))/(a*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x**2*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.365 $\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=373

$$\frac{c\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+e\cdots}}{a\cdots}$$

```
[Out] -Sqrt[d + e*x^2]/(3*a*x^3) + (2*e*Sqrt[d + e*x^2])/(3*a*d*x) + ((b*d - a*e)
*Sqrt[d + e*x^2])/(a^2*d*x) + (c*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (c*(b*d - a*e - (b^2*d - 2*a*c*d -
 a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*
x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 2.52711, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1295, 271, 264, 6728, 1692, 377, 205}

$$\frac{c\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+e\cdots}}{a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(3*a*x^3) + (2*e*Sqrt[d + e*x^2])/(3*a*d*x) + ((b*d - a*e)
*Sqrt[d + e*x^2])/(a^2*d*x) + (c*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (c*(b*d - a*e - (b^2*d - 2*a*c*d -
 a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*
x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
```

```
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{bd-ae+cdx^2}{x^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a} + \frac{d\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} - \frac{\int \left(\frac{bd-ae}{ax^2\sqrt{d+ex^2}} + \frac{-b^2d+acd+abe-c(bd-ae)x^2}{a\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx}{a} - \frac{(2e)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{3a}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} - \frac{\int \frac{-b^2d+acd+abe-c(bd-ae)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a^2} - \frac{(bd-ae)\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} - \frac{\int \left(\frac{-c(bd-ae)-\frac{c\left(b^2d-2acd-abe\right)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{-c(bd-ae)+\frac{c\left(b^2d-2ac\right.}{\sqrt{b^2-}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right.}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} + \frac{\left(c\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right.}}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} + \frac{\left(c\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{}{b+\sqrt{b^2-}}\right.}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} + \frac{c\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b\right.}}}{\sqrt{b-\sqrt{b^2-4ac}}}\right.}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4\right.}}}$$

Mathematica [B] time = 6.40871, size = 7777, normalized size = 20.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.03, size = 322, normalized size = 0.9

$$-\frac{b}{a^2}\sqrt{e}\ln\left(\sqrt{ex^2+d}-\sqrt{e}x\right)+\frac{1}{2\,a^2}\sqrt{e}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+\left(16\,ae^2-8\,deb+6\,cd^2\right)\_Z^2+\left(4\,bd^2e-4\,cd^3\right)\_Z+cd^4\right)}\frac{c\,(ae-bd)\_R^2}{\_R^3c+3\_R^2be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a^2*e^(1/2)*b*ln((e*x^2+d)^(1/2)-e^(1/2)*x)+1/2/a^2*e^(1/2)*sum((c*(a*e-
b*d)*_R^2+2*(2*a*b*e^2+a*c*d*e-2*b^2*d*e+b*c*d^2)*_R+e*d^2*c*a-b*c*d^3)/(_R
^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*
ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(1
6*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))+1/a^2*b/d/x*(e
*x^2+d)^(3/2)-1/a^2*b*e/d*x*(e*x^2+d)^(1/2)-1/a^2*b*e^(1/2)*ln(e^(1/2)*x+(e
```

```
*x^2+d)^(1/2))-1/3/a/d/x^3*(e*x^2+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [B] time = 75.0038, size = 8227, normalized size = 22.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b
^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c
+ 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2
)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(((a^5*b^2*c^2 - 4*a
^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^
2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + 2*(a*b^4
*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 - 2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*d*e - (
(b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*a^2*b^2*c^3 + 4
*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e
*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a
^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2
)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b
*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt(-((b^5 - 5*a*b
^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*
a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d
^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 -
 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6
*c)))/x^2) - 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d
 - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6
*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2
*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*
b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(((a^5*b^2*c^
2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*
e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + 2
*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 - 2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*
d*e - ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*a^2*b^2*
c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 - 2*sqrt(1/2)
*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*
```

```
c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c
^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 -
4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt(-((b^5
- 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b
^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^
2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2
- 4*a^6*c)))/x^2) + 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b
*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + (a^5*b^2 - 4*a^6*c)*sqrt(((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7
- 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c +
 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(-((a^
5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b
*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11
*c)) - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 + 2*(a^2*b^3*c^2 - 2*a^3
*b*c^3)*d*e + ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c^2 - 14*
a^2*b^2*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x^2 + 2*s
qrt(1/2)*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*
a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a
^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^
3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)*x)*sqrt
(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e
+ (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d
*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(
a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + (a^5*b^2 - 4*a^6*c)
*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2
*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3
*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*l
og(-((a^5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 -
 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2
- 4*a^11*c)) - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 + 2*(a^2*b^3*c^2
 - 2*a^3*b*c^3)*d*e + ((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2 - (5*a*b^4*c
^2 - 14*a^2*b^2*c^3 + 4*a^3*c^4)*d*e + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*e^2)*x
^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqr
t(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*
b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4
*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^2*b^5*c + 1
3*a^3*b^3*c^2 - 4*a^4*b*c^3)*d - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*e)
*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3
*c^2)*e + (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*
b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^1
1*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) + 4*((3*b*d - a*e)*x^2 - a*d)*sqrt(e*x^2
+ d))/(a^2*d*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.366 $\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=512

$$-\frac{\sqrt{d+ex^2}\left(-abe-acd+b^2d\right)}{a^3dx}-\frac{c\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{2a^2ce-ab^2e-3abc}{\sqrt{b^2-4ac}}\right.}{a^3\sqrt{}}$$

```
[Out] -Sqrt[d + e*x^2]/(5*a*x^5) + (4*e*Sqrt[d + e*x^2])/(15*a*d*x^3) + ((b*d - a
*e)*Sqrt[d + e*x^2])/(3*a^2*d*x^3) - (8*e^2*Sqrt[d + e*x^2])/(15*a*d^2*x) -
 (2*e*(b*d - a*e)*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - ((b^2*d - a*c*d - a*b*e)
*Sqrt[d + e*x^2])/(a^3*d*x) - (c*(b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*
d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sq
rt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (c*(b^
2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b
^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 4.94253, antiderivative size = 512, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1295, 271, 264, 6728, 1692, 377, 205}

$$-\frac{\sqrt{d+ex^2}\left(-abe-acd+b^2d\right)}{a^3dx}-\frac{c\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{2a^2ce-ab^2e-3abc}{\sqrt{b^2-4ac}}\right.}{a^3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
```


```
[Out] -Sqrt[d + e*x^2]/(5*a*x^5) + (4*e*Sqrt[d + e*x^2])/(15*a*d*x^3) + ((b*d - a
*e)*Sqrt[d + e*x^2])/(3*a^2*d*x^3) - (8*e^2*Sqrt[d + e*x^2])/(15*a*d^2*x) -
 (2*e*(b*d - a*e)*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - ((b^2*d - a*c*d - a*b*e)
*Sqrt[d + e*x^2])/(a^3*d*x) - (c*(b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*
d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[
b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sq
rt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (c*(b^
2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b
^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e])
```


Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}dx = -\frac{\int \frac{bd-ae+cdx^2}{x^4\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx}{a} + \frac{d\int \frac{1}{x^6\sqrt{d+ex^2}}dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} - \frac{\int \left(\frac{bd-ae}{ax^4\sqrt{d+ex^2}} + \frac{-b^2d+acd+abe}{a^2x^2\sqrt{d+ex^2}} + \frac{b^3d-2abcd-ab^2e+a^2ce+c\left(b^2d-acd-abe\right)x^2}{a^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx}{a} - \frac{(4e)\int \frac{}{x^4\sqrt{}}}{5a}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} - \frac{\int \frac{b^3d-2abcd-ab^2e+a^2ce+c\left(b^2d-acd-abe\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx}{a^3} + \frac{\left(8e^2\right)\int \frac{1}{x^2\sqrt{d+ex^2}}dx}{15ad} - \frac{(b}{}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{\left(b^2d-acd-abe\right)\sqrt{d+ex^2}}{a^3dx}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-ae)\sqrt{d+ex^2}}{3a^2d^2x} - \frac{\left(b^2d\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-ae)\sqrt{d+ex^2}}{3a^2d^2x} - \frac{\left(b^2d\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-ae)\sqrt{d+ex^2}}{3a^2d^2x} - \frac{\left(b^2d\right.}{}$$

Mathematica [B] time = 6.597, size = 10933, normalized size = 21.35

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.034, size = 503, normalized size = 1.

$$-\frac{c}{a^2}\sqrt{e}\ln\left(\sqrt{ex^2+d}-\sqrt{e}x\right) + \frac{b^2}{a^3}\sqrt{e}\ln\left(\sqrt{ex^2+d}-\sqrt{e}x\right) - \frac{1}{2a^3}\sqrt{e}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/a^2*e^(1/2)*ln((e*x^2+d)^(1/2)-e^(1/2)*x)*c+1/a^3*e^(1/2)*ln((e*x^2+d)^(
1/2)-e^(1/2)*x)*b^2-1/2/a^3*e^(1/2)*sum((c*(a*b*e+a*c*d-b^2*d)*_R^2+2*(-2*a
^2*c*e^2+2*a*b^2*e^2+3*a*b*c*d*e-a*c^2*d^2-2*b^3*d*e+b^2*c*d^2)*_R+a*b*c*d^
2*e+a*c^2*d^3-b^2*c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*
e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(
c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^
3)*_Z+c*d^4))+1/a^2/d/x*(e*x^2+d)^(3/2)*c-1/a^3/d/x*(e*x^2+d)^(3/2)*b^2-1/a
```

```
^2*e/d*x*(e*x^2+d)^(1/2)*c+1/a^3*e/d*x*(e*x^2+d)^(1/2)*b^2-1/a^2*e^(1/2)*ln
(e^(1/2)*x+(e*x^2+d)^(1/2))*c+1/a^3*e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))*b
^2-1/5/a/d/x^5*(e*x^2+d)^(3/2)+2/15/a*e/d^2/x^3*(e*x^2+d)^(3/2)+1/3/a^2*b/d
/x^3*(e*x^2+d)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2+d}}{(cx^4+bx^2+a)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x^2 + d)/((c*x^4 + b*x^2 + a)*x^6), x)
```

Fricas [B] time = 126.104, size = 11867, normalized size = 23.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/60*(15*sqrt(1/2)*a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 -
7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e - (a^7
*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c
 + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a
^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^
2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(-((a^7*b^2*c^3 - 4*a^8*
c^4)*d*x^2*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46
*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29
*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^
10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a
^14*b^2 - 4*a^15*c)) + 2*(a*b^6*c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c
^6)*d^2 - 2*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e - ((b^7*c^3 - 5
*a*b^5*c^4 + 6*a^2*b^3*c^5 - a^3*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4
 + 27*a^3*b^2*c^5 - 4*a^4*c^6)*d*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4
*b*c^5)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12
*a^10*b*c^2)*x*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c
+ 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^
2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2
)/(a^14*b^2 - 4*a^15*c)) + ((a*b^10 - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^
4*b^4*c^3 + 29*a^5*b^2*c^4 - 4*a^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4
*b^5*c^2 - 31*a^5*b^3*c^3 + 12*a^6*b*c^4)*e)*x)*sqrt(-((b^7 - 7*a*b^5*c + 1
4*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a
^4*c^3)*e - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2
- 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^
11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6
*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9
*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c)))/x^2) - 15*
sqrt(1/2)*a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^
```

3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e - (a^7*b^2 - 4*a
^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*
b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*
b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 -
8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b
^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(-((a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2
*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c
^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c
^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3
*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 -
4*a^15*c)) + 2*(a*b^6*c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 -
2*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e - ((b^7*c^3 - 5*a*b^5*c^4
 + 6*a^2*b^3*c^5 - a^3*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*
b^2*c^5 - 4*a^4*c^6)*d*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*e^
2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^
2)*x*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b
^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b
^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8
*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^
2 - 4*a^15*c)) + ((a*b^10 - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3
+ 29*a^5*b^2*c^4 - 4*a^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 -
 31*a^5*b^3*c^3 + 12*a^6*b*c^4)*e)*x)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*
c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e
- (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b
^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2
*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*
e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c
^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c)))/x^2) + 15*sqrt(1/2)*
a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*
b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 - 4*a^8*c)*sqrt
(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 -
40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*
c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^1
5*c)))/(a^7*b^2 - 4*a^8*c))*log(((a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2*sqrt(((b^1
2 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5
*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4
*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22
*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c))
- 2*(a*b^6*c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 + 2*(a^2*b^5*
c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + ((b^7*c^3 - 5*a*b^5*c^4 + 6*a^2*b^
3*c^5 - a^3*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*b^2*c^5 - 4
*a^4*c^6)*d*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*e^2)*x^2 + 2*
sqrt(1/2)*sqrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*x*sqrt((
(b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12
*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40
*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c
+ 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*
c)) - ((a*b^10 - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3 + 29*a^5*b^
2*c^4 - 4*a^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 - 31*a^5*b^3
*c^3 + 12*a^6*b*c^4)*e)*x)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3
*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2
- 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46
*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29
*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^
10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a
^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c)))/x^2) - 15*sqrt(1/2)*a^3*d^2*x^5
*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2
*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 1

```
0*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*
c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*
c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*
b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7
*b^2 - 4*a^8*c))*log(((a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2*sqrt(((b^12 - 10*a*b^
10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 +
22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^
2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)) - 2*(a*b^6*
c^3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 + 2*(a^2*b^5*c^3 - 4*a^3
*b^3*c^4 + 3*a^4*b*c^5)*d*e + ((b^7*c^3 - 5*a*b^5*c^4 + 6*a^2*b^3*c^5 - a^3
*b*c^6)*d^2 - (5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*b^2*c^5 - 4*a^4*c^6)*d
*e + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*e^2)*x^2 - 2*sqrt(1/2)*s
qrt(e*x^2 + d)*((a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*x*sqrt(((b^12 - 10*
a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^
5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^
3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^
6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)) - ((a*b
^10 - 10*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3 + 29*a^5*b^2*c^4 - 4*a
^6*c^5)*d - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 - 31*a^5*b^3*c^3 + 12*a
^6*b*c^4)*e)*x)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d -
 (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 - 4*a^8*c)*
sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^
4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^
2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*
b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4
*a^15*c)))/(a^7*b^2 - 4*a^8*c)))/x^2) - 4*((5*a*b*d*e + 2*a^2*e^2 - 15*(b^2
 - a*c)*d^2)*x^4 - 3*a^2*d^2 + (5*a*b*d^2 - a^2*d*e)*x^2)*sqrt(e*x^2 + d))/
(a^3*d^2*x^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.367 $\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=460

$$\frac{\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e^2\right)\tanh^{-1}}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] ((c*d - b*e)*Sqrt[d + e*x^2])/c^2 + (d + e*x^2)^(3/2)/(3*c) + ((b^3*e^2 - b
^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt
[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e))
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*
a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) - ((b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c*(c*d^2
- e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sq
rt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 5.08444, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 824, 826, 1166, 208}

$$\frac{\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e^2\right)\tanh^{-1}}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```


```
[Out] ((c*d - b*e)*Sqrt[d + e*x^2])/c^2 + (d + e*x^2)^(3/2)/(3*c) + ((b^3*e^2 - b
^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt
[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e))
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*
a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) - ((b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c*(c*d^2
- e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sq
rt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3 \left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{\left(d+ex^2\right)^{3/2}}{3c} + \frac{\operatorname{Subst}\left(\int \frac{\sqrt{d+ex}(-ae+(cd-be)x)}{a+bx+cx^2}\,dx,x,x^2\right)}{2c}\\
&= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{\left(d+ex^2\right)^{3/2}}{3c} + \frac{\operatorname{Subst}\left(\int \frac{-ae(2cd-be)+\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx,x,x^2\right)}{2c^2}\\
&= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{\left(d+ex^2\right)^{3/2}}{3c} + \frac{\operatorname{Subst}\left(\int \frac{-ae^2(2cd-be)-d\left(c^2d^2+b^2e^2-ce(2bd+ae)\right)+\left(c^2d^2+b^2e^2-ce(2bd+\right.}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\right.}{c^2}\\
&= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{\left(d+ex^2\right)^{3/2}}{3c} - \frac{\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}e\right)+c\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{\right.\right.\right.}{}\\
&= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{\left(d+ex^2\right)^{3/2}}{3c} + \frac{\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}e\right)+c\left(a\sqrt{b^2-4ac}e^2-cd\left(\sqrt{\right.\right.\right.}{\sqrt{2}c^{5/2}\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.988296, size = 457, normalized size = 0.99

$$
-\frac{\left(-bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(cd\left(d\sqrt{b^2-4ac}-4ae\right)-ae^2\sqrt{b^2-4ac}\right)+b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3\left(-e^2\right.\right.}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d - b*e)*Sqrt[d + e*x^2])/c^2 + (d + e*x^2)^(3/2)/(3*c) - ((-(b^3*e^2)
+ b^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d
*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 3
*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b
^2 - 4*a*c]*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt
[b^2 - 4*a*c])*e]) - ((b^3*e^2 + b^2*e*(-2*c*d + Sqrt[b^2 - 4*a*c]*e) + b*c
*(c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) + c*(-(a*Sqrt[b^2 - 4*a*c]*e^2
) + c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e
*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2
- 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Maple [C] time = 0.03, size = 490, normalized size = 1.1

$$-\frac{x^3}{6c}e^{\frac{3}{2}} + \frac{ex^2}{8c}\sqrt{ex^2+d} - \frac{3\,dx}{4c}\sqrt{e} + \frac{1}{24c}\left(ex^2+d\right)^{\frac{3}{2}} + \frac{bx}{2c^2}e^{\frac{3}{2}} - \frac{be}{2c^2}\sqrt{ex^2+d} + \frac{5\,d}{8c}\sqrt{ex^2+d} - \frac{deb}{2c^2}\left(\sqrt{ex^2+d}-\sqrt{e}x\right)^{-1} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/6/c*e^(3/2)*x^3+1/8/c*e*(e*x^2+d)^(1/2)*x^2-3/4/c*e^(1/2)*x*d+1/24*(e*x^
2+d)^(3/2)/c+1/2/c^2*e^(3/2)*x*b-1/2/c^2*(e*x^2+d)^(1/2)*b*e+5/8/c*(e*x^2+d
)^(1/2)*d-1/2/c^2*d/((e*x^2+d)^(1/2)-e^(1/2)*x)*b*e+5/8/c*d^2/((e*x^2+d)^(1
/2)-e^(1/2)*x)+1/24/c*d^3/((e*x^2+d)^(1/2)-e^(1/2)*x)^3+1/4/c^2*sum(((-a*c*
e^2+b^2*e^2-2*b*c*d*e+c^2*d^2)*_R^6+(4*a*b*e^3-5*a*c*d*e^2-3*b^2*d*e^2+6*b*
c*d^2*e-3*c^2*d^3)*_R^4+d*(-4*a*b*e^3+5*a*c*d*e^2+3*b^2*d*e^2-6*b*c*d^2*e+3
*c^2*d^3)*_R^2+a*c*d^3*e^2-b^2*d^3*e^2+2*b*c*d^4*e-c^2*d^5)/(_R^7*c+3*_R^5*
b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*
ln((e*x^2+d)^(1/2)-e^(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*
e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*x^3/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.368 $\int \frac{x\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=327

$$-\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+\right.\right.}{\sqrt{2}}$$

```
[Out] (e*Sqrt[d + e*x^2])/c - ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e
*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2
])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*
a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((2*c^2*d^2 + b*(b + Sqrt[b
^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[
2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 1.45632, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1247, 703, 826, 1166, 208}

$$-\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+\right.\right.}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*Sqrt[d + e*x^2])/c - ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e
*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2
])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*
a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((2*c^2*d^2 + b*(b + Sqrt[b
^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[
2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{e\sqrt{d+ex^2}}{c} + \frac{\mathrm{Subst}\left(\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}\left(a+bx+cx^2\right)}\,dx, x, x^2\right)}{2c}$$

$$= \frac{e\sqrt{d+ex^2}}{c} + \frac{\mathrm{Subst}\left(\int \frac{-de(2cd-be)+e\left(cd^2-ae^2\right)+e(2cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{c}$$

$$= \frac{e\sqrt{d+ex^2}}{c} + \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\mathrm{Subst}\left(\int \frac{}{-\frac{1}{2}\sqrt{b^2-4ac}e\cdots}\right.}{2c\sqrt{b^2-4ac}}$$

$$= \frac{e\sqrt{d+ex^2}}{c} - \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+e\cdots}}{\sqrt{2cd-\left(b-\sqrt{b^2\cdots}\right.}}\right.}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 0.581587, size = 324, normalized size = 0.99

$$\frac{\left(2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}-b\right)-2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+\cdots\right.\right.}{\sqrt{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*Sqrt[d + e*x^2])/c + ((-2*c^2*d^2 + b*(-b + Sqrt[b^2 - 4*a*c])*e^2 + 2*c
*e*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 -
4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + ((2*c^2*d^2 + b*(b + Sqr
```

```
t[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Maple [C] time = 0.022, size = 279, normalized size = 0.9

$$-\frac{x}{2c}e^{\frac{3}{2}}+\frac{e}{2c}\sqrt{ex^2+d}+\frac{de}{2c}\left(\sqrt{ex^2+d}-\sqrt{ex}\right)^{-1}+\frac{e}{4c}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+(4be-4cd)\_Z^6+\left(16ae^2-8deb+6cd^2\right)\_Z^4+\left(4bd^2e-4cd^3\right)\_Z^2+cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*e^(3/2)/c*x+1/2*e*(e*x^2+d)^(1/2)/c+1/2*e/c*d/((e*x^2+d)^(1/2)-e^(1/2)
*x)+1/4*e/c*sum(((-b*e+2*c*d)*_R^6+(-4*a*e^2+3*b*d*e-2*c*d^2)*_R^4+d*(4*a*e
^2-3*b*d*e+2*c*d^2)*_R^2+b*d^3*e-2*c*d^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_
R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)
-e^(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^
2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(d+ex^2\right)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.369 $\int \frac{\left(d+ex^2\right)^{3/2}}{x\left(a+bx^2+cx^4\right)} dx$

Optimal. Leaf size=346

$$-\frac{\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\cdots\right.}{\sqrt{2}a\sqrt{c}\sqrt{b^2\cdots}}$$

```
[Out] -((d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) - ((a*Sqrt[b^2 - 4*a*c]*e^2
 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]
*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((a
*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e
^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e])
```

Rubi [A] time = 1.74378, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 897, 1287, 206, 1166, 208}

$$-\frac{\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\cdots\right.}{\sqrt{2}a\sqrt{c}\sqrt{b^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) - ((a*Sqrt[b^2 - 4*a*c]*e^2
 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]
*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((a
*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e
^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
```

```
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{d^2e}{a\left(d-x^2\right)}+\frac{e\left(d\left(cd^2-bde+ae^2\right)-\left(cd^2-ae^2\right)x^2\right)}{a\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{d\left(cd^2-bde+ae^2\right)+\left(-cd^2+ae^2\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex^2}\right)}{a}$$

$$= -\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b\left(cd^2+ae^2\right)\right)\text{Subst}\left(\int \cdots\right.}{2a\sqrt{b^2-4ac}}$$

$$= -\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} - \frac{\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b\left(cd^2+ae^2\right)\right)\tanh^{-1}\left(\cdots\right.}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [A] time = 1.37316, size = 333, normalized size = 0.96

$$-\frac{\frac{\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\left(cd\left(d\sqrt{b^2-4ac}-4ae\right)-ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}}{\sqrt{2}a\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((-(((-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) + b*(
c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e
+ Sqrt[b^2 - 4*a*c]*e]])/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + ((a*Sq
rt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e^2)
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])/(Sqrt[2]*a*Sqrt[c]*Sqrt
[b^2 - 4*a*c])) + (d^(3/2)*Log[x])/a - (d^(3/2)*Log[d + Sqrt[d]*Sqrt[d + e*
x^2]])/a
```

Maple [C] time = 0.027, size = 388, normalized size = 1.1

$$\frac{x^3}{6a}e^{\frac{3}{2}}-\frac{ex^2}{8a}\sqrt{ex^2+d}+\frac{3dx}{4a}\sqrt{e}+\frac{7}{24a}\left(ex^2+d\right)^{\frac{3}{2}}+\frac{3d}{8a}\sqrt{ex^2+d}-\frac{5d^2}{8a}\left(\sqrt{ex^2+d}-\sqrt{e}x\right)^{-1}-\frac{d^3}{24a}\left(\sqrt{ex^2+d}-\sqrt{e}x\right)^{-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/6/a*e^(3/2)*x^3-1/8/a*e*(e*x^2+d)^(1/2)*x^2+3/4/a*e^(1/2)*x*d+7/24/a*(e*x
^2+d)^(3/2)+3/8/a*(e*x^2+d)^(1/2)*d-5/8/a*d^2/((e*x^2+d)^(1/2)-e^(1/2)*x)-1
/24/a*d^3/((e*x^2+d)^(1/2)-e^(1/2)*x)^3-1/4/a*sum(((-a*e^2+c*d^2)*_R^6+d*(-
5*a*e^2+4*b*d*e-3*c*d^2)*_R^4+d^2*(5*a*e^2-4*b*d*e+3*c*d^2)*_R^2+a*d^3*e^2-
c*d^5)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2
+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*
b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+c*
d^4))-1/a*d^(3/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{\left(cx^4+bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^2\right)^{\frac{3}{2}}}{x\left(a + bx^2 + cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d + e*x**2)**(3/2)/(x*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.370 $\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=417

$$-\frac{\sqrt{c}\left(-2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(d\sqrt{b^2-4ac}-2ae\right)+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-2a\left(cd^2-e\left(\right.\right.\right.}{}$$

```
[Out] -(d*Sqrt[d + e*x^2])/(2*a*x^2) + (Sqrt[d]*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]
])/(2*a) + (Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a^2 - (
Sqrt[c]*(b^2*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2*a*(c*d^2 + e*(Sqrt
[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[c]*(b^2*d^2 - b*d*(Sqrt[b^2 - 4*a*c]
*d + 2*a*e) - 2*a*(c*d^2 - e*(Sqrt[b^2 - 4*a*c]*d + a*e)))*ArcTanh[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2
]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 3.24265, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1251, 897, 1287, 199, 206, 1166, 208}

$$-\frac{\sqrt{c}\left(bd\left(d\sqrt{b^2-4ac}-2ae\right)-2ae\left(d\sqrt{b^2-4ac}-ae\right)-2acd^2+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-bd\left(d\sqrt{b^2-4ac}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(d*Sqrt[d + e*x^2])/(2*a*x^2) + (Sqrt[d]*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]
])/(2*a) + (Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a^2 - (
Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2*a*e*(S
qrt[b^2 - 4*a*c]*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + 2*a*e*(S
qrt[b^2 - 4*a*c]*d + a*e) - b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(Sqr
t[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sq
rt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{d^2e^2}{a\left(d-x^2\right)^2}-\frac{de(-bd+2ae)}{a^2\left(d-x^2\right)}+\frac{e\left(-(bd-ae)\left(cd^2-bde+ae^2\right)+cd(bd-2ae)x^2\right)}{a^2\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{-(bd-ae)\left(cd^2-bde+ae^2\right)+cd(bd-2ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a^2} + \frac{\left(d^2e\right)\text{Subst}\left(\int \frac{1}{\left(d-x^2\right)^2}\,dx, x, \sqrt{d+\cdots}\right.}{a}$$

$$= -\frac{d\sqrt{d+ex^2}}{2ax^2} + \frac{\sqrt{d}(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2} + \frac{(de)\,\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+ex^2}\right)}{2a} + \frac{\left(c\left(b^2 \cdots\right.\right.}{\cdots}$$

$$= -\frac{d\sqrt{d+ex^2}}{2ax^2} + \frac{\sqrt{d}e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a} + \frac{\sqrt{d}(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2} - \frac{\sqrt{c}\left(b^2d^2-2acd^2+b\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.62938, size = 380, normalized size = 0.91

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(\frac{\left(2a\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(2ae-d\sqrt{b^2-4ac}\right)-b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\left(bd\left(d\sqrt{b^2-4ac}+2ae\right)-2ae\left(d\sqrt{b^2-4ac}+ae\right)+2acd^2-b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}} - \cdots}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*d*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(((-(b^2*d^2) + b*d*(-(Sqr
t[b^2 - 4*a*c]*d) + 2*a*e) + 2*a*(c*d^2 + e*(Sqrt[b^2 - 4*a*c]*d - a*e)))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*
c]*e]])/Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e] - ((-(b^2*d^2) + 2*a*c*d^2
 - 2*a*e*(Sqrt[b^2 - 4*a*c]*d + a*e) + b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*A
rcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/Sqrt[b^2 - 4*a*c] - Sqrt[
d]*(2*b*d - 3*a*e)*Log[x] + Sqrt[d]*(2*b*d - 3*a*e)*Log[d + Sqrt[d]*Sqrt[d
+ e*x^2]])/(2*a^2)
```

Maple [C] time = 0.03, size = 555, normalized size = 1.3

$$-\frac{x^3b}{6a^2}e^{\frac{3}{2}} + \frac{x^2eb}{8a^2}\sqrt{ex^2+d} - \frac{3\,bdx}{4\,a^2}\sqrt{e} - \frac{7\,b}{24\,a^2}\left(ex^2+d\right)^{\frac{3}{2}} + \frac{x}{2\,a}e^{\frac{3}{2}} + \frac{e}{a}\sqrt{ex^2+d} - \frac{3\,bd}{8\,a^2}\sqrt{ex^2+d} - \frac{de}{2\,a}\left(\sqrt{ex^2+d}-\sqrt{e}x\right)^{-1}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/6/a^2*e^(3/2)*x^3*b+1/8/a^2*e*(e*x^2+d)^(1/2)*x^2*b-3/4/a^2*e^(1/2)*x*b*
d-7/24/a^2*(e*x^2+d)^(3/2)*b+1/2/a*e^(3/2)*x+1/a*(e*x^2+d)^(1/2)*e-3/8/a^2*
(e*x^2+d)^(1/2)*b*d-1/2/a*d/((e*x^2+d)^(1/2)-e^(1/2)*x)*e+5/8/a^2*d^2/((e*x
^2+d)^(1/2)-e^(1/2)*x)*b+1/24/a^2*b*d^3/((e*x^2+d)^(1/2)-e^(1/2)*x)^3+1/4/a
^2*sum((c*d*(-2*a*e+b*d)*_R^6+(4*a^2*e^3-8*a*b*d*e^2+2*a*c*d^2*e+4*b^2*d^2*
e-3*b*c*d^3)*_R^4+d*(-4*a^2*e^3+8*a*b*d*e^2-2*a*c*d^2*e-4*b^2*d^2*e+3*b*c*d
^3)*_R^2+2*a*c*d^4*e-b*c*d^5)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*
_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_
R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b
*d^2*e-4*c*d^3)*_Z^2+c*d^4))+1/a^2*b*d^(3/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1
/2))/x)-1/2/a/d/x^2*(e*x^2+d)^(5/2)+1/2/a*e/d*(e*x^2+d)^(3/2)-3/2/a*e*d^(1/
2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{\left(cx^4+bx^2+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.371 $\int \frac{x^4\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=595

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^{\cdots}}\right.}}{\cdots}$$

```
[Out] ((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*c^2) + (x*(d + e*x^2)^(3/2))/(4*c) -
 (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d
 - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (
b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]
)/(2*c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) + (d*(3*c*d - 4*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c^2*Sqrt[e]
) - (Sqrt[e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*
c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3) - (Sq
rt[e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sq
rt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3)
```

Rubi [A] time = 3.28462, antiderivative size = 595, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.31, Rules used = {1291, 388, 195, 217, 206, 1692, 402, 377, 205}

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^{\cdots}}\right.}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*c^2) + (x*(d + e*x^2)^(3/2))/(4*c) -
 (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d
 - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (
b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]
)/(2*c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) + (d*(3*c*d - 4*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c^2*Sqrt[e]
) - (Sqrt[e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*
c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3) - (Sq
rt[e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sq
rt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3)
```

Rule 1291

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
```

```
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[((f*x)^(m - 4)*(d +
 e*x^2)^(q - 1)*Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x])/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4 \left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{\int \sqrt{d+ex^2}\left(cd-be+cex^2\right)dx}{c^2} - \frac{\int \frac{\sqrt{d+ex^2}\left(a(cd-be)+\left(bcd-b^2e+ace\right)x^2\right)}{a+bx^2+cx^4}\,dx}{c^2}$$

$$= \frac{x\left(d+ex^2\right)^{3/2}}{4c} - \frac{\int \left(\frac{\left(bcd-b^2e+ace+\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(bcd-b^2e+ace-\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\right)}{c^2}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x\left(d+ex^2\right)^{3/2}}{4c} + \frac{(d(3cd-4be))\int \frac{1}{\sqrt{d+ex^2}}\,dx}{8c^2} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-}{}\right.}{}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x\left(d+ex^2\right)^{3/2}}{4c} + \frac{(d(3cd-4be))\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{8c^2} - \frac{\left(e\left(bc\right.\right.}{}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x\left(d+ex^2\right)^{3/2}}{4c} + \frac{d(3cd-4be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2\sqrt{e}} - \frac{\left(e\left(bcd-b^2e+ace\right.\right.}{}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x\left(d+ex^2\right)^{3/2}}{4c} - \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(bcd-b^2e+ace-\frac{b^2cd-2ac}{\sqrt{}}\right.}{2c^3\sqrt{b-\sqrt{b^2-4}}}$$

Mathematica [B] time = 6.51279, size = 18689, normalized size = 31.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.034, size = 516, normalized size = 0.9

$$\frac{x}{4c}\left(ex^2+d\right)^{\frac{3}{2}} + \frac{3dx}{8c}\sqrt{ex^2+d} + \frac{3d^2}{8c}\ln\left(\sqrt{e}x+\sqrt{ex^2+d}\right)\frac{1}{\sqrt{e}} + \frac{bx^2}{4c^2}e^{\frac{3}{2}} - \frac{xeb}{4c^2}\sqrt{ex^2+d} + \frac{bd}{8c^2}\sqrt{e} + \frac{a}{c^2}e^{\frac{3}{2}}\ln\left(\sqrt{ex^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*x*(e*x^2+d)^(3/2)/c+3/8/c*d*x*(e*x^2+d)^(1/2)+3/8/c*d^2/e^(1/2)*ln(e^(1
/2)*x+(e*x^2+d)^(1/2))+1/4/c^2*e^(3/2)*b*x^2-1/4/c^2*e*b*(e*x^2+d)^(1/2)*x+
1/8/c^2*e^(1/2)*b*d+1/c^2*e^(3/2)*ln((e*x^2+d)^(1/2)-e^(1/2)*x)*a-1/c^3*e^(
3/2)*ln((e*x^2+d)^(1/2)-e^(1/2)*x)*b^2+3/2/c^2*e^(1/2)*ln((e*x^2+d)^(1/2)-e
^(1/2)*x)*b*d-1/8/c^2*e^(1/2)*b*d^2/((e*x^2+d)^(1/2)-e^(1/2)*x)^2-1/2/c^3*e
^(1/2)*sum((((2*a*b*c*e^2-2*a*c^2*d*e-b^3*e^2+2*b^2*c*d*e-b*c^2*d^2)*_R^2+2*
(2*a^2*c*e^3-2*a*b^2*e^3+2*a*b*c*d*e^2+b^3*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)*_
R+2*a*b*c*d^2*e^2-2*a*c^2*d^3*e-b^3*d^2*e^2+2*b^2*c*d^3*e-c^2*d^4*b)/(_R^3*
c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(
((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a
```

```
*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*x^4/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\left(d+ex^2\right)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.43415, size = 140, normalized size = 0.24

$$\frac{1}{8}\sqrt{x^2e+d}\left(\frac{2\,x^2e}{c}+\frac{\left(5\,c^5de^2-4\,bc^4e^3\right)e^{(-2)}}{c^6}\right)x-\frac{\left(3\,c^2d^2-12\,bcde+8\,b^2e^2-8\,ace^2\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^2*e + d)*(2*x^2*e/c + (5*c^5*d*e^2 - 4*b*c^4*e^3)*e^(-2)/c^6)*x
- 1/16*(3*c^2*d^2 - 12*b*c*d*e + 8*b^2*e^2 - 8*a*c*e^2)*e^(-1/2)*log((x*e^(
1/2) - sqrt(x^2*e + d))^2)/c^3
```

3.372 $\int \frac{x^2\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=491

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{2ace+b^2}{\sqrt{b^2}}\right.}{2c^2}$$

```
[Out] (e*x*Sqrt[d + e*x^2])/(2*c) + (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d
 - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])
])/(2*c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(2*c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (d*Sqrt[e]*ArcTanh[(Sqr
t[e]*x)/Sqrt[d + e*x^2]])/(2*c) + (Sqrt[e]*(c*d - b*e - (b*c*d - b^2*e + 2*
a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2) + (
Sqrt[e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(
Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2)
```

Rubi [A] time = 1.80219, antiderivative size = 491, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1293, 195, 217, 206, 1692, 402, 377, 205}

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{2ace+b^2}{\sqrt{b^2}}\right.}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*x*Sqrt[d + e*x^2])/(2*c) + (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d
 - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])
])/(2*c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(2*c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (d*Sqrt[e]*ArcTanh[(Sqr
t[e]*x)/Sqrt[d + e*x^2]])/(2*c) + (Sqrt[e]*(c*d - b*e - (b*c*d - b^2*e + 2*
a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2) + (
Sqrt[e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(
Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2)
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
```

```
] && LeQ[m, 3]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2 \left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4}\,dx = -\frac{\int \frac{\sqrt{d+ex^2}\left(ae-(cd-be)x^2\right)}{a+bx^2+cx^4}dx}{c} + \frac{e\int \sqrt{d+ex^2}\,dx}{c}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} - \frac{\int \left(\frac{\left(-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(-cd+be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{c} + \frac{(de)\int \frac{1}{\sqrt{d+ex^2}}dx}{2c}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{(de)\,\text{Subst}\left(\int \frac{1}{1-ex^2}dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}dx}{c}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\left(e\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{d+ex^2}}dx}{2c^2} + \frac{\left(\left(2cd-\left(b-\right.\right.\right.}{}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\left(e\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{1-ex^2}dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}} +$$

Mathematica [B] time = 6.27491, size = 14032, normalized size = 28.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.029, size = 382, normalized size = 0.8

$$-\frac{x^2}{4c}e^{\frac{3}{2}} + \frac{ex}{4c}\sqrt{ex^2+d} - \frac{d}{8c}\sqrt{e} + \frac{d^2}{8c}\sqrt{e}\left(\sqrt{ex^2+d}-\sqrt{e}x\right)^{-2} + \frac{b}{c^2}e^{\frac{3}{2}}\ln\left(\sqrt{ex^2+d}-\sqrt{e}x\right) - \frac{3d}{2c}\sqrt{e}\ln\left(\sqrt{ex^2+d}-\sqrt{e}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*e^(3/2)/c*x^2+1/4*e*x*(e*x^2+d)^(1/2)/c-1/8*e^(1/2)/c*d+1/8*e^(1/2)/c*
d^2/((e*x^2+d)^(1/2)-e^(1/2)*x)^2+e^(3/2)/c^2*ln((e*x^2+d)^(1/2)-e^(1/2)*x)
*b-3/2*e^(1/2)/c*ln((e*x^2+d)^(1/2)-e^(1/2)*x)*d+1/2*e^(1/2)/c^2*sum(((a*c*
e^2-b^2*e^2+2*b*c*d*e-c^2*d^2)*_R^2+2*(-2*a*b*e^3+3*a*c*d*e^2+b^2*d*e^2-2*b
*c*d^2*e+c^2*d^3)*_R+a*c*d^2*e^2-b^2*d^2*e^2+2*b*c*d^3*e-c^2*d^4)/(_R^3*c+3
*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e
*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^
2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}x^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\left(d+ex^2\right)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.43818, size = 78, normalized size = 0.16

$$\frac{\sqrt{x^2e+d}\,xe}{2\,c}-\frac{\left(3\,cde-2\,be^2\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2*e + d)*x*e/c - 1/4*(3*c*d*e - 2*b*e^2)*e^(-1/2)*log((x*e^(1/2)
 - sqrt(x^2*e + d))^2)/c^2
```

3.373 $\int \frac{\left(d+ex^2\right)^{3/2}}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=487

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae\cdots\right.\right.}{c\sqrt{b^2-\cdots}}$$

```
[Out] ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c
]*d + a*e))*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sq
rt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((2*c^2*d^2 + b*(b + S
qrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])]])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[e]*(3*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c*Sqrt[b^2 - 4*a*c]) - (Sqrt[e]*(
3*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*c*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 1.57217, antiderivative size = 487, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1174, 416, 523, 217, 206, 377, 205}

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae\cdots\right.\right.}{c\sqrt{b^2-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c
]*d + a*e))*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sq
rt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((2*c^2*d^2 + b*(b + S
qrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])]])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[e]*(3*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c*Sqrt[b^2 - 4*a*c]) - (Sqrt[e]*(
3*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*c*Sqrt[b^2 - 4*a*c])
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{(d+ex^2)^{3/2}}{b-\sqrt{b^2-4ac}+2cx^2}dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^2)^{3/2}}{b+\sqrt{b^2-4ac}+2cx^2}dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\int \frac{d\left(4cd-\left(b-\sqrt{b^2-4ac}\right)e\right)+2e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}dx}{2\sqrt{b^2-4ac}} - \frac{\int \frac{d\left(4cd-\left(b+\sqrt{b^2-4ac}\right)e\right)+2e\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}dx}{2\sqrt{b^2-4ac}}$$

$$= \frac{\left(e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\right)\int \frac{1}{\sqrt{d+ex^2}}dx}{2c\sqrt{b^2-4ac}} - \frac{\left(e\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\right)\int \frac{1}{\sqrt{d+ex^2}}dx}{2c\sqrt{b^2-4ac}} + \frac{\left(2c^2d\right.}{}$$

$$= \frac{\left(e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\right)\text{Subst}\left(\int \frac{1}{1-ex^2}dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{b^2-4ac}} - \frac{\left(e\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\right)\text{Su}}{2c\sqrt{b^2-4}}$$

$$= \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(2c^2\right.}{}$$

Mathematica [B] time = 6.16337, size = 9290, normalized size = 19.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.021, size = 217, normalized size = 0.5

$$-\frac{1}{c}e^{\frac{3}{2}}\ln\left(\sqrt{ex^2+d}-\sqrt{e}x\right)+\frac{1}{2c}e^{\frac{3}{2}}\sum_{\_R=\text{RootOf}\left(c\_Z^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z+cd^4\right)}\frac{(be-}{\_R^3c+3\_R^2be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -e^(3/2)/c*ln((e*x^2+d)^(1/2)-e^(1/2)*x)+1/2*e^(3/2)/c*sum(((b*e-2*c*d)*_R^
2+2*e*(2*a*e-b*d)*_R+b*d^2*e-2*c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*
e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_
R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b
*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^{\frac{3}{2}}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex^{2}\right)^{\frac{3}{2}}}{a + bx^{2} + cx^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

Giac [A] time = 1.35158, size = 36, normalized size = 0.07

$$-\frac{e^{\frac{3}{2}} \log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^{2}e + d}\right)^{2}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*e^(3/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

3.374 $\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=260

$$-\frac{\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{3/2}}+\frac{\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{3/2}}-$$

```
[Out] -((d*Sqrt[d + e*x^2])/(a*x)) - ((2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)^(3/2)*A
rcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*
c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(3/2)) +
((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)^(3/2)*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^
2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(3/2))
```

Rubi [A] time = 0.854973, antiderivative size = 432, normalized size of antiderivative = 1.66, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1295, 277, 217, 206, 1692, 402, 377, 205}

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{2a\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d*Sqrt[d + e*x^2])/(a*x)) - (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(d
+ (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(d - (b*d - 2*
a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/
(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a*Sqrt[b + Sqrt[b^2 - 4*
a*c]]) + (d*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/a - (Sqrt[e]*(d -
 (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*
a) - (Sqrt[e]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]])/(2*a)
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{\left(bd-ae+cdx^2\right)\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx}{a} + \frac{d\int \frac{\sqrt{d+ex^2}}{x^2}\,dx}{a}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax} - \frac{\int \left(\frac{\left(cd+\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(cd-\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{a} + \frac{(de)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{a}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax} + \frac{(de)\,\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{a} - \cdots$$

$$= -\frac{d\sqrt{d+ex^2}}{ax} + \frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a} - \frac{\left(e\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2a} - \frac{\left(\left(2cd-\left(b+\sqrt{b^2-\cdots}\right.\right.\right.}{\cdots}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax} + \frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a} - \frac{\left(e\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2a} - \cdots$$

$$= -\frac{d\sqrt{d+ex^2}}{ax} - \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2c\cdots}}{\cdots}$$

Mathematica [B] time = 6.32235, size = 7789, normalized size = 29.96

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.03, size = 360, normalized size = 1.4

$$\frac{x^2}{4a}e^{\frac{3}{2}} + \frac{5ex}{4a}\sqrt{ex^2+d} + \frac{d}{8a}\sqrt{e} - \frac{d^2}{8a}\sqrt{e}\left(\sqrt{ex^2+d}-\sqrt{e}x\right)^{-2} + \frac{3d}{2a}\sqrt{e}\ln\left(\sqrt{ex^2+d}-\sqrt{e}x\right) - \frac{1}{2a}\sqrt{e}\sum_{\_R=\text{RootOf}(c\_Z^4+(4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/a*e^(3/2)*x^2+5/4/a*e*(e*x^2+d)^(1/2)*x+1/8/a*e^(1/2)*d-1/8/a*e^(1/2)*d
^2/((e*x^2+d)^(1/2)-e^(1/2)*x)^2+3/2/a*e^(1/2)*d*ln((e*x^2+d)^(1/2)-e^(1/2)
*x)-1/2/a*e^(1/2)*sum(((a*e^2-c*d^2)*_R^2+2*d*(3*a*e^2-2*b*d*e+c*d^2)*_R+a*
d^2*e^2-c*d^4)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d
^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*
b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^
4))-1/a/d/x*(e*x^2+d)^(5/2)+1/a*e/d*x*(e*x^2+d)^(3/2)+3/2/a*e^(1/2)*d*ln(e^
(1/2)*x+(e*x^2+d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{\left(cx^4+bx^2+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [B] time = 28.9126, size = 8124, normalized size = 31.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3
- 3*(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 -
 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*
e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4
*c))*log(-(12*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2
*(a*b^3 + 2*a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 + ((a^3*b^2*c -
4*a^4*c^2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x
^2*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^
6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 -
 4*a^7*c)) + (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^
4 + 6*a*b^2*c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*
a^2*b^2 + 4*a^3*c)*d^3*e^3)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 -
4*a^5*b*c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d
^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5
*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c +
 4*a^3*c^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2
*e^2)*x)*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2
- 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*
e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^
2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2)
- sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*
(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a
^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e -
3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))
*log(-(12*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*
b^3 + 2*a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 + ((a^3*b^2*c - 4*a^
4*c^2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x^2*s
qrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 +
6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a
^7*c)) + (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 +
6*a*b^2*c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*
b^2 + 4*a^3*c)*d^3*e^3)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^
5*b*c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e
^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2
*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) - ((a*b^4 - 5*a^2*b^2*c + 4*a
^3*c^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2
)*x)*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*
```

```
a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4
- (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^
2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) - sq
rt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b
^2 - 2*a^2*c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d
^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5
*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log
(-(12*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*b^3
+ 2*a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 - ((a^3*b^2*c - 4*a^4*c^
2)*d^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x^2*sqrt(
-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a
*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c
)) + (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 + 6*a*
b^2*c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*b^2
+ 4*a^3*c)*d^3*e^3)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^5*b*
c)*d - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 -
 (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2
 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c
^2)*d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2)*x)
*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*
c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b
^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 -
2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) + sqrt(1
/2)*a*x*sqrt(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 -
 2*a^2*c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e
^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2
*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(1
2*a^3*b*d^3*e^3 - 6*a^4*d^2*e^4 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*(a*b^3 + 2*
a^2*b*c)*d^5*e - 4*(2*a^2*b^2 + a^3*c)*d^4*e^2 - ((a^3*b^2*c - 4*a^4*c^2)*d
^3 - (a^3*b^3 - 4*a^4*b*c)*d^2*e + (a^4*b^2 - 4*a^5*c)*d*e^2)*x^2*sqrt(-(18
*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3
 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) +
 (27*a^3*b*d^2*e^4 - 12*a^4*d*e^5 + (b^3*c - a*b*c^2)*d^6 - (b^4 + 6*a*b^2*
c - 4*a^2*c^2)*d^5*e + 2*(4*a*b^3 + 5*a^2*b*c)*d^4*e^2 - 2*(11*a^2*b^2 + 4*
a^3*c)*d^3*e^3)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3 - 4*a^5*b*c)*d
 - 2*(a^5*b^2 - 4*a^6*c)*e)*x*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^
4 - 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2
*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)) + ((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*
d^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*e^2)*x)*sqr
t(-(3*a^2*b*d*e^2 - 2*a^3*e^3 + (b^3 - 3*a*b*c)*d^3 - 3*(a*b^2 - 2*a^2*c)*d
^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - 9*a^4*d^2*e^4 - (b^4 -
 2*a*b^2*c + a^2*c^2)*d^6 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^
3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))/x^2) + 4*sqrt(e*x
^2 + d)*d)/(a*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^{\frac{3}{2}}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d + e*x**2)**(3/2)/(x**2*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.375 $\int \frac{\left(d+ex^2\right)^{3/2}}{x^4\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=523

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(-\frac{-abe-2ac}{\sqrt{b^2-4}}\right.}{2a^2\sqrt{\vphantom{b}}}$$

```
[Out] ((b*d - a*e)*Sqrt[d + e*x^2])/(a^2*x) - (d + e*x^2)^(3/2)/(3*a*x^3) + (Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt
[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e - (b^2*d - 2*a*c*
d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a^2*Sqrt[b + Sqrt[
b^2 - 4*a*c]]) - (Sqrt[e]*(b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/a^2 + (Sqrt[e]*(b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*A
rcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*a^2) + (Sqrt[e]*(b*d - a*e + (b^2*d
 - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
)/(2*a^2)
```

Rubi [A] time = 2.61745, antiderivative size = 523, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {1295, 264, 6728, 277, 217, 206, 1692, 402, 377, 205}

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(-\frac{-abe-2ac}{\sqrt{b^2-4}}\right.}{2a^2\sqrt{\vphantom{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*d - a*e)*Sqrt[d + e*x^2])/(a^2*x) - (d + e*x^2)^(3/2)/(3*a*x^3) + (Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt
[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e - (b^2*d - 2*a*c*
d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a^2*Sqrt[b + Sqrt[
b^2 - 4*a*c]]) - (Sqrt[e]*(b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/a^2 + (Sqrt[e]*(b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*A
rcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*a^2) + (Sqrt[e]*(b*d - a*e + (b^2*d
 - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
)/(2*a^2)
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
```

```
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^{3/2}}{x^4\left(a+bx^2+cx^4\right)}dx = -\frac{\int \frac{\left(bd-ae+cdx^2\right)\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}dx}{a} + \frac{d\int \frac{\sqrt{d+ex^2}}{x^4}dx}{a}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\int \left(\frac{(bd-ae)\sqrt{d+ex^2}}{ax^2} + \frac{\sqrt{d+ex^2}\left(-b^2d+acd+abe-c(bd-ae)x^2\right)}{a\left(a+bx^2+cx^4\right)}\right)dx}{a}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\int \frac{\sqrt{d+ex^2}\left(-b^2d+acd+abe-c(bd-ae)x^2\right)}{a+bx^2+cx^4}dx}{a^2} - \frac{(bd-ae)\int \frac{\sqrt{d+ex^2}}{x^2}dx}{a^2}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\int \left(\frac{\left(-c(bd-ae)-\frac{c\left(b^2d-2acd-abe\right)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(-c(bd-ae)+\frac{c\left(b^2d-2acd-abe\right.}{\sqrt{b^2-4ac}}\right.}{b+\sqrt{b^2-4ac}+2cx^2}\right.}{a^2}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{(e(bd-ae))\,\mathrm{Subst}\left(\int \frac{1}{1-ex^2}dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\left(c\left(bd-a\right.\right.}{}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\sqrt{e}(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\left(e\left(bd-ae-\frac{b^2d-2acd-}{\sqrt{b^2-4ac}}\right.\right.}{2a^2}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} - \frac{\sqrt{e}(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{\left(e\left(bd-ae-\frac{b^2d-2acd-}{\sqrt{b^2-4ac}}\right.\right.}{}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{\left(d+ex^2\right)^{3/2}}{3ax^3} + \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan}{2a^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [B] time = 6.43, size = 9321, normalized size = 17.82

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.031, size = 511, normalized size = 1.

$$-\frac{bx^2}{4a^2}e^{\frac{3}{2}} - \frac{5\,exb}{4a^2}\sqrt{ex^2+d} - \frac{bd}{8a^2}\sqrt{e} + \frac{1}{a}e^{\frac{3}{2}}\ln\left(\sqrt{ex^2+d}-\sqrt{e}x\right) - \frac{3\,bd}{2a^2}\sqrt{e}\ln\left(\sqrt{ex^2+d}-\sqrt{e}x\right) + \frac{bd^2}{8a^2}\sqrt{e}\left(\sqrt{ex^2+d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4/a^2*e^(3/2)*x^2*b-5/4/a^2*e*(e*x^2+d)^(1/2)*x*b-1/8/a^2*e^(1/2)*b*d+1/
a*e^(3/2)*ln((e*x^2+d)^(1/2)-e^(1/2)*x)-3/2/a^2*e^(1/2)*ln((e*x^2+d)^(1/2)-
e^(1/2)*x)*b*d+1/8/a^2*e^(1/2)*b*d^2/((e*x^2+d)^(1/2)-e^(1/2)*x)^2+1/2/a^2*
```

```
e^(1/2)*sum((c*d*(2*a*e-b*d)*_R^2+2*(-2*a^2*e^3+4*a*b*d*e^2-2*b^2*d^2*e+b*c
*d^3)*_R+2*a*c*d^3*e-c*d^4*b)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R
*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=Ro
otOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4
*c*d^3)*_Z+c*d^4))+1/a^2*b/d/x*(e*x^2+d)^(5/2)-1/a^2*b*e/d*x*(e*x^2+d)^(3/2
)-3/2/a^2*b*e^(1/2)*d*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/3/a/d/x^3*(e*x^2+d)^(
5/2)-2/3/a*e/d^2/x*(e*x^2+d)^(5/2)+2/3/a*e^2/d^2*x*(e*x^2+d)^(3/2)+1/a*e^2/
d*x*(e*x^2+d)^(1/2)+1/a*e^(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}}{\left(cx^4+bx^2+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [B] time = 167.91, size = 15790, normalized size = 30.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(1/2)*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a
*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^
3*b^2 - 2*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5
*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*
a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*
c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^
3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log((2*a^5
*b*c*d*e^5 - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^6 + 2*(a*b^5*c - 5*a
^3*b*c^3)*d^5*e - 4*(2*a^2*b^4*c - 3*a^3*b^2*c^2 - a^4*c^3)*d^4*e^2 + 4*(3*
a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 - 2*(4*a^4*b^2*c - 3*a^5*c^2)*d^2*e^4 + ((
a^5*b^2*c^2 - 4*a^6*c^3)*d^3 - (a^5*b^3*c - 4*a^6*b*c^2)*d^2*e + (a^6*b^2*c
 - 4*a^7*c^2)*d*e^2)*x^2*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2
 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^
5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(
5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5
)/(a^10*b^2 - 4*a^11*c)) + (4*a^5*b*c*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*
c^4)*d^6 - (b^6*c + 4*a*b^4*c^2 - 17*a^2*b^2*c^3 + 4*a^3*c^4)*d^5*e + 2*(4*
a*b^5*c - 3*a^2*b^3*c^2 - 11*a^3*b*c^3)*d^4*e^2 - 2*(11*a^2*b^4*c - 16*a^3*
b^2*c^2 - 4*a^4*c^3)*d^3*e^3 + 7*(4*a^3*b^3*c - 5*a^4*b*c^2)*d^2*e^4 - (17*
a^4*b^2*c - 12*a^5*c^2)*d*e^5)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4
 - 6*a^7*b^2*c + 8*a^8*c^2)*d - (a^7*b^3 - 4*a^8*b*c)*e)*x*sqrt((a^6*b^2*e^
6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a
*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20
*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b
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^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2
*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^
2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d^4 - (4*a^2*b^6 - 25*a^3*b^4*c + 3
7*a^4*b^2*c^2 - 4*a^5*c^3)*d^3*e + 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*c
^2)*d^2*e^2 - (4*a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^2)*d*e^3 + (a^5*b^3 - 4*
a^6*b*c)*e^4)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*
a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2
*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^
3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c
^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*
e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b
*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)
*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*
c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*
e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^
2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a
^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*
(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^
5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log((2*a^5*b*c*d*e^5 - 2*(a
*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^6 + 2*(a*b^5*c - 5*a^3*b*c^3)*d^5*e -
 4*(2*a^2*b^4*c - 3*a^3*b^2*c^2 - a^4*c^3)*d^4*e^2 + 4*(3*a^3*b^3*c - 4*a^4
*b*c^2)*d^3*e^3 - 2*(4*a^4*b^2*c - 3*a^5*c^2)*d^2*e^4 + ((a^5*b^2*c^2 - 4*a
^6*c^3)*d^3 - (a^5*b^3*c - 4*a^6*b*c^2)*d^2*e + (a^6*b^2*c - 4*a^7*c^2)*d*e
^2)*x^2*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d
^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 -
2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^
5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a
^11*c)) + (4*a^5*b*c*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6 - (b^6*c
 + 4*a*b^4*c^2 - 17*a^2*b^2*c^3 + 4*a^3*c^4)*d^5*e + 2*(4*a*b^5*c - 3*a^2*b
^3*c^2 - 11*a^3*b*c^3)*d^4*e^2 - 2*(11*a^2*b^4*c - 16*a^3*b^2*c^2 - 4*a^4*c
^3)*d^3*e^3 + 7*(4*a^3*b^3*c - 5*a^4*b*c^2)*d^2*e^4 - (17*a^4*b^2*c - 12*a^
5*c^2)*d*e^5)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4 - 6*a^7*b^2*c +
8*a^8*c^2)*d - (a^7*b^3 - 4*a^8*b*c)*e)*x*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*
c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a
^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c
^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3
 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) - ((a*b^7 - 7*a^2*b^5*c + 13*a^3*
b^3*c^2 - 4*a^4*b*c^3)*d^4 - (4*a^2*b^6 - 25*a^3*b^4*c + 37*a^4*b^2*c^2 - 4
*a^5*c^3)*d^3*e + 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*c^2)*d^2*e^2 - (4*
a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^2)*d*e^3 + (a^5*b^3 - 4*a^6*b*c)*e^4)*x)*
sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3
*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 + (a^
5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4
*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d
^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^
4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*
b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))/x^2) - 3*sqrt(1/2)*a^2*x^3*sqrt(-((
b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^
2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 -
4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*
c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*
d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 -
 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a
^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*
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a^11*c)))/(a^5*b^2 - 4*a^6*c))*log((2*a^5*b*c*d*e^5 - 2*(a*b^4*c^2 - 3*a^2*
b^2*c^3 + a^3*c^4)*d^6 + 2*(a*b^5*c - 5*a^3*b*c^3)*d^5*e - 4*(2*a^2*b^4*c -
 3*a^3*b^2*c^2 - a^4*c^3)*d^4*e^2 + 4*(3*a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 -
 2*(4*a^4*b^2*c - 3*a^5*c^2)*d^2*e^4 - ((a^5*b^2*c^2 - 4*a^6*c^3)*d^3 - (a^
5*b^3*c - 4*a^6*b*c^2)*d^2*e + (a^6*b^2*c - 4*a^7*c^2)*d*e^2)*x^2*sqrt((a^6
*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6
 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b
^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 3
0*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c
^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) + (4*a^5*
b*c*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6 - (b^6*c + 4*a*b^4*c^2 -
17*a^2*b^2*c^3 + 4*a^3*c^4)*d^5*e + 2*(4*a*b^5*c - 3*a^2*b^3*c^2 - 11*a^3*b
*c^3)*d^4*e^2 - 2*(11*a^2*b^4*c - 16*a^3*b^2*c^2 - 4*a^4*c^3)*d^3*e^3 + 7*(
4*a^3*b^3*c - 5*a^4*b*c^2)*d^2*e^4 - (17*a^4*b^2*c - 12*a^5*c^2)*d*e^5)*x^2
 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*d - (a
^7*b^3 - 4*a^8*b*c)*e)*x*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2
 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^
5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(
5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5
)/(a^10*b^2 - 4*a^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b
*c^3)*d^4 - (4*a^2*b^6 - 25*a^3*b^4*c + 37*a^4*b^2*c^2 - 4*a^5*c^3)*d^3*e +
 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*c^2)*d^2*e^2 - (4*a^4*b^4 - 19*a^5*
b^2*c + 12*a^6*c^2)*d*e^3 + (a^5*b^3 - 4*a^6*b*c)*e^4)*x)*sqrt(-((b^5 - 5*a
*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(
a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 - 4*a^6*c)*
sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3
*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^
3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c
+ 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))
/(a^5*b^2 - 4*a^6*c)))/x^2) + 3*sqrt(1/2)*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c +
 5*a^2*b*c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3
- 3*a^3*b*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 - 4*a^6*c)*sqrt((a^
6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^
6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*
b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 -
30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*
c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^
2 - 4*a^6*c))*log((2*a^5*b*c*d*e^5 - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4
)*d^6 + 2*(a*b^5*c - 5*a^3*b*c^3)*d^5*e - 4*(2*a^2*b^4*c - 3*a^3*b^2*c^2 -
a^4*c^3)*d^4*e^2 + 4*(3*a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 - 2*(4*a^4*b^2*c -
 3*a^5*c^2)*d^2*e^4 - ((a^5*b^2*c^2 - 4*a^6*c^3)*d^3 - (a^5*b^3*c - 4*a^6*b
*c^2)*d^2*e + (a^6*b^2*c - 4*a^7*c^2)*d*e^2)*x^2*sqrt((a^6*b^2*e^6 + (b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a
^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c
 + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*
a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(
a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)) + (4*a^5*b*c*e^6 + (b^5*c^
2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6 - (b^6*c + 4*a*b^4*c^2 - 17*a^2*b^2*c^3 +
4*a^3*c^4)*d^5*e + 2*(4*a*b^5*c - 3*a^2*b^3*c^2 - 11*a^3*b*c^3)*d^4*e^2 - 2
*(11*a^2*b^4*c - 16*a^3*b^2*c^2 - 4*a^4*c^3)*d^3*e^3 + 7*(4*a^3*b^3*c - 5*a
^4*b*c^2)*d^2*e^4 - (17*a^4*b^2*c - 12*a^5*c^2)*d*e^5)*x^2 - 2*sqrt(1/2)*sq
rt(e*x^2 + d)*(((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*d - (a^7*b^3 - 4*a^8*b*
c)*e)*x*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d
^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 -
2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^
5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a
```

```
^11*c)) + ((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d^4 - (4*a^
2*b^6 - 25*a^3*b^4*c + 37*a^4*b^2*c^2 - 4*a^5*c^3)*d^3*e + 3*(2*a^3*b^5 - 1
1*a^4*b^3*c + 12*a^5*b*c^2)*d^2*e^2 - (4*a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^
2)*d*e^3 + (a^5*b^3 - 4*a^6*b*c)*e^4)*x)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*
c^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b
*c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 - (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6
 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*
b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*
a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^
3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*
e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6
*c)))/x^2) + 4*((3*b*d - 4*a*e)*x^2 - a*d)*sqrt(e*x^2 + d))/(a^2*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.376 $\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=281

$$\frac{\left(-\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[Out] -((b*Sqrt[1 - x^2])/c^2) - (1 - x^2)^(3/2)/(3*c) + ((b^2 - a*c + b*c - (b^3
 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*S
qrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b +
 2*c - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2
*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b +
 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*
c]])
```

Rubi [A] time = 7.33561, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 897, 1287, 1166, 208}

$$\frac{\left(-\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*Sqrt[1 - x^2])/c^2) - (1 - x^2)^(3/2)/(3*c) + ((b^2 - a*c + b*c - (b^3
 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*S
qrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b +
 2*c - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2
*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b +
 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*
c]])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}x^2}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{x^2\left(1-x^2\right)^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)\\
&= -\text{Subst}\left(\int \left(\frac{b}{c^2}+\frac{x^2}{c}-\frac{b(a+b+c)-\left(b^2-ac+bc\right)x^2}{c^2\left(a+b+c+(-b-2c)x^2+cx^4\right)}\right)dx, x, \sqrt{1-x^2}\right)\\
&= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}+\frac{\text{Subst}\left(\int \frac{b(a+b+c)+\left(-b^2+ac-bc\right)x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{c^2}\\
&= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}-\frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x\right.}{2c^2}\\
&= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}+\frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{\left(b^2-ac\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.535399, size = 354, normalized size = 1.26

$$\frac{-\frac{3\sqrt{2}\left(b^2\left(\sqrt{b^2-4ac}+c\right)+bc\left(\sqrt{b^2-4ac}-3a\right)-ac\left(\sqrt{b^2-4ac}+2c\right)+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b-2c}}-\frac{3\sqrt{2}\left(b^2\left(\sqrt{b^2-4ac}-c\right)+bc\left(\sqrt{b^2-4ac}+3a\right)+ac\left(2c-\sqrt{b^2-4ac}\right)-b^3\right.}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b-2c}}}{6c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-6*b*Sqrt[c]*Sqrt[1 - x^2] - 2*c^(3/2)*(1 - x^2)^(3/2) - (3*Sqrt[2]*(b^3 +
 b*c*(-3*a + Sqrt[b^2 - 4*a*c]) + b^2*(c + Sqrt[b^2 - 4*a*c]) - a*c*(2*c +
Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c -
```

```
Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]])
 - (3*Sqrt[2]*(-b^3 + a*c*(2*c - Sqrt[b^2 - 4*a*c]) + b*c*(3*a + Sqrt[b^2 -
 4*a*c]) + b^2*(-c + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x
^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c
+ Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

Maple [B] time = 0.081, size = 2134, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/3*(-x^2+1)^(3/2)/c+4/c*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/
2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^
2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2
*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)-2/c^2*a/(8*a*c-2*b^2
)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*a
rctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a
*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2*(-
4*a*c+b^2)^(1/2)-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*
b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a
+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*
a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b+8/c*a^2/(8*a*c-2*b^2)/(4*
a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan
(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*
b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b+8*a^2/(8*
a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b
)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-
2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2
))-2/c^2*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^
2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b
^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/
2)-2*a*b)^(1/2))*b^3-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*
a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x
^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*
(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2+4/c*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(
-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^
2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+
b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)-2/c^2*
a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-
2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+
2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^
(1/2))*b^2*(-4*a*c+b^2)^(1/2)-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^
2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2
)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2
)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b-8/c*a^2/(8*a*
c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^
(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b
)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*
b-8*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^
(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^
(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2
*a*b)^(1/2))+2/c^2*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*
(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*
(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c
```

```
+b^2)^(1/2)-2*a*b)^(1/2))*b^3+2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^
2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2
)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2
)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2-2/c^2*b/(2/x^2-2/x^2*(-x^2+1)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}x^5}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^5/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 33.2244, size = 7294, normalized size = 25.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(1/2)*c^2*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*
a*b^3 - b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2
)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 +
b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)
)*log(-(2*a^3*b^4 + (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*
b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4
- 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^
5 - 2*a^4*b)*c^2 + (a^2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^
4)*c)*x^2 - 2*(3*a^4*b^2 - a^3*b^3)*c + sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 +
 12*a^2*b*c^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b
^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*
b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a
^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*
x^2)*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c -
(b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*
b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a
*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 +
(a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 3*sqr
t(1/2)*c^2*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4
)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(
3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 -
2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a
^3*b^4 + (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b
^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5
+ b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b
)*c^2 + (a^2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 -
 2*(3*a^4*b^2 - a^3*b^3)*c - sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c
^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*
b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*
c)/(b^2*c^10 - 4*a*c^11)) + (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14
```

```
*a^2*b^3)*c^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((
b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c - (b^2*c^5 -
4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2
*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)
*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^
4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 3*sqrt(1/2)*c^2*
sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*
c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 -
 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6
- b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a^3*b^4 - (a
^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2
*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2
- 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b)*c^2 + (a^
2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 - 2*(3*a^4*b
^2 - a^3*b^3)*c + sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*x^2*sqr
t((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b
^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^1
0 - 4*a*c^11)) - (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14*a^2*b^3)*c
^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((b^5 + 2*a^2
*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c^6)*sq
rt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*
b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^
10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^4*b)*c^2 -
(3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) + 3*sqrt(1/2)*c^2*sqrt((b^5 +
 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c
^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3
+ 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(
b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a^3*b^4 - (a^2*b^2*c^5
- 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2
 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^
6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b)*c^2 + (a^2*b^5 + (a^
4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 - 2*(3*a^4*b^2 - a^3*b^
3)*c - sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*x^2*sqrt((b^8 + (a
^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (
11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^1
1)) - (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^
2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((b^5 + 2*a^2*c^3 + (5*a
^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (
a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 +
(11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^
11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2
- a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 2*(c*x^2 - 3*b - c)*sqrt(-x^2 + 1))/c^
2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**5*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

<u>**Giac**</u> **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.377 $\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=229

$$-\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{\sqrt{1-x^2}}{c}$$

```
[Out] Sqrt[1 - x^2]/c - ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[
(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*c^(3/2)*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b + c + (b^2 - 2*a*c + b*c
)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.75111, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1251, 824, 826, 1166, 208}

$$-\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{\sqrt{1-x^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[1 - x^2]/c - ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[
(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*c^(3/2)*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b + c + (b^2 - 2*a*c + b*c
)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 824

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 826

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
```

```
a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{1-x}x}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{\sqrt{1-x^2}}{c}+\frac{\text{Subst}\left(\int \frac{a+(b+c)x}{\sqrt{1-x}\left(a+bx+cx^2\right)}\,dx,x,x^2\right)}{2c}\\
&= \frac{\sqrt{1-x^2}}{c}+\frac{\text{Subst}\left(\int \frac{-a-b-c+(b+c)x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx,x,\sqrt{1-x^2}\right)}{c}\\
&= \frac{\sqrt{1-x^2}}{c}+\frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,\sqrt{1-x^2}\right)}{2c}+\frac{\left(b+c+\frac{b^2-2ac+}{\sqrt{b^2-4a}}\right.}{}\\
&= \frac{\sqrt{1-x^2}}{c}-\frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+2c-\sqrt{b^2-4ac}}}-\frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x}}{\sqrt{b+2c+\sqrt{b^2-}}}\right.}{\sqrt{2}c^{3/2}\sqrt{b+2c+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.394853, size = 276, normalized size = 1.21

$$\frac{\frac{\left(b\left(c-\sqrt{b^2-4ac}\right)-c\left(\sqrt{b^2-4ac}+2a\right)+b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(b\left(\sqrt{b^2-4ac}+c\right)+c\left(\sqrt{b^2-4ac}-2a\right)+b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\sqrt{1-x^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[1 - x^2] + ((b^2 + b*(c - Sqrt[b^2 - 4*a*c]) - c*(2*a + Sqrt[b^2 - 4*
a*c]))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]
]) - ((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))*ArcT
anh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]))/c
```

Maple [B] time = 0.043, size = 1223, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1
/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(
1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*
a*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b+4*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b
^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1
/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1
/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)-8*a^2/(8*a*c-
2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1
/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)
/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))+2
/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/
2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1
/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a
*b)^(1/2))*b^2+2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*
(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*
(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c
+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)*b+4*a/(8*a*c-2*b^2)/(4*a*c-2*b
^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2
*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-
4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*(-4*a*c+b^2)^(1/2)+
8*a^2/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1
/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1
/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a
*b)^(1/2))-2/c*a/(8*a*c-2*b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*
a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*
a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2
)^(1/2)-2*a*b)^(1/2))*b^2+2/c/(1/x^2*(-x^2+1)-2/x^2*(-x^2+1)^(1/2)+1/x^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^3/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 12.4537, size = 4096, normalized size = 17.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4
)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*sqr
```

```
t((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7))
+ (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + sqrt(1/2)*((b^4*c^3
 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*
b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*a*b^
3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4
)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 + 1))/
x^2) - sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a
*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*s
qrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)
) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c - sqrt(1/2)*((b^4*c
^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(
a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*a*
b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a
*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 + 1)
)/x^2) - sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4*a
*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 - (a*b^2*c^3 - 4*a^2*c^4)*x^2
*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^
7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + sqrt(1/2)*((b^4
*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2
*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) - (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*
a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4*a
*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 +
1))/x^2) + sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 - (a*b^2*c^3 - 4*a^2*c^4)*x
^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*
c^7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c - sqrt(1/2)*((b
^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 -
 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) - (b^5 + 4*(a^2*b - a*b^2)*c^2 - (
5*a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2
+ 1))/x^2) + 2*sqrt(-x^2 + 1))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**3*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.378 $\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=182

$$\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -((Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]))
+ (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.267961, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1247, 699, 1130, 208}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]))
+ (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\sqrt{1-x}}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= -\,\mathrm{Subst}\left(\int \frac{x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)$$
$$= \frac{1}{2}\left(-1-\frac{b+2c}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-x^2}\right)-\frac{1}{2}\left(1-\frac{b+2c}{\sqrt{b^2-4ac}}\right.$$
$$= -\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.245174, size = 169, normalized size = 0.93

$$\frac{\sqrt{-\sqrt{b^2-4ac}-b-2c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}-b-2c}}\right)-\sqrt{\sqrt{b^2-4ac}-b-2c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}-b-2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/
Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]] - Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]*Ar
cTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.036, size = 1167, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -2*a/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)
-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2
)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b
)^(1/2))*(-4*a*c+b^2)^(1/2)-1/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)
*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/
x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b
*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b*(-4*a*c+b^2)^(1/2)-4*a/(4*a*c-b^2)/(4*a
*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(
1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b
^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*c+1/(4*a*c-b
^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)
*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4
*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2-
2*a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-
2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+
2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^
```

```
(1/2))*(-4*a*c+b^2)^(1/2)-1/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a
-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2
*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-
4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b*(-4*a*c+b^2)^(1/2)+4*a/(4*a*c-b^2)/(4*a*c-
2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2
*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2
*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*c-1/(4*a*c-b^2)/
(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arc
tan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-
2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 5.87307, size = 1868, normalized size = 10.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(
b^2*c - 4*a*c^2))*log((b*x^2 + (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3
) + sqrt(1/2)*((b^2 - 4*a*c)*x^2 + (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4
*a*c^3))*sqrt((b + 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2) + 1/2*sqrt(1/2)*sqrt((b + 2*c
- (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((b*x^2
+ (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3) - sqrt(1/2)*((b^2 - 4*a*c)*
x^2 + (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((b + 2*c - (b^2
*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt(-x^2 + 1
)*a + 2*a)/x^2) - 1/2*sqrt(1/2)*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*
c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((b*x^2 - (b^2*c - 4*a*c^2)*x^2/sqrt(
b^2*c^2 - 4*a*c^3) + sqrt(1/2)*((b^2 - 4*a*c)*x^2 - (b^3*c - 4*a*b*c^2)*x^2
/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 -
4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2) + 1/2*sqrt(1/
2)*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*
c^2))*log((b*x^2 - (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3) - sqrt(1/2
)*((b^2 - 4*a*c)*x^2 - (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqr
t((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))
- 2*sqrt(-x^2 + 1)*a + 2*a)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.379 $\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=241

$$\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\tanh^{-1}\left(\sqrt{\right.}}{a}$$

```
[Out] -(ArcTanh[Sqrt[1 - x^2]]/a) + (Sqrt[c]*(2*a + b + Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*a
+ b - Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c + Sqrt[
b^2 - 4*a*c]])
```

Rubi [A] time = 1.64543, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 897, 1287, 207, 1166, 208}

$$\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\tanh^{-1}\left(\sqrt{\right.}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```


```
[Out] -(ArcTanh[Sqrt[1 - x^2]]/a) + (Sqrt[c]*(2*a + b + Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*a
+ b - Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c + Sqrt[
b^2 - 4*a*c]])
```


Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{1-x}}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= -\,\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b+c+(-b-2c)x^2+cx^4\right)}\,dx, x, \sqrt{1-x^2}\right)\\
&= -\,\text{Subst}\left(\int \left(-\frac{1}{a\left(-1+x^2\right)}+\frac{-a-b-c+cx^2}{a\left(a+b+c-(b+2c)x^2+cx^4\right)}\right)dx, x, \sqrt{1-x^2}\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1-x^2}\right)}{a} - \frac{\text{Subst}\left(\int \frac{-a-b-c+cx^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{a}\\
&= -\frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a} + \frac{\left(c\left(2a+b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-x^2}\right)}{2a\sqrt{b^2-4ac}}\\
&= -\frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a} + \frac{\sqrt{c}\left(2a+b+\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b^2-4ac}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(2a+b-\sqrt{b^2-\right.}{\sqrt{2}a\sqrt{b^2-4a}}
\end{aligned}
$$

Mathematica [A] time = 0.442067, size = 212, normalized size = 0.88

$$
\frac{\frac{\sqrt{2}\left(\sqrt{-\sqrt{b^2-4ac}+b+2c}\left(\sqrt{b^2-4ac}+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)+\left(\sqrt{b^2-4ac}-b\right)\sqrt{\sqrt{b^2-4ac}+b+2c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)\right)}{\sqrt{c}\sqrt{b^2-4ac}}-4\tanh^{-1}\left(\sqrt{1-x^2}\right)}{4a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-4*ArcTanh[Sqrt[1 - x^2]] + (Sqrt[2]*(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*(b
 + Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c
- Sqrt[b^2 - 4*a*c]]] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[b + 2*c + Sqrt[b^2 -
4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4
*a*c]]]))/(Sqrt[c]*Sqrt[b^2 - 4*a*c]))/(4*a)
```

Maple [B] time = 0.049, size = 2099, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*
a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2
*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(
1/2))*b*(-4*a*c+b^2)^(1/2)-2/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*
a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x
^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*
(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*c*(-4*a*c+b^2)^(1/2)+1/a/(4*a*c-b^2)/(4*a*
c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1
/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^
2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2*(-4*a*c+b
^2)^(1/2)+4*a/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b
^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+
b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1
/2)-2*a*b)^(1/2))*c-1/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(
-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*
(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c
+b^2)^(1/2)-2*a*b)^(1/2))*b^2+4/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/
2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^
2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2
*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b*c-1/a/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*
a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+
1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^
2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^3+1/(4*a*c-b^2)/(4*a*c-2*
b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(
2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(
-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b*(-4*a*c+b^2)^(1/
2)-2/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)
-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)
+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)
^(1/2))*c*(-4*a*c+b^2)^(1/2)+1/a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1
/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^
2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2
*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2*(-4*a*c+b^2)^(1/2)-4*a/(4*a*c-b^2)/
(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arc
tan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-
2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*c+1/(4*a*
c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1
/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/
(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^
2-4/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-
2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+
2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^
(1/2))*b*c+1/a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+
```

```
b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+
b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1
/2)-2*a*b)^(1/2))*b^3-2/a/(2/x^2-2/x^2*(-x^2+1)^(1/2))+1/a*(-x^2+1)^(1/2)-1
/a*arctanh(1/(-x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{\left(cx^4+bx^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x), x)
```

Fricas [B] time = 23.6363, size = 2604, normalized size = 10.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 +
2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log((2*sqrt(1/2)*(a
^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a
*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 -
4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b
 + b^2)/(a^4*b^2 - 4*a^5*c)) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a^2 + a
*b)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 -
 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c
))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*
b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2
*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - (a^2*b^2 - 4*a^3*c
)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)) - (a*b + b^2)*x^2 - 2*a
^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*a*sqrt((a*b + b
^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*
c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a
^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 -
 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c
)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))
- (a*b + b^2)*x^2 - 2*a^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) - sq
rt(1/2)*a*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b +
 b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log((2*sqrt(1/2)*(a^3*b^2
- 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^
2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c
)))/(a^2*b^2 - 4*a^3*c)) - (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)
/(a^4*b^2 - 4*a^5*c)) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a^2 + a*b)*sqr
t(-x^2 + 1))/x^2) + 2*log((sqrt(-x^2 + 1) - 1)/x))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{x(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.380 $\int \frac{\sqrt{1-x^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=290

$$-\frac{\sqrt{c}\left(\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a^2\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{(a+2b)\tanh^{-1}(}{2a^2}$$

```
[Out] -1/(4*a*(1 - Sqrt[1 - x^2])) + 1/(4*a*(1 + Sqrt[1 - x^2])) + ((a + 2*b)*Arc
Tanh[Sqrt[1 - x^2]])/(2*a^2) - (Sqrt[c]*(a + b + (b^2 + a*(b - 2*c))/Sqrt[b
^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(a
 + b - (b^2 + a*(b - 2*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt
[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c +
Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.36223, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1251, 897, 1287, 207, 1166, 208}

$$-\frac{\sqrt{c}\left(\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a^2\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{(a+2b)\tanh^{-1}(}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*a*(1 - Sqrt[1 - x^2])) + 1/(4*a*(1 + Sqrt[1 - x^2])) + ((a + 2*b)*Arc
Tanh[Sqrt[1 - x^2]])/(2*a^2) - (Sqrt[c]*(a + b + (b^2 + a*(b - 2*c))/Sqrt[b
^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(a
 + b - (b^2 + a*(b - 2*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt
[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c +
Sqrt[b^2 - 4*a*c]])
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{1-x}}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= -\operatorname{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2\left(a+b+c+(-b-2c)x^2+cx^4\right)}\,dx, x, \sqrt{1-x^2}\right)\\
&= -\operatorname{Subst}\left(\int \left(\frac{1}{4a(-1+x)^2}+\frac{1}{4a(1+x)^2}+\frac{a+2b}{2a^2\left(-1+x^2\right)}+\frac{b(a+b+c)-(a+b)cx^2}{a^2\left(a+b+c-(b+2c)x^2+cx^4\right.}\right.\right.\\
&= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)}-\frac{\operatorname{Subst}\left(\int \frac{b(a+b+c)-(a+b)cx^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{a^2}-\frac{(a}{\quad}\\
&= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)}+\frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}+\frac{\left(c\left(a+b-\frac{b^2+a(b-2}{\sqrt{b^2-4ac}}\right.\right.}{\quad}\\
&= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)}+\frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}-\frac{\sqrt{c}\left(a+b+\frac{b^2+a(b-}{\sqrt{b^2-4a}}\right.}{\sqrt{2}a^2\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.770169, size = 292, normalized size = 1.01

$$\frac{\frac{\sqrt{2}\sqrt{c}\left(-\frac{\left(b\left(\sqrt{b^2-4ac}+b\right)+a\left(\sqrt{b^2-4ac}+b-2c\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(b\left(\sqrt{b^2-4ac}-b\right)+a\left(\sqrt{b^2-4ac}-b+2c\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{b^2-4ac}}+(a+2b)\log\left(\sqrt{1-x^2}\right.}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*Sqrt[1 - x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(-(((b*(b + Sqrt[b^2 - 4*a*c])
 + a*(b - 2*c + Sqrt[b^2 - 4*a*c]))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])
/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (
(b*(-b + Sqrt[b^2 - 4*a*c]) + a*(-b + 2*c + Sqrt[b^2 - 4*a*c]))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c] - (a + 2*b)*Log[x] + (a + 2*b)*
Log[1 + Sqrt[1 - x^2]])/(2*a^2)
```

Maple [B] time = 0.059, size = 2770, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 2/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*
a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2
*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(
1/2))*c*(-4*a*c+b^2)^(1/2)-1/a/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2
)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2
/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*
b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2*(-4*a*c+b^2)^(1/2)+3/a/(4*a*c-b^2)/(
4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arct
an(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-
2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b*c*(-4*a
*c+b^2)^(1/2)-1/a^2/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4
*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-
4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b
^2)^(1/2)-2*a*b)^(1/2))*b^3*(-4*a*c+b^2)^(1/2)-4/(4*a*c-b^2)/(4*a*c-2*b^2+2
*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((
-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a
*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b*c+4/(4*a*c-b^2)/(4*a
*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(
1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b
^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*c^2+1/a/(4*a
*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(
1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)-2*a-2*b
)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*
b^3-5/a/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1
/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(
1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*
a*b)^(1/2))*b^2*c+1/a^2/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b
*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*((-x^2+1)^(1/2)-1)^2/x^2*a+
2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a
*c+b^2)^(1/2)-2*a*b)^(1/2))*b^4+2/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(
1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)
^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-
2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*c*(-4*a*c+b^2)^(1/2)-1/a/(4*a*c-b^2)/(
4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arct
an(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2
*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2*(-4*a*
c+b^2)^(1/2)+3/a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*
```

```
c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*
c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^
(1/2)-2*a*b)^(1/2))*b*c*(-4*a*c+b^2)^(1/2)-1/a^2/(4*a*c-b^2)/(4*a*c-2*b^2-2
*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-
x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*
c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^3*(-4*a*c+b^2)^(1/2)+
4/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*
a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*
a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1
/2))*b*c-4/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)
^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)
^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-
2*a*b)^(1/2))*c^2-1/a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(
-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^2/x^2*a+2*(
-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+
b^2)^(1/2)-2*a*b)^(1/2))*b^3+5/a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1
/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x^2+1)^(1/2)-1)^
2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2
*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^2*c-1/a^2/(4*a*c-b^2)/(4*a*c-2*b^2-2*
(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*((-x
^2+1)^(1/2)-1)^2/x^2*a+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c
+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))*b^4+2/a^2*b/(2/x^2-2/x^2
*(-x^2+1)^(1/2))-1/a^2*b*(-x^2+1)^(1/2)+1/a^2*b*arctanh(1/(-x^2+1)^(1/2))-1
/2/a/x^2*(-x^2+1)^(3/2)-1/2/a*(-x^2+1)^(1/2)+1/2/a*arctanh(1/(-x^2+1)^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{(cx^4+bx^2+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^3), x)
```

Fricas [B] time = 67.3797, size = 5650, normalized size = 19.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)
*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4
*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/
(a^4*b^2 - 4*a^5*c))*log(((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 + 2*a*b
^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b
^4)*c)/(a^8*b^2 - 4*a^9*c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b^2)*c^
2 - (a*b^3 + b^4)*c)*x^2 - 2*(a^2*b^2 + a*b^3)*c + sqrt(1/2)*((a^5*b^3 - 4*
a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^
2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + (a^2*b^4 +
```

```
a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((a*b^3
 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*
b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*
b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 2*((a^3 + 2*
a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*a^2*x^2*
sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)
*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b
^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log
(((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*
b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*
c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2
 - 2*(a^2*b^2 + a*b^3)*c - sqrt(1/2)*((a^5*b^3 - 4*a^6*b*c)*x^2*sqrt((a^2*b
^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b
^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b
)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*
a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a
^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b
^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 +
a*b^3)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^
2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5
 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4
)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-((a^4*b^2*c - 4*a^5*c^
2)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*
(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - 2*(a^3 + 2*a^2*b)
*c^2 - ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2 + 2*(a^2*b^2 + a*b^3)*
c + sqrt(1/2)*((a^5*b^3 - 4*a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a
^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b
^2 - 4*a^9*c)) - (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*
a^2*b^3)*c)*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a
^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^
2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^
2 - 4*a^5*c)) + 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1
))/x^2) - sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*
b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b
 + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c
)))/(a^4*b^2 - 4*a^5*c))*log(-((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 +
2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 +
2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - 2*(a^3 + 2*a^2*b)*c^2 - ((a^2*b + 2*a*b^
2)*c^2 - (a*b^3 + b^4)*c)*x^2 + 2*(a^2*b^2 + a*b^3)*c - sqrt(1/2)*((a^5*b^3
 - 4*a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^
2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - (a^2*b
^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((
a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt(
(a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3
*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 2*((a^3
 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1))/x^2) + (a + 2*b)*x^2
*log((sqrt(-x^2 + 1) - 1)/x) + sqrt(-x^2 + 1)*a)/(a^2*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/x**3/(c*x**4+b*x**2+a),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.381 $\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=325

$$-\frac{\left(-\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

```
[Out] (x*Sqrt[1 - x^2])/(2*c) + ((2*b + c)*ArcSin[x])/(2*c^2) - ((b^2 - a*c + b*c
 - (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*
c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(c^
2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b^2 -
a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^
2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 5.39064, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1291, 388, 216, 1692, 377, 205}

$$-\frac{\left(-\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-3abc-2ac^2+b^2c+b^3}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[1 - x^2])/(2*c) + ((2*b + c)*ArcSin[x])/(2*c^2) - ((b^2 - a*c + b*c
 - (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*
c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(c^
2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b^2 -
a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^
2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 1291

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[((f*x)^(m - 4)*(d +
 e*x^2)^(q - 1)*Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x])/(a +
b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\int \frac{b+c-cx^2}{\sqrt{1-x^2}}\,dx}{c^2} - \frac{\int \frac{a(b+c)+\left(b^2-ac+bc\right)x^2}{\sqrt{1-x^2}\left(a+bx^2+cx^4\right)}\,dx}{c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} - \frac{\int \left(\frac{b^2-ac+bc+\frac{-b^3+3abc-b^2c+2ac^2}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} + \frac{b^2-ac+bc-\frac{-b^3+3abc-b^2c+2ac^2}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right)dx}{c^2} + \frac{(2b+c)\int \frac{1}{\sqrt{1-x^2}}\,dx}{2c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{c^2} - \cdots$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-b-2c+\sqrt{b^2-\cdots}\right.}\cdots\right.}{c^2}\cdots$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \cdots$$

Mathematica [B] time = 6.30397, size = 10606, normalized size = 32.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.033, size = 222, normalized size = 0.7

$$\frac{x}{2c}\sqrt{-x^2+1}+\frac{\arcsin(x)}{2c}+\frac{1}{4c^2}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)}\frac{a(b+c)\_R^6+\left(3ab-ac+4b^2\right.}{\_R^7a+3\_R^5a+3\_R^5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*x*(-x^2+1)^(1/2)/c+1/2*arcsin(x)/c+1/4/c^2*sum((a*(b+c)*_R^6+(3*a*b-a*c
+4*b^2+4*b*c)*_R^4+(3*a*b-a*c+4*b^2+4*b*c)*_R^2+a*b+a*c)/(_R^7*a+3*_R^5*a+3
*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*ln(((-x^2+1)^(1/2)-1)/x-_R),_
R=RootOf(a*_Z^8+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))-2/c^2
*b*arctan(((-x^2+1)^(1/2)-1)/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-x^2+1}x^4}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^4/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 12.4666, size = 5638, normalized size = 17.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c +
(b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*
b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*
c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3
*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c + sqrt(1/2)*((b^6 + 4*a^2*b*
c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^
6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x - ((b^4*
c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 +
8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 -
6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4
+ (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 +
 a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 -
a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*c^2*sqrt(
-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt(
(b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2
- 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a
^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*
(2*a^3*b - a^2*b^2)*c - sqrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^
```

```
3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^2*
b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x - ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2
*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 +
 a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (
4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a
*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8
 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*
b^2)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c
^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2
*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b
^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a
^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c + s
qrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*
c)*sqrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a
*b^4 - b^5)*c)*x + ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x -
(b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a
*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8
 - 4*a*c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c
^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 -
6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 -
4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x
^2) - sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c -
(b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*
b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*
c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3
*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c - sqrt(1/2)*((b^6 + 4*a^2*b*
c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^
6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x + ((b^4*
c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 +
8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 -
6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4
+ (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^6 +
 a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 -
a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(-x^2 + 1)*c*x
+ 2*(2*b + c)*arctan((sqrt(-x^2 + 1) - 1)/x))/c^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.382 $\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=263

$$\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sin^{-1}(x)}{c}$$

```
[Out] -(ArcSin[x]/c) + ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x
^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) +
((b + c + (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqr
t[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(c*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.13482, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1293, 216, 1692, 377, 205}

$$\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sin^{-1}(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(ArcSin[x]/c) + ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x
^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) +
((b + c + (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqr
t[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(c*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx &= -\frac{\int \frac{1}{\sqrt{1-x^2}}\,dx}{c} - \frac{\int \frac{-a-(b+c)x^2}{\sqrt{1-x^2}\left(a+bx^2+cx^4\right)}\,dx}{c}\\
&= -\frac{\sin^{-1}(x)}{c} - \frac{\int \left(\frac{-b-c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} + \frac{-b-c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right)dx}{c}\\
&= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{c} + \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+\right.}
\\
&= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{c} + \frac{\left(b+c+\frac{b^2-2a}{\sqrt{b^2-}}\right.}
\\
&= -\frac{\sin^{-1}(x)}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c+\sqrt{b^2-4}}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{}}\right.}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4}}}
\end{aligned}$$

Mathematica [B] time = 6.14809, size = 7543, normalized size = 28.68

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.02, size = 175, normalized size = 0.7

$$-\frac{1}{4c}\sum_{\_R=\text{RootOf}\left(a\_Z^8+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)} \frac{\_R^6a+(4c+3a+4b)\_R^4+(4c+3a+4b)\_R^2+}{\_R^7a+3\_R^5a+3\_R^5b+3\_R^3a+4\_R^3b+8\_R^3c+\_R\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4/c*sum((_R^6*a+(4*c+3*a+4*b)*_R^4+(4*c+3*a+4*b)*_R^2+a)/(_R^7*a+3*_R^5*
a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*ln(((-x^2+1)^(1/2)-1)/x-_R
),_R=RootOf(a*_Z^8+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))+2/
c*arctan(((-x^2+1)^(1/2)-1)/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}x^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 5.62704, size = 2969, normalized size = 11.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3)*sqrt((b^2
+ 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-(2*(a*b + a*
c)*x^2 - 2*a*b - 2*a*c + sqrt(1/2)*((b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*sqrt(
-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*x - ((b^3*c^2 - 4*a*b*c^3)*
sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c
^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3)*sqrt((b^2 +
2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) + 2*(a*b + a*c)*sqr
t(-x^2 + 1))/x^2) - sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c
^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log
(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c - sqrt(1/2)*((b^3 - 4*a*c^2 - (4*a*b -
 b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*x - ((b^3*c^2
 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((b^2 + 2*b*c
 + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3
)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) + 2*(
a*b + a*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c - (b
^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 -
 4*a*c^3))*log(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c + sqrt(1/2)*((b^3 - 4*a*
c^2 - (4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)
*x + ((b^3*c^2 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqr
t((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c - (b^2
*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4
*a*c^3)) + 2*(a*b + a*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*c*sqrt(-(b^2 - (2
*a - b)*c - (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c - sqrt(1/2
)*((b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4
*a*b - b^2)*c)*x + ((b^3*c^2 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a
*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a
 - b)*c - (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5))
)/(b^2*c^2 - 4*a*c^3)) + 2*(a*b + a*c)*sqrt(-x^2 + 1))/x^2) - 4*arctan((sqr
t(-x^2 + 1) - 1)/x))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.383 $\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=220

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]
]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b
 + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.294096, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1174, 402, 216, 377, 205}

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]
]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b
 + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{\sqrt{1-x^2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\sqrt{1-x^2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(b+2c-\sqrt{b^2-4ac}\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(b+2c+\sqrt{b^2-4ac}\right)\int \frac{1}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(b+2c-\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}} - \frac{\left(b+2c+\sqrt{b^2-4ac}\right)\text{Su}}{}$$

$$= \frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [B] time = 6.06606, size = 2959, normalized size = 13.45

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Log[-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x]/Sqrt[1 + (b/c - Sqr
t[b^2 - 4*a*c]/c)/2] - Log[2 - Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*x
 + 2*Sqrt[1 + (b/c - Sqrt[b^2 - 4*a*c]/c)/2]*Sqrt[1 - x^2]]/Sqrt[1 + (b/c -
 Sqrt[b^2 - 4*a*c]/c)/2])/(Sqrt[2]*c*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(
Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*
c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) +
Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) - (Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(Log[-
(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x]/Sqrt[1 + (b/c - Sqrt[b^2
- 4*a*c]/c)/2] - Log[2 - Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*x + 2*S
qrt[1 + (b/c - Sqrt[b^2 - 4*a*c]/c)/2]*Sqrt[1 - x^2]]/Sqrt[1 + (b/c - Sqrt[
b^2 - 4*a*c]/c)/2]))/(2*Sqrt[2]*c*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqr
t[2]) + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2
 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) - (Log[
Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x]/Sqrt[1 + (b/c - Sqrt[b^2 -
4*a*c]/c)/2] - Log[2 + Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*x + 2*Sqr
t[1 + (b/c - Sqrt[b^2 - 4*a*c]/c)/2]*Sqrt[1 - x^2]]/Sqrt[1 + (b/c - Sqrt[b^
2 - 4*a*c]/c)/2])/(Sqrt[2]*c*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b
/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sq
rt[2])*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2
 - 4*a*c]/c]/Sqrt[2])) + (Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(Log[Sqrt[-(b/
c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x]/Sqrt[1 + (b/c - Sqrt[b^2 - 4*a*c]/c)
```

```
/2] - Log[2 + Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*x + 2*Sqrt[1 + (b/
c - Sqrt[b^2 - 4*a*c]/c)/2]*Sqrt[1 - x^2]]/Sqrt[1 + (b/c - Sqrt[b^2 - 4*a*c
]/c)/2]))/(2*Sqrt[2]*c*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqr
t[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c
]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) + (Log[-(Sqrt[-(b/
c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x]/Sqrt[1 + (b/c + Sqrt[b^2 - 4*a*c]/c
)/2] - Log[2 - Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*x + 2*Sqrt[1 + (b
/c + Sqrt[b^2 - 4*a*c]/c)/2]*Sqrt[1 - x^2]]/Sqrt[1 + (b/c + Sqrt[b^2 - 4*a*
c]/c)/2])/(Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Sqrt[-(b/c) - Sqrt
[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqr
t[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c
]/Sqrt[2])) - (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Log[-(Sqrt[-(b/c) - Sqrt
[b^2 - 4*a*c]/c]/Sqrt[2]) + x]/Sqrt[1 + (b/c + Sqrt[b^2 - 4*a*c]/c)/2] - Lo
g[2 - Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*x + 2*Sqrt[1 + (b/c + Sqrt
[b^2 - 4*a*c]/c)/2]*Sqrt[1 - x^2]]/Sqrt[1 + (b/c + Sqrt[b^2 - 4*a*c]/c)/2])
)/(2*Sqrt[2]*c*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) +
Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] +
 Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) - (Log[Sqrt[-(b/c) - Sqrt[b^2
 - 4*a*c]/c]/Sqrt[2] + x]/Sqrt[1 + (b/c + Sqrt[b^2 - 4*a*c]/c)/2] - Log[2 +
 Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*x + 2*Sqrt[1 + (b/c + Sqrt[b^2
- 4*a*c]/c)/2]*Sqrt[1 - x^2]]/Sqrt[1 + (b/c + Sqrt[b^2 - 4*a*c]/c)/2])/(Sqr
t[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c
]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c)
- Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
])) + (Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Log[Sqrt[-(b/c) - Sqrt[b^2 - 4*a
*c]/c]/Sqrt[2] + x]/Sqrt[1 + (b/c + Sqrt[b^2 - 4*a*c]/c)/2] - Log[2 + Sqrt[
2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*x + 2*Sqrt[1 + (b/c + Sqrt[b^2 - 4*a*
c]/c)/2]*Sqrt[1 - x^2]]/Sqrt[1 + (b/c + Sqrt[b^2 - 4*a*c]/c)/2]))/(2*Sqrt[2
]*c*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2
 - 4*a*c]/c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt
[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]))
```

Maple [C] time = 0.011, size = 130, normalized size = 0.6

$$-\frac{1}{4}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{8}+(4a+4b)\_Z^{6}+(6a+8b+16c)\_Z^{4}+(4a+4b)\_Z^{2}+a\right)}\frac{\_R^{6}-\_R^{4}-\_R^{2}+1}{\_R^{7}a+3\_R^{5}a+3\_R^{5}b+3\_R^{3}a+4\_R^{3}b+8\_R^{3}c+\_R$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*sum((_R^6-_R^4-_R^2+1)/(_R^7*a+3*_R^5*a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_
R^3*c+_R*a+_R*b)*ln(((-x^2+1)^(1/2)-1)/x-_R),_R=RootOf(a*_Z^8+(4*a+4*b)*_Z^
6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] time = 2.47431, size = 1651, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2 + 1)*x
 - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4
*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1) - 1)/x
^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3
*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 - sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2
+ 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b
^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1)
- 1)/x^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 -
 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt
(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt
(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2
 + 1) - 1)/x^2) + 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2
*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 - sqrt(1/2)*((a*b^2 - 4*a^2*c
)*sqrt(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c
)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqr
t(-x^2 + 1) - 1)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.384 $\int \frac{\sqrt{1-x^2}}{x^2(a+bx^2+cx^4)} dx$

Optimal. Leaf size=265

$$-\frac{c\left(\frac{2a+b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{1-x^2}}{ax}$$

```
[Out] -(Sqrt[1 - x^2]/(a*x)) - (c*(1 + (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])
])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (c*(
1 - (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*
x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a
*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.779562, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1295, 264, 1692, 377, 205}

$$-\frac{c\left(\frac{2a+b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{1-x^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[1 - x^2]/(a*x)) - (c*(1 + (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])
])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (c*(
1 - (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*
x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a
*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 1295

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[((f*x)^(m + 2)*(d + e*x^2)^(q - 1)*Simp[b*d - a*e
+ c*d*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
```

```
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx=\frac{\int \frac{1}{x^2\sqrt{1-x^2}}\,dx}{a}-\frac{\int \frac{a+b+cx^2}{\sqrt{1-x^2}\left(a+bx^2+cx^4\right)}\,dx}{a}$$

$$=-\frac{\sqrt{1-x^2}}{ax}-\frac{\int \left(\frac{c+\frac{(2a+b)c}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}+\frac{c-\frac{(2a+b)c}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right)dx}{a}$$

$$=-\frac{\sqrt{1-x^2}}{ax}-\frac{\left(c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{a}-\frac{\left(c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+\right.}}{a}$$

$$=-\frac{\sqrt{1-x^2}}{ax}-\frac{\left(c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-b-2c-\sqrt{b^2-4ac}\right)x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)}{a}-\frac{\left(c\left(1+\frac{2a}{\sqrt{b^2}}\right.\right.}{}$$

$$=-\frac{\sqrt{1-x^2}}{ax}-\frac{c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}}-\frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1-x^2}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4a}}}$$

Mathematica [B] time = 5.47949, size = 2661, normalized size = 10.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(4*Sqrt[b^2 - 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sq
rt[b^2 - 4*a*c]))/c^2]*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c +
Sqrt[b^2 - 4*a*c]))/c^2)]*Sqrt[1 - x^2] + Sqrt[2]*(2*a + b + Sqrt[b^2 - 4*a
*c])*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c])
)/c^2)]*x*Log[-(Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]/Sqrt[2]) + x] - Sqrt[2]*(2
*a + b + Sqrt[b^2 - 4*a*c])*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*
(c + Sqrt[b^2 - 4*a*c]))/c^2)]*x*Log[Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]/Sqrt[
2] + x] - 2*Sqrt[2]*a*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sq
rt[b^2 - 4*a*c]))/c^2]*x*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2])
+ x] - Sqrt[2]*b*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^
2 - 4*a*c]))/c^2]*x*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2]) + x]
+ Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-
c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqr
t[2]) + x] + 2*Sqrt[2]*a*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c +
```

```
 Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2]
+ x] + Sqrt[2]*b*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^
2 - 4*a*c]))/c^2]*x*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x] - S
qrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c +
 Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2]
+ x] - 2*Sqrt[2]*a*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt
[b^2 - 4*a*c]))/c^2)]*x*Log[2 - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x
+ Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[2]*b*
Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2
)]*x*Log[2 - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b +
 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqr
t[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*
x*Log[2 - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*
c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] + 2*Sqrt[2]*a*Sqrt[-((b^2 + c*(-2*
a + Sqrt[b^2 - 4*a*c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*x*Log[2 + Sqrt[2]
*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2 - 4*
a*c])/c]*Sqrt[1 - x^2]] + Sqrt[2]*b*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*
c]) + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*x*Log[2 + Sqrt[2]*Sqrt[(-b + Sqrt[b^
2 - 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x
^2]] + Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[-((b^2 + c*(-2*a + Sqrt[b^2 - 4*a*c])
 + b*(c + Sqrt[b^2 - 4*a*c]))/c^2)]*x*Log[2 + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 -
 4*a*c])/c]*x + Sqrt[2]*Sqrt[(b + 2*c - Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]
] + 2*Sqrt[2]*a*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2
 - 4*a*c]))/c^2]*x*Log[2 - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + S
qrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] + Sqrt[2]*b*Sqr
t[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*
Log[2 - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*
c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(
-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log
[2 - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*c +
 Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]] - 2*Sqrt[2]*a*Sqrt[(-b^2 + c*(2*a + S
qrt[b^2 - 4*a*c]) + b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[2 + Sqrt[2]*Sqrt
[-((b + Sqrt[b^2 - 4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c
])/c]*Sqrt[1 - x^2]] - Sqrt[2]*b*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) +
 b*(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[2 + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 -
4*a*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]
] + Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(-b^2 + c*(2*a + Sqrt[b^2 - 4*a*c]) + b*
(-c + Sqrt[b^2 - 4*a*c]))/c^2]*x*Log[2 + Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a
*c])/c)]*x + Sqrt[2]*Sqrt[(b + 2*c + Sqrt[b^2 - 4*a*c])/c]*Sqrt[1 - x^2]])/
(2*a*Sqrt[b^2 - 4*a*c]*Sqrt[((b + 2*c - Sqrt[b^2 - 4*a*c])*(-b + Sqrt[b^2 -
 4*a*c]))/c^2]*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(b + 2*c + Sqrt[b^2 - 4*a*c]
))/c^2)]*x)
```

Maple [C] time = 0.023, size = 217, normalized size = 0.8

$$\frac{1}{4a}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)}\frac{(a+b)\_R^6+(3a+3b+4c)\_R^4+(3a+3b+4c)\_R^2}{\_R^7a+3\_R^5a+3\_R^5b+3\_R^3a+4\_R^3b+8\_R^3c+\_}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/a*sum(((a+b)*_R^6+(3*a+3*b+4*c)*_R^4+(3*a+3*b+4*c)*_R^2+a+b)/(_R^7*a+3*
_R^5*a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*ln(((-x^2+1)^(1/2)-1)
/x-_R),_R=RootOf(a*_Z^8+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a
))-2/a*arctan(((-x^2+1)^(1/2)-1)/x)-1/a/x*(-x^2+1)^(3/2)-1/a*x*(-x^2+1)^(1/
```

```
2)-1/a*arcsin(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}}{(cx^4+bx^2+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [B] time = 3.45911, size = 4091, normalized size = 15.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c + sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c - sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) + sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c + sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x + ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
```

```
)*x^2 - 2*(a*b + b^2)*c - sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x + ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - 2*sqrt(-x^2 + 1))/(a*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}}{x^2(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x**2*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

3.385 $\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx$

Optimal. Leaf size=96

$$\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right)-\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}x}{\sqrt{1-x^2}}\right)-\sin^{-1}(x)$$

```
[Out] -ArcSin[x] + Sqrt[(2 + Sqrt[5])/5]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*x)/Sqrt[1
- x^2]] - Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[(-1 + Sqrt[5])/2]*x)/Sqrt[1
- x^2]]
```

Rubi [A] time = 0.200399, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1293, 216, 1692, 377, 207, 203}

$$\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right)-\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}x}{\sqrt{1-x^2}}\right)-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
```

```
[Out] -ArcSin[x] + Sqrt[(2 + Sqrt[5])/5]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*x)/Sqrt[1
- x^2]] - Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[(-1 + Sqrt[5])/2]*x)/Sqrt[1
- x^2]]
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx &= -\int \frac{1}{\sqrt{1-x^2}}\,dx - \int \frac{1-2x^2}{\sqrt{1-x^2}\left(-1+x^2+x^4\right)}\,dx \\
&= -\sin^{-1}(x) - \int \left(\frac{-2+\frac{4}{\sqrt{5}}}{\sqrt{1-x^2}\left(1-\sqrt{5}+2x^2\right)} + \frac{-2-\frac{4}{\sqrt{5}}}{\sqrt{1-x^2}\left(1+\sqrt{5}+2x^2\right)}\right) dx \\
&= -\sin^{-1}(x) + \frac{1}{5}\left(2\left(5-2\sqrt{5}\right)\right)\int \frac{1}{\sqrt{1-x^2}\left(1-\sqrt{5}+2x^2\right)}\,dx + \frac{1}{5}\left(2\left(5+2\sqrt{5}\right)\right)\int \frac{1}{\sqrt{1-x^2}\left(1+\right.} \\
&= -\sin^{-1}(x) + \frac{1}{5}\left(2\left(5-2\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{5}-\left(-3+\sqrt{5}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right) + \frac{1}{5}\left(2\left(5+2\sqrt{5}\right.\right. \\
&= -\sin^{-1}(x) + \sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right) - \sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.509611, size = 743, normalized size = 7.74

$$\frac{i\sqrt{5\left(\sqrt{5}-2\right)}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}\sqrt{1-x^2}-i\sqrt{2\left(1+\sqrt{5}\right)}x+2\right)+2i\sqrt{\sqrt{5}-2}\log\left(\sqrt{2\left(3+\sqrt{5}\right)}\sqrt{1-x^2}-i\sqrt{2\left(1+\sqrt{5}\right.\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
```

```
[Out] (-2*Sqrt[5]*ArcSin[x] + (-2 + Sqrt[5])*Sqrt[2 + Sqrt[5]]*Log[-Sqrt[(-1 + Sq
rt[5])/2] + x] + 2*Sqrt[2 + Sqrt[5]]*Log[Sqrt[(-1 + Sqrt[5])/2] + x] - Sqrt
[5*(2 + Sqrt[5])]*Log[Sqrt[(-1 + Sqrt[5])/2] + x] - (2*I)*Sqrt[-2 + Sqrt[5]
]*Log[(-I)*Sqrt[(1 + Sqrt[5])/2] + x] - I*Sqrt[5*(-2 + Sqrt[5])]*Log[(-I)*S
qrt[(1 + Sqrt[5])/2] + x] + (2*I)*Sqrt[-2 + Sqrt[5]]*Log[I*Sqrt[(1 + Sqrt[5
])/2] + x] + I*Sqrt[5*(-2 + Sqrt[5])]*Log[I*Sqrt[(1 + Sqrt[5])/2] + x] + (2
*I)*Sqrt[-2 + Sqrt[5]]*Log[2 - I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3 + Sqrt
[5])]*Sqrt[1 - x^2]] + I*Sqrt[5*(-2 + Sqrt[5])]*Log[2 - I*Sqrt[2*(1 + Sqrt[
5])]*x + Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 - x^2]] - (2*I)*Sqrt[-2 + Sqrt[5]]*Lo
g[2 + I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 - x^2]] - I*
Sqrt[5*(-2 + Sqrt[5])]*Log[2 + I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3 + Sqrt
[5])]*Sqrt[1 - x^2]] + 2*Sqrt[2 + Sqrt[5]]*Log[2 - Sqrt[2*(-1 + Sqrt[5])]*x
 + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]] - Sqrt[5*(2 + Sqrt[5])]*Log[2 -
 Sqrt[2*(-1 + Sqrt[5])]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]] - 2*Sq
rt[2 + Sqrt[5]]*Log[2 + Sqrt[2*(-1 + Sqrt[5])]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[
5])*(-1 + x^2)]] + Sqrt[5*(2 + Sqrt[5])]*Log[2 + Sqrt[2*(-1 + Sqrt[5])]*x +
 Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]])/(2*Sqrt[5])
```

Maple [B] time = 0.069, size = 160, normalized size = 1.7

$$-\frac{\sqrt{5}}{5\sqrt{2+\sqrt{5}}}\operatorname{Artanh}\left(\frac{1}{x\sqrt{2+\sqrt{5}}}\left(\sqrt{-x^2+1}-1\right)\right)-\frac{\sqrt{5}}{5\sqrt{-2+\sqrt{5}}}\arctan\left(\frac{1}{x\sqrt{-2+\sqrt{5}}}\left(\sqrt{-x^2+1}-1\right)\right)-\frac{\sqrt{2+\sqrt{5}}\sqrt{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x)
```

```
[Out] -1/5*5^(1/2)/(2+5^(1/2))^(1/2)*arctanh(((-x^2+1)^(1/2)-1)/x/(2+5^(1/2))^(1/
2))-1/5*5^(1/2)/(-2+5^(1/2))^(1/2)*arctan(((-x^2+1)^(1/2)-1)/x/(-2+5^(1/2))
^(1/2))-1/5*(2+5^(1/2))^(1/2)*5^(1/2)*arctan(((-x^2+1)^(1/2)-1)/x/(2+5^(1/2
))^(1/2))+1/5*5^(1/2)*(-2+5^(1/2))^(1/2)*arctanh(((-x^2+1)^(1/2)-1)/x/(-2+5
^(1/2))^(1/2))+2*arctan(((-x^2+1)^(1/2)-1)/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2+1}x^2}{x^4+x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^2/(x^4 + x^2 - 1), x)
```

Fricas [B] time = 1.7074, size = 790, normalized size = 8.23

$$\frac{2}{5}\sqrt{5}\sqrt{\sqrt{5}+2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{-x^2+1}\left(\sqrt{5}-3\right)+\sqrt{5}-3\right)\sqrt{\sqrt{5}+2}\sqrt{\frac{x^4-4x^2-\sqrt{5}\left(x^4-2x^2\right)-2\left(\sqrt{5}x^2-x^2+2\right)\sqrt{-x^2+1}+4}{x^4}}+2\sqrt{-x^2+}}{4x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(sqrt(5) + 2)*arctan(1/4*(sqrt(2)*(sqrt(-x^2 + 1)*(sqrt(5)
- 3) + sqrt(5) - 3)*sqrt(sqrt(5) + 2)*sqrt((x^4 - 4*x^2 - sqrt(5)*(x^4 - 2*
x^2) - 2*(sqrt(5)*x^2 - x^2 + 2)*sqrt(-x^2 + 1) + 4)/x^4) + 2*sqrt(-x^2 + 1
)*sqrt(sqrt(5) + 2)*(sqrt(5) - 3))/x) + 1/10*sqrt(5)*sqrt(sqrt(5) - 2)*log(
-(2*x^2 + (sqrt(-x^2 + 1)*(sqrt(5)*x + x) - sqrt(5)*x - x)*sqrt(sqrt(5) - 2
) + 2*sqrt(-x^2 + 1) - 2)/x^2) - 1/10*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(2*x^2
 - (sqrt(-x^2 + 1)*(sqrt(5)*x + x) - sqrt(5)*x - x)*sqrt(sqrt(5) - 2) + 2*s
qrt(-x^2 + 1) - 2)/x^2) + 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-(x-1)(x+1)}}{x^4+x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+1)**(1/2)/(x**4+x**2-1),x)
```

```
[Out] Integral(x**2*sqrt(-(x - 1)*(x + 1))/(x**4 + x**2 - 1), x)
```

Giac [B] time = 1.20677, size = 282, normalized size = 2.94

$$-\frac{1}{2}\pi\operatorname{sgn}(x)-\frac{1}{5}\sqrt{5\sqrt{5}+10}\arctan\left(-\frac{\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}}{\sqrt{2\sqrt{5}+2}}\right)-\frac{1}{10}\sqrt{5\sqrt{5}-10}\log\left(\left|\sqrt{2\sqrt{5}-2}-\frac{x}{\sqrt{-x^2+1}-1}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*pi*sgn(x) - 1/5*sqrt(5*sqrt(5) + 10)*arctan(-(x/(sqrt(-x^2 + 1) - 1) -
 (sqrt(-x^2 + 1) - 1)/x)/sqrt(2*sqrt(5) + 2)) - 1/10*sqrt(5*sqrt(5) - 10)*l
og(abs(sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/
x)) + 1/10*sqrt(5*sqrt(5) - 10)*log(abs(-sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2
 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x)) - arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)
^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))
```

3.386 $\int \frac{x^8}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=479

$$-\frac{\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{(b^2}{}$$

```
[Out] (-3*d*x*Sqrt[d + e*x^2])/(8*c*e^2) - (b*x*Sqrt[d + e*x^2])/(2*c^2*e) + (x^3
*Sqrt[d + e*x^2])/(4*c*e) - ((b^3 - 2*a*b*c - (b^4 - 4*a*b^2*c + 2*a^2*c^2)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqr
t[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]
]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b^3 - 2*a*b*c + (b^4 - 4*a*b
^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt[b +
Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + (3*d^2*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c*e^(5/2)) + (b*d*ArcTanh[(Sqrt[e]*x)/Sq
rt[d + e*x^2]])/(2*c^2*e^(3/2)) + ((b^2 - a*c)*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]])/(c^3*Sqrt[e])
```

Rubi [A] time = 1.85821, antiderivative size = 479, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1303, 217, 206, 321, 1692, 377, 205}

$$-\frac{\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{(b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-3*d*x*Sqrt[d + e*x^2])/(8*c*e^2) - (b*x*Sqrt[d + e*x^2])/(2*c^2*e) + (x^3
*Sqrt[d + e*x^2])/(4*c*e) - ((b^3 - 2*a*b*c - (b^4 - 4*a*b^2*c + 2*a^2*c^2)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqr
t[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]
]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b^3 - 2*a*b*c + (b^4 - 4*a*b
^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt[b +
Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + (3*d^2*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c*e^(5/2)) + (b*d*ArcTanh[(Sqrt[e]*x)/Sq
rt[d + e*x^2]])/(2*c^2*e^(3/2)) + ((b^2 - a*c)*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]])/(c^3*Sqrt[e])
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && !IntegerQ[q] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^8}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx = \int \left(\frac{b^2-ac}{c^3\sqrt{d+ex^2}} - \frac{bx^2}{c^2\sqrt{d+ex^2}} + \frac{x^4}{c\sqrt{d+ex^2}} - \frac{a\left(b^2-ac\right)+b\left(b^2-2ac\right)x^2}{c^3\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx$$

$$= -\frac{\int \frac{a(b^2-ac)+b(b^2-2ac)x^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}dx}{c^3} - \frac{b\int \frac{x^2}{\sqrt{d+ex^2}}dx}{c^2} + \frac{\int \frac{x^4}{\sqrt{d+ex^2}}dx}{c} + \frac{\left(b^2-ac\right)\int \frac{1}{\sqrt{d+ex^2}}dx}{c^3}$$

$$= -\frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} - \frac{\int \left(\frac{b(b^2-2ac)+\frac{-b^4+4ab^2c-2a^2c^2}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{b(b^2-2ac)-\frac{-b^4+4ab^2c-2a^2c^2}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c^3} +$$

$$= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} + \frac{\left(b^2-ac\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}} - \frac{\left(b^3-2abc\right.}{}$$

$$= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} + \frac{bd\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c^2e^{3/2}} + \frac{\left(b^2-ac\right)\tanh^{-1}}{c^3\sqrt{e}}$$

$$= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} - \frac{\left(b^3-2abc-\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd}}{\sqrt{b-}}\right.}{c^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b\right.}}}$$

Mathematica [A] time = 1.93862, size = 461, normalized size = 0.96

$$-\frac{\frac{8\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} - \frac{8\left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{8\left(b^2-ac\right)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} + \frac{4bcd}{}}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-4*b*c*x*Sqrt[d + e*x^2])/e + (2*c^2*x^3*Sqrt[d + e*x^2])/e - (8*(b^3 - 2
*a*b*c - (b^4 - 4*a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
]) - (8*(b^3 - 2*a*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*
c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e]) + (4*b*c*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(3/2)
+ (8*(b^2 - a*c)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e] + (3*c^2*d*(
-(Sqrt[e]*x*Sqrt[d + e*x^2]) + d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]))/e^(
5/2))/(8*c^3)
```

Maple [C] time = 0.029, size = 377, normalized size = 0.8

$$\frac{x^3}{4ce}\sqrt{ex^2+d} - \frac{3dx}{8ce^2}\sqrt{ex^2+d} + \frac{3d^2}{8c}\ln\left(\sqrt{e}x+\sqrt{ex^2+d}\right)e^{-\frac{5}{2}} - \frac{bx}{2c^2e}\sqrt{ex^2+d} + \frac{bd}{2c^2}\ln\left(\sqrt{e}x+\sqrt{ex^2+d}\right)e^{-\frac{3}{2}} - \frac{a}{c^2}\ln\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/4*x^3*(e*x^2+d)^(1/2)/c/e-3/8*d*x*(e*x^2+d)^(1/2)/c/e^2+3/8/c*d^2/e^(5/2)
*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/2*b*x*(e*x^2+d)^(1/2)/c^2/e+1/2/c^2*b*d/e^
(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/c^2*a*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e
^(1/2)+1/c^3*b^2*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e^(1/2)-1/2/c^3*e^(1/2)*sum(
(b*(2*a*c-b^2)*_R^2+2*(2*a^2*c*e-2*a*b^2*e-2*a*b*c*d+b^3*d)*_R+2*a*b*c*d^2-
b^3*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d
^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*
c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\left(cx^4 + bx^2 + a\right)\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^8}{\sqrt{d + ex^2}\left(a + bx^2 + cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**8/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [A] time = 1.23256, size = 142, normalized size = 0.3

$$\frac{1}{8}\sqrt{x^2 e + d}\left(\frac{2\,x^2 e^{(-1)}}{c} - \frac{\left(3\,c^5 de + 4\,bc^4 e^2\right)e^{(-3)}}{c^6}\right)x - \frac{\left(3\,c^2 d^2 + 4\,bcde + 8\,b^2 e^2 - 8\,ace^2\right)e^{\left(-\frac{5}{2}\right)}\log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2 e + d}\right)^2\right)}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^2*e + d)*(2*x^2*e^(-1)/c - (3*c^5*d*e + 4*b*c^4*e^2)*e^(-3)/c^6)
*x - 1/16*(3*c^2*d^2 + 4*b*c*d*e + 8*b^2*e^2 - 8*a*c*e^2)*e^(-5/2)*log((x*e
^(1/2) - sqrt(x^2*e + d))^2)/c^3
```

3.387 $\int \frac{x^6}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=366

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex}}\right.}{c^2\sqrt{e}}$$

```
[Out] (x*Sqrt[d + e*x^2])/(2*c*e) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^
2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*c*e^(3/2)) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c^2*Sqrt[e])
```

Rubi [A] time = 1.1727, antiderivative size = 366, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1303, 217, 206, 321, 1692, 377, 205}

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex}}\right.}{c^2\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c*e) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^
2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*c*e^(3/2)) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c^2*Sqrt[e])
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \int \left(-\frac{b}{c^2\sqrt{d+ex^2}}+\frac{x^2}{c\sqrt{d+ex^2}}+\frac{ab+\left(b^2-ac\right)x^2}{c^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{ab+\left(b^2-ac\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c^2}-\frac{b\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c^2}+\frac{\int \frac{x^2}{\sqrt{d+ex^2}}\,dx}{c}$$

$$= \frac{x\sqrt{d+ex^2}}{2ce}+\frac{\int \left(\frac{b^2-ac+\frac{b\left(-b^2+3ac\right)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{b^2-ac-\frac{b\left(-b^2+3ac\right)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c^2}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\cdots\right.}{c^2}\cdots$$

$$= \frac{x\sqrt{d+ex^2}}{2ce}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}}+\frac{\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c^2}+\cdots$$

$$= \frac{x\sqrt{d+ex^2}}{2ce}-\frac{d\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}}+\frac{\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int\cdots\right.}{\cdots}$$

$$= \frac{x\sqrt{d+ex^2}}{2ce}+\frac{\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\left(b^2-ac+\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\tan\cdots}{c^2\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2\cdots}}$$

Mathematica [A] time = 1.14884, size = 355, normalized size = 0.97

$$\frac{\frac{2\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{2\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}-\frac{cd\tanh^{-1}\left(\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right)}{e^{3/2}}+cx}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((c*x*Sqrt[d + e*x^2])/e + (2*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e]) + (2*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e]) - (c*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(3/2
) - (2*b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e])/(2*c^2)
```

Maple [C] time = 0.023, size = 269, normalized size = 0.7

$$\frac{x}{2\,ce}\sqrt{ex^2+d}-\frac{d}{2\,c}\ln\left(\sqrt{e}x+\sqrt{ex^2+d}\right)e^{-\frac{3}{2}}-\frac{b}{c^2}\ln\left(\sqrt{e}x+\sqrt{ex^2+d}\right)\frac{1}{\sqrt{e}}+\frac{1}{2\,c^2}\sqrt{e}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+\left(16\,ae^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/2*x*(e*x^2+d)^(1/2)/c/e-1/2/c*d/e^(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/c
^2*b*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e^(1/2)+1/2/c^2*e^(1/2)*sum(((a*c-b^2)*_
R^2+2*(-2*a*b*e-a*c*d+b^2*d)*_R+a*c*d^2-b^2*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*
c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(
1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2
)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^6}{\left(cx^4+bx^2+a\right)\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**6/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [A] time = 1.22523, size = 74, normalized size = 0.2

$$\frac{\sqrt{x^2e+d}xe^{(-1)}}{2\,c}+\frac{(cd+2\,be)e^{\left(-\frac{3}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2*e + d)*x*e^(-1)/c + 1/4*(c*d + 2*b*e)*e^(-3/2)*log((x*e^(1/2)
- sqrt(x^2*e + d))^2)/c^2
```

3.388 $\int \frac{x^4}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=298

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) - ((b + (b^2
 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]]/(c*Sqrt[e])
```

Rubi [A] time = 0.723755, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1303, 217, 206, 1692, 377, 205}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) - ((b + (b^2
 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]]/(c*Sqrt[e])
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{1}{c\sqrt{d+ex^2}} - \frac{a+bx^2}{c\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c} - \frac{\int \frac{a+bx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c}$$

$$= -\frac{\int \left(\frac{b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{b-\frac{-b^2+2ac}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c} + \frac{\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{}{\left(b+\sqrt{b^2-4\ldots\right.}}}{c}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{c}$$

$$= -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)\ldots}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+e\ldots}}\right)}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4\ldots\right.}}}$$

Mathematica [A] time = 0.633138, size = 292, normalized size = 0.98

$$\frac{-\frac{\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} - \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[
b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])) - ((b + (b^
2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(Sqrt[b + Sqrt[b^2 -
4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x)/Sqrt
[d + e*x^2]]/Sqrt[e])/c
```

Maple [C] time = 0.02, size = 200, normalized size = 0.7

$$\frac{1}{c}\ln\left(\sqrt{e}x+\sqrt{ex^2+d}\right)\frac{1}{\sqrt{e}}+\frac{1}{2c}\sqrt{e}\sum_{\_R=RootOf\left(c\_Z^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z+cd^4\right)}\overline{\_R^3c+3\_R^2be}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/c*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e^(1/2)+1/2/c*e^(1/2)*sum((_R^2*b+2*(2*a*
e-b*d)*_R+b*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c
*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(
4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*
d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(cx^4+bx^2+a\right)\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Fricas [B] time = 125.849, size = 21835, normalized size = 73.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*
c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^
2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c
)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4
- 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a
^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*
```

```
c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*c^3 - 4
*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)
*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3
- a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e +
(b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d
*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b
*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 + 2*sqrt(1/2)
*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a
*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d
*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^
2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*
(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 -
 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^
5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e
+ (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c
)*e - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4
*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b
^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e
 + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5
)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*
c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sqrt(1/2)*c*e*
sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^2 -
 (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2
 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4
*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^
2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c
^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^
2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b
^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^
2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^
2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c
^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4
- 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*
b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((
b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c
^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2
- 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 -
 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6
)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^
2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^5 - 5*a*b^3*c + 4*a^
2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b
*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*
a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt(
(a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(
(b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^
2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c
^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e
 + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*
c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^
3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c
+ a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3
*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(
a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3
 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))
*log((2*a^3*b*d*e - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3
)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2
*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4
 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*
e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) -
```

```
2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^
2 - 4*a^3*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^
4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4
*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*
x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)
*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 -
 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^
2*b^2*c^4 - 4*a^3*c^5)*e^4)) - ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*
b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((
b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^
2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4
- 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*
d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d
^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)
))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^
2*c^3)*e^2)))/x^2) + sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^
2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2
- 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(
a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^
3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*
c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b
^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e -
((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c
^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*
d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*
b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 -
 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)
*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^
2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 -
 (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3
 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2
+ (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*
a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2
*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^
5)*e^4)) - ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c +
4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (
a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e +
 (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^
2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4
*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^
4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c
^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) +
 2*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d))/(c*e), 1/4*(sqr
t(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a
*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((
a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((
b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2
*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^
4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e
+ (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*c^3 - 4*a^2*c^4)
*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x
^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c
)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4
- 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a
^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a
*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x
^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3
+ 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a
^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2
```

```
*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5
- 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3
*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^5 - 5*a*b
^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^
3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b
^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)
*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*
b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c
^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 +
 (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a
*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sqrt(1/2)*c*e*sqrt(-((b
^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2
 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 -
 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d
^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^
2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4))
)/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2
*c^3)*e^2))*log((2*a^3*b*d*e + ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 -
4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2
 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4
*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^
2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c
^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2
- (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 -
 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e
 + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b
*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3
 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e +
 (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*
d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) + ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*
d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*
x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2*c^3 - 4*a*c^4)*d^
2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*
e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6
- 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8
*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^
3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2
*c^2 - 4*a^2*c^3)*e^2)))/x^2) - sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a
*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e +
(a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2
)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*
a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4
 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^
4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a
^3*b*d*e - ((a*b^2*c^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e +
 (a^2*b^2*c^2 - 4*a^3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c
+ a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3
*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(
a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b
^2 - a^3*c)*d^2 + (4*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3
*c)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2
*c^5)*d^3 - (b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*
a^2*b^2*c^3 + 4*a^3*c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a
^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b
^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*
c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4
 - 4*a^3*c^5)*e^4)) - ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a
^2*b^2*c + 4*a^3*c^2)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a
*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b
```

```
*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2
*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(
b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 -
2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*
c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^
2)))/x^2) + sqrt(1/2)*c*e*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e +
((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c
^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a
^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^
4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^
3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 -
4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*log((2*a^3*b*d*e - ((a*b^2*c
^3 - 4*a^2*c^4)*d^3 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*e + (a^2*b^2*c^2 - 4*a^
3*c^3)*d*e^2)*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(
a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^
3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*
c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)) - 2*(a^2*b^2 - a^3*c)*d^2 + (4
*a^3*b*e^2 + (a*b^3 - a^2*b*c)*d^2 - (5*a^2*b^2 - 4*a^3*c)*d*e)*x^2 - 2*sqr
t(1/2)*sqrt(e*x^2 + d)*(((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3 - (b^5*c^2
 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*
c^4)*d*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*e^3)*x*sqrt((a^2*b^2*e^2 + (b^4 -
2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^
4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2
*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4))
- ((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^2 - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2
)*d*e + (a^2*b^3 - 4*a^3*b*c)*e^2)*x)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2
*a^2*c)*e + ((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c
^2 - 4*a^2*c^3)*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 -
2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)
*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2
*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 -
 (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)))/x^2) - 4*sqrt(-
e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(c*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [A] time = 1.20107, size = 36, normalized size = 0.12

$$-\frac{e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

3.389 $\int \frac{x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Arc
Tan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]
]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e])
```

Rubi [A] time = 0.304258, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1303, 377, 205}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Arc
Tan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]
]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e])
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx = \int \left(\frac{1-\frac{b}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{1+\frac{b}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}dx+\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{}{\left(b+\sqrt{\right.}}$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}dx, x, \frac{x}{\sqrt{d+}}\right.$$

$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4a\right.}}}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+}}\right.}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)}}$$

Mathematica [A] time = 0.473695, size = 227, normalized size = 0.95

$$\frac{\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c
]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/Sqrt[2*c*d + (-b +
Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])
])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])/Sqrt[b^2 - 4*a*c]
```

Maple [C] time = 0.017, size = 161, normalized size = 0.7

$$-\frac{1}{2}\sqrt{e}\sum_{\_R=\text{RootOf}\left(c\_Z^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z+cd^4\right)}\frac{\_R^2-2\_Rd+d^2}{\_R^3c+3\_R^2be-3\_R^2cd+8\_Rae^2-4\_Rbd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -1/2*e^(1/2)*sum((_R^2-2*_R*d+d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2
-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),
_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^
2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(cx^4 + bx^2 + a\right)\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Fricas [B] time = 31.4936, size = 6973, normalized size = 29.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)
*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c
- 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a
^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 -
4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log((((b^2*c - 4*a*c^2)*d^3 - (b^3 -
 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^
4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2
*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 + 2*a*d^2 - (b*d
^2 - 4*a*d*e)*x^2 + 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x - ((b^3*c - 4*a*b*c^2)
*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 -
2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4
*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*
b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*
e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*s
qrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a
*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^
3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c
)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 -
 (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)
*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2
- 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^
2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log((((b^2*c - 4*a*c
^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c
^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*
c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2
+ 2*a*d^2 - (b*d^2 - 4*a*d*e)*x^2 - 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x - ((b^
3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a
^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^
4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2
*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*s
qrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 -
 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^
3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 +
 (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e +
(a*b^2 - 4*a^2*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - ((b^2*c
- 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^
2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a
```

```
^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/
((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log(
-(((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2 - 4*a^2*c)*d*e^2)
*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2
*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*
a^3*c)*e^4))*x^2 - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2 + 2*sqrt(1/2)*((b^2 - 4*
a*c)*d^2*x + ((b^3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
 + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3)*sqrt(d^2/((b^2*
c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2
*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*x)*
sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*
c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*
c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4
*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3
- 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2) + 1/4*sqrt(1/2)*sqrt(-(b*d -
 2*a*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e
^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4
- 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 -
 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*
a^2*c)*e^2))*log(-(((b^2*c - 4*a*c^2)*d^3 - (b^3 - 4*a*b*c)*d^2*e + (a*b^2
- 4*a^2*c)*d*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)
*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^
3 + (a^2*b^2 - 4*a^3*c)*e^4))*x^2 - 2*a*d^2 + (b*d^2 - 4*a*d*e)*x^2 - 2*sqr
t(1/2)*((b^2 - 4*a*c)*d^2*x + ((b^3*c - 4*a*b*c^2)*d^3 - (b^4 - 2*a*b^2*c -
 8*a^2*c^2)*d^2*e + 3*(a*b^3 - 4*a^2*b*c)*d*e^2 - 2*(a^2*b^2 - 4*a^3*c)*e^3
)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 -
2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4
*a^3*c)*e^4))*x)*sqrt(e*x^2 + d)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^
2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c
^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a
*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.390 $\int \frac{1}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=243

$$\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

```
[Out] (2*c*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]
*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (2*c*ArcTan[(Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e])
```

Rubi [A] time = 0.178283, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1174, 377, 205}

$$\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2c\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]
*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (2*c*ArcTan[(Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e])
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && !IntegerQ[q]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx = \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}dx}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\,\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-(\cdots}\right.}{\sqrt{\cdots}}$$

$$= \frac{2c\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{2c\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\cdots\right.}}{\sqrt{b+\sqrt{b^2-\cdots}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2c\cdots}}$$

Mathematica [A] time = 0.396951, size = 229, normalized size = 0.94

$$\frac{2c\left(\frac{\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} - \frac{\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*c*(ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])]/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e]) - ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*
x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]/(Sqrt[b + Sqrt[b^2 - 4*a*
c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])))/Sqrt[b^2 - 4*a*c]
```

Maple [C] time = 0.012, size = 151, normalized size = 0.6

$$-2\,e^{3/2}\sum_{\_R=\text{RootOf}\left(c\_Z^4+(4\,be-4\,cd)\_Z^3+\left(16\,ae^2-8\,deb+6\,cd^2\right)\_Z^2+\left(4\,bd^2e-4\,cd^3\right)\_Z+cd^4\right)}\frac{\_R\ln\left(\left(\sqrt{ex^2+d}-\sqrt{e}x\right)^2-\cdots\right.}{\_R^3c+3\_R^2be-3\_R^2cd+8\_R\,ae^2-4\_R\,bd\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -2*e^(3/2)*sum(_R/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*
c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+
(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c
*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)\sqrt{ex^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)), x)
```

Fricas [B] time = 75.4489, size = 9038, normalized size = 37.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 -
 (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d
*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*
d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*
c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 -
 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e
 + ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*
c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^
2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b
^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^
5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 + 2*sqrt
(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*
a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4
*a^4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3
*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*
a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)
) - ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d -
 (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (
a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2
- 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2
*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*
c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 -
 4*a^3*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*
b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)
*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a
^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2
 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4
*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2
*a*c^2*d^2 - 2*a*b*c*d*e + ((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*
b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e
 + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^
3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)
*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*
a*c^2)*d*e)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)
*d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c
^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^
```

```
2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^
2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 +
(a^4*b^2 - 4*a^5*c)*e^4)) - ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c
)*e^2)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*
b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e +
 b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*
e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d
*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a
^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*c*d
- (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e +
(a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2
 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^
2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5
*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2
- 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e - ((a*b^2*c^2 - 4*a^2*c^3)
*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*e^2)*x^2*s
qrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2
*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 -
 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) - (b*c^2*d^2 + 4
*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*((2*(
a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*e + (a^2*b^4
 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^4*b*c)*e^3)*x*sqrt((c^2*
d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c -
4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b
^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) + ((a*b^2*c - 4*a^2*c^2)*
d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*
c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqr
t((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b
^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2
*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2
*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)))/x^2) + 1/4
*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a
*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e
+ b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3
*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*
d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*
a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log(-(2*a*c^2*d^2 - 2*a*b*c*d*e -
((a*b^2*c^2 - 4*a^2*c^3)*d^3 - (a*b^3*c - 4*a^2*b*c^2)*d^2*e + (a^2*b^2*c -
 4*a^3*c^2)*d*e^2)*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 -
 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*
c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c
)*e^4)) - (b*c^2*d^2 + 4*a*b*c*e^2 - (b^2*c + 4*a*c^2)*d*e)*x^2 - 2*sqrt(1/
2)*sqrt(e*x^2 + d)*((2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3 - 3*(a^2*b^3*c - 4*a^3
*b*c^2)*d^2*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*e^2 - (a^3*b^3 - 4*a^
4*b*c)*e^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^
3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4
*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)) +
 ((a*b^2*c - 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*x)*sqrt(-(b*c*d - (b
^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2
*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4
*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c
- 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*
e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*
a^3*c)*e^2)))/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.391 $\int \frac{1}{x^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=280

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{adx}$$

```
[Out] -(Sqrt[d + e*x^2]/(a*d*x)) - (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b + Sqrt
[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rubi [A] time = 0.602196, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1303, 264, 1692, 377, 205}

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```


```
[Out] -(Sqrt[d + e*x^2]/(a*d*x)) - (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b + Sqrt
[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```


Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
```

```
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{1}{ax^2\sqrt{d+ex^2}} + \frac{-b-cx^2}{a\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a} + \frac{\int \frac{-b-cx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} + \frac{\int \left(\frac{-c-\frac{bc}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{-c+\frac{bc}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \cdots}{\cdots}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\mathrm{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+\cdots}}\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} - \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{\cdots}}{a\sqrt{b+\sqrt{b^2-4ac}}\sqrt{\cdots}}$$

Mathematica [A] time = 1.0439, size = 271, normalized size = 0.97

$$-\frac{\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{\sqrt{d+ex^2}}{dx}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[d + e*x^2]/(d*x) + (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
- b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2
])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])
 + (c*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2
```

```
- 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a)
```

Maple [C] time = 0.023, size = 197, normalized size = 0.7

$$\frac{1}{2a}\sqrt{e}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z+cd^4\right)}\frac{c\_R^2+2\,(2be-cd)\_R+}{\_R^3c+3\_R^2be-3\_R^2cd+8\_R\,ae^2-4\_R\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] 1/2/a*e^(1/2)*sum((c*_R^2+2*(2*b*e-c*d)*_R+c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2
*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^
(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^
2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))-(e*x^2+d)^(1/2)/a/d/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2+a\right)\sqrt{ex^2+d}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^2), x)
```

Fricas [B] time = 37.89, size = 12658, normalized size = 45.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a
^2*c^2)*e - ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4
*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c
 - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/
((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b
^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^
8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c
)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log((((a^3*b^2*c^3 - 4*a^4*c^4)*d^3 - (a^
3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*a^5*c^3)*d*e^2)*x^2*sqrt(
((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c
^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)
*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c
^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)) + 2
*(a*b^2*c^3 - a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 2*a^2*b*c^3)*d*e - ((b^3*c^3 -
a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e + 4*(a*b^3*c^2 - 2*a
^2*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3*c^2 - 4*a^5*b*c
```

```
^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*a^6*c^3)*d^2*e + (a^4*b^5 - 5*a^
5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6*a^6*b^2*c + 8*a^7*c^2)*e^3)*x*s
qrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2
*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*
c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a
^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4))
 + ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^2 - (2*a*b^5*c - 11*a^2*b^3*c
^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4*c + 8*a^3*b^2*c^2)*e^2)*x)*sqrt
(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - ((a^3*b^2*c -
4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(
((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c
^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)
*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c
^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((
a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c
)*e^2)))/x^2) - sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b
^2*c + 2*a^2*c^2)*e - ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*
d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2
- 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*
c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*
e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d
*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 -
 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log((((a^3*b^2*c^3 - 4*a^4*c^4)
*d^3 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*a^5*c^3)*d*e^2)
*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 +
 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 -
4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c
 - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)
*e^4)) + 2*(a*b^2*c^3 - a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 2*a^2*b*c^3)*d*e - ((
b^3*c^3 - a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e + 4*(a*b^3
*c^2 - 2*a^2*b*c^3)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^4*b^3*c^2 -
 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*a^6*c^3)*d^2*e + (a^4*
b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6*a^6*b^2*c + 8*a^7*c^2
)*e^3)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c
^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^
2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b
^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^
9*c)*e^4)) + ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^2 - (2*a*b^5*c - 11
*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4*c + 8*a^3*b^2*c^2)*e^
2)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e - ((a^
3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*
e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 +
 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 -
4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c
 - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)
*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2
 - 4*a^5*c)*e^2)))/x^2) + sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b
^4 - 4*a*b^2*c + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4
*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2
*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c +
4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b
*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*
a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 -
(a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log(-(((a^3*b^2*c^3 -
 4*a^4*c^4)*d^3 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c^2 - 4*a^5*
c^3)*d*e^2)*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*
a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6
*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 -
2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2
```

```
 - 4*a^9*c)*e^4)) - 2*(a*b^2*c^3 - a^2*c^4)*d^2 + 2*(a*b^3*c^2 - 2*a^2*b*c^
3)*d*e + ((b^3*c^3 - a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*e
 + 4*(a*b^3*c^2 - 2*a^2*b*c^3)*e^2)*x^2 + 2*sqrt(1/2)*sqrt(e*x^2 + d)*(((a^
4*b^3*c^2 - 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*a^6*c^3)*d^
2*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6*a^6*b^2*c
+ 8*a^7*c^2)*e^3)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c
- 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(
(a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^
4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8
*b^2 - 4*a^9*c)*e^4)) - ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^2 - (2*a
*b^5*c - 11*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4*c + 8*a^3*
b^2*c^2)*e^2)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^
2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2
- 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*
a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6
*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 -
2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2
 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e
 + (a^4*b^2 - 4*a^5*c)*e^2)))/x^2) - sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*
c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (
a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^
2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4
*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*
c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a
^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*
c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log(-(((a^
3*b^2*c^3 - 4*a^4*c^4)*d^3 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*e + (a^4*b^2*c
^2 - 4*a^5*c^3)*d*e^2)*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*
(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e +
(a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3
 + (a^8*b^2 - 4*a^9*c)*e^4)) - 2*(a*b^2*c^3 - a^2*c^4)*d^2 + 2*(a*b^3*c^2 -
 2*a^2*b*c^3)*d*e + ((b^3*c^3 - a*b*c^4)*d^2 - (b^4*c^2 + 2*a*b^2*c^3 - 4*a
^2*c^4)*d*e + 4*(a*b^3*c^2 - 2*a^2*b*c^3)*e^2)*x^2 - 2*sqrt(1/2)*sqrt(e*x^2
 + d)*(((a^4*b^3*c^2 - 4*a^5*b*c^3)*d^3 - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*
a^6*c^3)*d^2*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*e^2 - (a^5*b^4 - 6
*a^6*b^2*c + 8*a^7*c^2)*e^3)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2
- 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*
c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*
e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d
*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)) - ((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)
*d^2 - (2*a*b^5*c - 11*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*e + (a*b^6 - 6*a^2*b^4
*c + 8*a^3*b^2*c^2)*e^2)*x)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c
 + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e
+ (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*
(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)
*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e +
(a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3
 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a
^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)))/x^2) + 4*sqrt(e*x^2 + d))/(a*d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.392 $\int \frac{1}{x^4\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=341

$$\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{b\sqrt{d+ex^2}}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3ad^2x}$$

```
[Out] -Sqrt[d + e*x^2]/(3*a*d*x^3) + (b*Sqrt[d + e*x^2])/(a^2*d*x) + (2*e*Sqrt[d
+ e*x^2])/(3*a*d^2*x) + (c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 -
 4*a*c])*e]) + (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2
])])/(a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e])
```

Rubi [A] time = 0.741386, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1303, 271, 264, 1692, 377, 205}

$$\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{b\sqrt{d+ex^2}}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3ad^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```


```
[Out] -Sqrt[d + e*x^2]/(3*a*d*x^3) + (b*Sqrt[d + e*x^2])/(a^2*d*x) + (2*e*Sqrt[d
+ e*x^2])/(3*a*d^2*x) + (c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 -
 4*a*c])*e]) + (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2
])])/(a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e])
```


Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{1}{ax^4\sqrt{d+ex^2}} - \frac{b}{a^2x^2\sqrt{d+ex^2}} + \frac{b^2-ac+bcx^2}{a^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{b^2-ac+bcx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a^2} + \frac{\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{a} - \frac{b\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{\int \left(\frac{bc+\frac{c\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{bc-\frac{c\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{a^2} \cdots$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+e \cdots}}}{a^2} \cdots$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{\cdots}{b+\sqrt{b^2-4ac}-(\cdots}\right.}{a^2} \cdots$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4a\cdots}\right.}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+\cdots}}\right.}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-\cdots}\right.}} \cdots$$

Mathematica [A] time = 0.72963, size = 320, normalized size = 0.94

$$\frac{\frac{3c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{3c\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} - \frac{a\left(d-2ex^2\right)\sqrt{d+ex^2}}{d^2x^3} + \frac{3b\sqrt{d+ex^2}}{dx}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((3*b*Sqrt[d + e*x^2])/(d*x) - (a*(d - 2*e*x^2)*Sqrt[d + e*x^2])/(d^2*x^3)
+ (3*c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqr
t[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(Sqrt[
b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (3*c*(b
+ (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(Sqrt[b + Sqrt[
b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(3*a^2)
```

Maple [C] time = 0.024, size = 248, normalized size = 0.7

$$-\frac{1}{2a^2}\sqrt{e}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z+cd^4\right)}\frac{bc\_R^2+2\left(-2ace+2b^2e-bc\right.}{\_R^3c+3\_R^2be-3\_R^2cd+8\_Rae^2-4\_R}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -1/2/a^2*e^(1/2)*sum((b*c*_R^2+2*(-2*a*c*e+2*b^2*e-b*c*d)*_R+b*c*d^2)/(_R^3
*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln
(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*
a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))+b*(e*x^2+d)^(1/2
)/a^2/d/x-1/3*(e*x^2+d)^(1/2)/a/d/x^3+2/3*e*(e*x^2+d)^(1/2)/a/d^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)\sqrt{ex^2 + d}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.393 $\int \frac{1}{x^6\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=443

$$-\frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx}-\frac{c\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

```
[Out] -Sqrt[d + e*x^2]/(5*a*d*x^5) + (b*Sqrt[d + e*x^2])/(3*a^2*d*x^3) + (4*e*Sqr
t[d + e*x^2])/(15*a*d^2*x^3) - ((b^2 - a*c)*Sqrt[d + e*x^2])/(a^3*d*x) - (2
*b*e*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - (8*e^2*Sqrt[d + e*x^2])/(15*a*d^3*x)
- (c*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]
)])/(a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
]) - (c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e])
```

Rubi [A] time = 1.43202, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1303, 271, 264, 1692, 377, 205}

$$-\frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx}-\frac{c\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -Sqrt[d + e*x^2]/(5*a*d*x^5) + (b*Sqrt[d + e*x^2])/(3*a^2*d*x^3) + (4*e*Sqr
t[d + e*x^2])/(15*a*d^2*x^3) - ((b^2 - a*c)*Sqrt[d + e*x^2])/(a^3*d*x) - (2
*b*e*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - (8*e^2*Sqrt[d + e*x^2])/(15*a*d^3*x)
- (c*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2]
)])/(a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
]) - (c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e])
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{1}{ax^6\sqrt{d+ex^2}} - \frac{b}{a^2x^4\sqrt{d+ex^2}} + \frac{b^2-ac}{a^3x^2\sqrt{d+ex^2}} + \frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x}{a^3\sqrt{d+ex^2}\left(a+bx^2+cx^4\right.}\right.$$

$$= \frac{\int \frac{-b\left(b^2-2ac\right)-c\left(b^2-ac\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{a^3} + \frac{\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx}{a} - \frac{b\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{a^2} + \frac{\left(b^2-ac\right)\int \frac{1}{x^2\sqrt{d+ex^2}}}{a^3}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} - \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx} + \frac{\int \left(\frac{-\frac{bc\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-c\left(b^2-ac\right)}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \right.}{a^3}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx} - \frac{2be\sqrt{d+ex^2}}{3a^2d^2x} -$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx} - \frac{2be\sqrt{d+ex^2}}{3a^2d^2x} - 8$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{\left(b^2-ac\right)\sqrt{d+ex^2}}{a^3dx} - \frac{2be\sqrt{d+ex^2}}{3a^2d^2x} - 8$$

Mathematica [A] time = 1.79496, size = 383, normalized size = 0.86

$$-\frac{\frac{a^2\sqrt{d+ex^2}\left(3d^2-4dex^2+8e^2x^4\right)}{d^3x^5}+\frac{15\left(b^2-ac\right)\sqrt{d+ex^2}}{dx}+\frac{15c\left(\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{e\sqrt{b^2-4ac}-be+2cd}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}+\frac{15c\left(-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}\cdots\right.}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+\cdots}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\cdots}{15a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((15*(b^2 - a*c)*Sqrt[d + e*x^2])/(d*x) - (5*a*b*(d - 2*e*x^2)*Sqrt[d + e*
x^2])/(d^2*x^3) + (a^2*Sqrt[d + e*x^2]*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^4))/(d^
3*x^5) + (15*c*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*
a*c])*e]) + (15*c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sq
rt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]))/(15*a^3)
```

Maple [C] time = 0.027, size = 350, normalized size = 0.8

$$-\frac{1}{2a^3}\sqrt{e}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4be-4cd)\_Z^3+\left(16ae^2-8deb+6cd^2\right)\_Z^2+\left(4bd^2e-4cd^3\right)\_Z+cd^4\right)}\frac{c\left(ac-b^2\right)\_R^2+2\left(4abce-ac^2d-2b^3e+\cdots\right.}{\_R^3c+3\_R^2be-3\_R^2cd+8\_Rae^2-4\_R\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x)
```

```
[Out] -1/2/a^3*e^(1/2)*sum((c*(a*c-b^2)*_R^2+2*(4*a*b*c*e-a*c^2*d-2*b^3*e+b^2*c*d
)*_R+a*c^2*d^2-b^2*c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d
*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf
(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d
^3)*_Z+c*d^4))-(-a*c+b^2)*(e*x^2+d)^(1/2)/a^3/d/x-1/5*(e*x^2+d)^(1/2)/a/d/x
^5+4/15*e*(e*x^2+d)^(1/2)/a/d^2/x^3-8/15*e^2*(e*x^2+d)^(1/2)/a/d^3/x+1/3*b*
(e*x^2+d)^(1/2)/a^2/d/x^3-2/3*b*e*(e*x^2+d)^(1/2)/a^2/d^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2+a\right)\sqrt{ex^2+d}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(e*x^2 + d)*x^6), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.394 $\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=350

$$\frac{2\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}}+\frac{2\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}-\frac{d^2x}{e\sqrt{d+ex^2}\left(ae^2\right.}$$

```
[Out] -((d^2*x)/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + (2*(b^2 - a*c - (b
*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)^(3/2)) + (2*(b^2 - a*c +
 (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b + S
qrt[b^2 - 4*a*c]]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)^(3/2)) + ArcTanh[(Sqr
t[e]*x)/Sqrt[d + e*x^2]]/(c*e^(3/2))
```

Rubi [A] time = 4.3278, antiderivative size = 507, normalized size of antiderivative = 1.45, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1297, 288, 217, 206, 1692, 377, 205}

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}+\frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\tan^{-1}\left(\frac{x\sqrt{2c\ldots}}{\sqrt{b\ldots}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d^2*x)/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + ((b^2*d - a*c*d -
a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*S
qrt[d + e*x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a*b*e + (b^3*d -
 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]
)/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c
*d^2 - b*d*e + a*e^2)) + (d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(e^(3/2
)*(c*d^2 - b*d*e + a*e^2)) - ((b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(c*Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 1297

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(d^2*f^4)/(c*d^2 - b*d*e + a*e^2), Int[(f
*x)^(m - 4)*(d + e*x^2)^q, x], x] - Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(
(f*x)^(m - 4)*(d + e*x^2)^(q + 1)*Simp[a*d + (b*d - a*e)*x^2, x])/(a + b*x^
2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[q] && LtQ[q, -1] && GtQ[m, 3]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^6}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{x^2\left(ad+(bd-ae)x^2\right)}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2} + \frac{d^2\int \frac{x^2}{\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{\int \left(\frac{bd-ae}{c\sqrt{d+ex^2}} - \frac{a(bd-ae)+\left(b^2d-acd-abe\right)x^2}{c\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx}{cd^2-bde+ae^2} + \frac{d^2\int \frac{1}{\sqrt{d}}\cdots}{e\left(cd^2-b\cdots\right.}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\int \frac{a(bd-ae)+\left(b^2d-acd-abe\right)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{c\left(cd^2-bde+ae^2\right)} + \frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{1-ex^2}\,dx,\cdots\right.}{e\left(cd^2-bde+\cdots\right.}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)} + \frac{\int \left(\frac{b^2d-acd-abe+\frac{-b^3d+3abcd+ab^2e\cdots}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex\cdots}}\cdots\right.}{c\left(\cdots\right.}$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)} - \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}\left(cd^2-bde+ae^2\right)}\cdots$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(cd^2-bde+ae^2\right)} - \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}\left(cd^2-bde+ae^2\right)}\cdots$$

$$= -\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd\cdots}}{\sqrt{b-\cdots}}\right.}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd^2\cdots\right.}$$

Mathematica [B] time = 11.2763, size = 10968, normalized size = 31.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.034, size = 480, normalized size = 1.4

$$-\frac{x}{ce}\frac{1}{\sqrt{ex^2+d}} + \frac{1}{c}\ln\left(\sqrt{e}x+\sqrt{ex^2+d}\right)e^{-\frac{3}{2}} - \frac{bx}{c^2d}\frac{1}{\sqrt{ex^2+d}} + 8\frac{e^{3/2}ab}{c^2\left(4\,ae^2-4\,deb+4\,cd^2\right)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex^2+d}x+2\,d\right)} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/c*x/e/(e*x^2+d)^(1/2)+1/c/e^(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))-1/c^2*b*
x/d/(e*x^2+d)^(1/2)+8/c^2*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1
/2)*(e*x^2+d)^(1/2)*x+2*d)*a*b+8/c*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x
```

```
^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x+2*d)*a*d-8/c^2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*
d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x+2*d)*b^2*d+2/c*e^(1/2)/(4*a*e^2-4
*b*d*e+4*c*d^2)*sum(((a*b*e+a*c*d-b^2*d)*_R^2+2*(2*a^2*e^2-3*a*b*d*e-a*c*d^
2+b^2*d^2)*_R+a*b*d^2*e+a*c*d^3-b^2*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R
*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^
2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(
4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(d+ex^2)^{\frac{3}{2}}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**6/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Giac [A] time = 1.19012, size = 101, normalized size = 0.29

$$-\frac{c^2d^2x}{(c^3d^2e-bc^2de^2+ac^2e^3)\sqrt{x^2e+d}}-\frac{e^{\left(-\frac{3}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -c^2*d^2*x/((c^3*d^2*e - b*c^2*d*e^2 + a*c^2*e^3)*sqrt(x^2*e + d)) - 1/2*e^
(-3/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

3.395 $\int \frac{x^4}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=360

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+cd^2\right)}$$

```
[Out] (d*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((b*d - a*e - (b^2*d - 2*
a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^
2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)
) - ((b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d
+ e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 1.26706, antiderivative size = 360, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1297, 191, 1692, 377, 205}

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (d*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((b*d - a*e - (b^2*d - 2*
a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^
2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)
) - ((b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d
+ e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 1297

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(d^2*f^4)/(c*d^2 - b*d*e + a*e^2), Int[(f
*x)^(m - 4)*(d + e*x^2)^q, x], x] - Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(
(f*x)^(m - 4)*(d + e*x^2)^(q + 1)*Simp[a*d + (b*d - a*e)*x^2, x])/(a + b*x^
2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[q] && LtQ[q, -1] && GtQ[m, 3]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{ad+(bd-ae)x^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2}+\frac{d^2\int \frac{1}{\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{dx}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{\int \left(\frac{bd-ae+\frac{-b^2d+2acd+abe}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{bd-ae-\frac{-b^2d+2acd+abe}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{cd^2-bde+ae^2}$$

$$= \frac{dx}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{cd^2-bde+ae^2}-\cdots$$

$$= \frac{dx}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{\cdots}\right.\right.}\cdots\right.}{cd^2-bde+ae^2}$$

$$= \frac{dx}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd^2-bde\cdots\right.}$$

Mathematica [B] time = 11.195, size = 7792, normalized size = 21.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.03, size = 338, normalized size = 0.9

$$\frac{x}{cd}\frac{1}{\sqrt{ex^2+d}}-8\frac{e^{3/2}a}{c\left(4\,ae^2-4\,deb+4\,cd^2\right)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex^2+dx}+2\,d\right)}+8\frac{\sqrt{e}bd}{c\left(4\,ae^2-4\,deb+4\,cd^2\right)\left(2\,ex^2-2\,\sqrt{e}\sqrt{ex}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/c*x/d/(e*x^2+d)^(1/2)-8/c*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^
(1/2)*(e*x^2+d)^(1/2)*x+2*d)*a+8/c*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x
^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x+2*d)*b*d-2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)
*sum(((a*e-b*d)*_R^2+2*d*(-3*a*e+b*d)*_R+a*d^2*e-b*d^3)/(_R^3*c+3*_R^2*b*e-
3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1
/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+
6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(cx^4+bx^2+a\right)\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(d+ex^2\right)^{\frac{3}{2}}\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.396 $\int \frac{x^2}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=333

$$\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}+\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+\right.}$$

```
[Out] -((e*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + (c*(d - (b*d - 2*a*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt
[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + (c*(d + (b*
d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.65782, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1299, 191, 1692, 377, 205}

$$\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}+\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((e*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + (c*(d - (b*d - 2*a*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt
[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + (c*(d + (b*
d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 1299

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> -Dist[(d*e*f^2)/(c*d^2 - b*d*e + a*e^2), Int[(
f*x)^(m - 2)*(d + e*x^2)^q, x], x] + Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[
((f*x)^(m - 2)*(d + e*x^2)^(q + 1)*Simp[a*e + c*d*x^2, x])/(a + b*x^2 + c*x
^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !Int
egerQ[q] && LtQ[q, -1] && GtQ[m, 1] && LeQ[m, 3]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^2}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}dx = \frac{\int \frac{ae+cdx^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}dx}{cd^2-bde+ae^2} - \frac{(de)\int \frac{1}{\left(d+ex^2\right)^{3/2}}dx}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\int \left(\frac{cd+\frac{c(-bd+2ae)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{cd-\frac{c(-bd+2ae)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right) d}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}dx}{cd^2-bde+ae^2} + \frac{\left(c\left(d\right.\right.}{}$$

$$= -\frac{ex}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right.\right.}\right.}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)ex}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd^2-bd\right.}$$

Mathematica [C] time = 10.9175, size = 2119, normalized size = 6.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((1 - b/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] +
2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + S
qrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
)^2))])/d - 45*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*
(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d +
```

```
(-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])
/d - (45*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/
(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*
a*c])*e)*x^4*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-
b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^
2)) + 4*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4
*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-
b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]
+ (4*e*x^2*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2
- 4*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d
*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))
)]])/d))/(15*(b - Sqrt[b^2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])
*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e
*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]) + ((1 + b/Sqrt[b^2 - 4*a*c]
)*x*(45*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e
)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*
Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d
 + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[
((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2))]] - (30*e*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30
*e*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)
/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])
*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2
, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2))] + (4*e*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x
^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
))])/d))/(15*(b + Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))
/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])
```

Maple [C] time = 0.027, size = 252, normalized size = 0.8

$$-8\,{\frac {d\sqrt {e}}{ \left(4\,a{e}^{2}-4\,deb+4\,c{d}^{2} \right) \left(2\,e{x}^{2}-2\,\sqrt {e}\sqrt {e{x}^{2}+dx}+2\,d \right) }}-2\,{\frac {\sqrt {e}}{4\,a{e}^{2}-4\,deb+4\,c{d}^{2}}}\sum _{\_R={\it RootOf} \left(c\_Z^{4}+ \left(4\,be-4\,cd \right) \_Z^{3}+ \left(16\,a{e}^{2}-8 \right. \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -8*e^(1/2)*d/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x
+2*d)-2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)*sum((_R^2*c*d+2*(2*a*e^2-c*d^2)*_
R+c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d
^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*
```

```
c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(cx^4+bx^2+a)(ex^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(d+ex^2)^{\frac{3}{2}}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.397 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=341

$$-\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}-\frac{c\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}(ae^2-bde}$$

```
[Out] (e^2*x)/(d*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - (c*(e - (2*c*d - b*e)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqr
t[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - (c*(e + (2
*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rubi [A] time = 0.773841, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1172, 191, 1692, 377, 205}

$$-\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}-\frac{c\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}(ae^2-bde}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (e^2*x)/(d*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - (c*(e - (2*c*d - b*e)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqr
t[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - (c*(e + (2
*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 1172

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x^2)^q, x], x] + Dist[1
/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x^2)^(q + 1)*(c*d - b*e - c*e*x^2))/(
a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q] && LtQ[q, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \frac{\int \frac{cd-be-cex^2}{\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2}+\frac{e^2\int \frac{1}{\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}+\frac{\int \left(\frac{-ce-\frac{c(-2cd+be)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}+\frac{-ce+\frac{c(-2cd+be)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right) d\ldots}{cd^2-bde+ae^2}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{cd^2-bde+ae^2}-\frac{\left(c\left(e-\ldots\right.\right.}{\ldots}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\ldots\right.\right.}\ldots\right.}{cd^2-bde+ae^2}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd^2-bd\ldots\right.}$$

Mathematica [C] time = 10.0012, size = 2112, normalized size = 6.19

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*c*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*
x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x
^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[
Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c]
```

```
)*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*
a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sq
rt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5
/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))
^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*
c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*Sqrt[b^2 -
 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*
x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sq
rt[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2
))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]) - (2*c*x*(45*Sqrt[-(((b + Sqrt[
b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((b + Sqrt[b^2 - 4*
a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]] - (30*e*x^2*ArcSin[
Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[((
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + Sqrt[b^2
 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))] + (4*e*x^2*((
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])/d))/(15*Sqrt[b^2 - 4*
a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(
d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2))])
```

Maple [C] time = 0.02, size = 246, normalized size = 0.7

$$32\,\frac{e^{3/2}}{\left(16\,ae^{2}-16\,deb+16\,cd^{2}\right)\left(2\,ex^{2}-2\,\sqrt{e}\sqrt{ex^{2}+d}\,x+2\,d\right)}+8\,\frac{e^{3/2}}{16\,ae^{2}-16\,deb+16\,cd^{2}}\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+(4\,be-4\,cd)\_Z^{3}+(1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 32*e^(3/2)/(16*a*e^2-16*b*d*e+16*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d)^(1/2)*
x+2*d)+8*e^(3/2)/(16*a*e^2-16*b*d*e+16*c*d^2)*sum((c*_R^2+2*(2*b*e-3*c*d)*_
R+c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d
^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*
c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)(ex^2 + d)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex^2)^{\frac{3}{2}} (a + bx^2 + cx^4)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(1/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.398 $\int \frac{1}{x^2(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=339

$$-\frac{2c^2\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}}-\frac{2c^2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}+\frac{ex(cd-}{ad\sqrt{d+ex^2}\left(e(ae\right.}$$

```
[Out] (e*(c*d - b*e)*x)/(a*d*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[d + e*x^2]) + (-d -
2*e*x^2)/(a*d^2*x*Sqrt[d + e*x^2]) - (2*c^2*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*
Sqrt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(2*c*d - (b - Sqrt[b^2 -
4*a*c])*e)^(3/2)) - (2*c^2*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]
)/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)^(3/2))
```

Rubi [A] time = 2.83666, antiderivative size = 462, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1301, 271, 191, 6728, 264, 1692, 377, 205}

$$-\frac{c\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}-\frac{c\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4\right.}}}{\sqrt{\sqrt{b^2-4ac}+b\sqrt{}}}\right.}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(e^2/(d*(c*d^2 - b*d*e + a*e^2)*x*Sqrt[d + e*x^2])) - (2*e^3*x)/(d^2*(c*d^
2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((c*d - b*e)*Sqrt[d + e*x^2])/(a*d*(c
*d^2 - b*d*e + a*e^2)*x) - (c*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - S
qrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - (c*(c*d - b*e -
 (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sq
rt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 -
b*d*e + a*e^2))
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(
d + e*x^2)^q, x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^m*(d + e*
x^2)^(q + 1)*Simp[c*d - b*e - c*e*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && L
tQ[q, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
```

```
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \frac{\int \frac{cd-be-cex^2}{x^2\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{x^2\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \frac{\int \left(\frac{cd-be}{ax^2\sqrt{d+ex^2}} + \frac{-bcd+b^2e-ace-c(cd-be)x^2}{a\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\right)dx}{cd^2-bde+ae^2} - \cdots$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\int \frac{-bcd+b^2e-\cdots}{\sqrt{d+ex^2}(\cdots}}{a\left(cd^2-\cdots\right.}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{(cd-be)\cdots}{ad\left(cd^2-b\cdots\right.}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{(cd-be)\cdots}{ad\left(cd^2-b\cdots\right.}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{(cd-be)\cdots}{ad\left(cd^2-b\cdots\right.}$$

$$= -\frac{e^2}{d\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{(cd-be)\cdots}{ad\left(cd^2-b\cdots\right.}$$

Mathematica [C] time = 8.77517, size = 2158, normalized size = 6.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d + 2*e*x^2)/(a*d^2*x*Sqrt[d + e*x^2])) - ((c + (b*c)/Sqrt[b^2 - 4*a*c])
*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*
Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(
d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt
[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] -
2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b + Sq
rt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)
*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[-
(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*
c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (-b +
 Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*
Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*
c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*
e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/
```

```
2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] -
 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*a*(b - Sqrt[b^
2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b
^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]))*Sq
rt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^2))]) + ((-c + (b*c)/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[-(((b
+ Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/
(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((b + Sqrt[b
^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]] - (30*e*x^2*
ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[
Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + S
qrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x
^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))] + (4*e
*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])/d))/(15*a*(b +
 Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))*
Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2))])
```

Maple [C] time = 0.032, size = 387, normalized size = 1.1

$$-8\,{\frac {{e}^{3/2}b}{a \left(4\,a{e}^{2}-4\,deb+4\,c{d}^{2} \right) \left(2\,e{x}^{2}-2\,\sqrt {e}\sqrt {e{x}^{2}+d}x+2\,d \right) }}+8\,{\frac {\sqrt {e}cd}{a \left(4\,a{e}^{2}-4\,deb+4\,c{d}^{2} \right) \left(2\,e{x}^{2}-2\,\sqrt {e}\sqrt {e{x}^{2}+d}x+2\,d \right) }}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -8/a*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x
+2*d)*b+8/a*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d)^
(1/2)*x+2*d)*c*d-2/a*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)*sum((c*(b*e-c*d)*_R^
2+2*(-2*a*c*e^2+2*b^2*e^2-3*b*c*d*e+c^2*d^2)*_R+b*c*d^2*e-c^2*d^3)/(_R^3*c+
3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((
e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e
^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))-1/a/d/x/(e*x^2+d)^(
1/2)-2/a*e/d^2*x/(e*x^2+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{{\left(c x^{4} + b x^{2} + a\right)} {\left(e x^{2} + d\right)}^{\frac{3}{2}} x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(d + ex^2\right)^{\frac{3}{2}} \left(a + bx^2 + cx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(1/(x**2*(d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.399 $\int \frac{1}{x^4\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=419

$$\frac{2c^2\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}}+\frac{2c^2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}-\frac{ex\left(ace+b^2(-e)\right.}{a^2d\sqrt{d+ex^2}\left(e(ae-\right.}$$

```
[Out] -1/(3*a*d*x^3*Sqrt[d + e*x^2]) + (3*b*d + 4*a*e)/(3*a^2*d^2*x*Sqrt[d + e*x^
2]) + (2*e*(3*b*d + 4*a*e)*x)/(3*a^2*d^3*Sqrt[d + e*x^2]) - (e*(b*c*d - b^2
*e + a*c*e)*x)/(a^2*d*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[d + e*x^2]) + (2*c^2*
(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(a^2*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)^(3/2)) + (2*c^2*(b
- (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(a^2*Sqrt[b + Sq
rt[b^2 - 4*a*c]]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)^(3/2))
```

Rubi [A] time = 5.56641, antiderivative size = 647, normalized size of antiderivative = 1.54, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1301, 271, 191, 6728, 264, 1692, 377, 205}

$$\frac{c\left(\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tan^{-1}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}+\frac{c\left(-\frac{3abce-2ac^2d+b^2cd+b^3(-e)}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right.}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -e^2/(3*d*(c*d^2 - b*d*e + a*e^2)*x^3*Sqrt[d + e*x^2]) + (4*e^3)/(3*d^2*(c*
d^2 - b*d*e + a*e^2)*x*Sqrt[d + e*x^2]) + (8*e^4*x)/(3*d^3*(c*d^2 - b*d*e +
 a*e^2)*Sqrt[d + e*x^2]) - ((c*d - b*e)*Sqrt[d + e*x^2])/(3*a*d*(c*d^2 - b*
d*e + a*e^2)*x^3) + (2*e*(c*d - b*e)*Sqrt[d + e*x^2])/(3*a*d^2*(c*d^2 - b*d
*e + a*e^2)*x) + ((b*c*d - b^2*e + a*c*e)*Sqrt[d + e*x^2])/(a^2*d*(c*d^2 -
b*d*e + a*e^2)*x) + (c*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*
e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(a^2*Sqrt[b - Sqr
t[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*
e^2)) + (c*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*
e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(S
qrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(a^2*Sqrt[b + Sqrt[b^2 - 4*a*
c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(
d + e*x^2)^q, x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^m*(d + e*
x^2)^(q + 1)*Simp[c*d - b*e - c*e*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && L
```

```
tQ[q, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx = \frac{\int \frac{cd-be-cex^2}{x^4\sqrt{d+ex^2}\left(a+bx^2+cx^4\right)}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{x^4\left(d+ex^2\right)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{\int\left(\frac{cd-be}{ax^4\sqrt{d+ex^2}} + \frac{-bcd+b^2e-ace}{a^2x^2\sqrt{d+ex^2}} + \frac{b^2cd-ac^2d-b^3e+2ab\cdots}{a^2\sqrt{d+ex^2}(\cdots}\cdots\right)}{cd^2-bde+ae^2}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \frac{\int \frac{b^2cd-ac^2\cdots}{\cdots}}{\cdots}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \frac{\cdots}{3d^3\left(cd^2\cdots\right.}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \frac{\cdots}{3d^3\left(cd^2\cdots\right.}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \frac{\cdots}{3d^3\left(cd^2\cdots\right.}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+ex^2}} + \frac{\cdots}{3d^3\left(cd^2\cdots\right.}$$

Mathematica [C] time = 8.59638, size = 2218, normalized size = 5.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (b*(d + 2*e*x^2))/(a^2*d^2*x*Sqrt[d + e*x^2]) - (d^2 - 4*d*e*x^2 - 8*e^2*x^
4)/(3*a*d^3*x^3*Sqrt[d + e*x^2]) + ((b*c + (c*(b^2 - 2*a*c))/Sqrt[b^2 - 4*a
*c])*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*
x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x
^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[
Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*
a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sq
rt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5
/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d
```

```
+ (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))
^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*
c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*a^2*(b - S
qrt[b^2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b +
Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
]))*Sqrt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2))]) + ((b*c - (c*(b^2 - 2*a*c))/Sqrt[b^2 - 4*a*c]
)*x*(45*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e
)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*
Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d
 + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[
((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2))]] - (30*e*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30
*e*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)
/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])
*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2
, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2))] + (4*e*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x
^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
))])/d))/(15*a^2*(b + Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x
^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])
```

Maple [C] time = 0.036, size = 541, normalized size = 1.3

$$-8\,{\frac{{e}^{3/2}c}{a \left(4\,a{e}^{2}-4\,deb+4\,c{d}^{2} \right) \left(2\,e{x}^{2}-2\,\sqrt{e}\sqrt{e{x}^{2}+d}x+2\,d \right) }}+8\,{\frac{{e}^{3/2}{b}^{2}}{{a}^{2} \left(4\,a{e}^{2}-4\,deb+4\,c{d}^{2} \right) \left(2\,e{x}^{2}-2\,\sqrt{e}\sqrt{e{x}^{2}+d}x+2\,d \right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -8/a*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d)^(1/2)*x
+2*d)*c+8/a^2*e^(3/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/2)*(e*x^2+d
)^(1/2)*x+2*d)*b^2-8/a^2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)/(2*e*x^2-2*e^(1/
2)*(e*x^2+d)^(1/2)*x+2*d)*b*c*d-2/a^2*e^(1/2)/(4*a*e^2-4*b*d*e+4*c*d^2)*sum
((c*(a*c*e-b^2*e+b*c*d)*_R^2+2*(4*a*b*c*e^2-3*a*c^2*d*e-2*b^3*e^2+3*b^2*c*d
*e-b*c^2*d^2)*_R+a*c^2*d^2*e-b^2*c*d^2*e+b*c^2*d^3)/(_R^3*c+3*_R^2*b*e-3*_R
^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-
e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*
d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+c*d^4))+1/a^2*b/d/x/(e*x^2+d)^(1/2)+2/a^2*
b*e/d^2*x/(e*x^2+d)^(1/2)-1/3/a/d/x^3/(e*x^2+d)^(1/2)+4/3/a*e/d^2/x/(e*x^2+
d)^(1/2)+8/3/a*e^2/d^3*x/(e*x^2+d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2+a\right)\left(ex^2+d\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(e*x^2 + d)^(3/2)*x^4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.400 $\int \frac{(fx)^m\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=243

$$\frac{2c(fx)^{m+1}\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{m+1}{2};1,-q;\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(fx)^{m+1}\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{m+1}{2};1,\right.}{f(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-\right.}}$$

```
[Out] (2*c*(f*x)^(1 + m)*(d + e*x^2)^q*AppellF1[(1 + m)/2, 1, -q, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt
[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^2)/d)^q) - (2*c*(f*x)^(1 + m)*(d + e*x^2
)^q*AppellF1[(1 + m)/2, 1, -q, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 +
 (e*x^2)/d)^q)
```

Rubi [A] time = 0.649722, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1305, 511, 510}

$$\frac{2c(fx)^{m+1}\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{m+1}{2};1,-q;\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(fx)^{m+1}\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{m+1}{2};1,\right.}{f(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(f*x)^(1 + m)*(d + e*x^2)^q*AppellF1[(1 + m)/2, 1, -q, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt
[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^2)/d)^q) - (2*c*(f*x)^(1 + m)*(d + e*x^2
)^q*AppellF1[(1 + m)/2, 1, -q, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 +
 (e*x^2)/d)^q)
```

Rule 1305

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q, 1/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2
- 4*a*c, 0] && !IntegerQ[q] && !IntegerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(fx)^m\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx = \int \left(\frac{2c(fx)^m\left(d+ex^2\right)^q}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} - \frac{2c(fx)^m\left(d+ex^2\right)^q}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right)dx$$

$$= \frac{(2c)\int \frac{(fx)^m\left(d+ex^2\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(fx)^m\left(d+ex^2\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{(fx)^m\left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{(fx)^m\left(1+\frac{ex^2}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{2c(fx)^{1+m}\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1+m}{2};1,-q;\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)f(1+m)} - \frac{2c(fx)^{1+m}\left(d+ex^2\right)^q \cdots}{\sqrt{b^2 \cdots}}$$

Mathematica [F] time = 0.212542, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F] time = 0.086, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(ex^2+d\right)^q}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q(fx)^m}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q\left(fx\right)^m}{cx^4+bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q\left(fx\right)^m}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.401 $\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=313

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q \ldots\right.}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+\ldots\right.\right.}$$

```
[Out] -((c*d + b*e)*(d + e*x^2)^(1 + q))/(2*c^2*e^2*(1 + q)) + (d + e*x^2)^(2 + q
)/(2*c*e^2*(2 + q)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c])
)*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/
(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e)*(1 + q)) + ((a - b^2/c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(d +
e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1
+ q))
```

Rubi [A] time = 0.938896, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1251, 1628, 68}

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q \ldots\right.}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+\ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((c*d + b*e)*(d + e*x^2)^(1 + q))/(2*c^2*e^2*(1 + q)) + (d + e*x^2)^(2 + q
)/(2*c*e^2*(2 + q)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c])
)*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/
(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e)*(1 + q)) + ((a - b^2/c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(d +
e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1
+ q))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^7\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(d+ex)^q}{a+bx+cx^2}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-cd-be)(d+ex)^q}{c^2e} + \frac{\left(\frac{b^2}{c^2}-\frac{a}{c}-\frac{b\left(b^2-3ac\right)}{c^2\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(\frac{b^2}{c^2}-\frac{a}{c}+\frac{b\left(b^2-3ac\right)}{c^2\sqrt{b^2-4ac}}\right)(d+e\ldots}{b+\sqrt{b^2-4ac}+2cx}\right.\right.$$

$$= -\frac{(cd+be)\left(d+ex^2\right)^{1+q}}{2c^2e^2(1+q)} + \frac{\left(d+ex^2\right)^{2+q}}{2ce^2(2+q)} - \frac{\left(a-\frac{b^2}{c}-\frac{b\left(b^2-3ac\right)}{c\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\,dx,x,x^2\right)}{2c}$$

$$= -\frac{(cd+be)\left(d+ex^2\right)^{1+q}}{2c^2e^2(1+q)} + \frac{\left(d+ex^2\right)^{2+q}}{2ce^2(2+q)} + \frac{\left(a-\frac{b^2}{c}+\frac{b\left(b^2-3ac\right)}{c\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\ldots\right.}{2c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

Mathematica [A] time = 0.762778, size = 272, normalized size = 0.87

$$\frac{\left(d+ex^2\right)^{q+1}\left(\frac{c\left(\frac{b\left(b^2-3ac\right)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{(q+1)\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)}+\frac{c\left(-\frac{b\left(b^2-3ac\right)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{be+cd}{e^2(q+1)}+\frac{c\left(d+ex^2\right)}{e^2(q+2)}\ldots\right.}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*(-((c*d + b*e)/(e^2*(1 + q))) + (c*(d + e*x^2))/(e^2*(
2 + q)) + (c*(a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*Hypergeo
metric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e)])/((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (c*(a - b^2/c - (
b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*Hypergeometric2F1[1, 1 + q, 2 + q,
(2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*(1 + q))))/(2*c^2)
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{x^7\left(ex^2+d\right)^q}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q x^7}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^2+d)^q x^7}{cx^4+bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q x^7}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

3.402 $\int \frac{x^5\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=256

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\Big(1,q+1;q+2;\frac{2c(}{2cd-(b}}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

```
[Out] (d + e*x^2)^(1 + q)/(2*c*e*(1 + q)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c]
)*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/
(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e)*(1 + q)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*H
ypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(2*c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rubi [A] time = 0.541668, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1251, 1628, 68}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\Big(1,q+1;q+2;\frac{2c(}{2cd-(b}}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (d + e*x^2)^(1 + q)/(2*c*e*(1 + q)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c]
)*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/
(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e)*(1 + q)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*H
ypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(2*c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^5\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(d+ex)^q}{c} + \frac{\left(-\frac{b}{c}+\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(-\frac{b}{c}-\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\right)dx, x, x^2\right)$$
$$= \frac{\left(d+ex^2\right)^{1+q}}{2ce(1+q)} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\,dx, x, x^2\right)}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\operatorname{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac-}}\right.}{2c}$$
$$= \frac{\left(d+ex^2\right)^{1+q}}{2ce(1+q)} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}{}_2F_1\left(1,1+q;2+q;\frac{2c\left(d+ex^2\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d\right.}{2c\left(\right.}$$

Mathematica [A] time = 0.384033, size = 211, normalized size = 0.82

$$\frac{\left(d+ex^2\right)^{q+1}\left(\frac{\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)} + \frac{1}{e}\right)}{2c(q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*(e^(-1) + ((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*Hype
rgeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 -
4*a*c])*e)])/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e) + ((b + (b^2 - 2*a*c)/Sqr
t[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)))/(2*c*
(1 + q))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \frac{x^5\left(ex^2+d\right)^q}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q x^5}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q x^5}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q x^5}{cx^4+bx^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

3.403 $\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=210

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

```
[Out] -((1 - b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q,
 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*(2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) - ((1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^
2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rubi [A] time = 0.328882, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1251, 830, 68}

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((1 - b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q,
 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*(2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) - ((1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^
2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x^3 \left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(d+ex)^q}{a+bx+cx^2}\,dx,x,x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\right)dx,x,x^2\right)$$
$$= \frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\,dx,x,x^2\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int -\right.$$
$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}-\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}\,2}{2\left(2cd-\left(b+\right.\right.}$$

Mathematica [A] time = 0.23669, size = 183, normalized size = 0.87

$$-\frac{\left(d+ex^2\right)^{q+1}\left(\left(d\sqrt{b^2-4ac}+2ae-bd\right){}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)+\left(d\sqrt{b^2-4ac}-2ae+bd\right){}_2F_1\left(1,q+1;\right.\right.}{4(q+1)\sqrt{b^2-4ac}\left(e(ae-bd)+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((d + e*x^2)^(1 + q)*((-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*Hypergeometri
c2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
)] + (b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, 1 + q, 2 + q,
 (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]))/(4*Sqrt[b^2 - 4*a
*c]*(c*d^2 + e*(-(b*d) + a*e))*(1 + q))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{x^3 \left(ex^2+d\right)^q}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q x^3}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q x^3}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

3.404 $\int \frac{x\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=198

$$\frac{c\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{c\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$

```
[Out] -((c*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2
))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)*(1 + q))) + (c*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1
, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sq
rt[b^2 - 4*a*c]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rubi [A] time = 0.357378, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1247, 711, 68}

$$\frac{c\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{c\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((c*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2
))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)*(1 + q))) + (c*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1
, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sq
rt[b^2 - 4*a*c]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 711

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{2c(d+ex)^q}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c(d+ex)^q}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx\right)}\right)dx, x, x^2\right)$$
$$= \frac{c\operatorname{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\,dx, x, x^2\right)}{\sqrt{b^2-4ac}} - \frac{c\operatorname{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\,dx, x, x^2\right)}{\sqrt{b^2-4ac}}$$
$$= -\frac{c(d+ex^2)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{c(d+ex^2)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+e\ldots}{2cd-\left(b+\sqrt{b\ldots}\right.}\right.}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1\ldots}$$

Mathematica [A] time = 0.299574, size = 168, normalized size = 0.85

$$\frac{c(d+ex^2)^{q+1}\left(\frac{{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)} - \frac{{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}\right)}{(q+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (c*(d + e*x^2)^(1 + q)*(-(Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^
2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])
*e)) + Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)))/(Sqrt[b^2 - 4*a
*c]*(1 + q))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q x}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q x}{cx^4+bx^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

3.405 $\int \frac{\left(d+ex^2\right)^q}{x\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=262

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(}{2cd-(b+}\right.}{2a(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q
, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*a*(2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(d +
 e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1
 + q)) - ((d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^2
)/d])/(2*a*d*(1 + q))
```

Rubi [A] time = 0.502102, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1251, 960, 65, 830, 68}

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(}{2cd-(b+}\right.}{2a(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q
, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*a*(2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(d +
 e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1
 + q)) - ((d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^2
)/d])/(2*a*d*(1 + q))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !RationalQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(d+ex^2\right)^q}{x\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(d+ex)^q}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{(d+ex)^q}{ax}+\frac{(-b-cx)(d+ex)^q}{a\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)\\
&= \frac{\mathrm{Subst}\left(\int \frac{(d+ex)^q}{x}\,dx, x, x^2\right)}{2a}+\frac{\mathrm{Subst}\left(\int \frac{(-b-cx)(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right)}{2a}\\
&= -\frac{\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2ad(1+q)}+\frac{\mathrm{Subst}\left(\int\left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(-c+\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex\right.}{b+\sqrt{b^2-4ac}+2cx}\right.}{2a}\\
&= -\frac{\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2ad(1+q)}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\mathrm{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\,dx, x,\right.}{2a}\\
&= \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c\left(d+ex^2\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)}{2a\left(2cd-\right.}
\end{aligned}
$$

Mathematica [A] time = 0.407428, size = 218, normalized size = 0.83

$$
\frac{\left(d+ex^2\right)^{q+1}\left(\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}-\frac{{}_2F_1\left(1,q+1;q+2;\frac{ex^2}{d}+1\right)}{d}\right)}{2a(q+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 +
 q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric
```

```
2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) - Hypergeometric2F1[1, 1 + q, 2 + q,
1 + (e*x^2)/d]/d))/(2*a*(1 + q))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{\left(e x^2+d\right)^q}{x\left(c x^4+b x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x)
```

```
[Out] int((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e x^2+d\right)^q}{\left(c x^4+b x^2+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(e x^2+d\right)^q}{c x^5+b x^3+a x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q/(c*x^5 + b*x^3 + a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x), x)
```

3.406 $\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=322

$$-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a^2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c}{2cd-(}\right.}{2a^2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

```
[Out] -(c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometri
c2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
])/(2*a^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) - (c*(b - (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 +
q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*a^2*(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (b*(d + e*x^2)^(1 + q)*Hypergeometric
2F1[1, 1 + q, 2 + q, 1 + (e*x^2)/d])/(2*a^2*d*(1 + q)) + (e*(d + e*x^2)^(1
+ q)*Hypergeometric2F1[2, 1 + q, 2 + q, 1 + (e*x^2)/d])/(2*a*d^2*(1 + q))
```

Rubi [A] time = 0.66344, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1251, 960, 65, 830, 68}

$$-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a^2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c}{2cd-(}\right.}{2a^2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometri
c2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
])/(2*a^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) - (c*(b - (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 +
q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*a^2*(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (b*(d + e*x^2)^(1 + q)*Hypergeometric
2F1[1, 1 + q, 2 + q, 1 + (e*x^2)/d])/(2*a^2*d*(1 + q)) + (e*(d + e*x^2)^(1
+ q)*Hypergeometric2F1[2, 1 + q, 2 + q, 1 + (e*x^2)/d])/(2*a*d^2*(1 + q))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^q}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{(d+ex)^q}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{(d+ex)^q}{ax^2} - \frac{b(d+ex)^q}{a^2x} + \frac{\left(b^2-ac+bcx\right)(d+ex)^q}{a^2\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)$$

$$= \frac{\operatorname{Subst}\left(\int \frac{\left(b^2-ac+bcx\right)(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2} + \frac{\operatorname{Subst}\left(\int \frac{(d+ex)^q}{x^2}\,dx, x, x^2\right)}{2a} - \frac{b\operatorname{Subst}\left(\int \frac{(d+ex)^q}{x}\,dx, \right.}{2a^2}$$

$$= \frac{b\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2a^2d(1+q)} + \frac{e\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(2,1+q;2+q;1+\frac{ex^2}{d}\right)}{2ad^2(1+q)} +$$

$$= \frac{b\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2a^2d(1+q)} + \frac{e\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(2,1+q;2+q;1+\frac{ex^2}{d}\right)}{2ad^2(1+q)} +$$

$$= -\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c\left(d+ex^2\right)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a^2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} - \frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+e\right.}{2a^2\left(2cd\right.}$$

Mathematica [A] time = 0.444902, size = 259, normalized size = 0.8

$$\frac{\left(d+ex^2\right)^{q+1}\left(-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd} - \frac{c\left(\frac{2ac-b^2}{\sqrt{b^2-4ac}}+b\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^2+d\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)} + \frac{ae\,{}_2F_1\left(2,q+1;q+2;\frac{ex^2}{d}+1\right)}{d^2} + \frac{b}{\ldots}\right.}{2a^2(q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*(-((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Hypergeome
tric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)])/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)) - (c*(b + (-b^2 + 2*a*c)/Sqrt[
b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) + (b*Hype
rgeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^2)/d])/d + (a*e*Hypergeometric2F1[2
, 1 + q, 2 + q, 1 + (e*x^2)/d])/d^2))/(2*a^2*(1 + q))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{\left(e x^2+d\right)^q}{x^3 \left(c x^4+b x^2+a\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] int((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e x^2+d\right)^q}{\left(c x^4+b x^2+a\right) x^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e x^2+d\right)^q}{c x^7+b x^5+a x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q/(c*x^7 + b*x^5 + a*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q}{\left(cx^4+bx^2+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^3), x)
```

3.407 $\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=339

$$\frac{x\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+\cdots\right.}{c^2\left(\sqrt{b^2-\cdots}\right.}$$

```
[Out] ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1
[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c^2*(
b - Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + ((b^2 - a*c + (b*(b^2 - 3*a*c))
/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c^2*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*
x^2)/d)^q) - (b*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d
)])/(c^2*(1 + (e*x^2)/d)^q) + (x^3*(d + e*x^2)^q*Hypergeometric2F1[3/2, -q,
 5/2, -((e*x^2)/d)])/(3*c*(1 + (e*x^2)/d)^q)
```

Rubi [A] time = 0.628195, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1303, 246, 245, 365, 364, 1692, 430, 429}

$$\frac{x\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+\cdots\right.}{c^2\left(\sqrt{b^2-\cdots}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1
[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c^2*(
b - Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + ((b^2 - a*c + (b*(b^2 - 3*a*c))
/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c^2*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*
x^2)/d)^q) - (b*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d
)])/(c^2*(1 + (e*x^2)/d)^q) + (x^3*(d + e*x^2)^q*Hypergeometric2F1[3/2, -q,
 5/2, -((e*x^2)/d)])/(3*c*(1 + (e*x^2)/d)^q)
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6 \left(d+e x^2\right)^q}{a+b x^2+c x^4} dx = \int \left(-\frac{b \left(d+e x^2\right)^q}{c^2}+\frac{x^2 \left(d+e x^2\right)^q}{c}+\frac{\left(a b+\left(b^2-a c\right) x^2\right) \left(d+e x^2\right)^q}{c^2 \left(a+b x^2+c x^4\right)}\right) dx$$

$$= \frac{\int \frac{\left(a b+\left(b^2-a c\right) x^2\right) \left(d+e x^2\right)^q}{a+b x^2+c x^4} dx}{c^2} - \frac{b \int \left(d+e x^2\right)^q dx}{c^2} + \frac{\int x^2 \left(d+e x^2\right)^q dx}{c}$$

$$= \frac{\int \left(\frac{\left(b^2-a c+\frac{b\left(-b^2+3 a c\right)}{\sqrt{b^2-4 a c}}\right)\left(d+e x^2\right)^q}{b-\sqrt{b^2-4 a c}+2 c x^2}+\frac{\left(b^2-a c-\frac{b\left(-b^2+3 a c\right)}{\sqrt{b^2-4 a c}}\right)\left(d+e x^2\right)^q}{b+\sqrt{b^2-4 a c}+2 c x^2}\right) dx}{c^2} - \frac{\left(b \left(d+e x^2\right)^q \left(1+\frac{e x^2}{d}\right)^{-q}\right) \int \left(1+\frac{e x^2}{d}\right)^q d}{c^2}$$

$$= -\frac{b x \left(d+e x^2\right)^q \left(1+\frac{e x^2}{d}\right)^{-q} {}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{e x^2}{d}\right)}{c^2} + \frac{x^3 \left(d+e x^2\right)^q \left(1+\frac{e x^2}{d}\right)^{-q} {}_2F_1\left(\frac{3}{2},-q;\frac{5}{2};-\frac{e x^2}{d}\right)}{3 c} + \cdots$$

$$= -\frac{b x \left(d+e x^2\right)^q \left(1+\frac{e x^2}{d}\right)^{-q} {}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{e x^2}{d}\right)}{c^2} + \frac{x^3 \left(d+e x^2\right)^q \left(1+\frac{e x^2}{d}\right)^{-q} {}_2F_1\left(\frac{3}{2},-q;\frac{5}{2};-\frac{e x^2}{d}\right)}{3 c} + \cdots$$

$$= \frac{\left(b^2-a c-\frac{b\left(b^2-3 a c\right)}{\sqrt{b^2-4 a c}}\right) x \left(d+e x^2\right)^q \left(1+\frac{e x^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{e x^2}{d}\right)}{c^2 \left(b-\sqrt{b^2-4 a c}\right)} + \frac{\left(b^2-a c+\frac{b\left(b^2-3 a c\right)}{\sqrt{b^2-4 a c}}\right.}{}$$

Mathematica [F] time = 0.536921, size = 0, normalized size = 0.

$$\int \frac{x^6 \left(d+e x^2\right)^q}{a+b x^2+c x^4} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \frac{x^6 \left(e x^2+d\right)^q}{c x^4+b x^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e x^2+d\right)^q x^6}{c x^4+b x^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2+d\right)^q x^6}{cx^4+bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q x^6}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

3.408 $\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=273

$$-\frac{x\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}-\frac{x\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-\right.}{c\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -
q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c*(b - Sqrt[b^2
 - 4*a*c])*(1 + (e*x^2)/d)^q)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(
d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
-((e*x^2)/d)])/(c*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (x*(d + e*x^
2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(c*(1 + (e*x^2)/d)^q)
```

Rubi [A] time = 0.53064, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1303, 246, 245, 1692, 430, 429}

$$-\frac{x\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}-\frac{x\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-\right.}{c\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -
q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c*(b - Sqrt[b^2
 - 4*a*c])*(1 + (e*x^2)/d)^q)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(
d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
-((e*x^2)/d)])/(c*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (x*(d + e*x^
2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(c*(1 + (e*x^2)/d)^q)
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx &= \int \left(\frac{\left(d+ex^2\right)^q}{c}-\frac{\left(a+bx^2\right)\left(d+ex^2\right)^q}{c\left(a+bx^2+cx^4\right)}\right)dx \\
&= \frac{\int \left(d+ex^2\right)^q\,dx}{c}-\frac{\int \frac{\left(a+bx^2\right)\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx}{c} \\
&= -\frac{\int \left(\frac{\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}+\frac{\left(b-\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{c}+\frac{\left(\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \left(1+\frac{ex^2}{d}\right)^q\,dx}{c} \\
&= \frac{x\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\,{}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{\left(d+ex^2\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\cdots}{\cdots} \\
&= \frac{x\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\,{}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c}-\frac{\left(\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^{\cdots}}{b-\sqrt{b^2-4a\cdots}}}{c} \\
&= -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x\left(d+\cdots\right.}{\cdots}
\end{aligned}
$$

Mathematica [F] time = 0.251013, size = 0, normalized size = 0.

$$\int \frac{x^4\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{x^4 \left(e x^2+d\right)^q}{c x^4+b x^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e x^2+d\right)^q x^4}{c x^4+b x^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e x^2+d\right)^q x^4}{c x^4+b x^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e x^2+d\right)^q x^4}{c x^4+b x^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

3.409 $\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4} dx$

Optimal. Leaf size=162

$$\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}-\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -((x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^2)/d)^q)) + (x*(d + e*x^2
)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)
/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^2)/d)^q)
```

Rubi [A] time = 0.313552, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1303, 430, 429}

$$\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}-\frac{x\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^2)/d)^q)) + (x*(d + e*x^2
)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)
/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^2)/d)^q)
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^2 \left(d + ex^2\right)^q}{a + bx^2 + cx^4} \, dx = \int \left(\frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) \left(d + ex^2\right)^q}{b - \sqrt{b^2 - 4ac} + 2cx^2} + \frac{\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) \left(d + ex^2\right)^q}{b + \sqrt{b^2 - 4ac} + 2cx^2} \right) dx$$

$$= \left(1 - \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{\left(d + ex^2\right)^q}{b - \sqrt{b^2 - 4ac} + 2cx^2} \, dx + \left(1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) \int \frac{\left(d + ex^2\right)^q}{b + \sqrt{b^2 - 4ac} + 2cx^2} \, dx$$

$$= \left(\left(1 - \frac{b}{\sqrt{b^2 - 4ac}}\right) \left(d + ex^2\right)^q \left(1 + \frac{ex^2}{d}\right)^{-q}\right) \int \frac{\left(1 + \frac{ex^2}{d}\right)^q}{b - \sqrt{b^2 - 4ac} + 2cx^2} \, dx + \left(\left(1 + \frac{b}{\sqrt{b^2 - 4ac}}\right) (d +$$

$$= -\frac{x \left(d + ex^2\right)^q \left(1 + \frac{ex^2}{d}\right)^{-q} F_1 \left(\frac{1}{2}; 1, -q; \frac{3}{2}; -\frac{2cx^2}{b-\sqrt{b^2-4ac}}, -\frac{ex^2}{d}\right)}{\sqrt{b^2 - 4ac}} + \frac{x \left(d + ex^2\right)^q \left(1 + \frac{ex^2}{d}\right)^{-q} F_1 \left(\frac{1}{2}; 1, -\right.}{\sqrt{b^2 - 4ac}}$$

Mathematica [F] time = 0.102823, size = 0, normalized size = 0.

$$\int \frac{x^2 \left(d + ex^2\right)^q}{a + bx^2 + cx^4} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{x^2 \left(ex^2 + d\right)^q}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2 + d\right)^q x^2}{cx^4 + bx^2 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ex^2 + d\right)^q x^2}{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q x^2}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

3.410 $\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=190

$$-\frac{2cx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\cdots\right.}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] (-2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 -
4*a*c]), -((e*x^2)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d
)^q) - (2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), -((e*x^2)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + (e*
x^2)/d)^q)
```

Rubi [A] time = 0.293837, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1174, 430, 429}

$$-\frac{2cx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\cdots\right.}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 -
4*a*c]), -((e*x^2)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d
)^q) - (2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), -((e*x^2)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + (e*
x^2)/d)^q)
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{2cx\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-\right.}{b^2-4ac+b\sqrt{b^2-}}$$

Mathematica [F] time = 0.0357463, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^2)^q/(a + b*x^2 + c*x^4),x]
```

```
[Out] Integrate[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x]
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int((e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^2+d)^q}{cx^4+bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(ex^2+d\right)^q}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q/(c*x^4 + b*x^2 + a), x)
```

3.411 $\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=264

$$-\frac{cx\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)}-\frac{cx\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1\cdots\right.}{a\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] -((c*(1 + b/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b - Sqrt[b^2 - 4*a*c])
*(1 + (e*x^2)/d)^q)) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF
1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b
 + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - ((d + e*x^2)^q*Hypergeometric2F1
[-1/2, -q, 1/2, -((e*x^2)/d)])/(a*x*(1 + (e*x^2)/d)^q)
```

Rubi [A] time = 0.552902, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1303, 365, 364, 1692, 430, 429}

$$-\frac{cx\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)}-\frac{cx\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1\cdots\right.}{a\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((c*(1 + b/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b - Sqrt[b^2 - 4*a*c])
*(1 + (e*x^2)/d)^q)) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF
1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b
 + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - ((d + e*x^2)^q*Hypergeometric2F1
[-1/2, -q, 1/2, -((e*x^2)/d)])/(a*x*(1 + (e*x^2)/d)^q)
```

Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^q}{x^2\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{\left(d+ex^2\right)^q}{ax^2}+\frac{\left(-b-cx^2\right)\left(d+ex^2\right)^q}{a\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{\left(d+ex^2\right)^q}{x^2}\,dx}{a}+\frac{\int \frac{\left(-b-cx^2\right)\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx}{a}$$

$$= \frac{\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}+\frac{\left(-c+\frac{bc}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{a}+\frac{\left(\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{x^2}\,dx}{a}$$

$$= -\frac{\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\,{}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\left(d+ex^2\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{a}-\frac{(c(\ldots}{\ldots}$$

$$= -\frac{\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\,{}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \ldots}{a}$$

$$= -\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)x\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\ldots}{\ldots}$$

Mathematica [F] time = 0.221173, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^q}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)), x]
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{\left(e x^{2}+d\right)^{q}}{x^{2} \left(c \,x^{4}+b \,x^{2}+a \right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] int((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(e x^{2} + d\right)}^{q}}{{\left(c x^{4} + b x^{2} + a\right)} x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{{\left(e x^{2} + d\right)}^{q}}{c x^{6} + b x^{4} + a x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q/(c*x^6 + b*x^4 + a*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(e x^{2} + d\right)}^{q}}{{\left(c x^{4} + b x^{2} + a\right)} x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^2), x)
```

3.412 $\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=328

$$\frac{cx\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{cx\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,\right.}{a^2\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] (c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -
q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a^2*(b - Sqrt[b
^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*
x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), -((e*x^2)/d)])/(a^2*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - ((d + e
*x^2)^q*Hypergeometric2F1[-3/2, -q, -1/2, -((e*x^2)/d)])/(3*a*x^3*(1 + (e*x
^2)/d)^q) + (b*(d + e*x^2)^q*Hypergeometric2F1[-1/2, -q, 1/2, -((e*x^2)/d)]
)/(a^2*x*(1 + (e*x^2)/d)^q)
```

Rubi [A] time = 0.620463, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1303, 365, 364, 1692, 430, 429}

$$\frac{cx\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{cx\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,\right.}{a^2\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x]
```


```
[Out] (c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -
q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a^2*(b - Sqrt[b
^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*
x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), -((e*x^2)/d)])/(a^2*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - ((d + e
*x^2)^q*Hypergeometric2F1[-3/2, -q, -1/2, -((e*x^2)/d)])/(3*a*x^3*(1 + (e*x
^2)/d)^q) + (b*(d + e*x^2)^q*Hypergeometric2F1[-1/2, -q, 1/2, -((e*x^2)/d)]
)/(a^2*x*(1 + (e*x^2)/d)^q)
```


Rule 1303

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^q}{x^4\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{\left(d+ex^2\right)^q}{ax^4}-\frac{b\left(d+ex^2\right)^q}{a^2x^2}+\frac{\left(b^2-ac+bcx^2\right)\left(d+ex^2\right)^q}{a^2\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{\int \frac{\left(b^2-ac+bcx^2\right)\left(d+ex^2\right)^q}{a+bx^2+cx^4}\,dx}{a^2}+\frac{\int \frac{\left(d+ex^2\right)^q}{x^4}\,dx}{a}-\frac{b\int \frac{\left(d+ex^2\right)^q}{x^2}\,dx}{a^2}$$

$$= \frac{\int \left(\frac{\left(bc+\frac{c\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}+\frac{\left(bc-\frac{c\left(b^2-2ac\right)}{\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{a^2}+\frac{\left(\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{x^4}\,dx}{a}-$$

$$= -\frac{\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(-\frac{3}{2},-q;-\frac{1}{2};-\frac{ex^2}{d}\right)}{3ax^3}+\frac{b\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\right.}{a^2x}$$

$$= -\frac{\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(-\frac{3}{2},-q;-\frac{1}{2};-\frac{ex^2}{d}\right)}{3ax^3}+\frac{b\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\right.}{a^2x}$$

$$= \frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x}{}$$

Mathematica [F] time = 0.516567, size = 0, normalized size = 0.

$$\int \frac{\left(d+ex^2\right)^q}{x^4\left(a+bx^2+cx^4\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Integrate[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)), x]
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q}{x^4(cx^4+bx^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] int((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2+d)^q}{(cx^4+bx^2+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^2+d)^q}{cx^8+bx^6+ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)^q/(c*x^8 + b*x^6 + a*x^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^q}{(cx^4 + bx^2 + a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^q/((c*x^4 + b*x^2 + a)*x^4), x)
```

3.413 $\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}} dx$

Optimal. Leaf size=40

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{1-c^4x^4}}{cx\sqrt{\frac{1}{c^2x^2}+1}}\right)}{c}$$

```
[Out] -(ArcTanh[Sqrt[1 - c^4*x^4]/(c*Sqrt[1 + 1/(c^2*x^2)]*x)]/c)
```

Rubi [A] time = 0.0722516, antiderivative size = 44, normalized size of antiderivative = 1.1, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1448, 1252, 848, 63, 208}

$$-\frac{x\sqrt{\frac{1}{c^2x^2}+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -((Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[Sqrt[1 - c^2*x^2]])/Sqrt[1 + c^2*x^2])
```

Rule 1448

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sym
bol] :> Dist[(e^IntPart[q]*(d + e*x^mn)^FracPart[q])/(x^(mn*FracPart[q])*(1
 + d/(x^mn*e))^FracPart[q]), Int[x^(mn*q)*(1 + d/(x^mn*e))^q*(a + c*x^n2)^p
, x], x] /; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] &&  !Integer
Q[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx &= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}x\right)\int \frac{\sqrt{1+c^2x^2}}{x\sqrt{1-c^4x^4}}\,dx}{\sqrt{1+c^2x^2}}\\ &= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}x\right)\text{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x\sqrt{1-c^4x^2}}\,dx,x,x^2\right)}{2\sqrt{1+c^2x^2}}\\ &= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{2\sqrt{1+c^2x^2}}\\ &= -\frac{\left(\sqrt{1+\frac{1}{c^2x^2}}x\right)\text{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)}{c^2\sqrt{1+c^2x^2}}\\ &= -\frac{\sqrt{1+\frac{1}{c^2x^2}}x\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{1+c^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0555167, size = 44, normalized size = 1.1

$$-\frac{x\sqrt{\frac{1}{c^2x^2}+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -((Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[Sqrt[1 - c^2*x^2]])/Sqrt[1 + c^2*x^2])
```

Maple [C] time = 0.061, size = 101, normalized size = 2.5

$$-\frac{x\text{csgn}\left(c^{-1}\right)}{\left(c^2x^2+1\right)c}\sqrt{\frac{c^2x^2+1}{c^2x^2}}\sqrt{-c^4x^4+1}\ln\left(2\frac{1}{xc^2}\left(\text{csgn}\left(c^{-1}\right)c\sqrt{-\frac{c^2x^2-1}{c^2}}+1\right)\right)\frac{1}{\sqrt{-\frac{c^2x^2-1}{c^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] -((c^2*x^2+1)/c^2/x^2)^(1/2)*x*(-c^4*x^4+1)^(1/2)*csgn(1/c)*ln(2*(csgn(1/c)
*c*(-1/c^2*(c^2*x^2-1))^(1/2)+1)/x/c^2)/(c^2*x^2+1)/(-1/c^2*(c^2*x^2-1))^(1
/2)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\frac{1}{c^2x^2}+1}}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(1/(c^2*x^2) + 1)/sqrt(-c^4*x^4 + 1), x)
```

Fricas [B] time = 1.8348, size = 255, normalized size = 6.38

$$-\frac{\log\left(\frac{c^2x^2+\sqrt{-c^4x^4+1}cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)-\log\left(-\frac{c^2x^2-\sqrt{-c^4x^4+1}cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c^2*x^2 + 1)) - log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)))/c
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{-(cx-1)(cx+1)\left(c^2x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c**2/x**2)**(1/2)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 + 1/(c**2*x**2))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
, x)
```

Giac [A] time = 1.1095, size = 78, normalized size = 1.95

$$\frac{\left(\log\left(\sqrt{2}+1\right)-\log\left(\sqrt{2}-1\right)-\log\left(\sqrt{-c^2x^2+1}+1\right)+\log\left(-\sqrt{-c^2x^2+1}+1\right)\right)|c|\operatorname{sgn}(x)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(log(sqrt(2) + 1) - log(sqrt(2) - 1) - log(sqrt(-c^2*x^2 + 1) + 1) + lo
g(-sqrt(-c^2*x^2 + 1) + 1))*abs(c)*sgn(x)/c^2
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
            print("both result and optimal complex");
         fi;
         #both result and optimal complex
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      else #result contains complex but optimal is not
         if debug then
            print("result contains complex but optimal is not");
         fi;
         return "C";
      end if
   else # result do not contain complex
        # this assumes optimal do not as well
        if debug then
            print("result do not contain complex, this assumes optimal do
   not as well");
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
           if debug then
              print("leaf_count_result<=2*leaf_count_optimal");
           fi;
           return "A";
        else
           if debug then
              print("leaf_count_result>2*leaf_count_optimal");
           fi;
           return "B";
        end if
   end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```